Optimal Multiserver Scheduling

with Unknown Job Sizes in Heavy Traffic

Ziv Scully
Isaac Grosof
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Carnegie Mellon University

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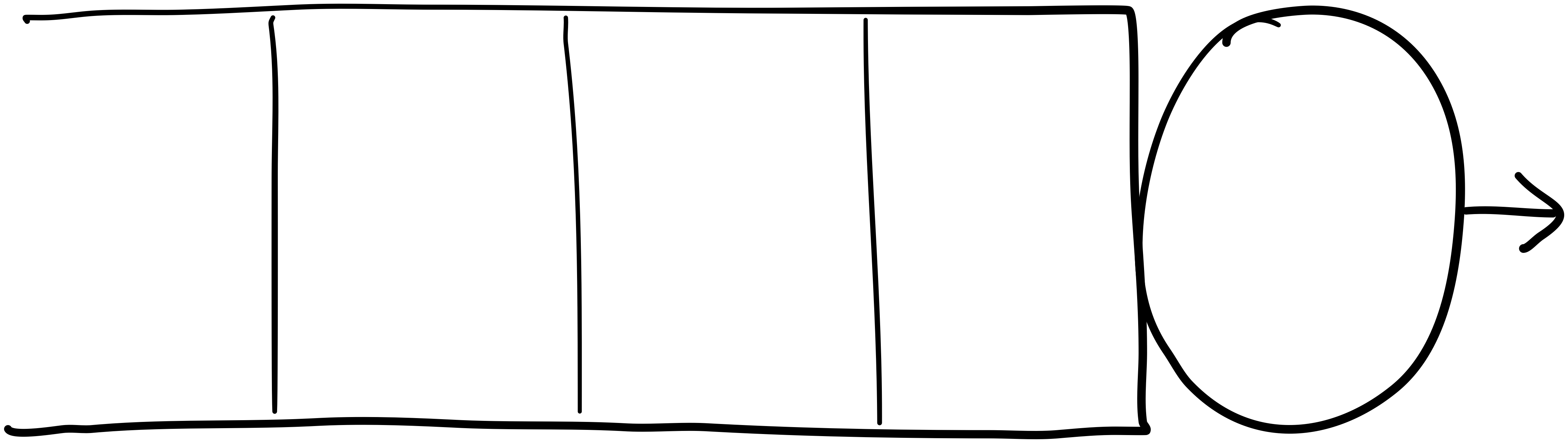
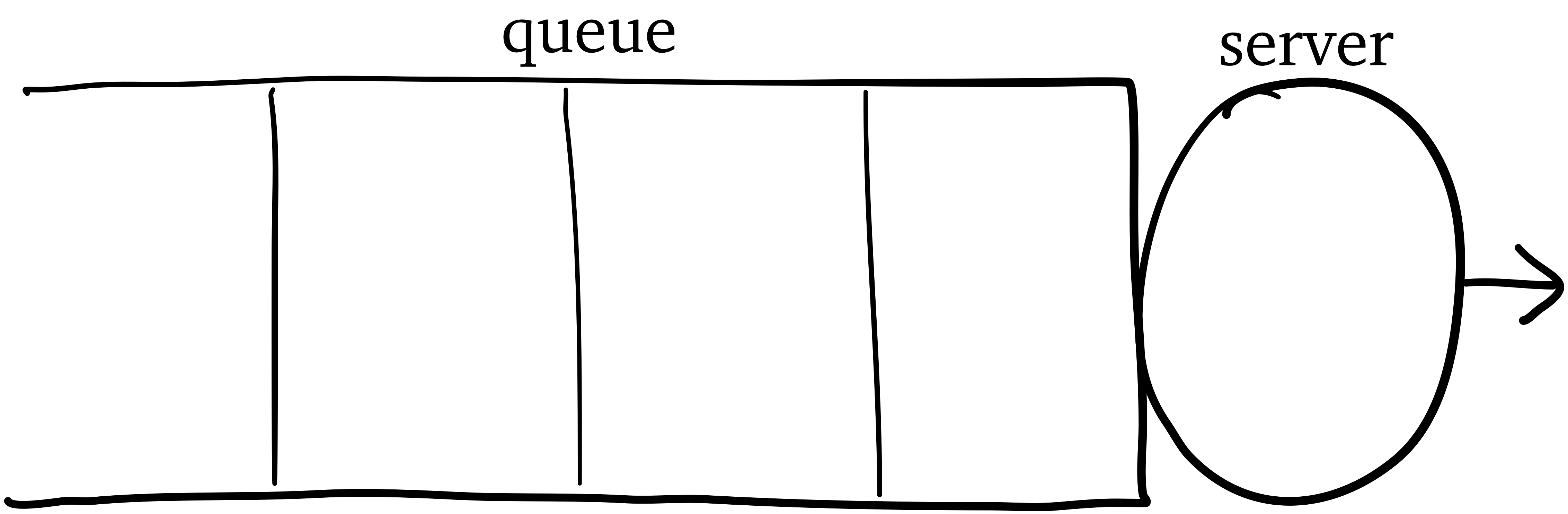
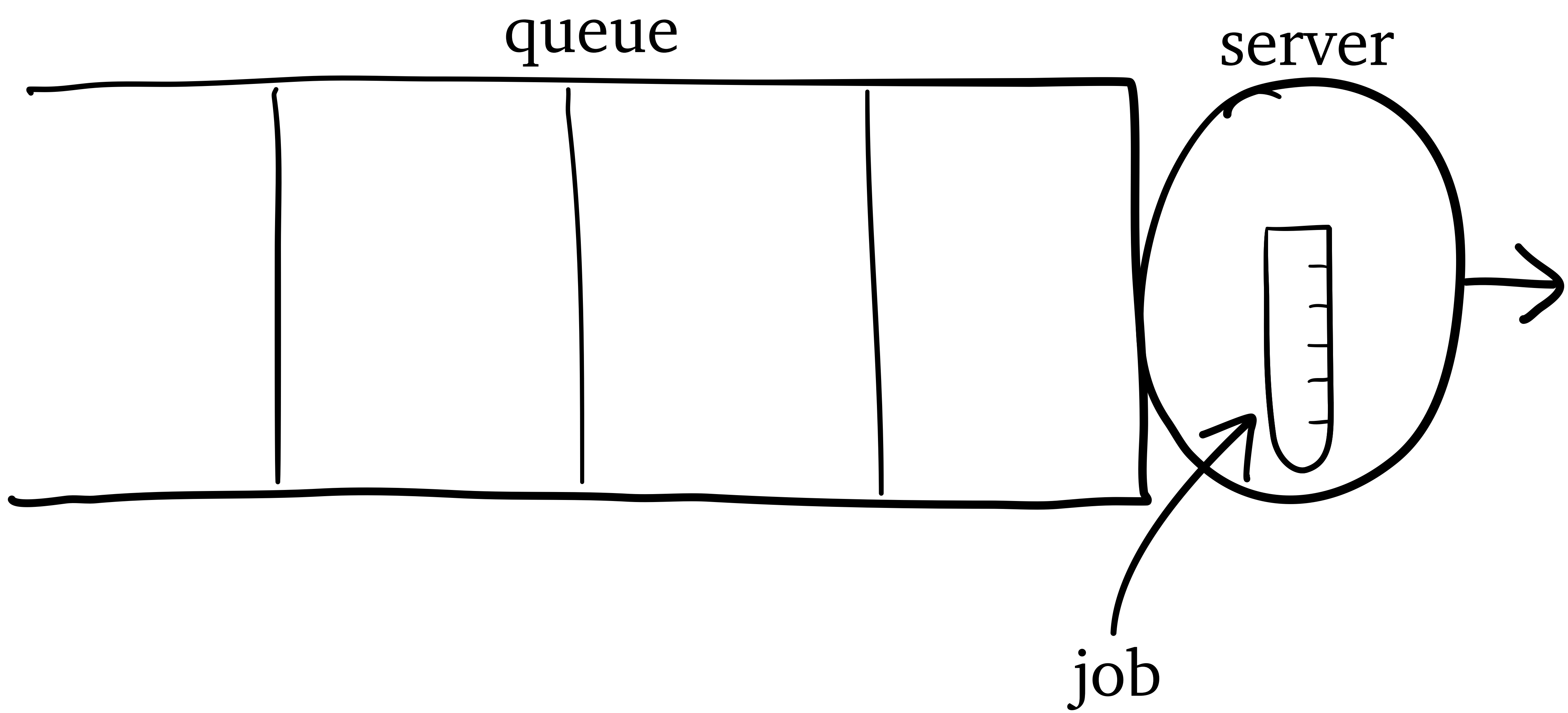
M/G/1 Queue



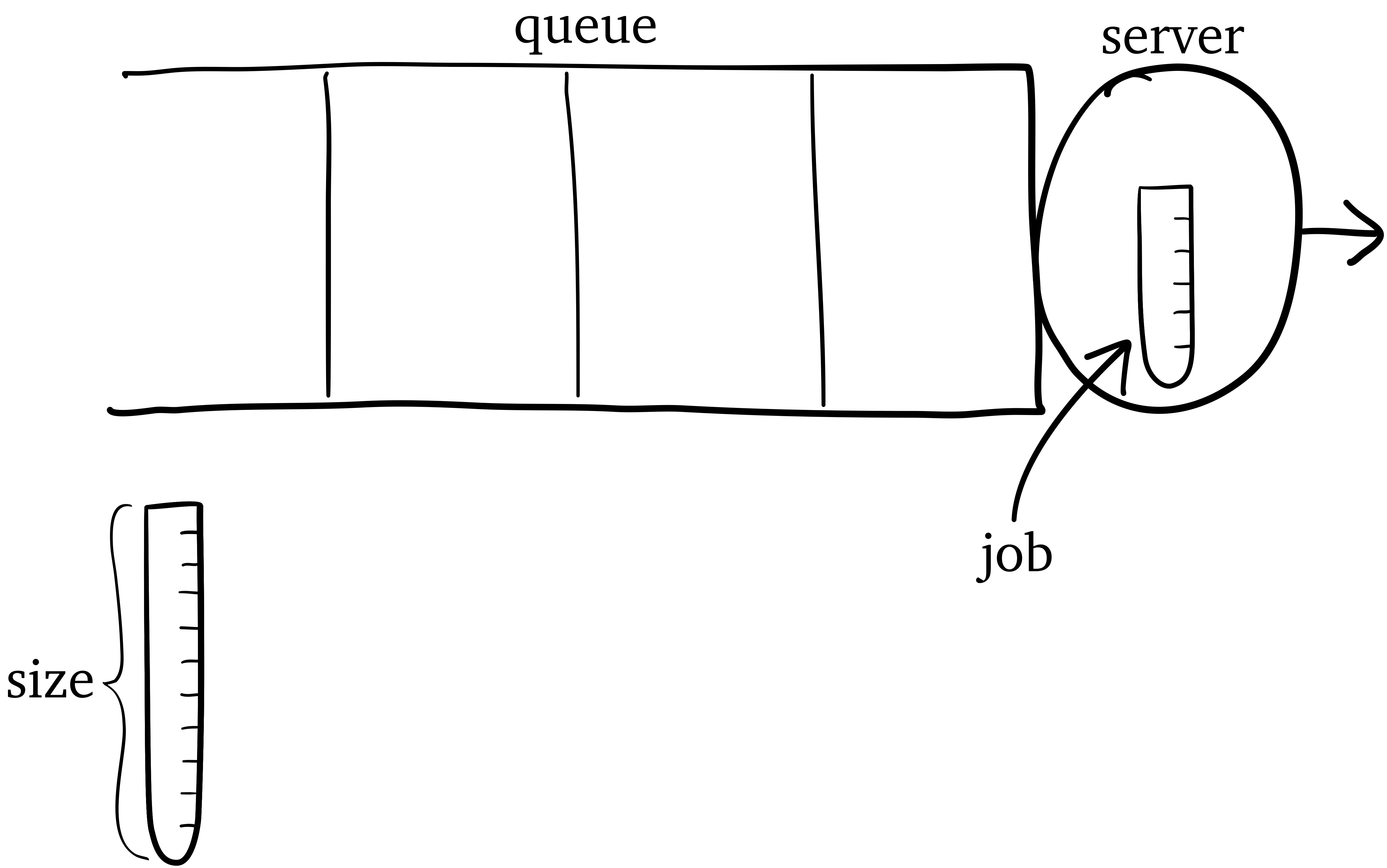
M/G/1 Queue



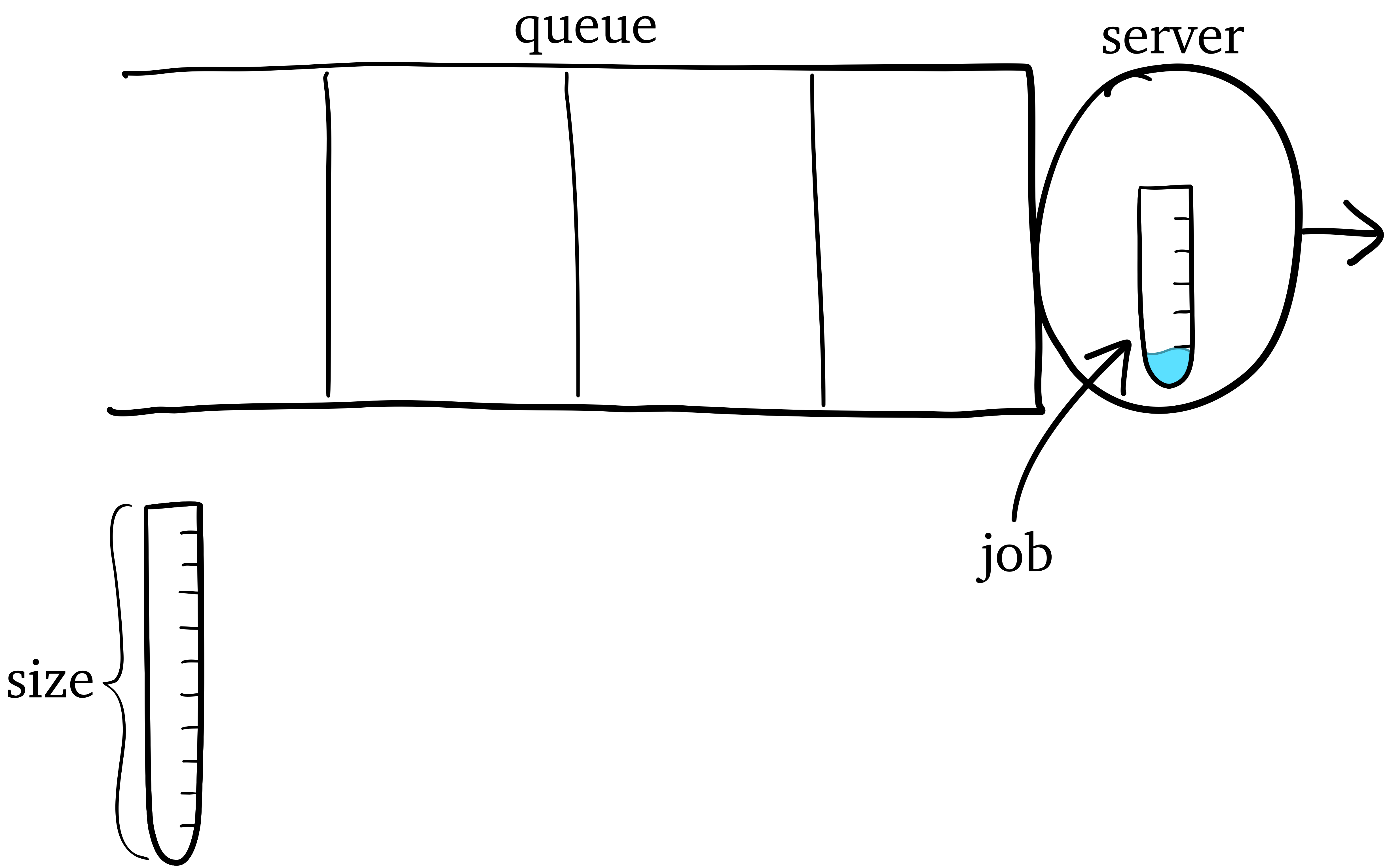
M/G/1 Queue



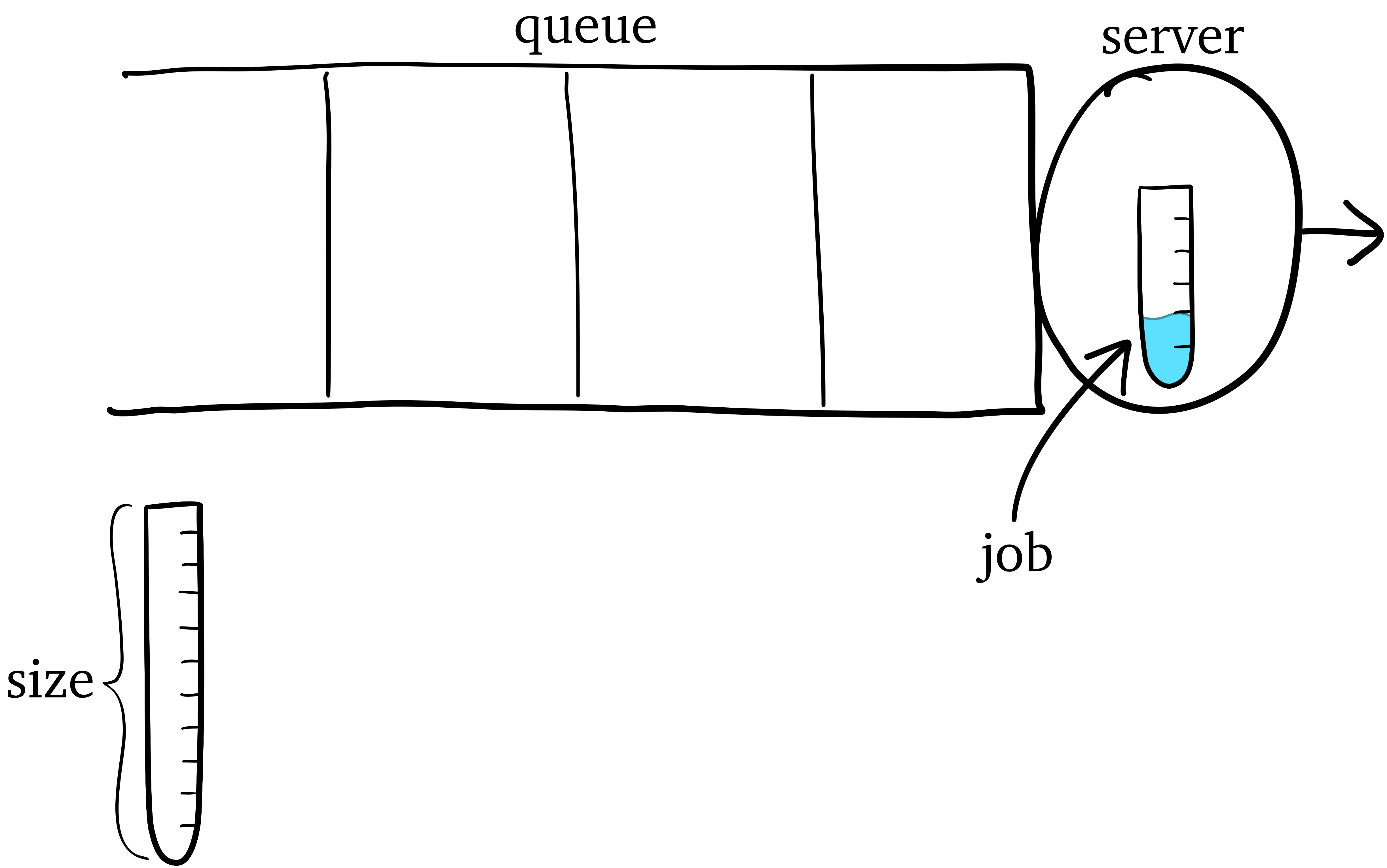
M/G/1 Queue



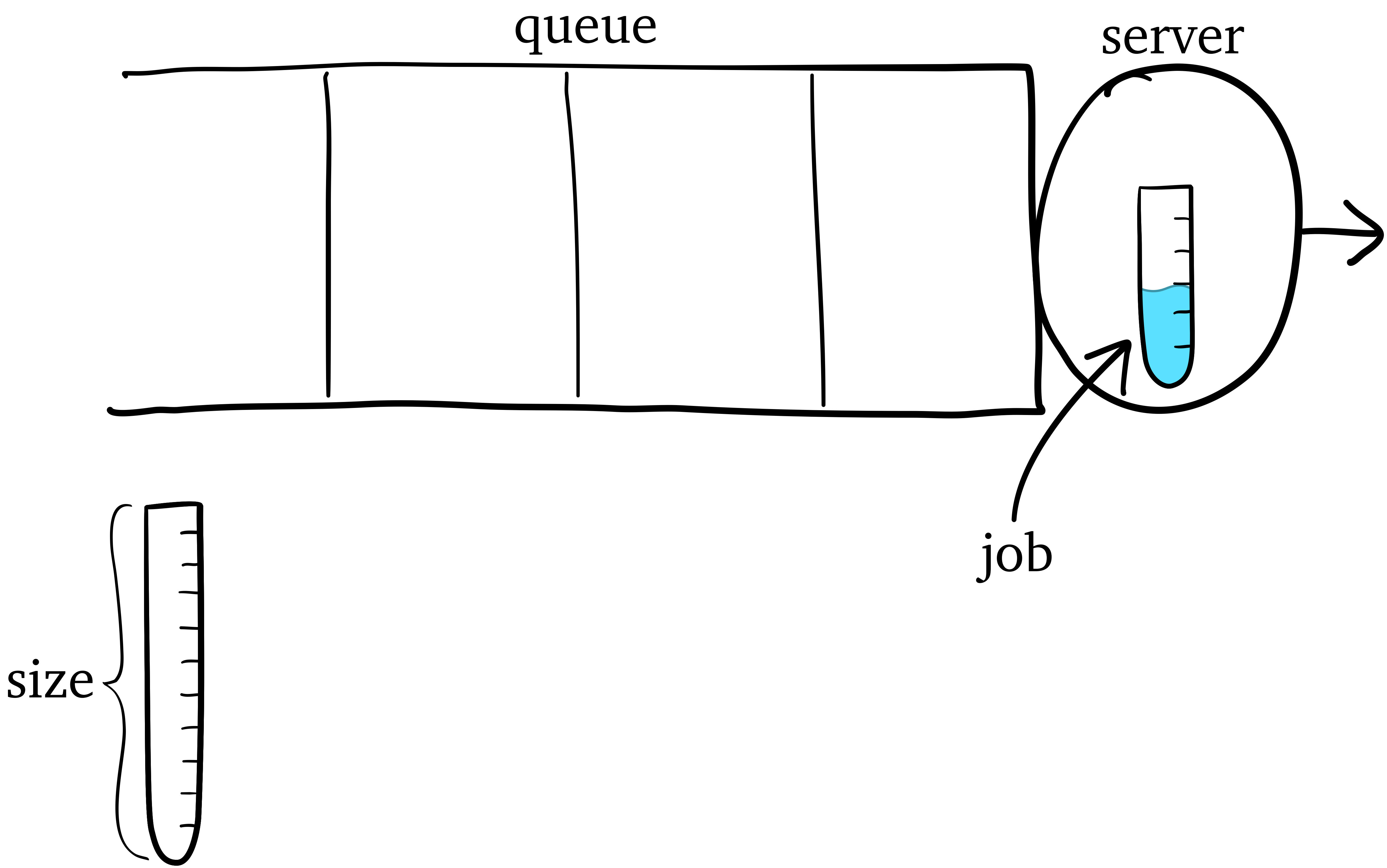
M/G/1 Queue



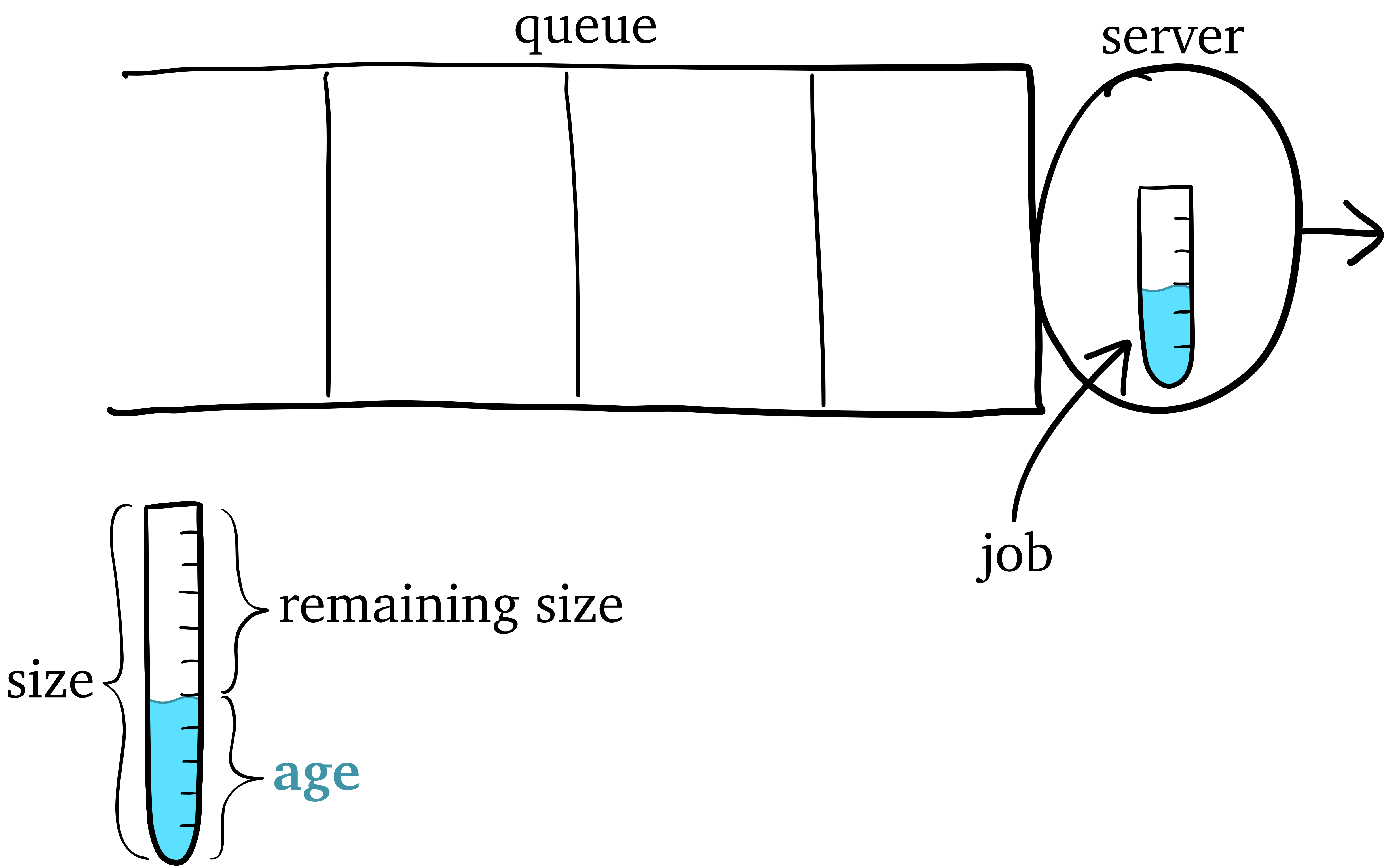
M/G/1 Queue



M/G/1 Queue



M/G/1 Queue



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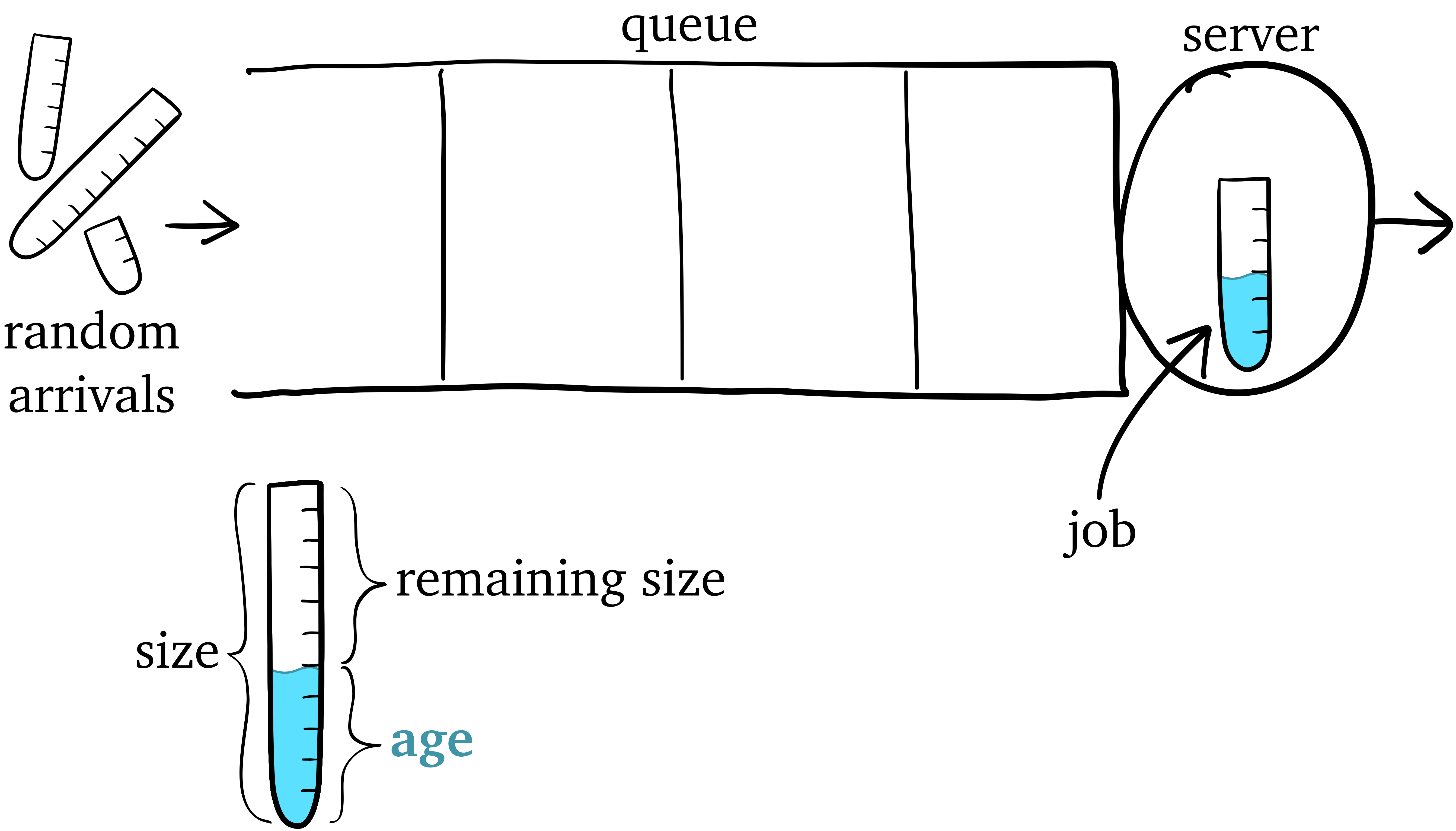

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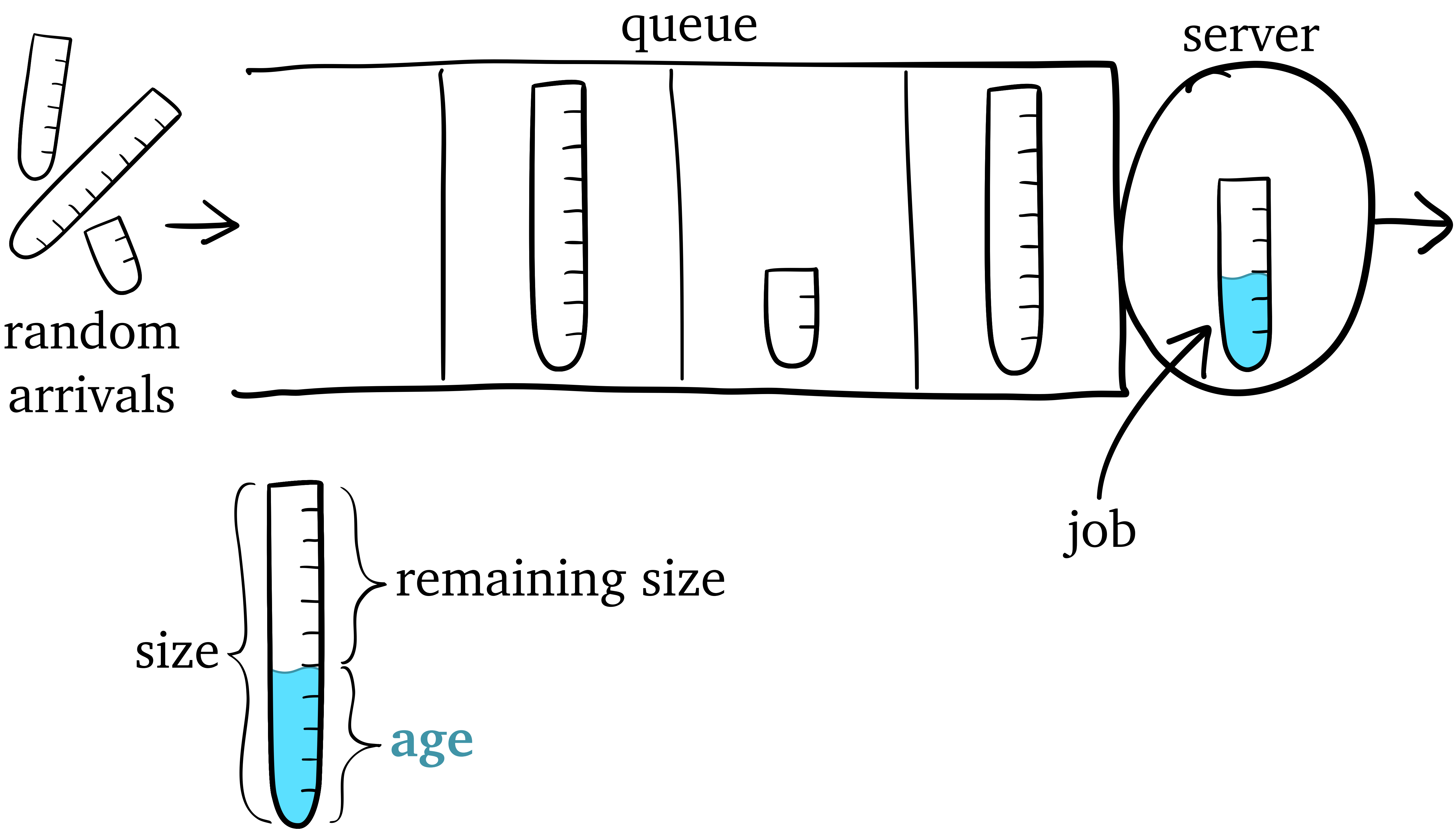
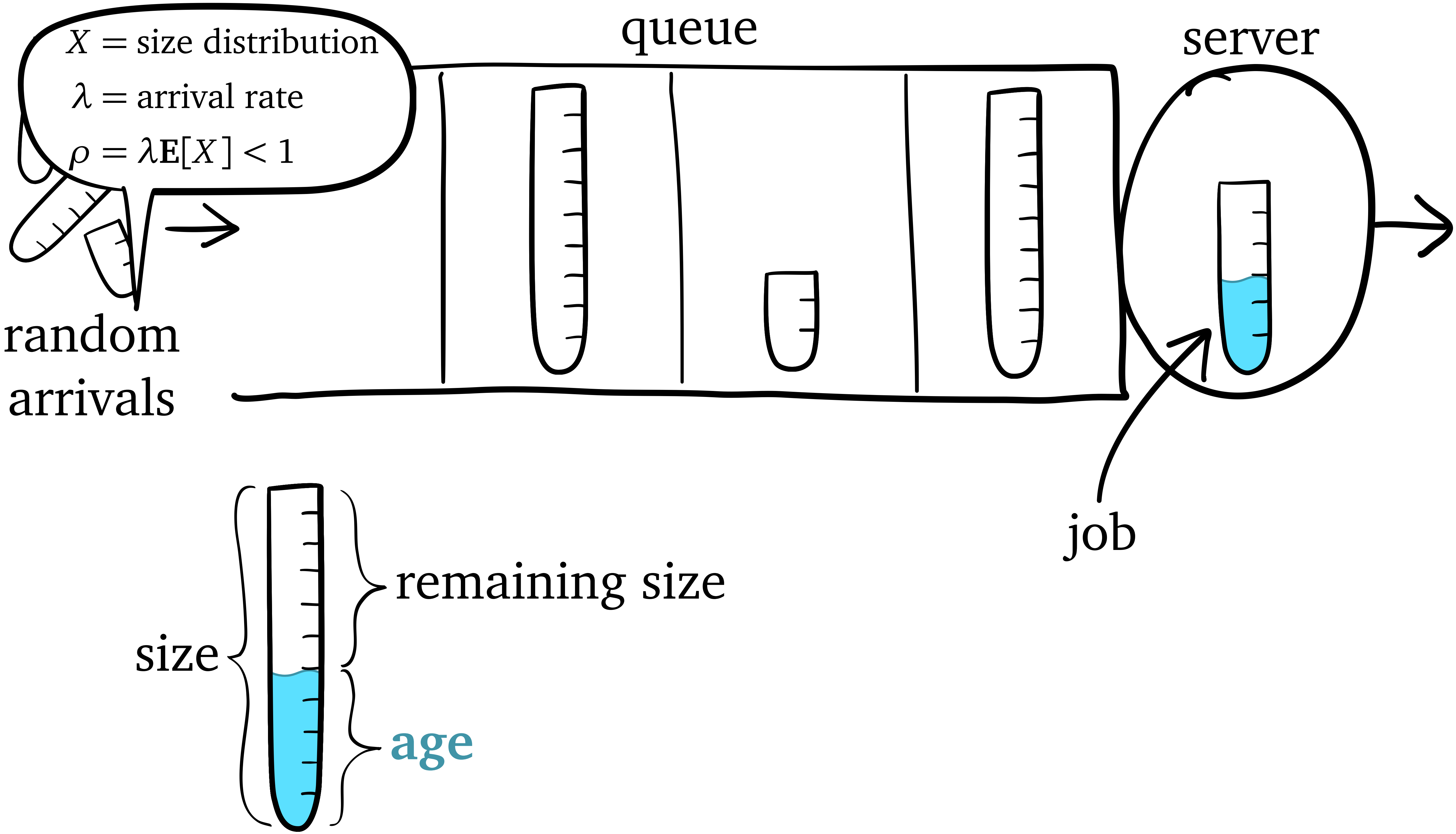
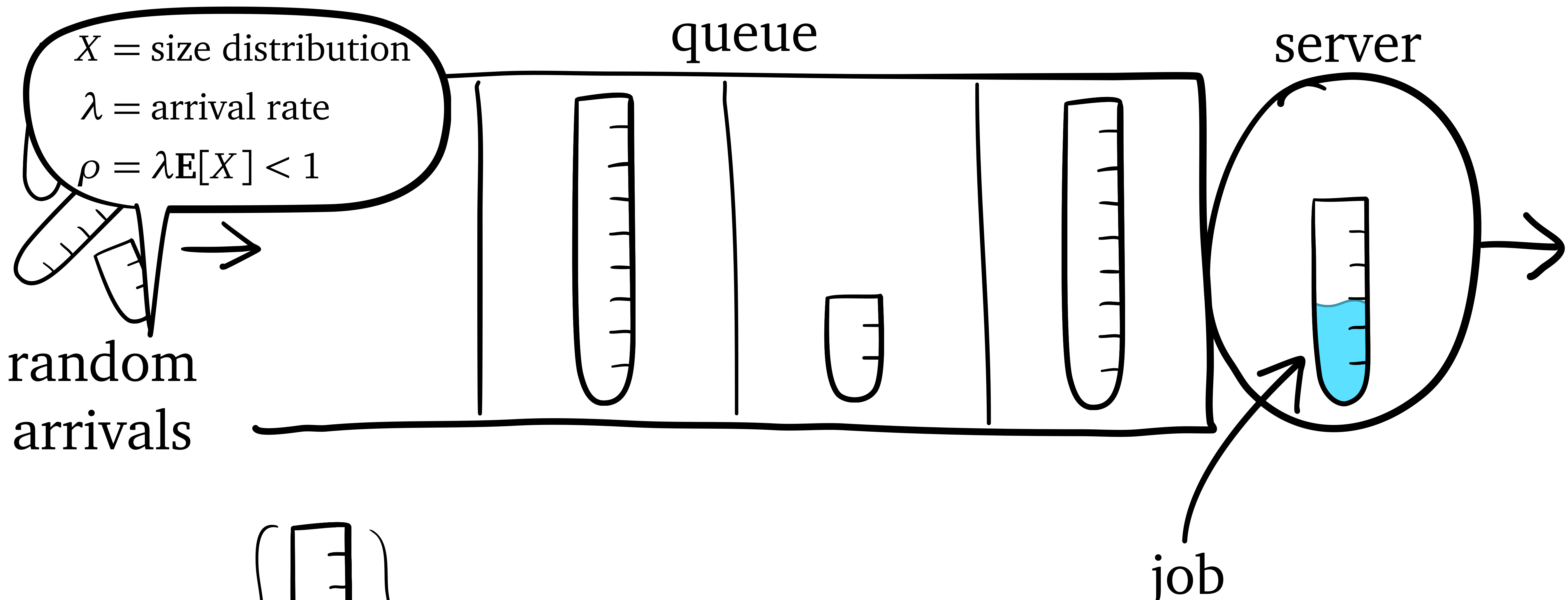


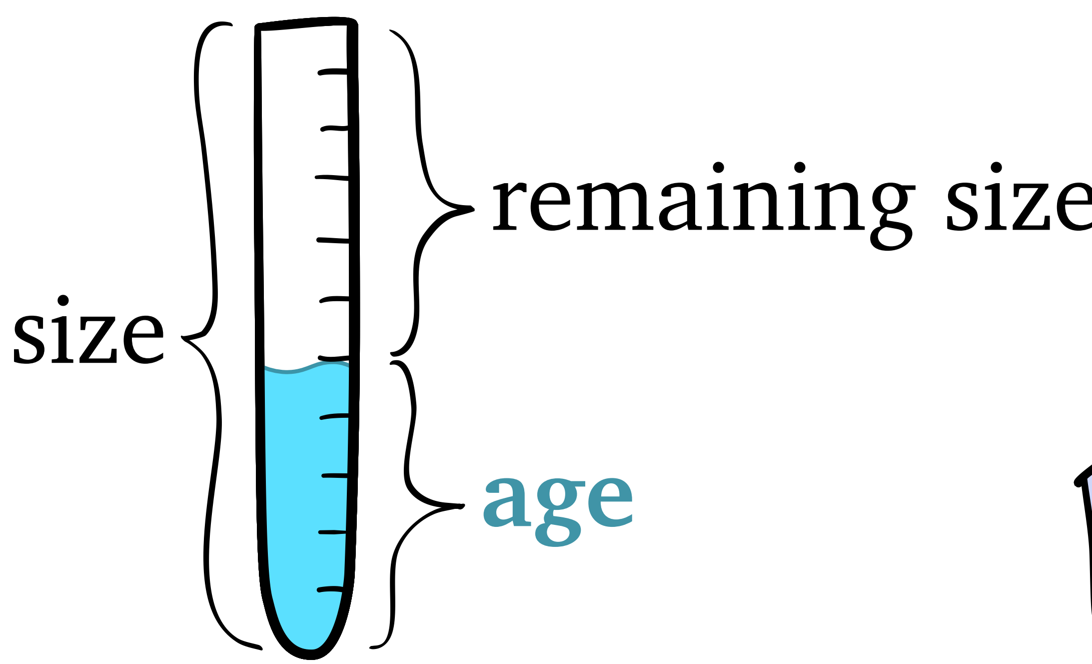
M/G/1 Queue

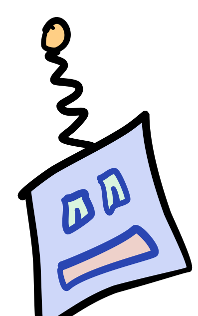


X = size distribution
λ = arrival rate
$\rho = \lambda \mathbf{E}[X] < 1$
random arrivals
queue
server
job
size
remaining size
age

M/G/1 Queue

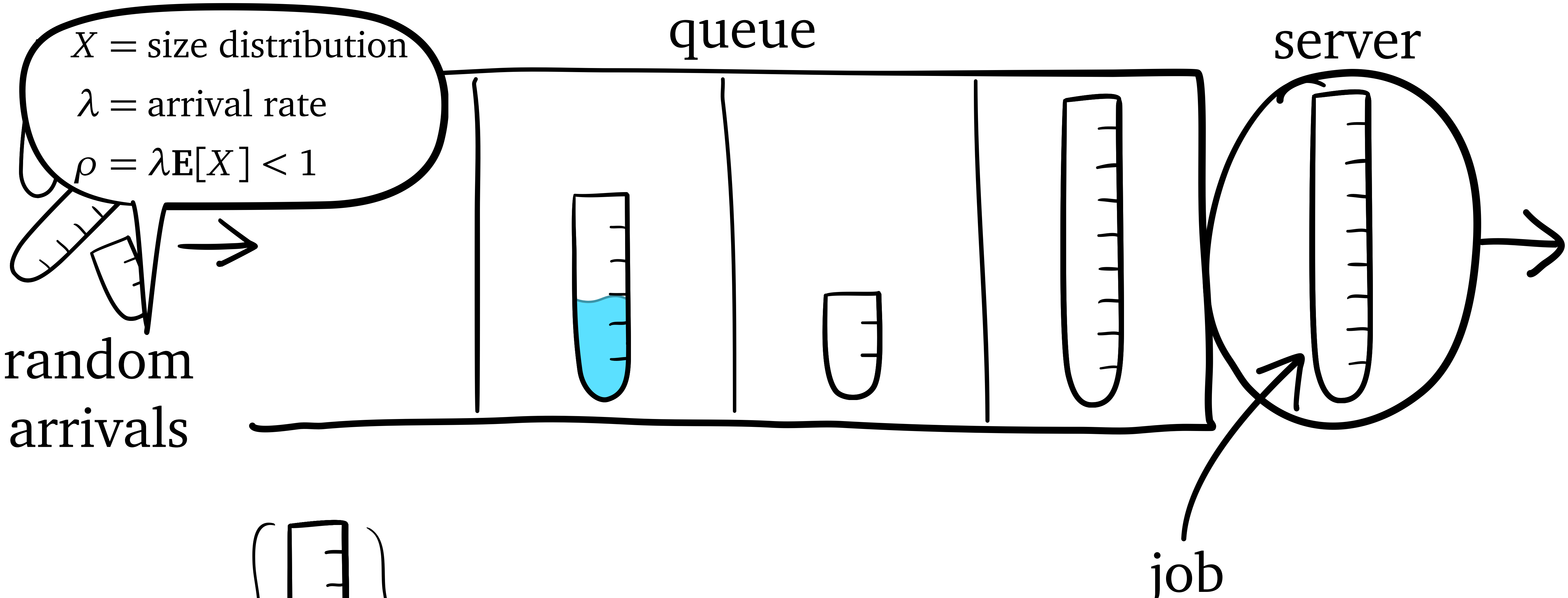


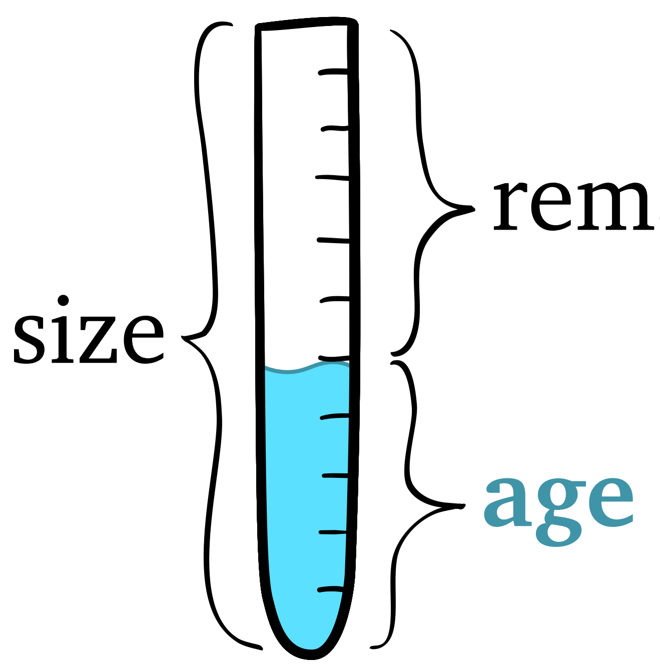


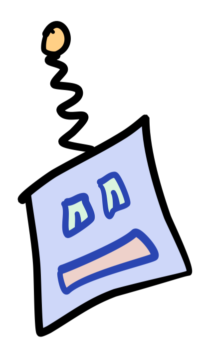


Scheduling policy:
picks which job to serve

M/G/1 Queue







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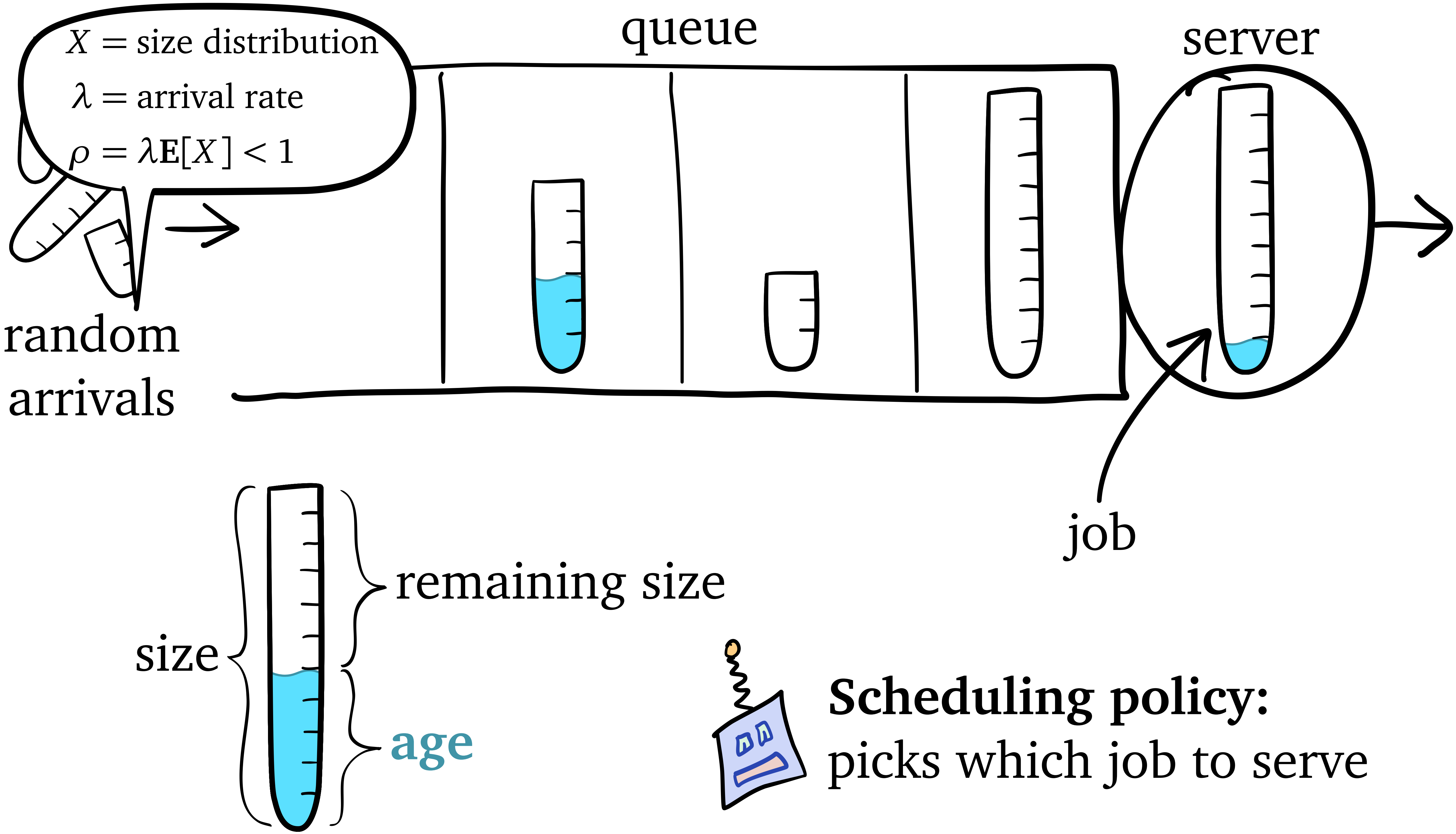

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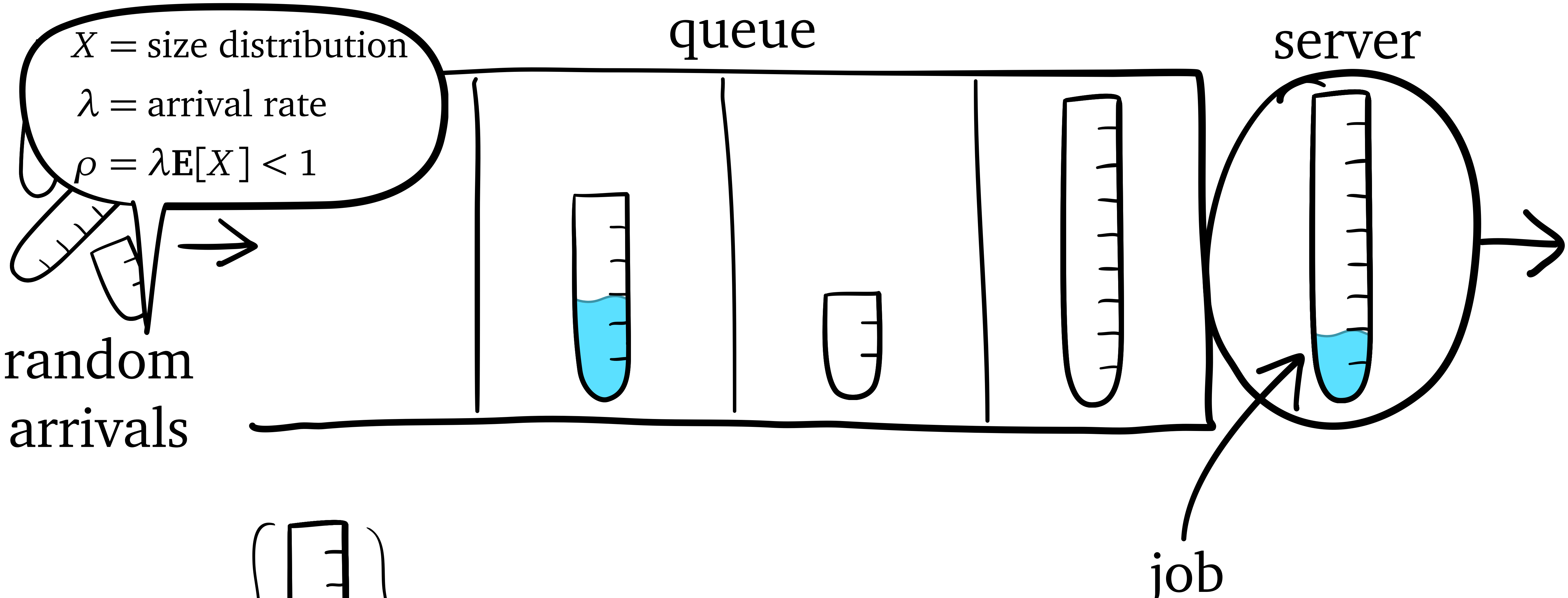

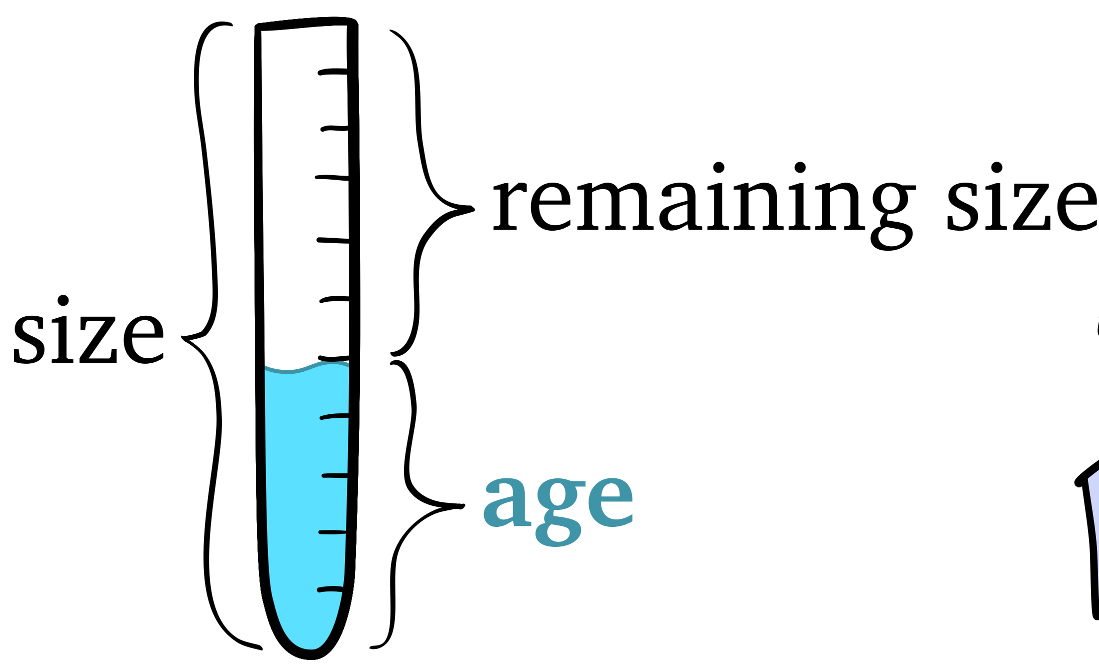

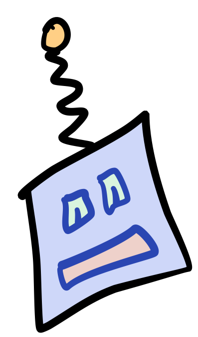

Scheduling policy:
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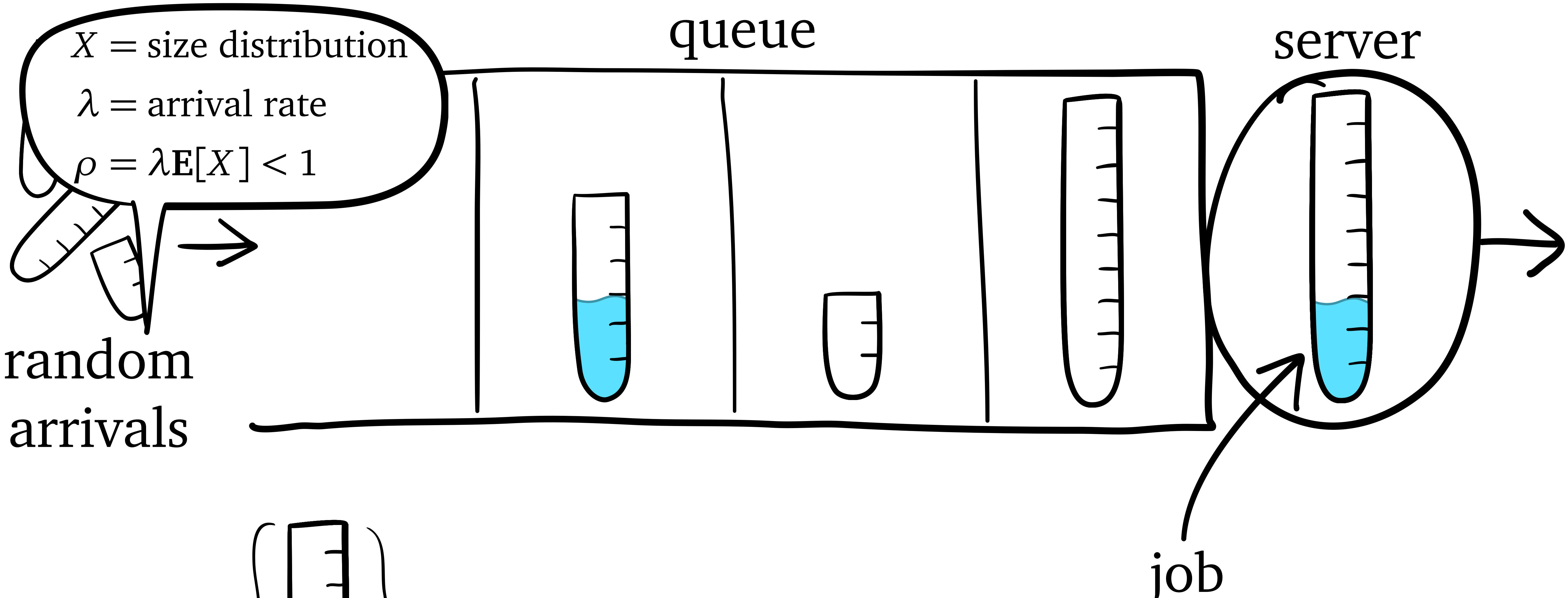
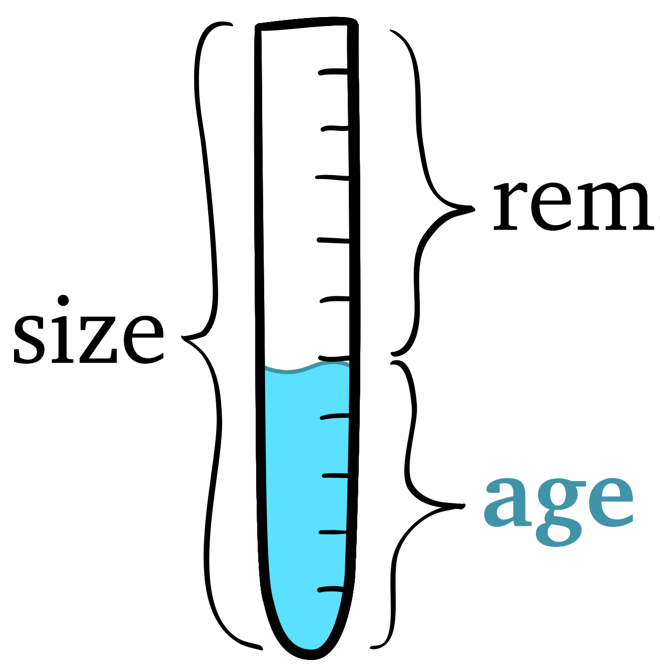
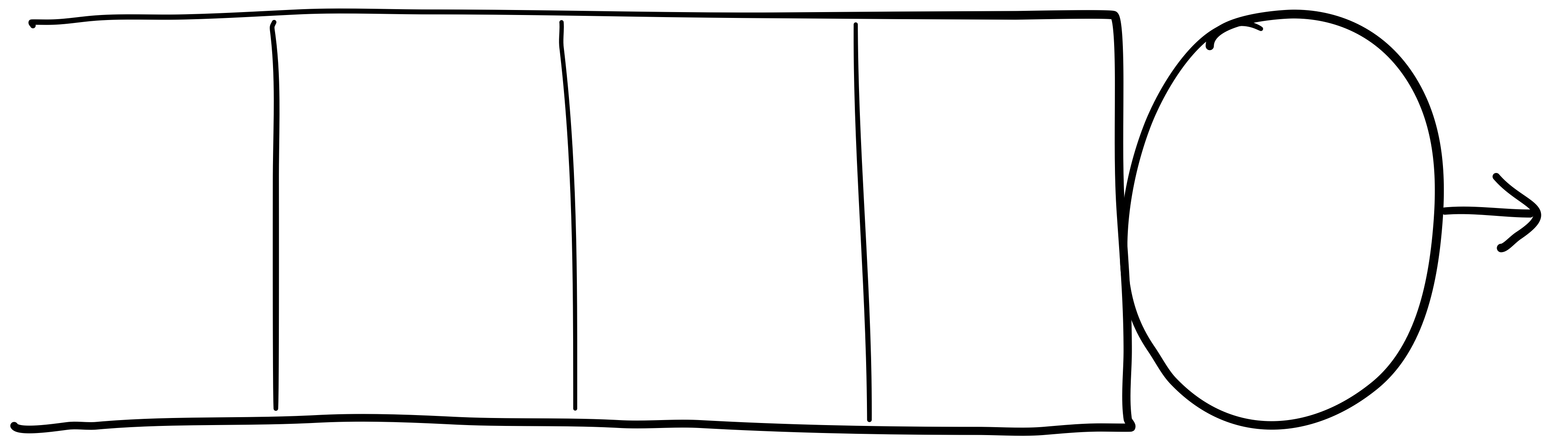
M/G/1 Queue



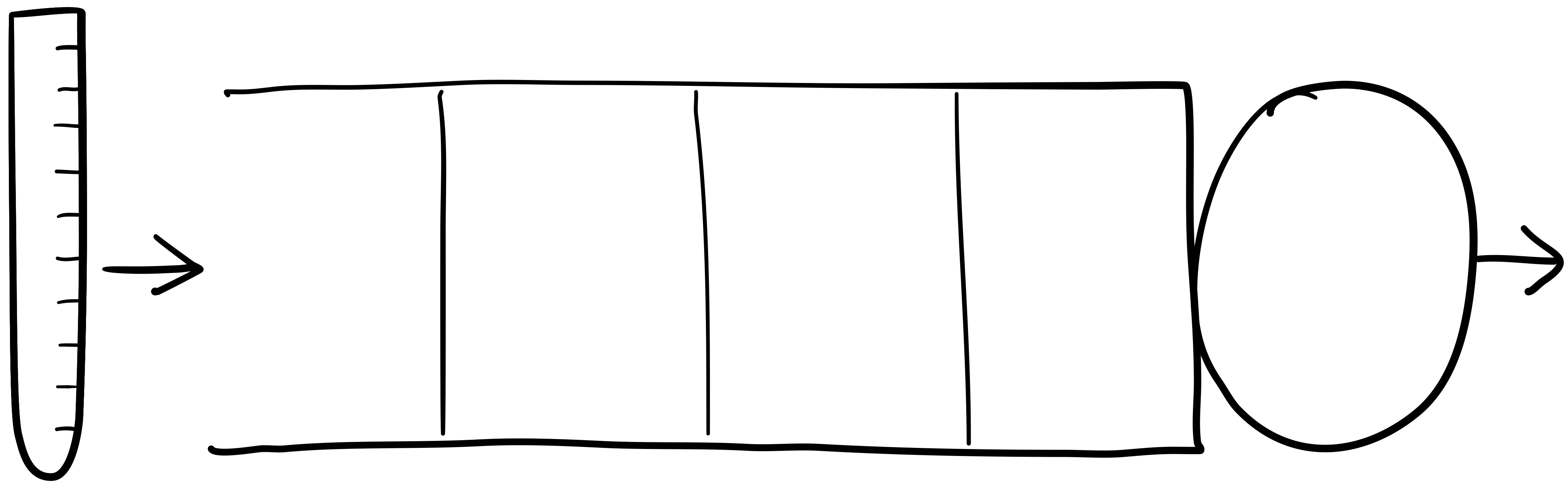


Scheduling policy:
picks which job to serve

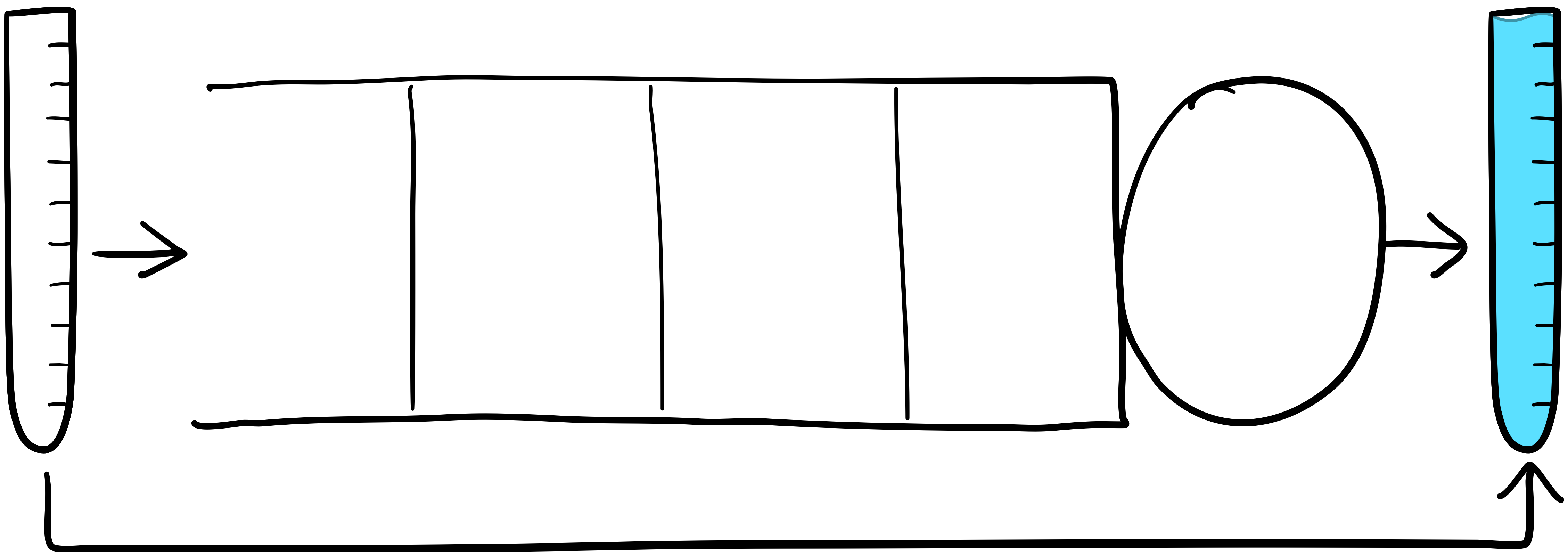
Response Time

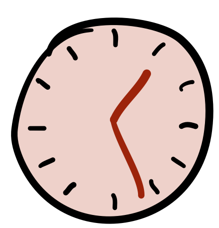


Response Time

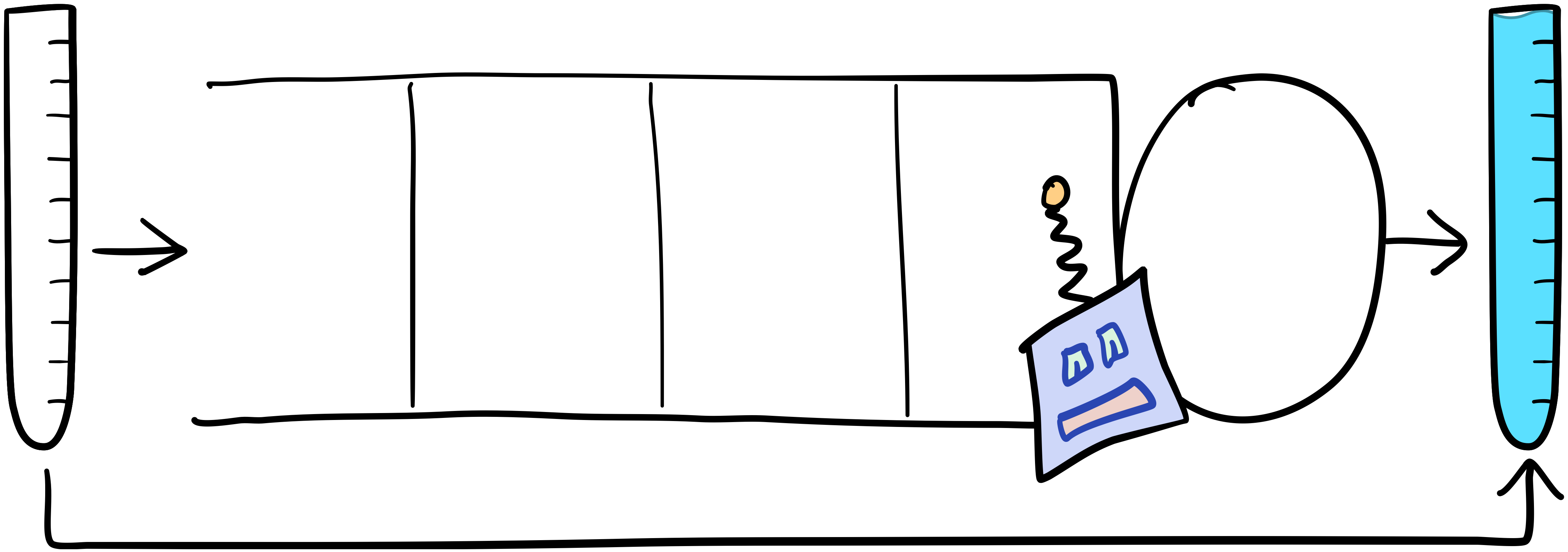


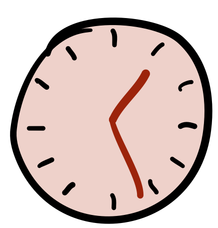
Response Time



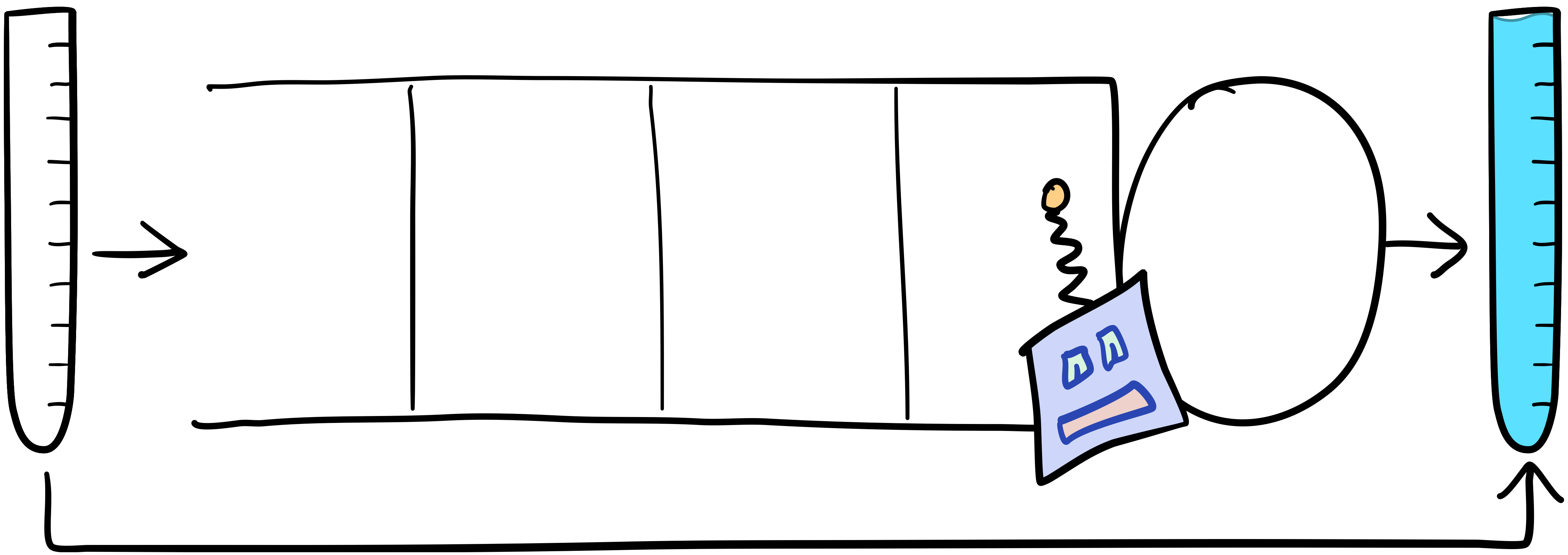
 $= T =$ *response time*

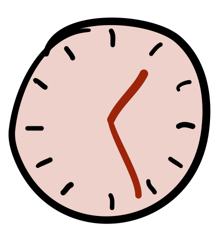
Response Time



 $= T = \textit{response time}$

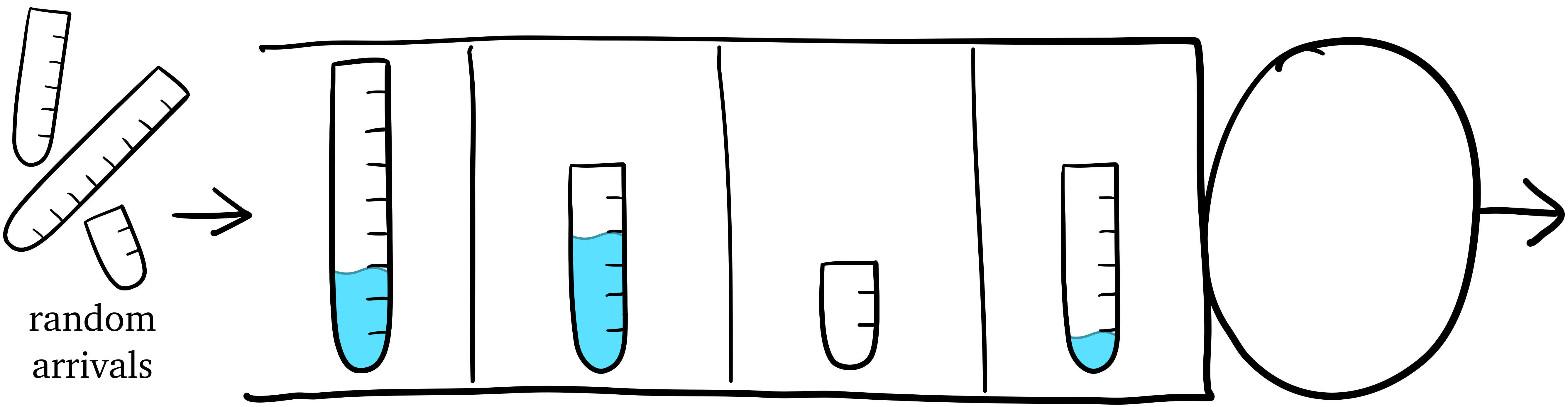
Response Time



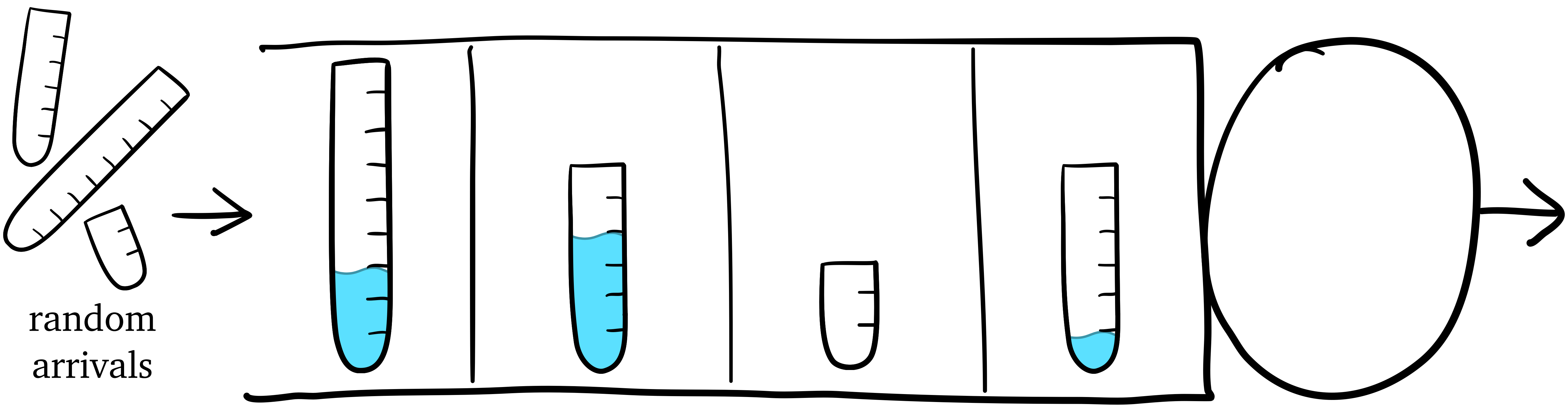
 = T = *response time*

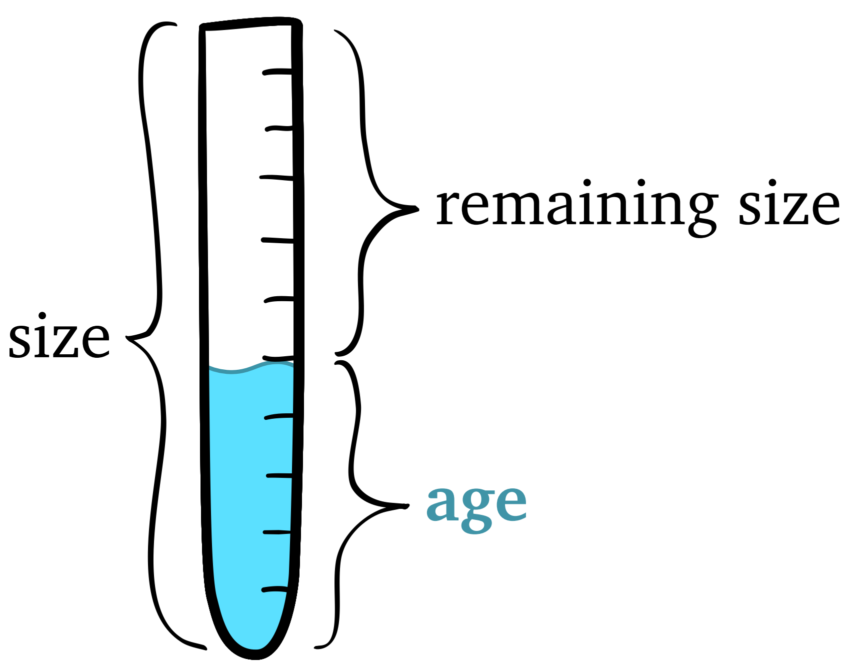
Goal: schedule to minimize
mean response time $\mathbf{E}[T]$

How to Schedule?

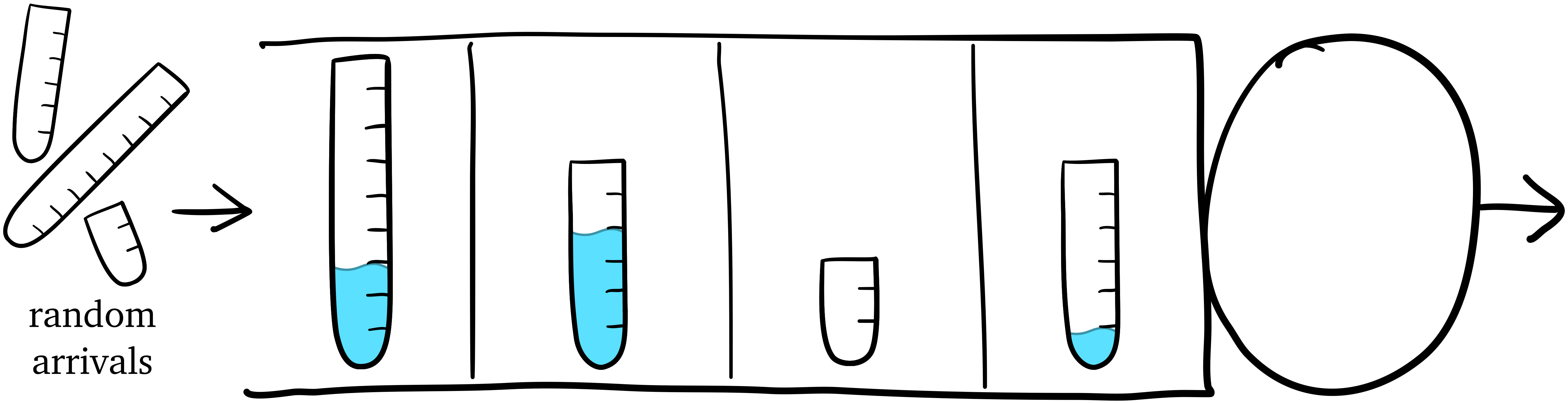


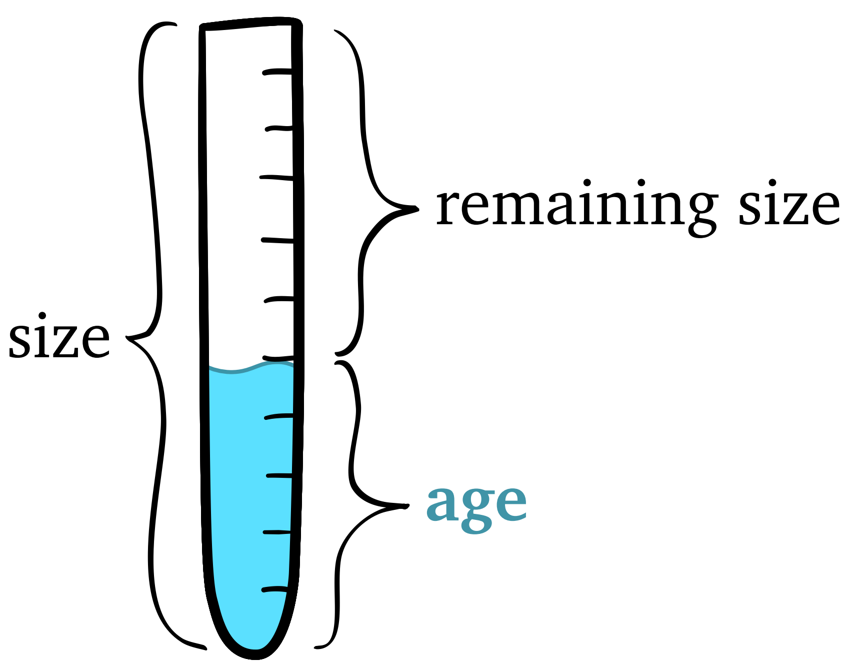
How to Schedule?

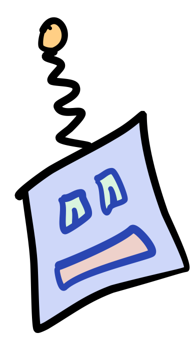




How to Schedule?

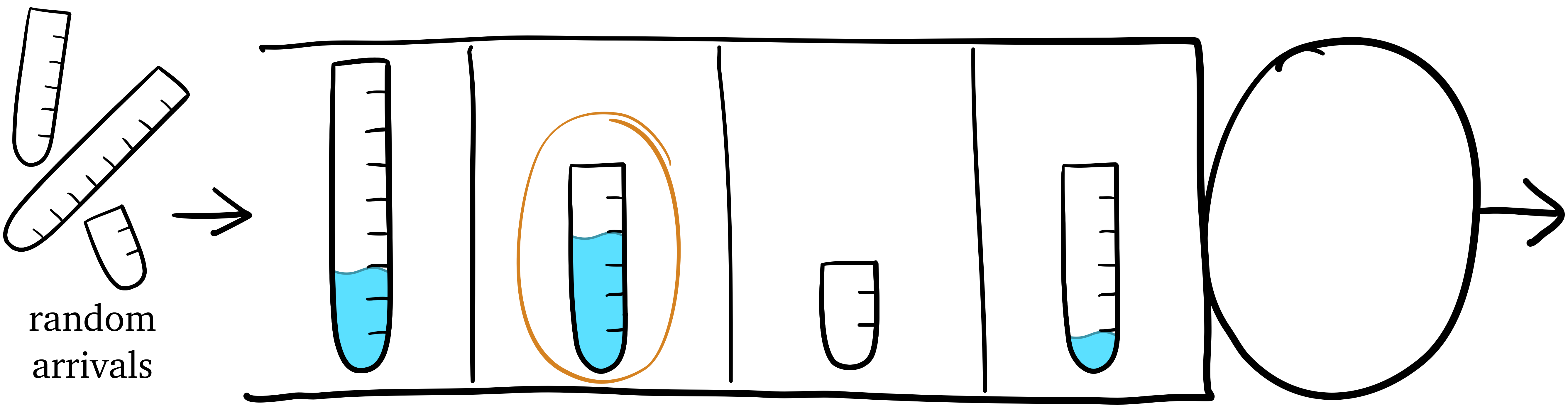


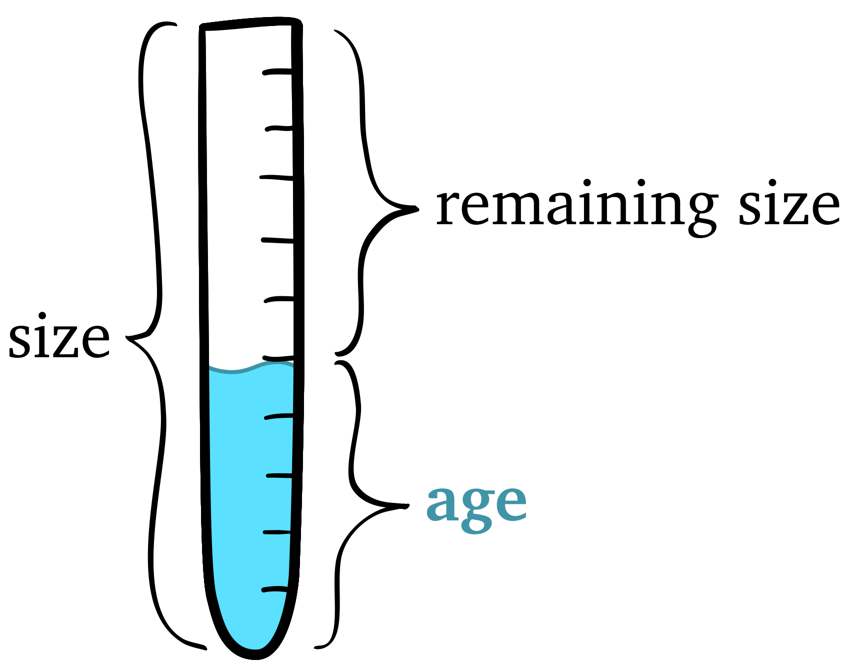


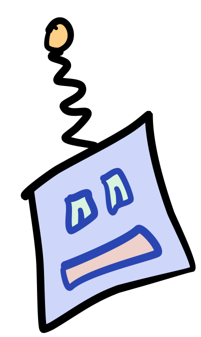


SRPT: always serve job of *least remaining size*

How to Schedule?







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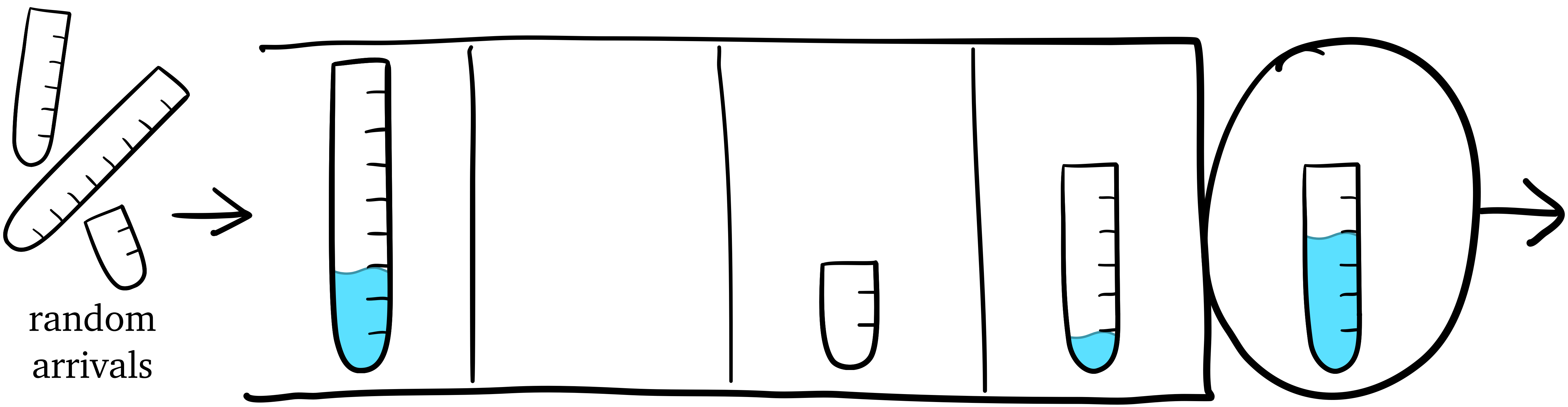
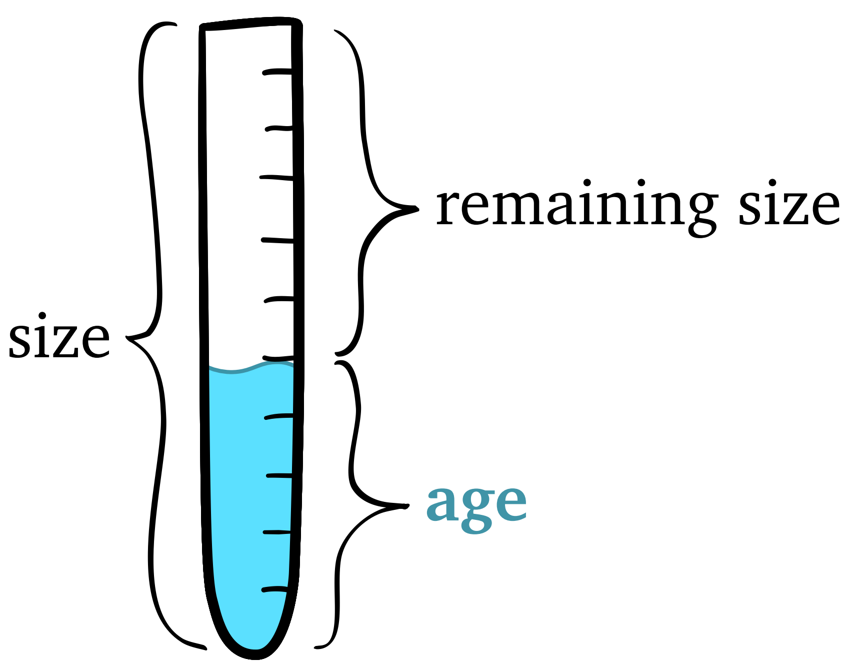
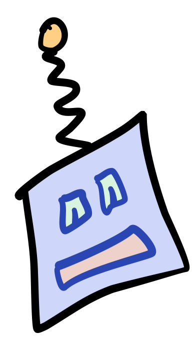
How to Schedule?







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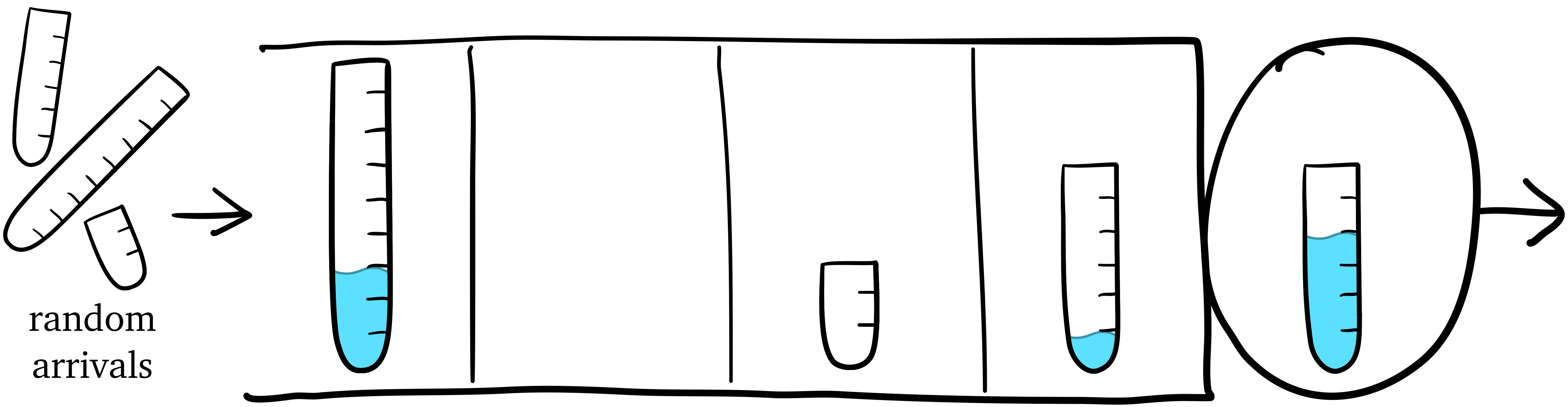
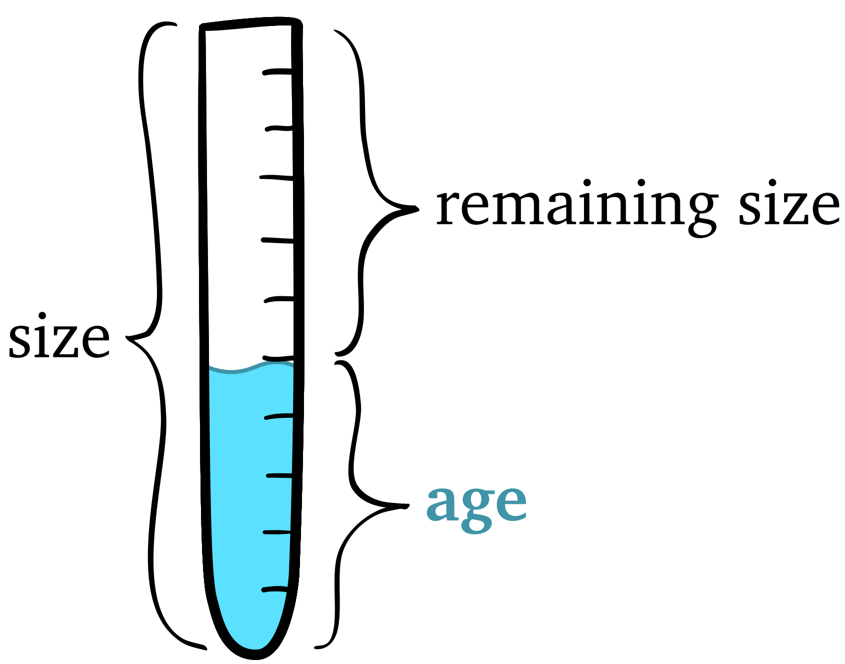
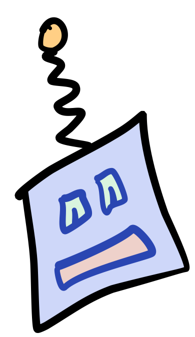
How to Schedule?



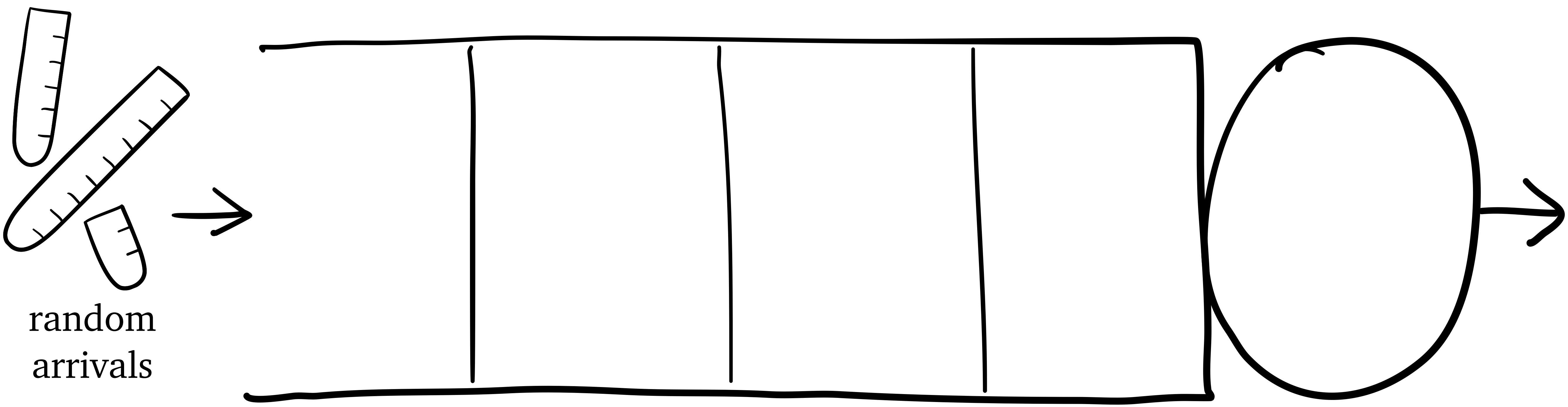




SRPT: always serve job of *least remaining size*

SRPT minimizes $\mathbf{E}[T]$

M/G/k Queue



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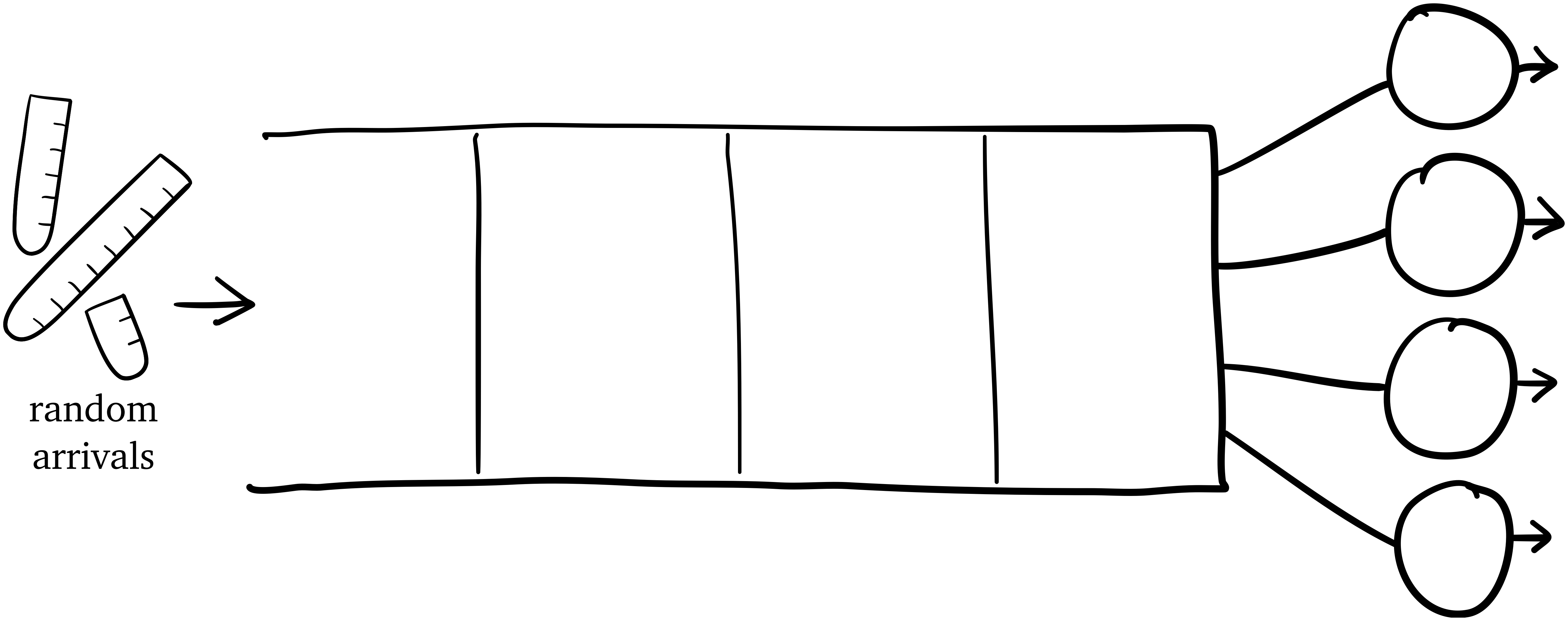
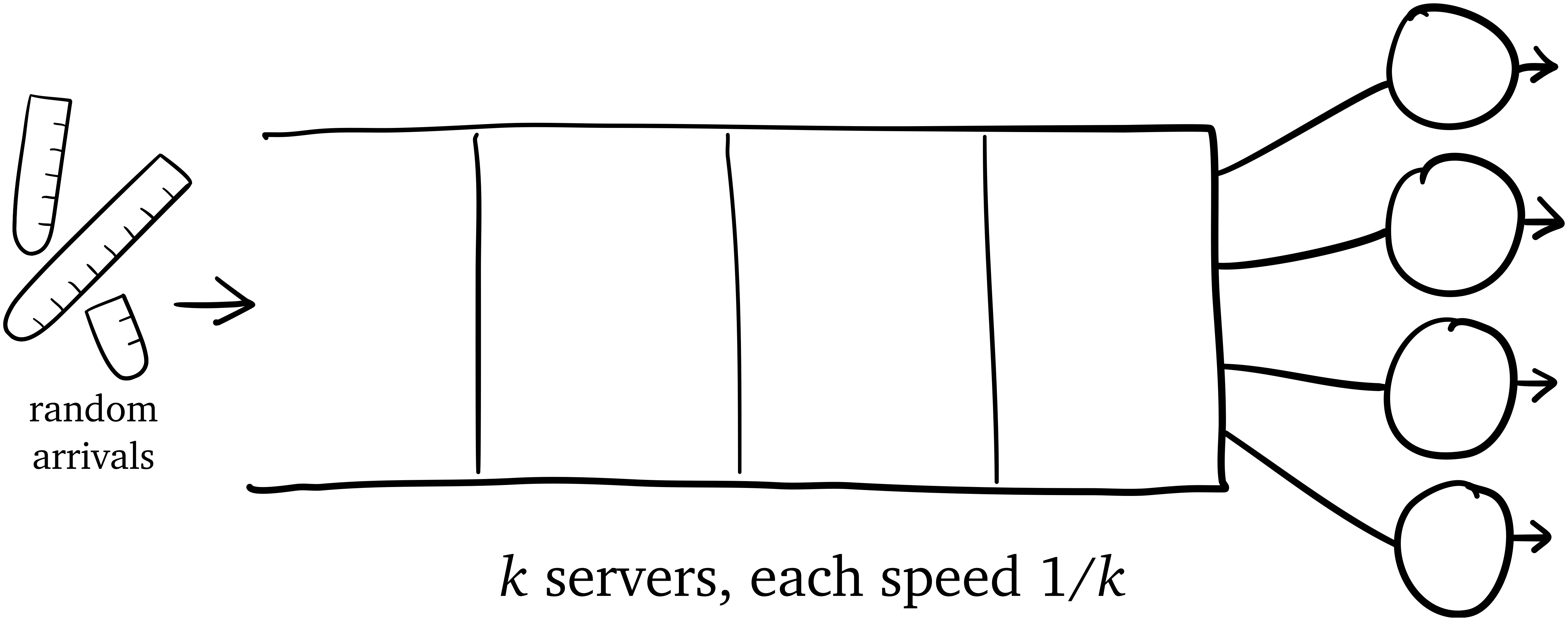
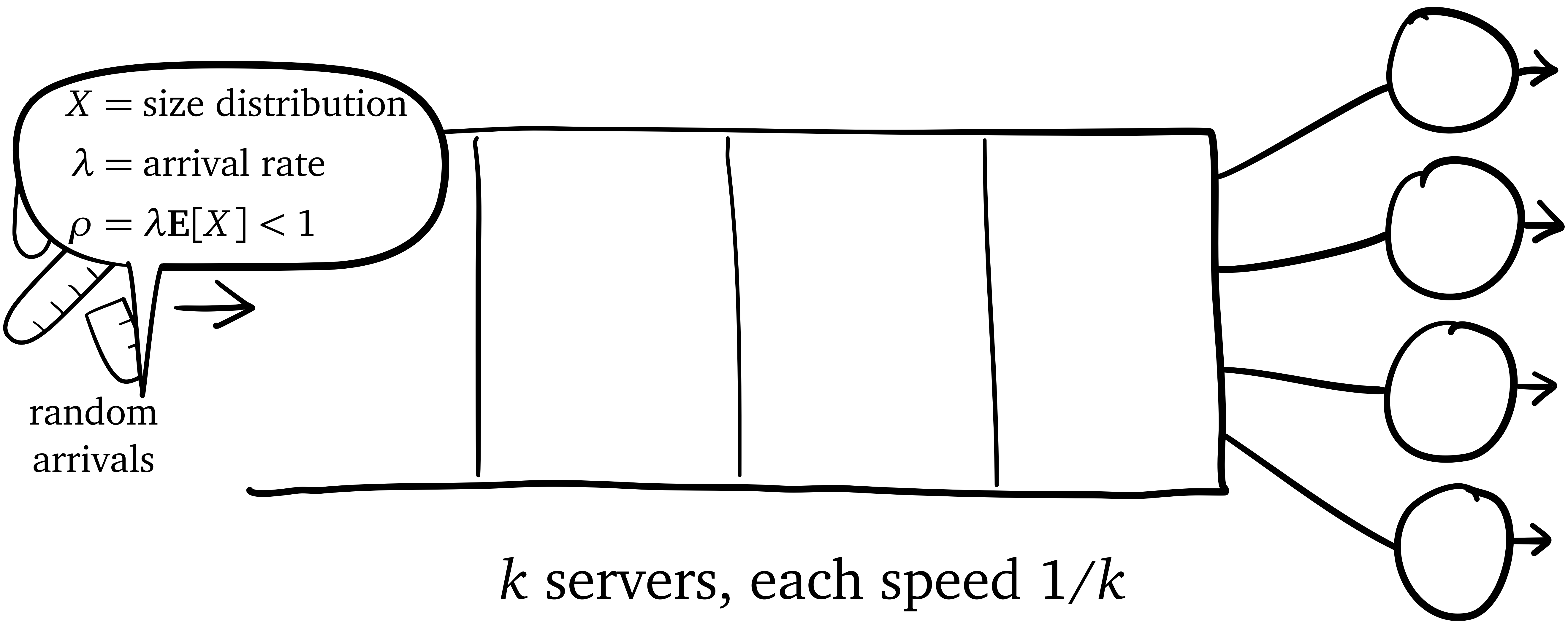


M/G/k Queue



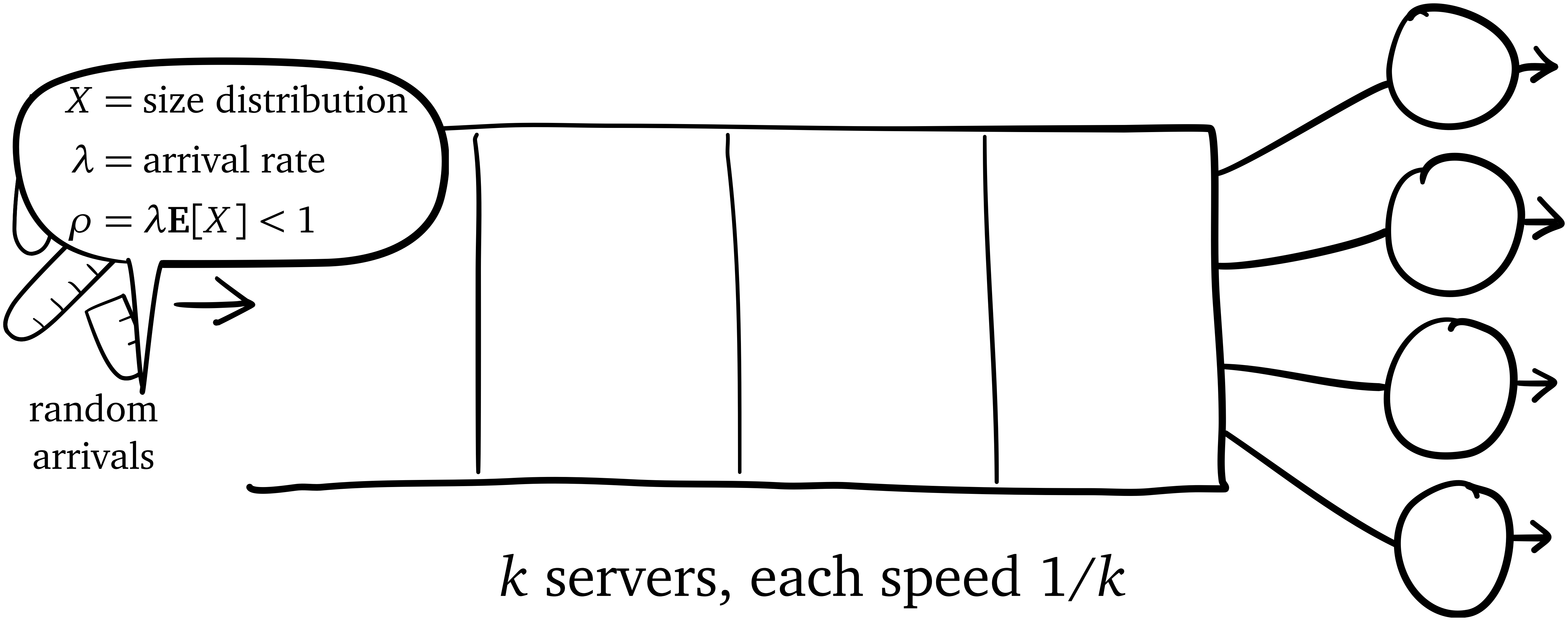
k servers, each speed $1/k$

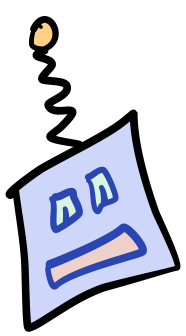
M/G/k Queue



X = size distribution
λ = arrival rate
$\rho = \lambda \mathbf{E}[X] < 1$
random arrivals
k servers, each speed $1/k$

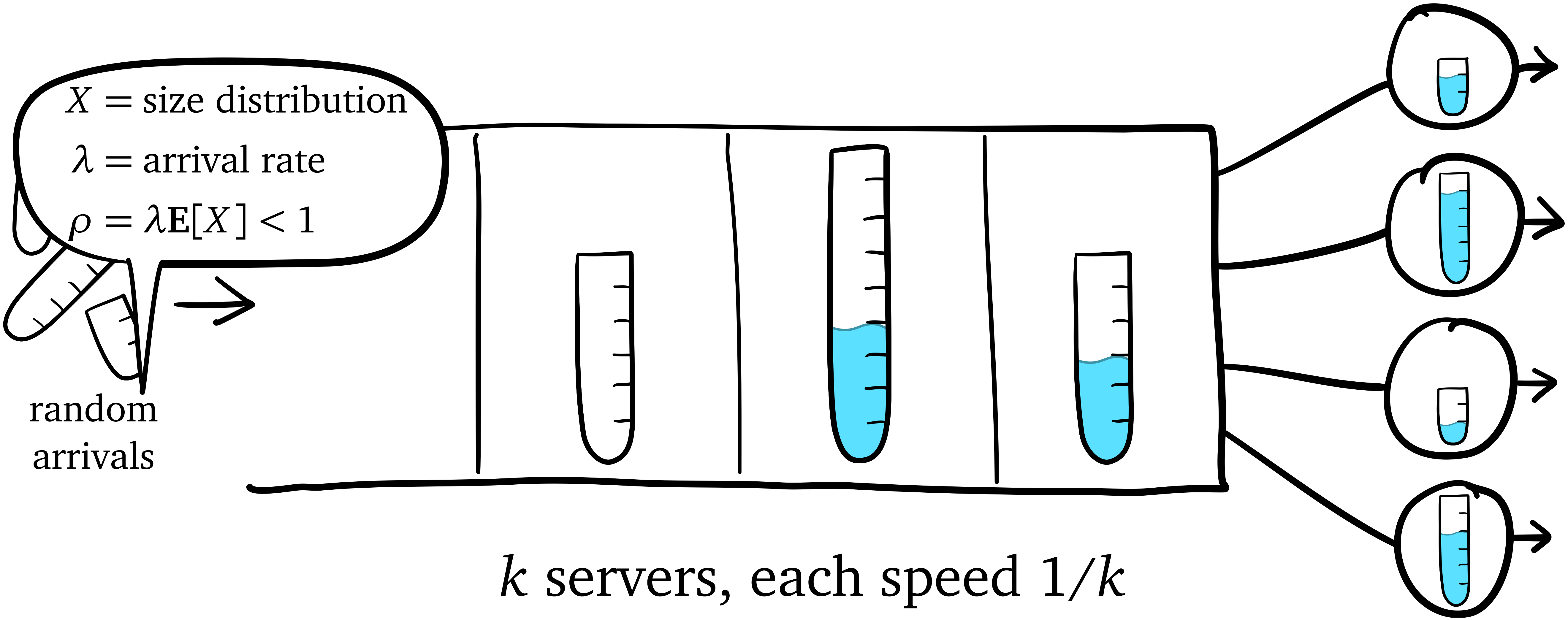
M/G/k Queue

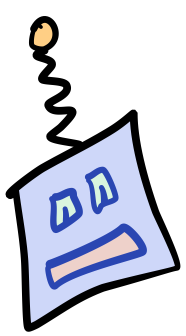




SRPT-*k*: always serve k jobs of *least remaining size*

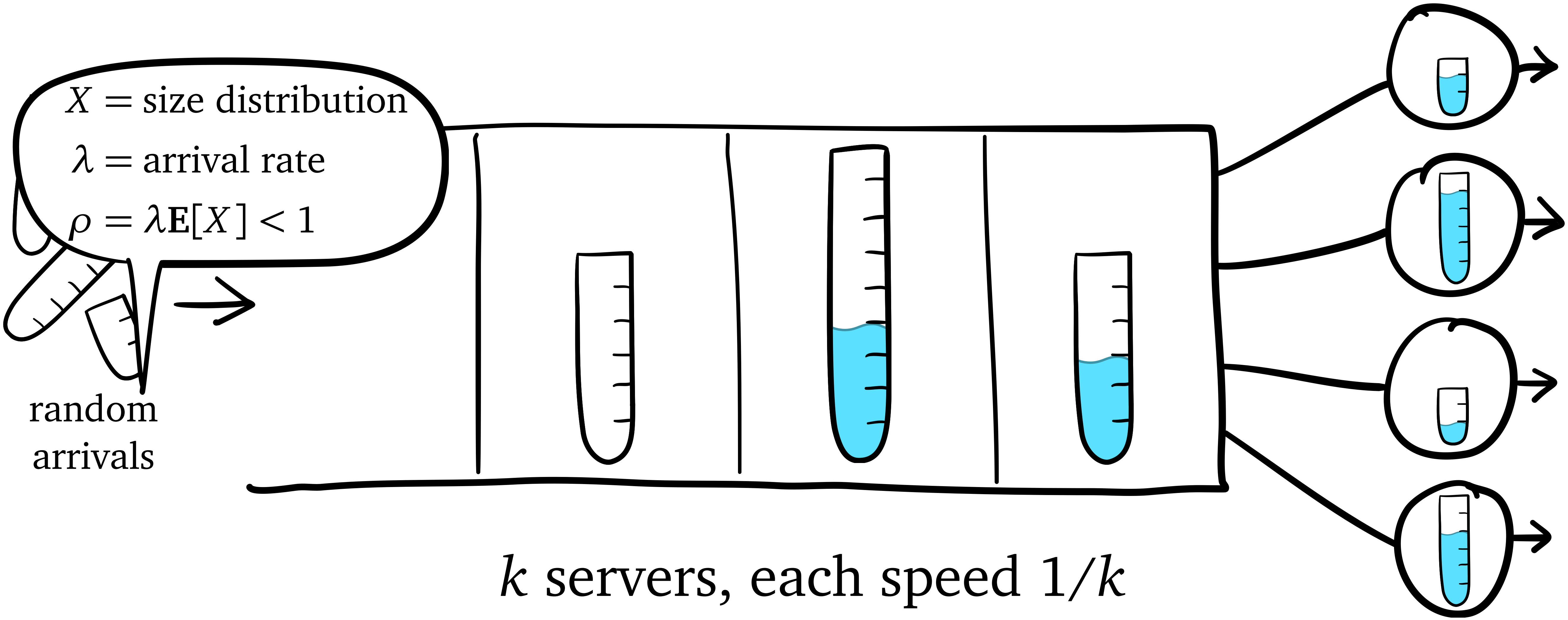
M/G/k Queue

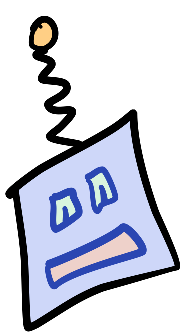




SRPT-k: always serve k jobs of *least remaining size*

M/G/k Queue

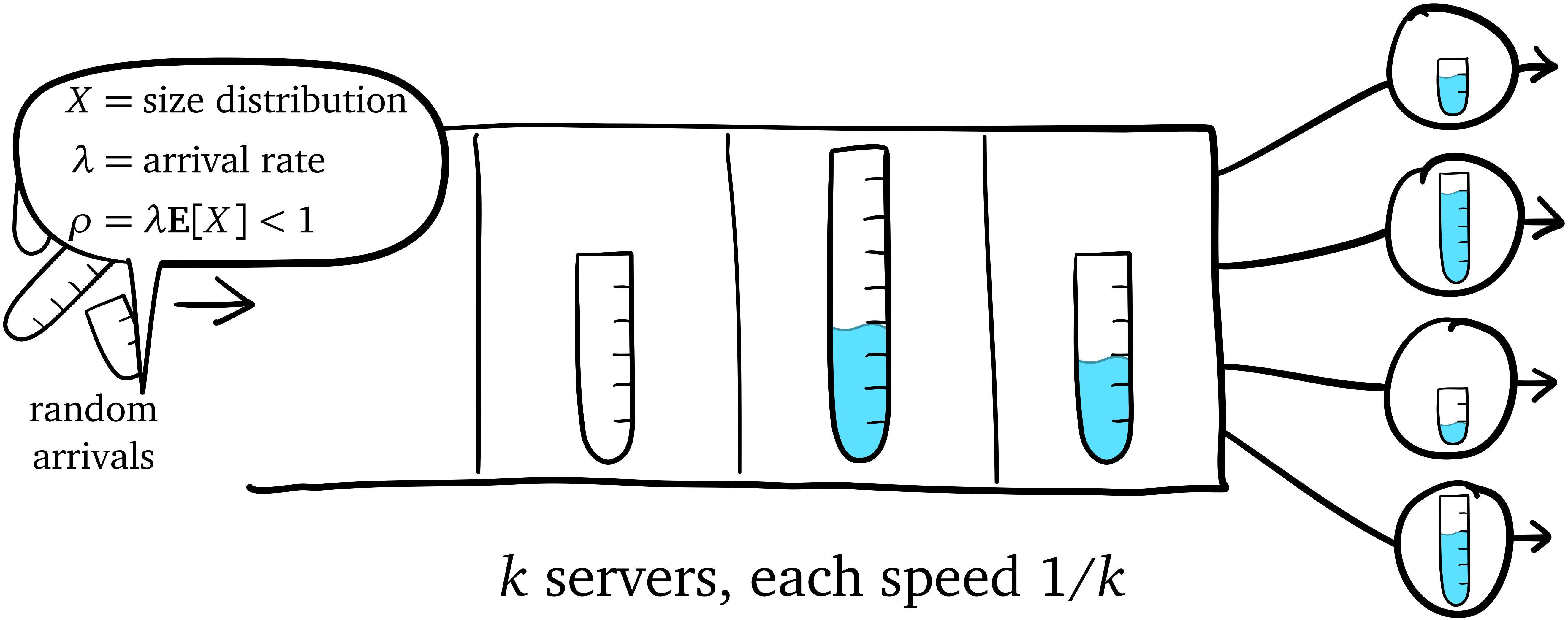


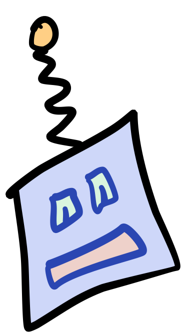


SRPT-k: always serve k jobs of *least remaining size*

SRPT-k minimizes $\mathbf{E}[T]$ in heavy traffic

M/G/k Queue



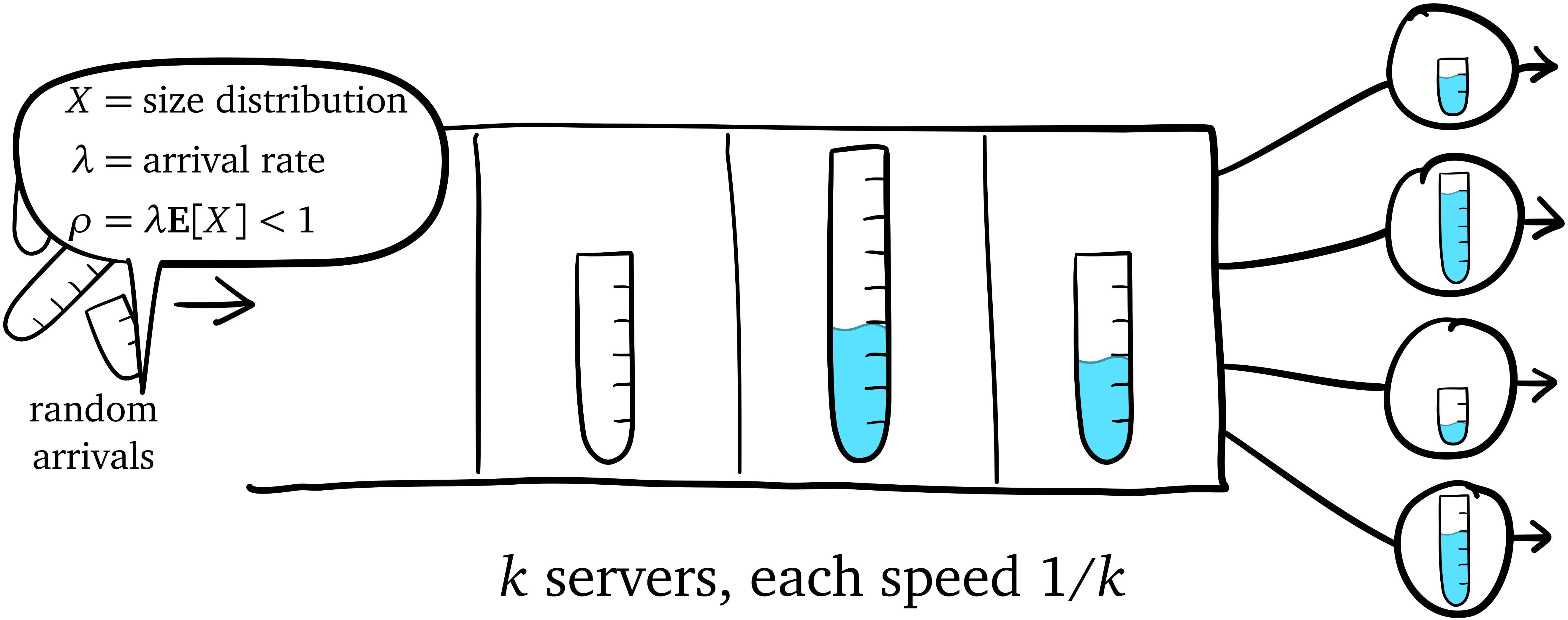


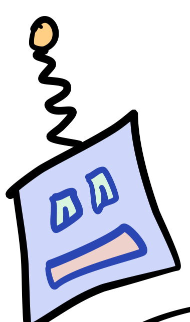
SRPT-k: always serve k jobs of *least remaining size*

SRPT-k minimizes $\mathbf{E}[T]$ in heavy traffic

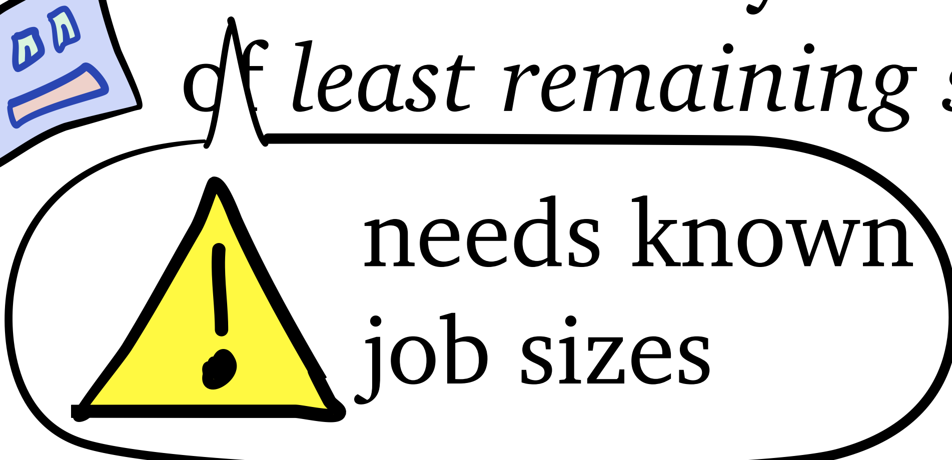
$$\lim_{\rho \to 1} \frac{\mathbf{E}[T_{\text{SRPT-}k}]}{\mathbf{E}[T_{\text{SRPT-1}}]} = 1$$

M/G/k Queue





SRPT-k: always serve k jobs of *least remaining size*



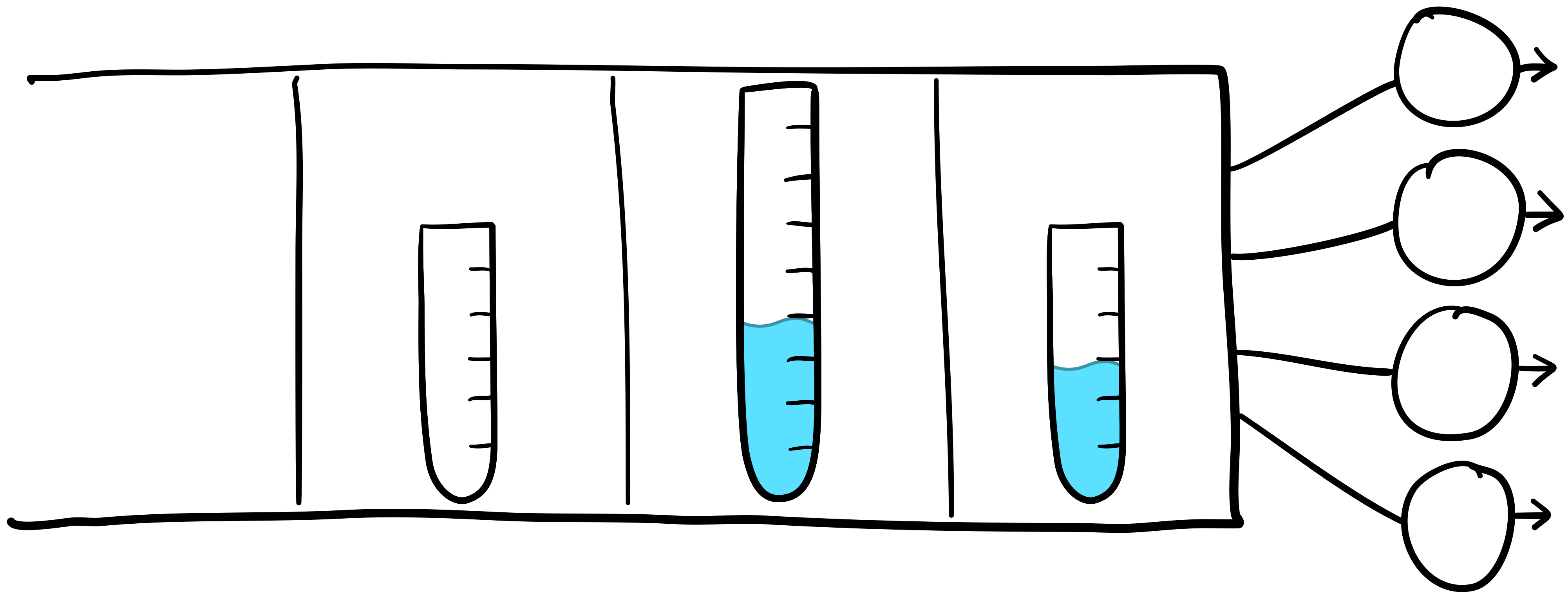
SRPT-k minimizes $\mathbf{E}[T]$ in heavy traffic

$$\lim_{\rho \to 1} \frac{\mathbf{E}[T_{\text{SRPT-}k}]}{\mathbf{E}[T_{\text{SRPT-1}}]} = 1$$

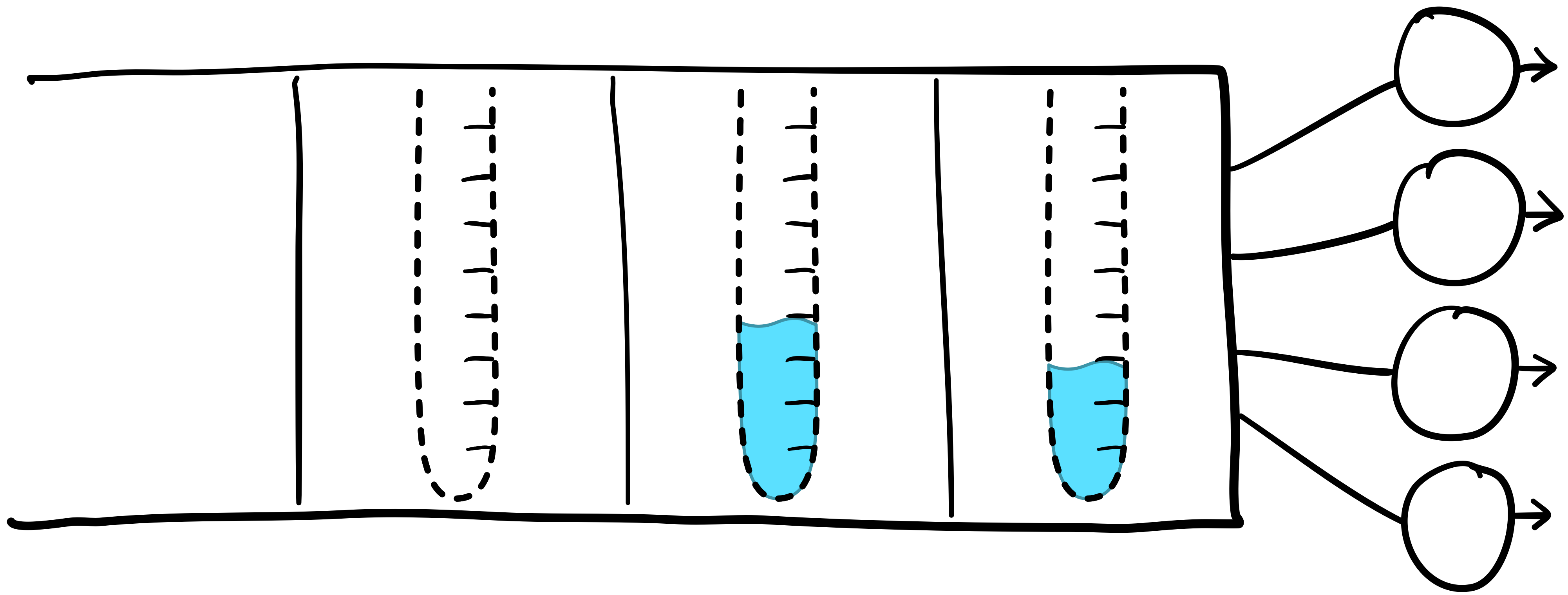
This talk:

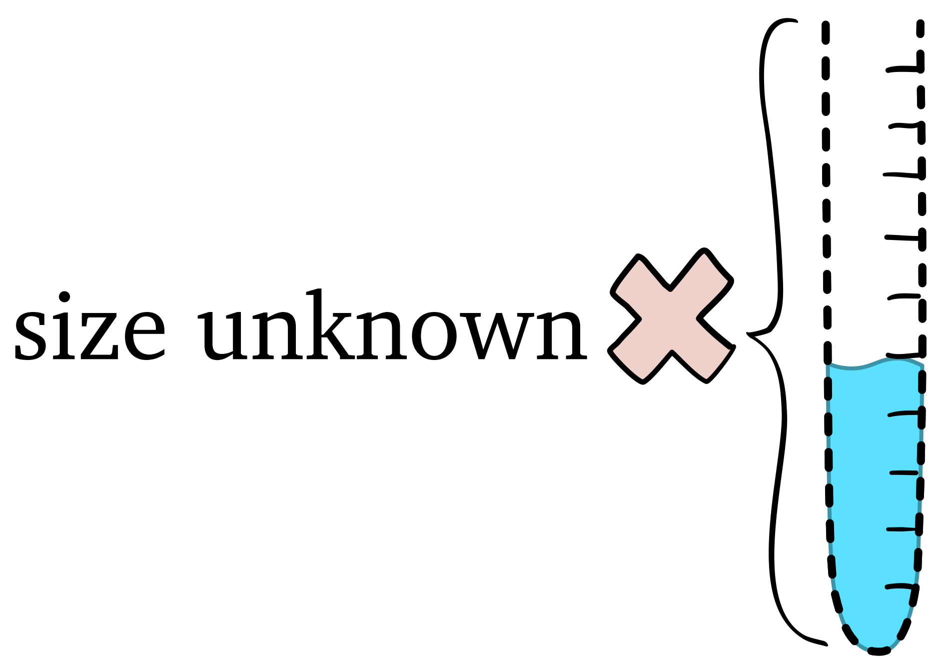
unknown job sizes

Unknown Job Sizes

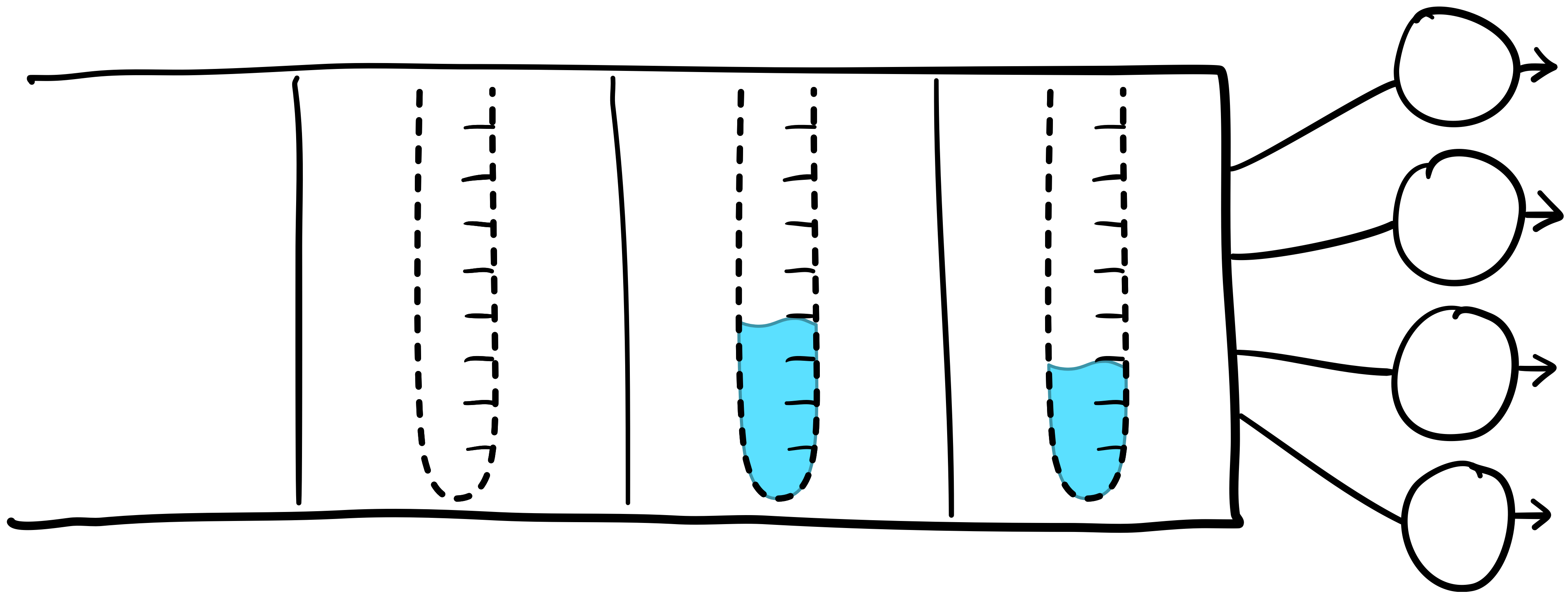


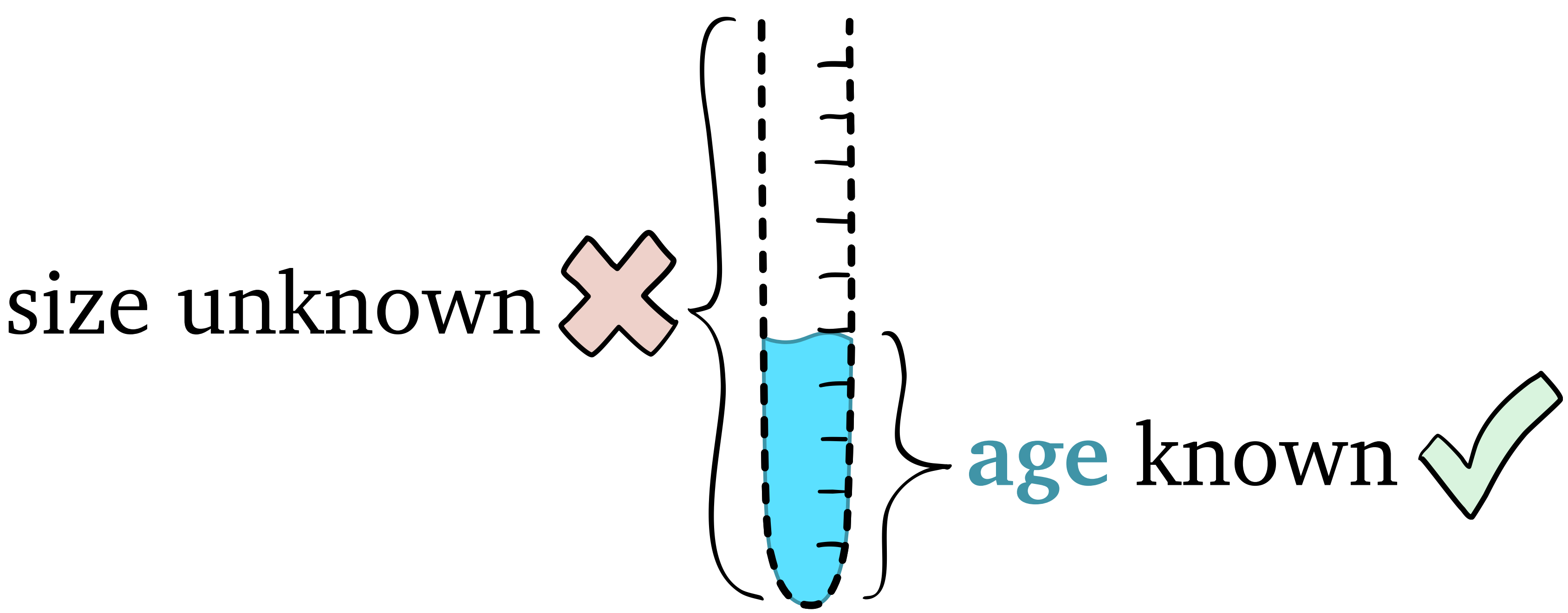
Unknown Job Sizes



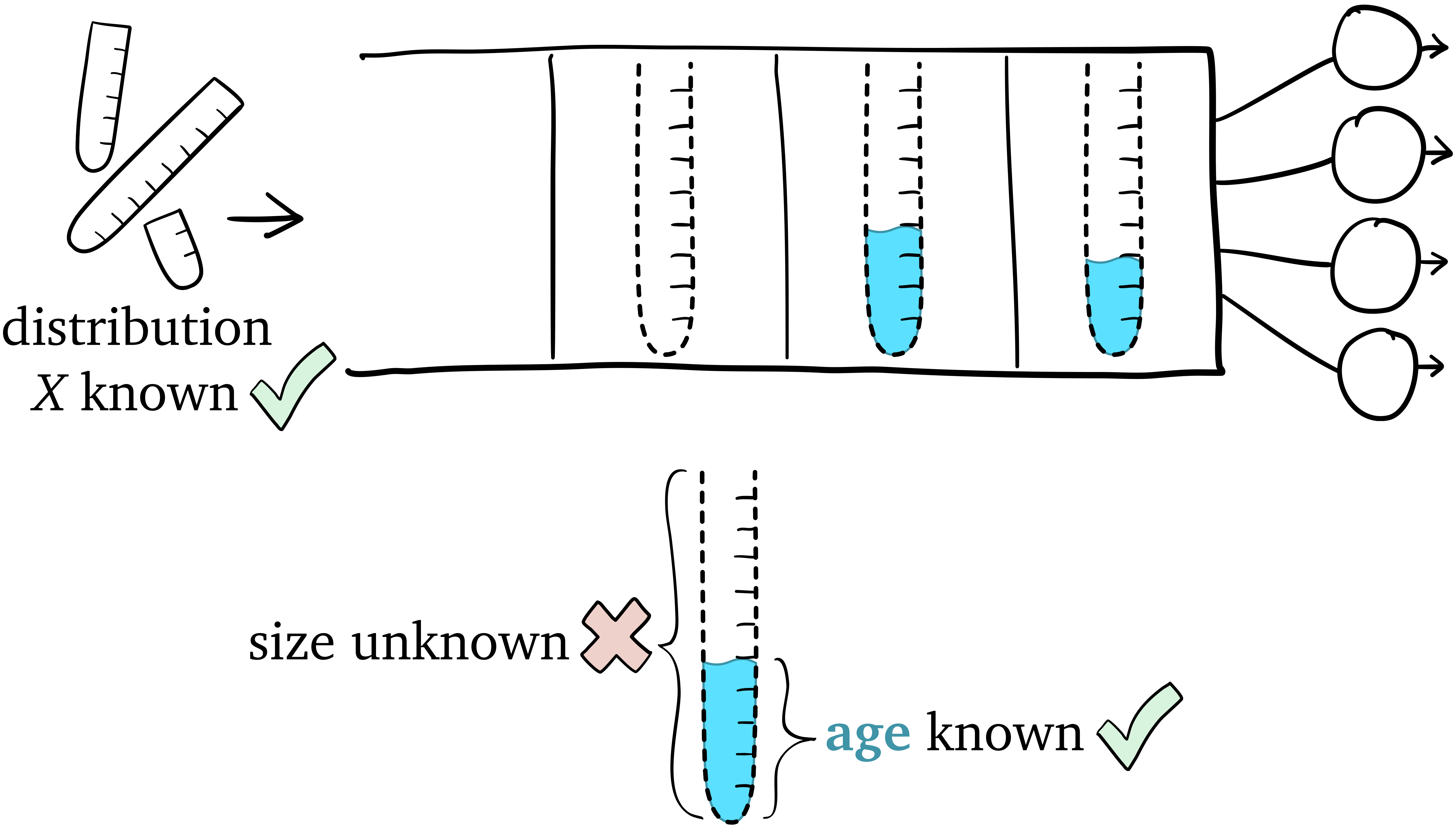


Unknown Job Sizes





Unknown Job Sizes



Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1		
M/G/k		

Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	
M/G/k	**SRPT-k**	

Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	
M/G/k	**SRPT-*k***	

in heavy traffic,
X "finite variance"

Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	**Gittins**
M/G/k	**SRPT-k**	

in heavy traffic,
X "finite variance"

Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	**Gittins**
M/G/k	**SRPT-k**	**???**

in heavy traffic,
X "finite variance"

What is **Gittins**?

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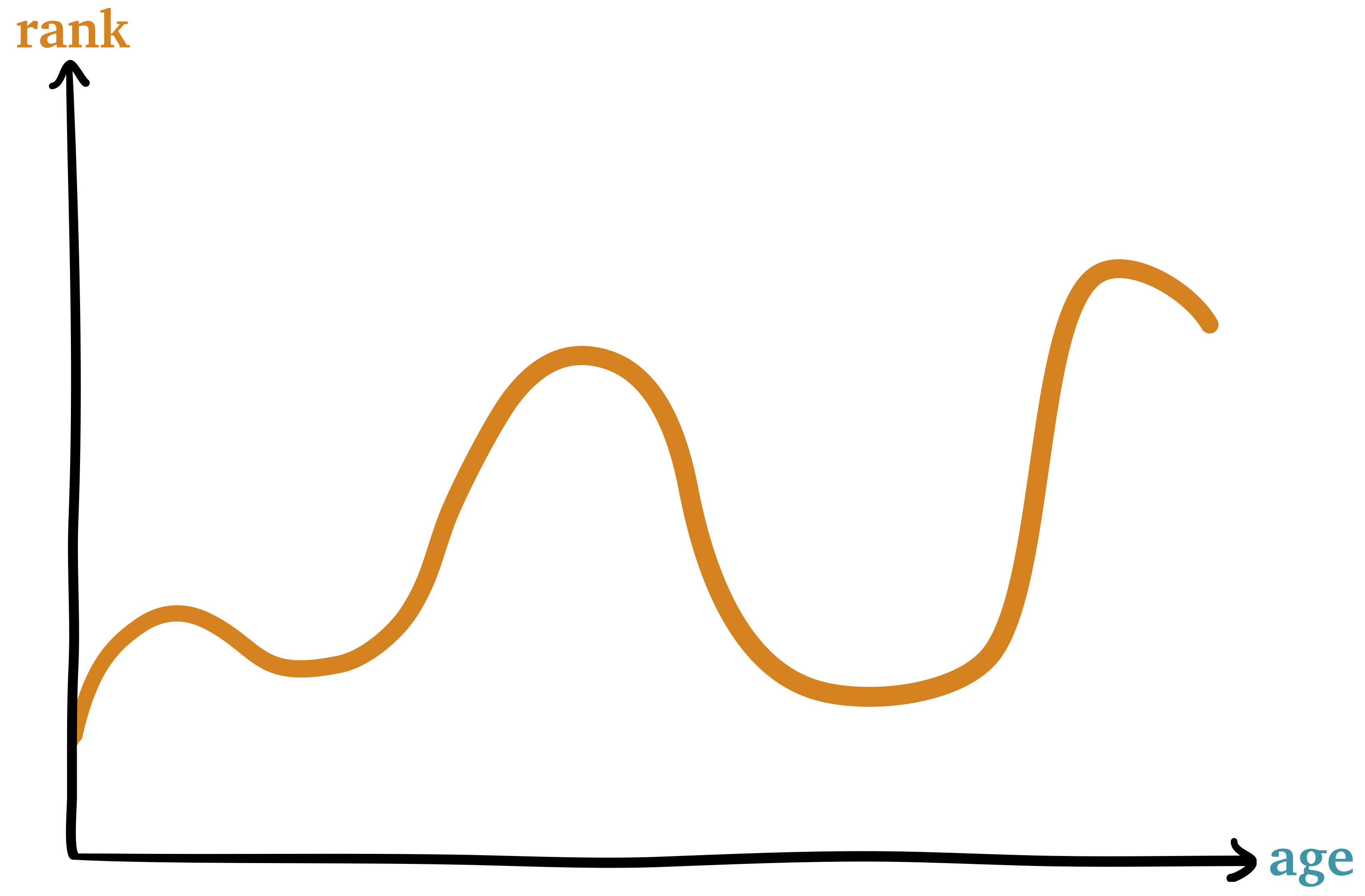

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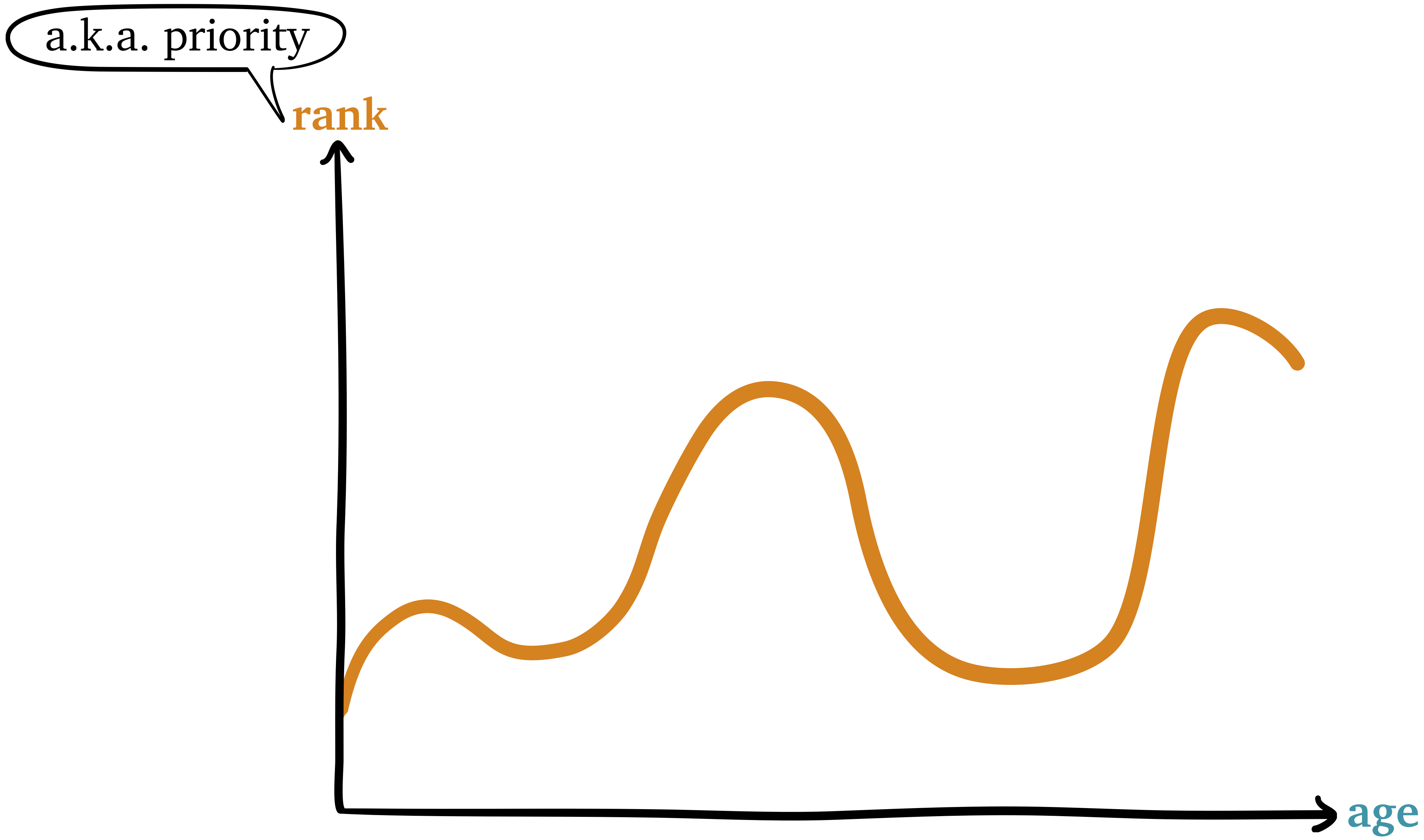

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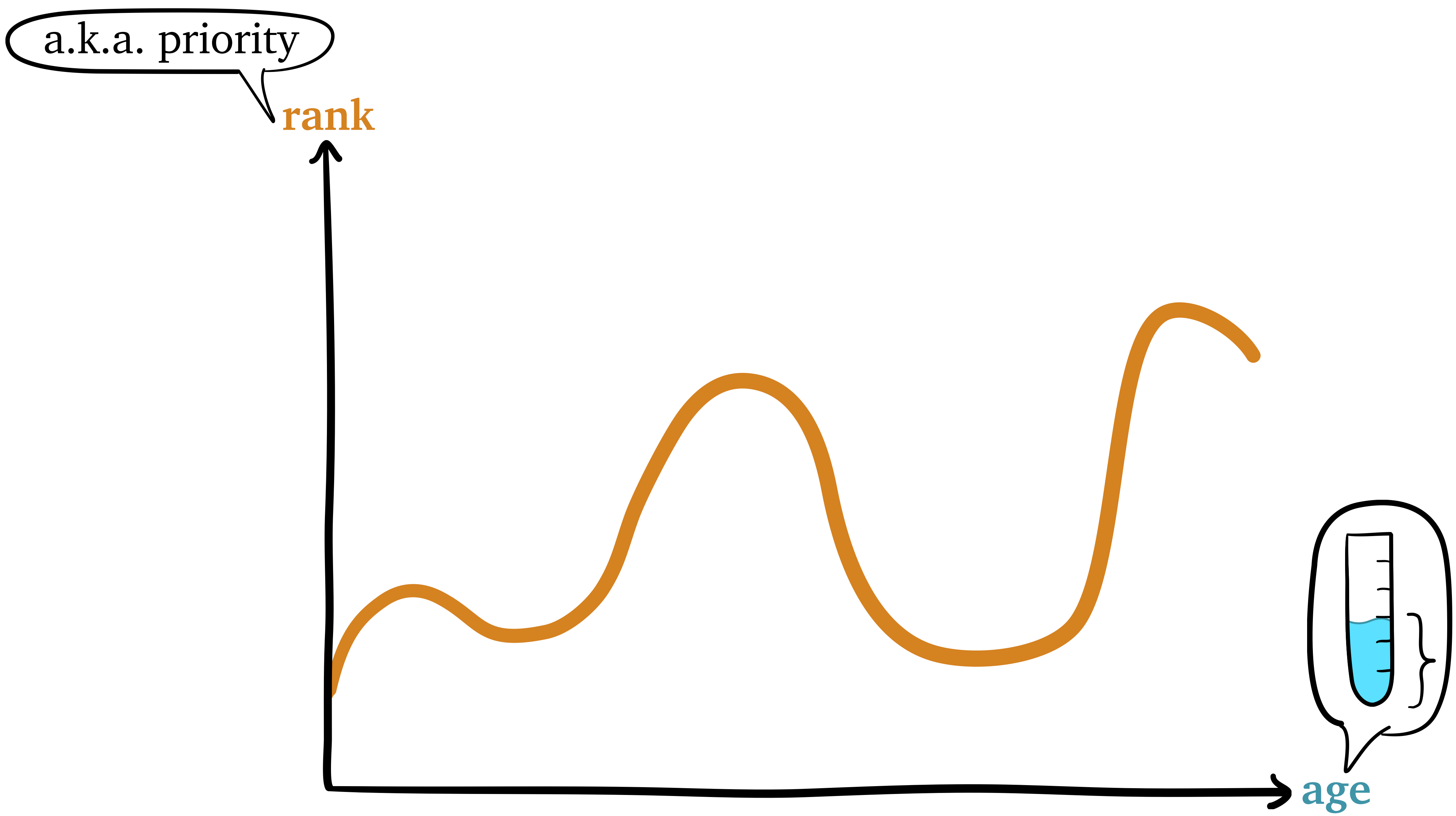

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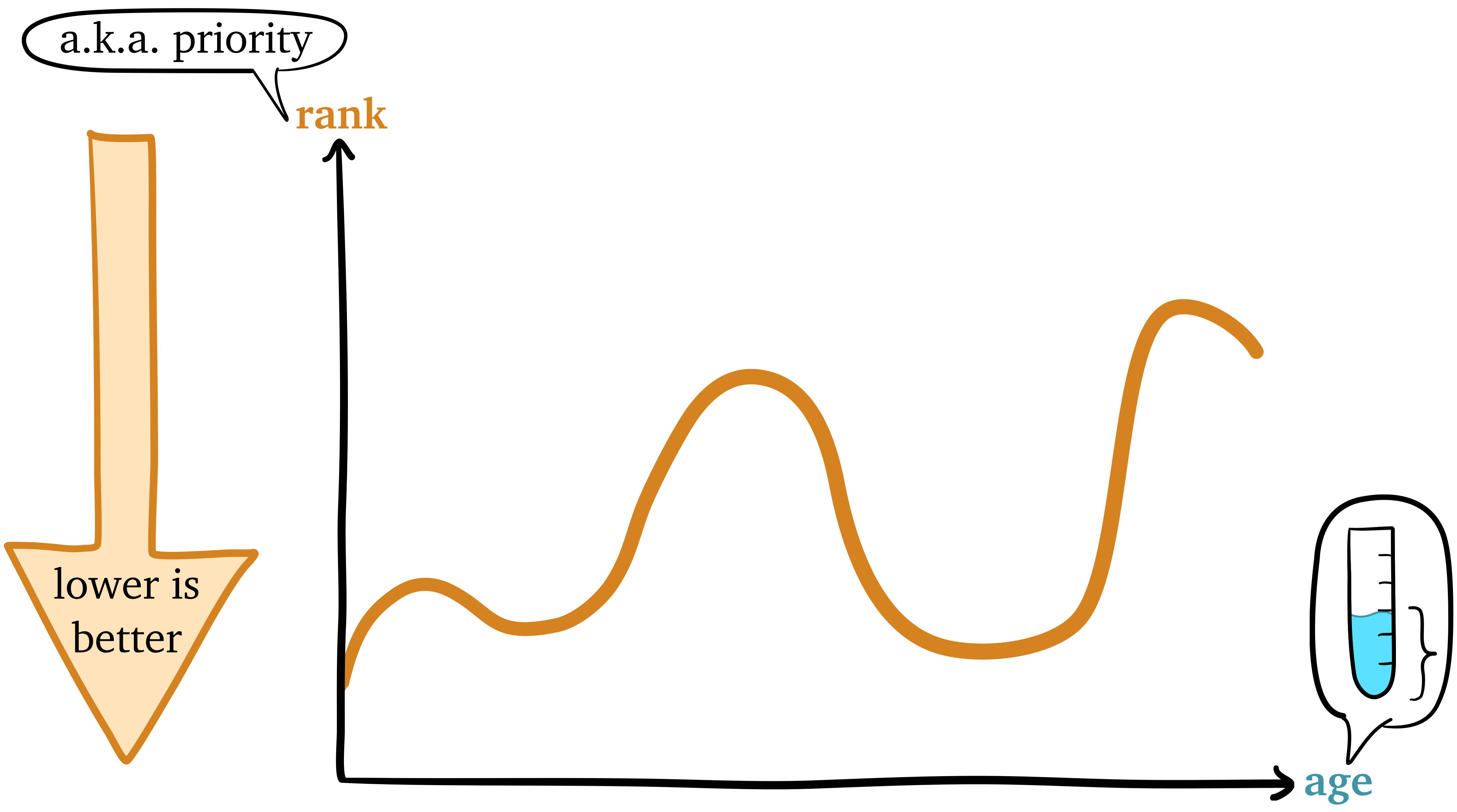
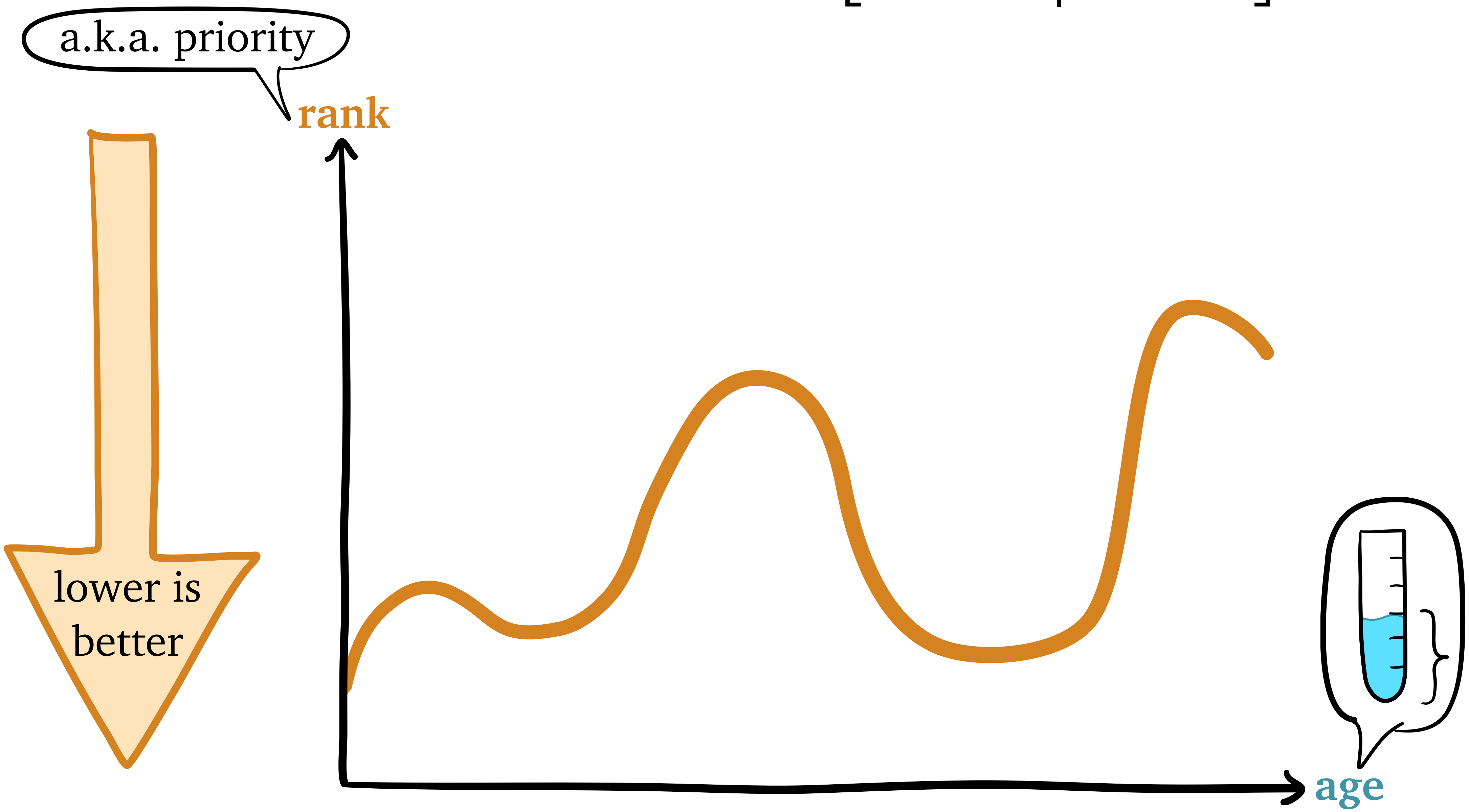


What is Gittins?

$$r_{\text{Gittins}}(a) = \inf_{b>a} \frac{\mathbf{E}[\min\{X, b\} \mid X > a]}{\mathbf{P}[X \le b \mid X > a]}$$



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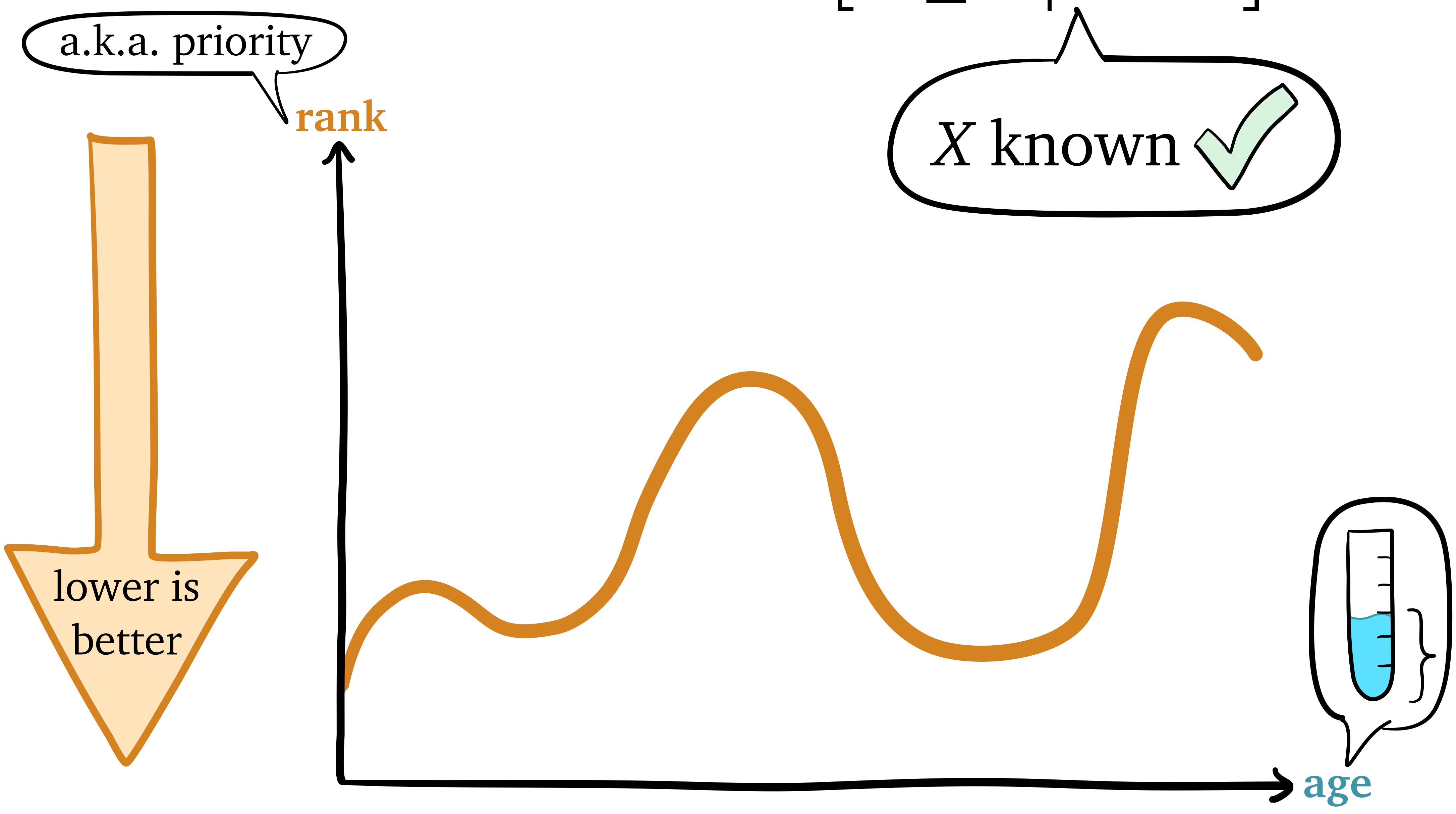

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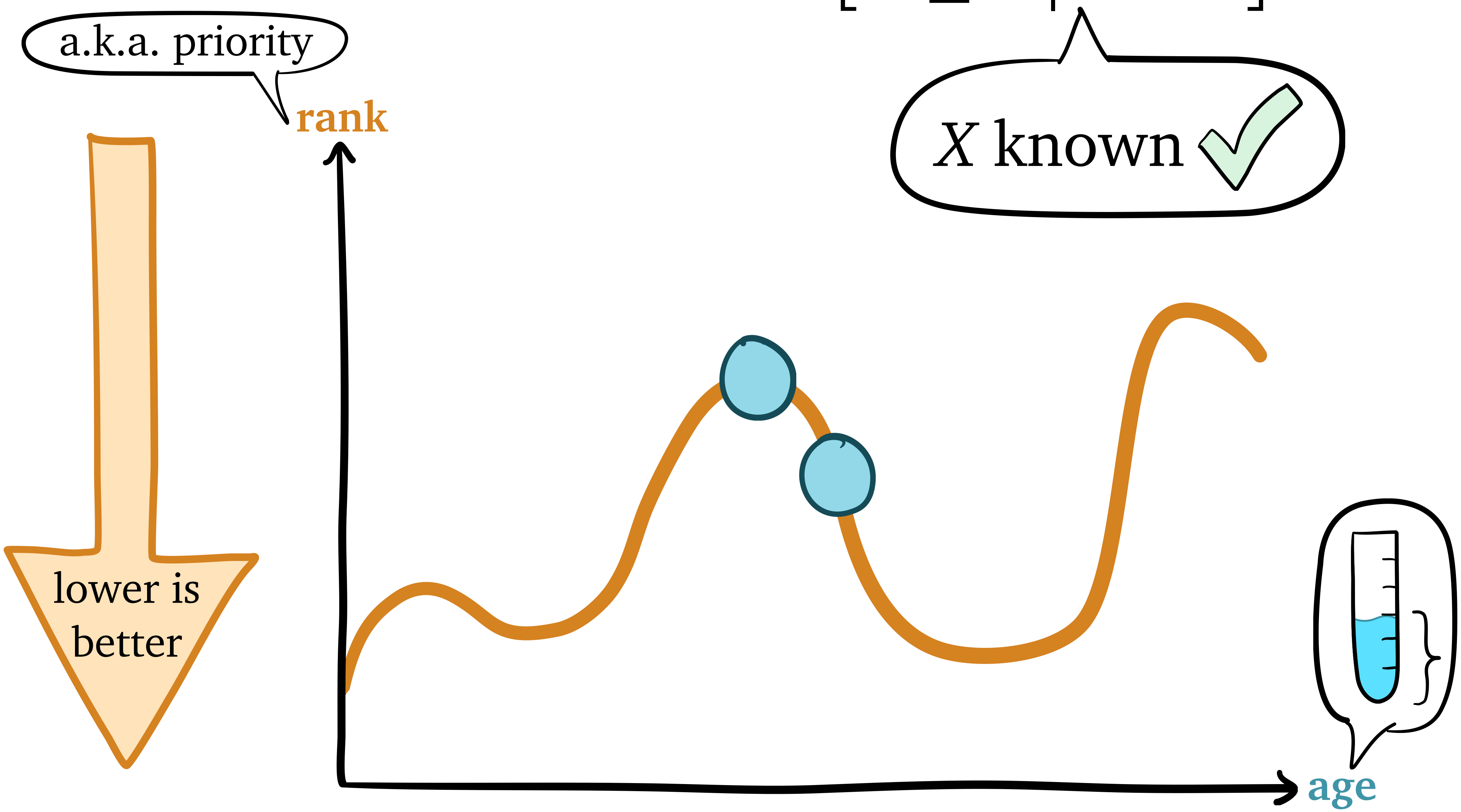

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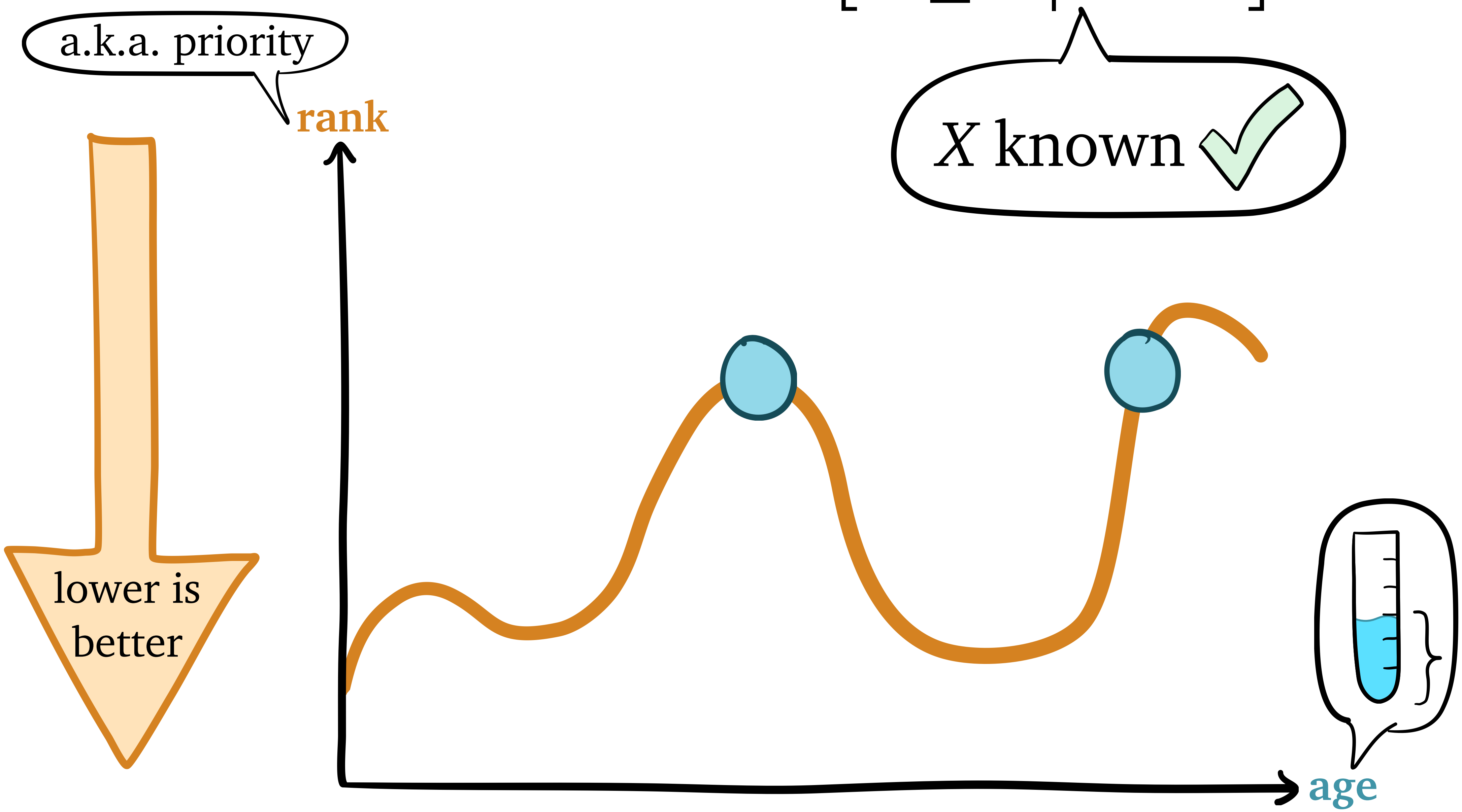

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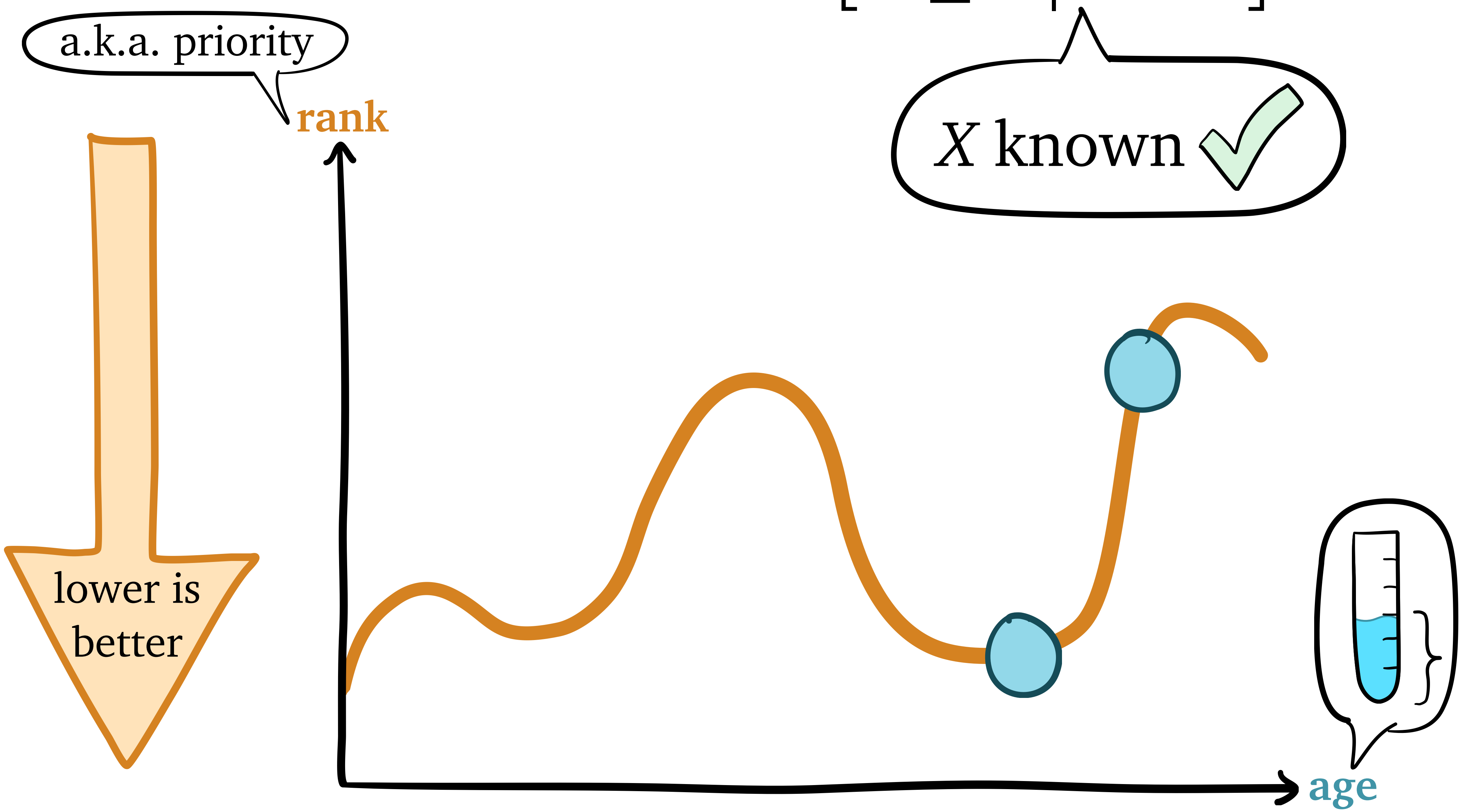


Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	**Gittins**
M/G/k	**SRPT-*k***	**???**

in heavy traffic,
X “finite variance”

Minimizing $\mathbf{E}[T]$

	Known job sizes	Unknown job sizes
M/G/1	**SRPT**	**Gittins**
M/G/k	**SRPT-*k***	**???**

in heavy traffic,
X "finite variance"

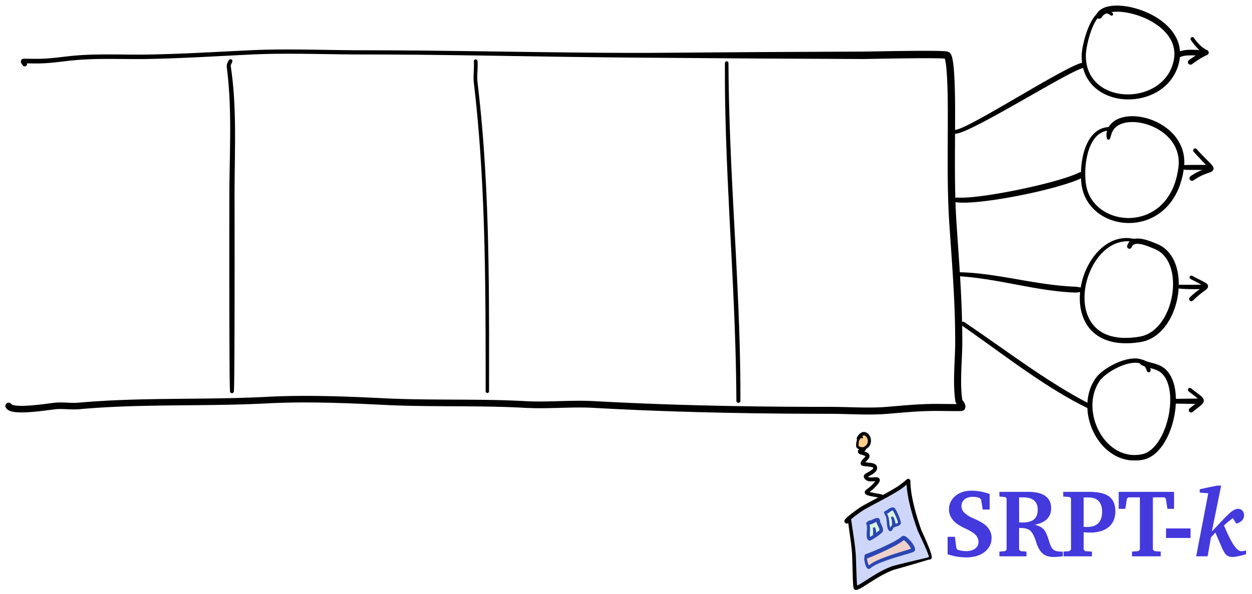
Does **Gittins-*k*** work?

Background:
SRPT-*k* optimality

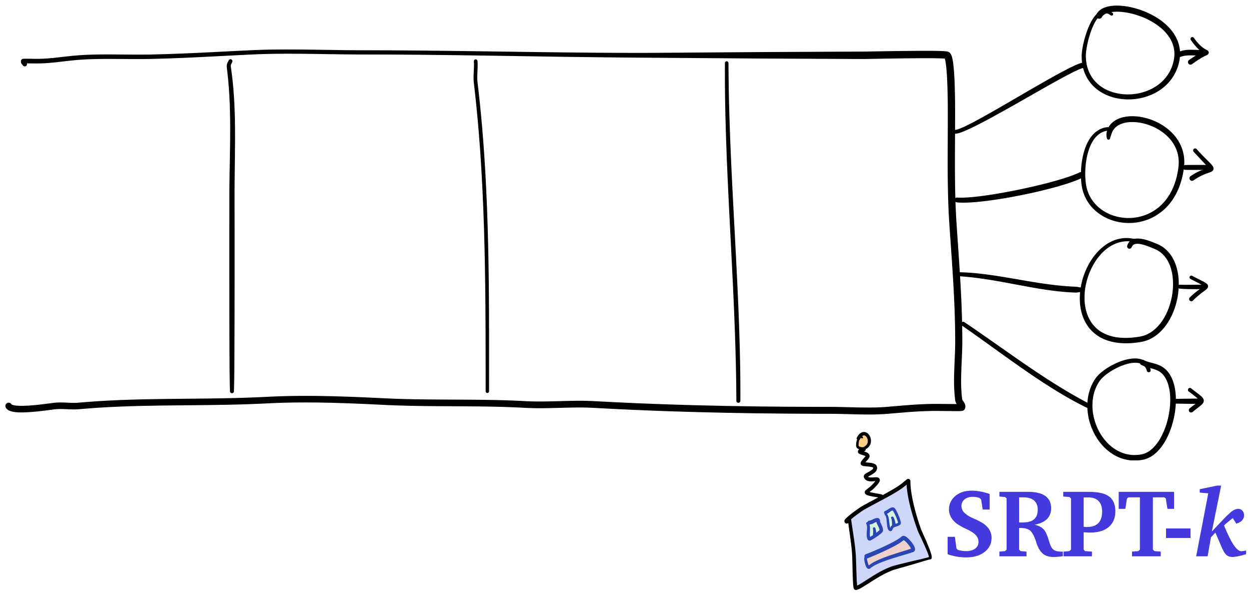
Background:
SRPT-*k* optimality

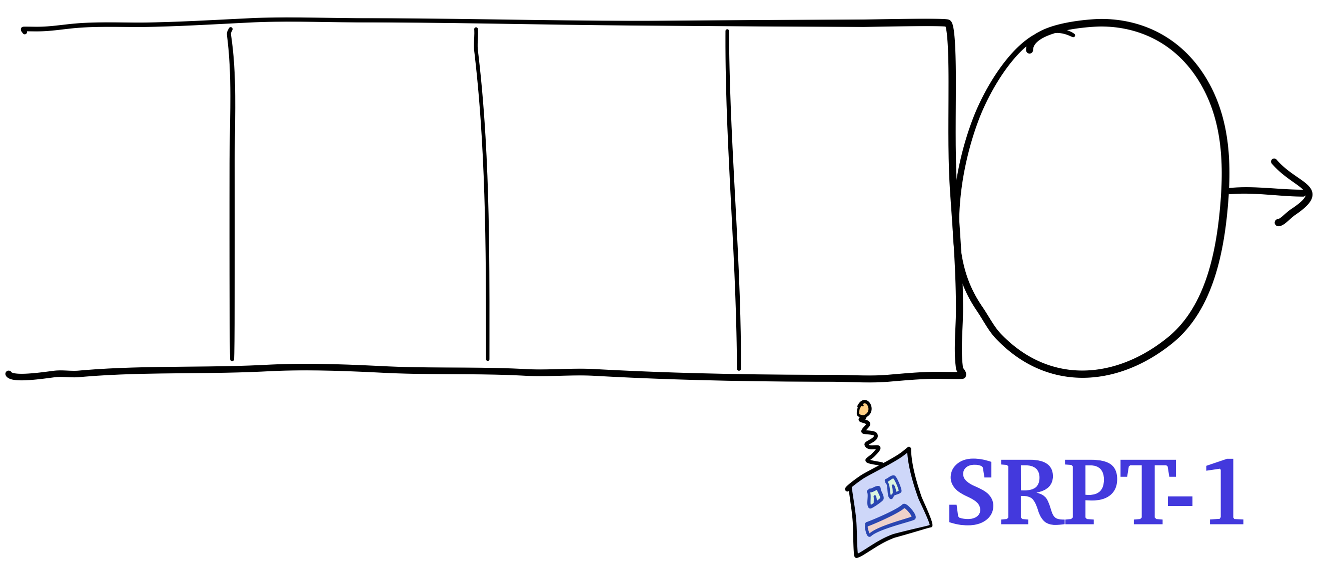
(Grosof, Scully, & Harchol-Balter, 2018)

SRPT-*k* Proof Sketch

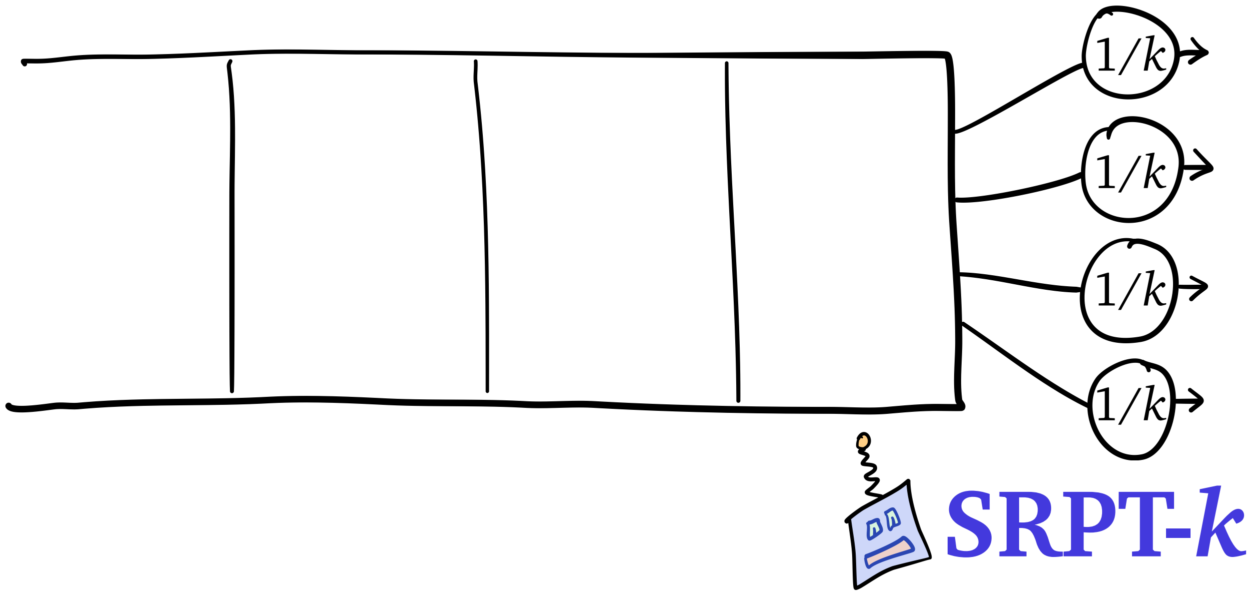


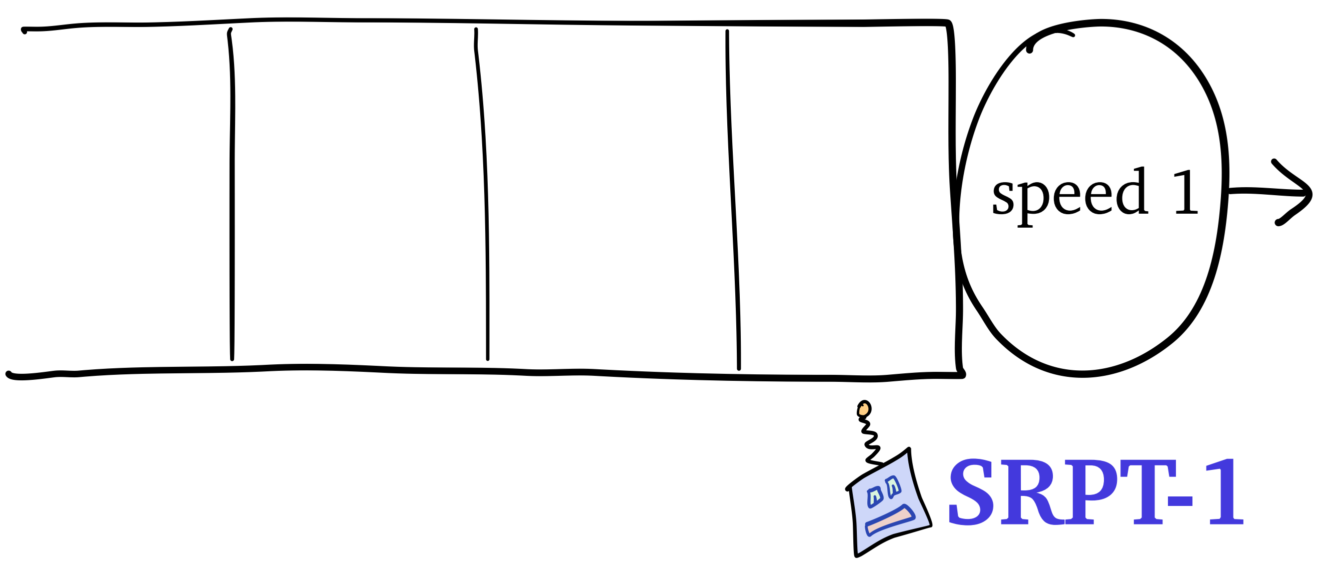
SRPT-*k* Proof Sketch



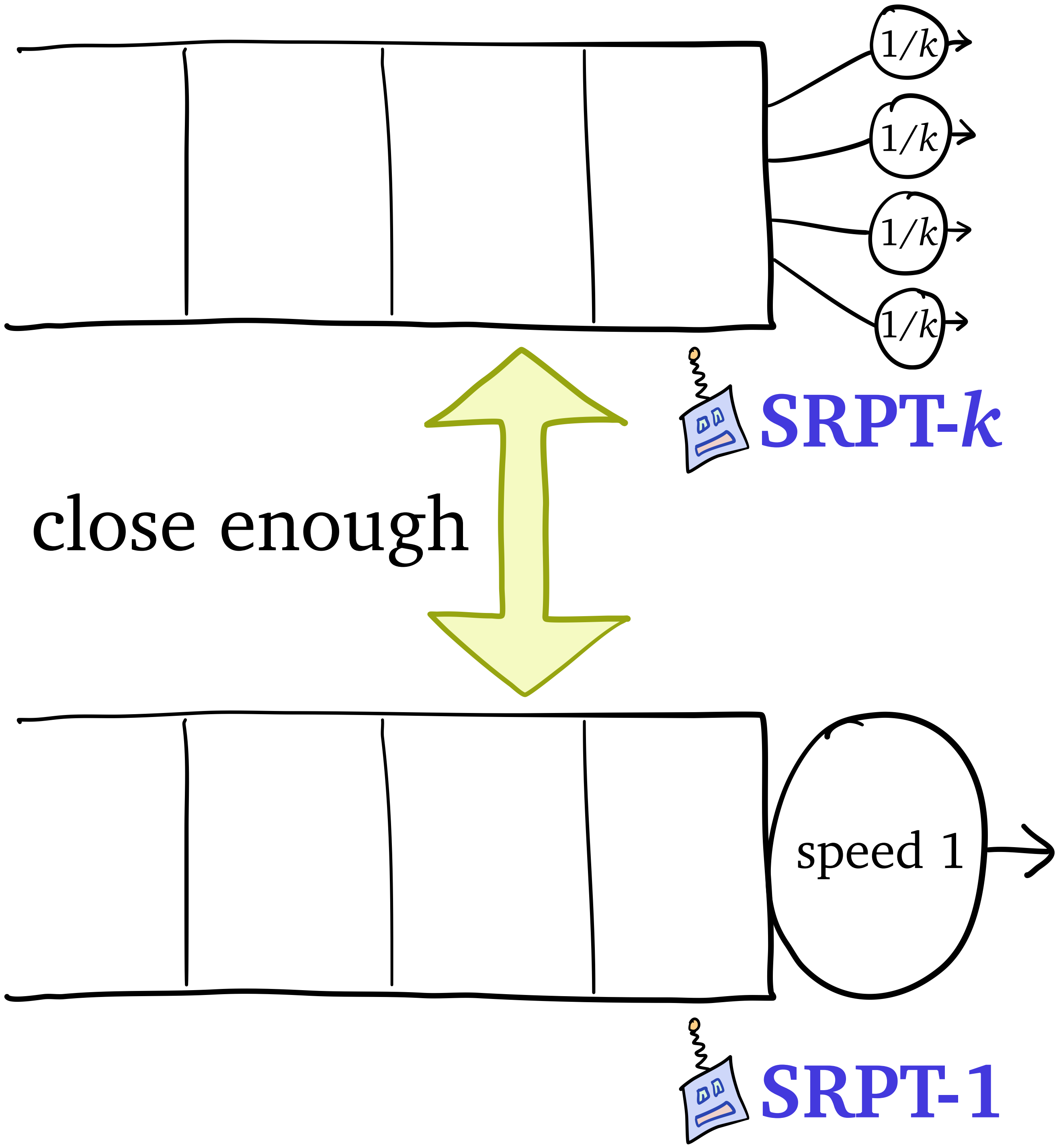


SRPT-k Proof Sketch

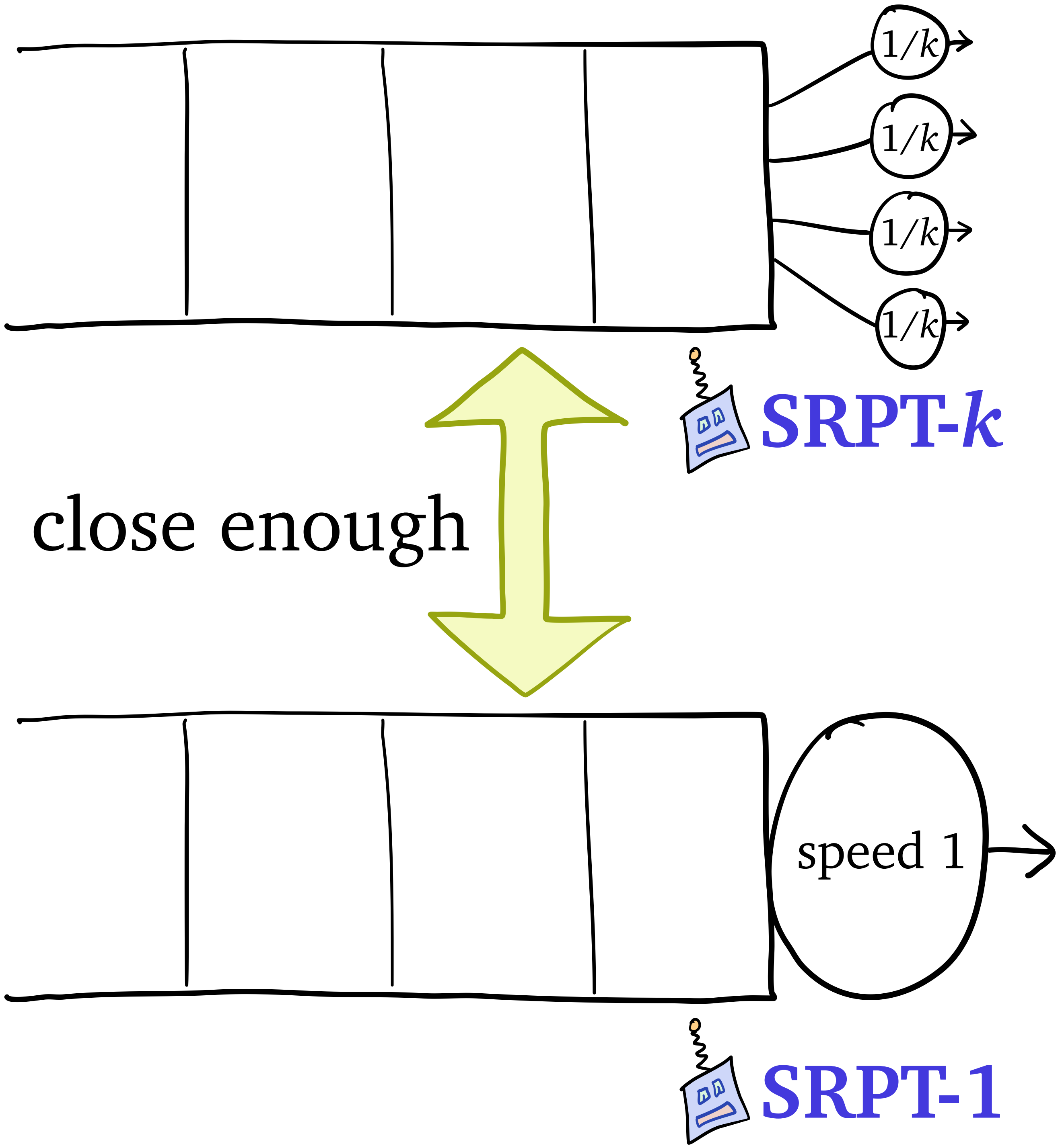




SRPT-*k* Proof Sketch



SRPT-*k* Proof Sketch

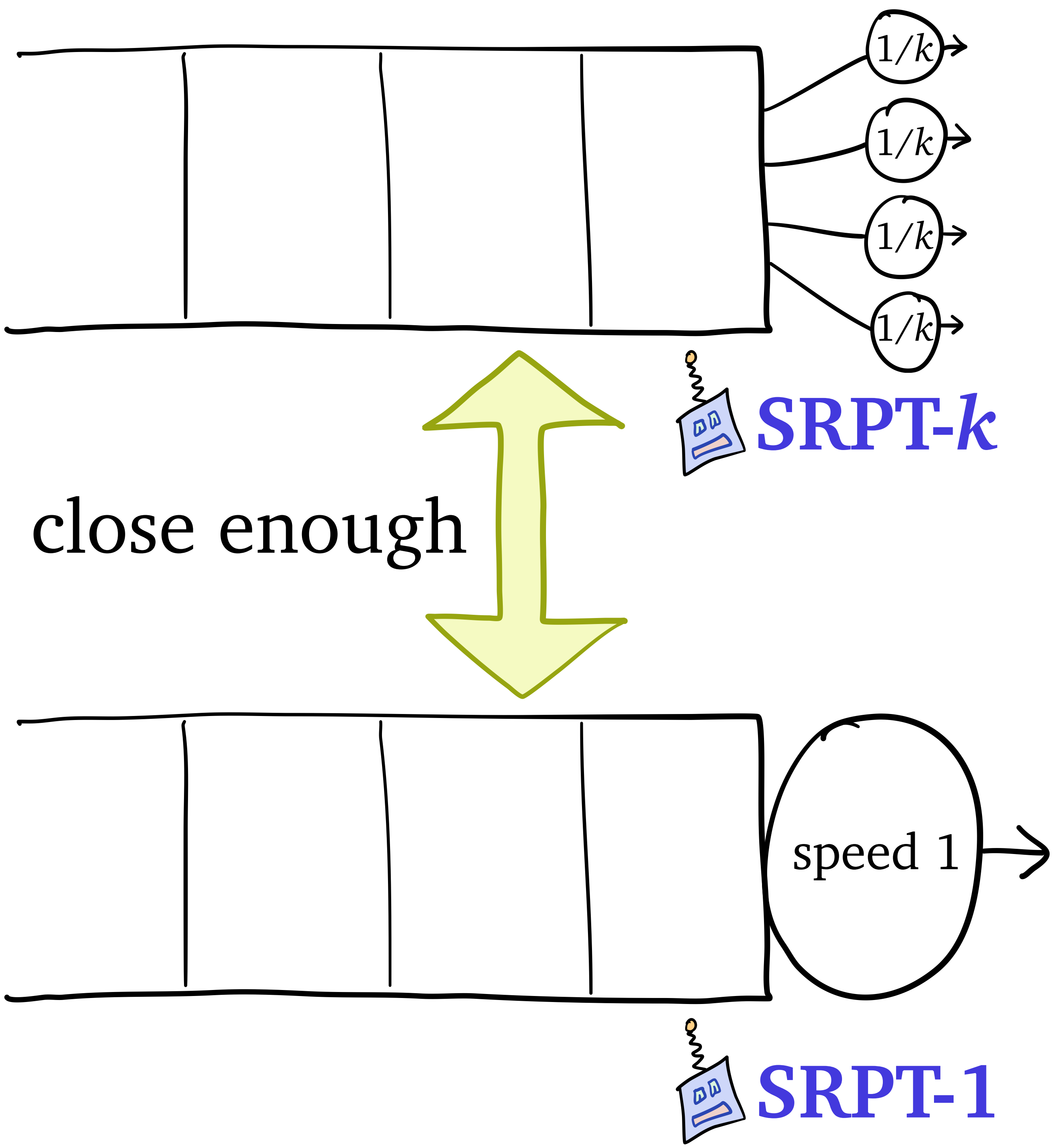


Step 1:

link **SRPT-*k*** to **SRPT-1**

$$\mathbf{E}[T_{\text{SRPT-}k}] \le \mathbf{E}[T_{\text{SRPT-1}}] + k \cdot O\left(\log \frac{1}{1-\rho}\right)$$

SRPT-k Proof Sketch



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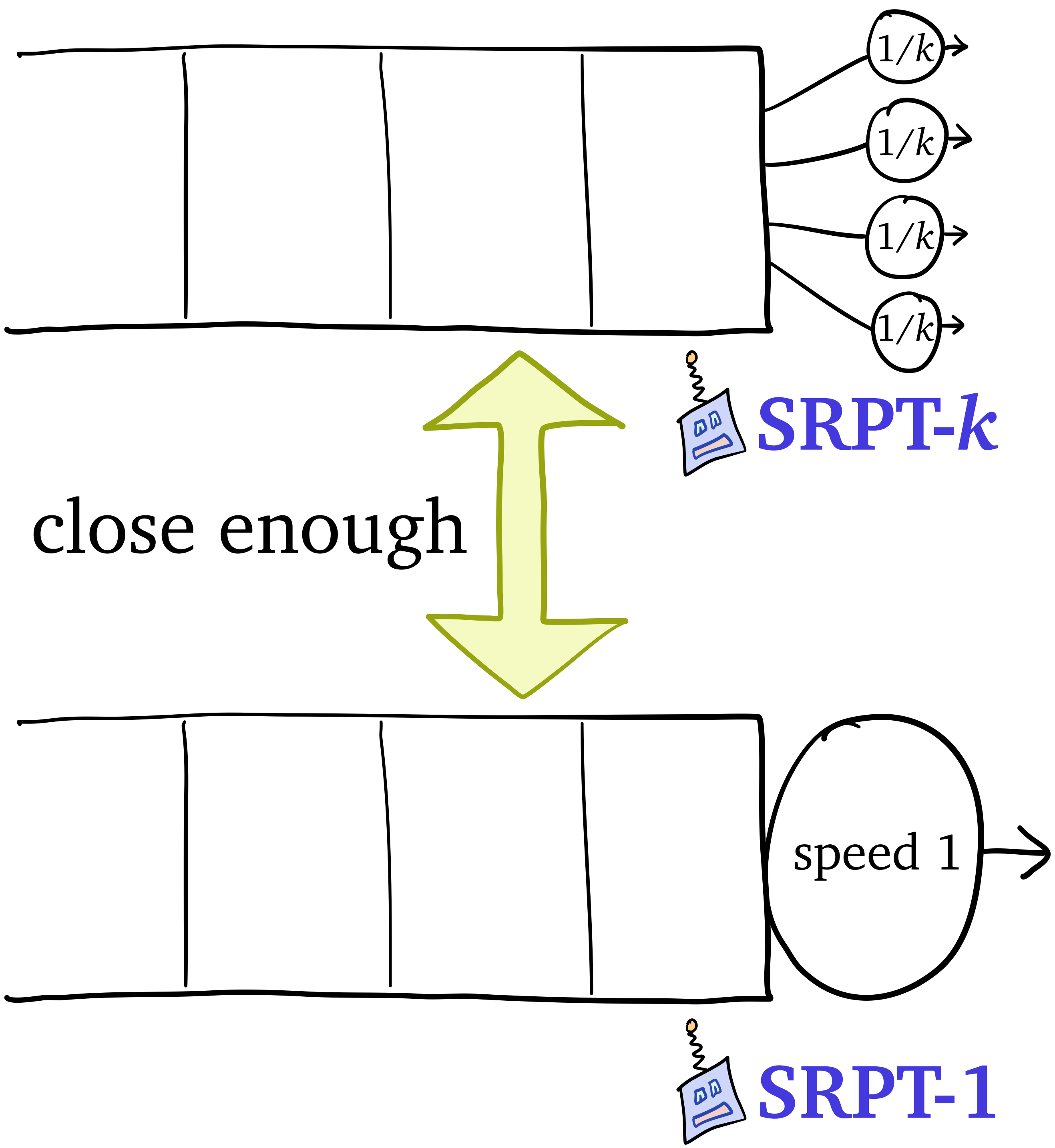
Step 2:

analyze heavy-traffic **SRPT-1**

$$\mathbf{E}[T_{\text{SRPT-1}}] = \omega\left(\log \frac{1}{1-\rho}\right)$$

SRPT-k Proof Sketch



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(Lin, Wierman, & Zwart, 2011)

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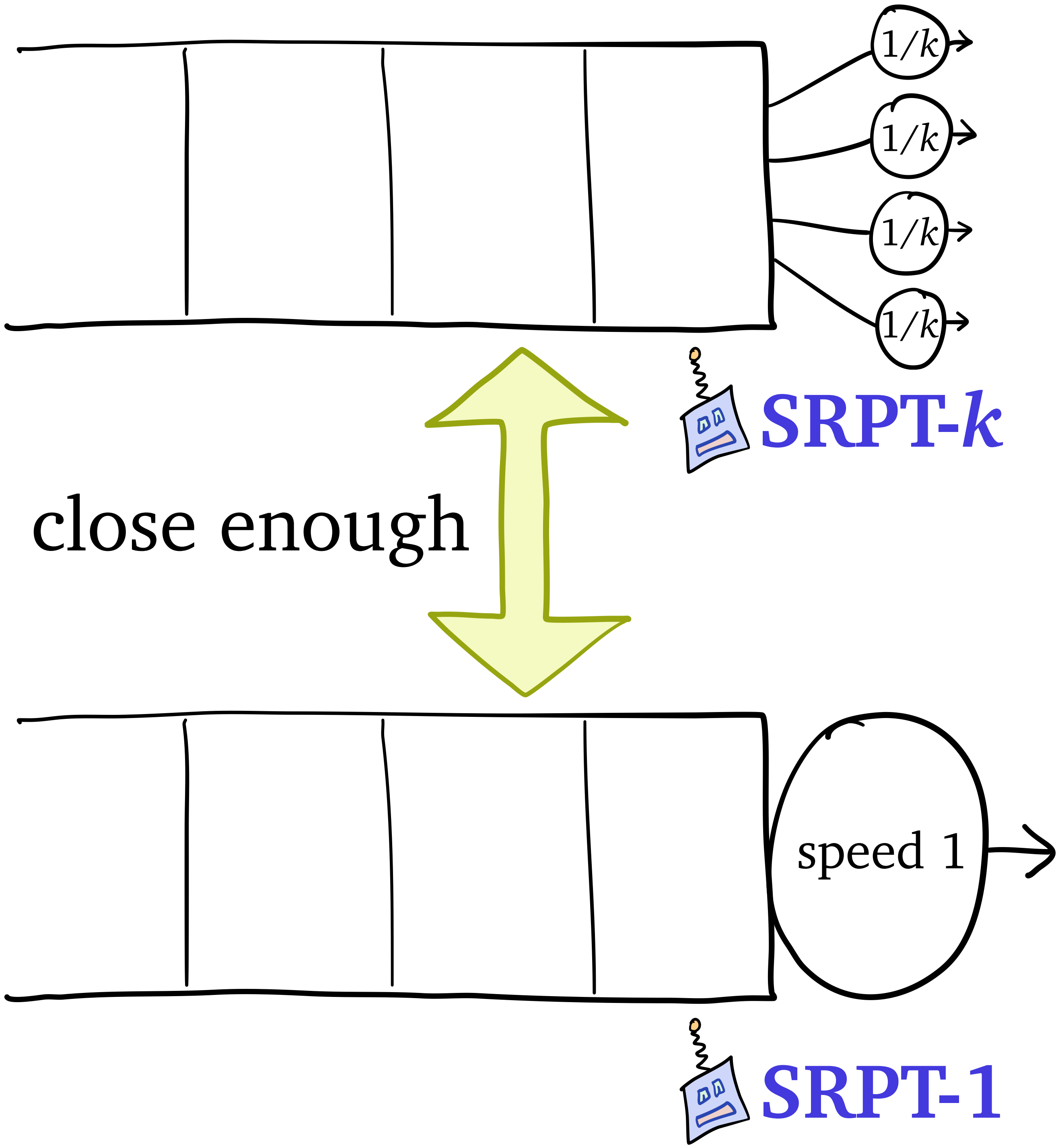
SRPT-*k* Proof Sketch



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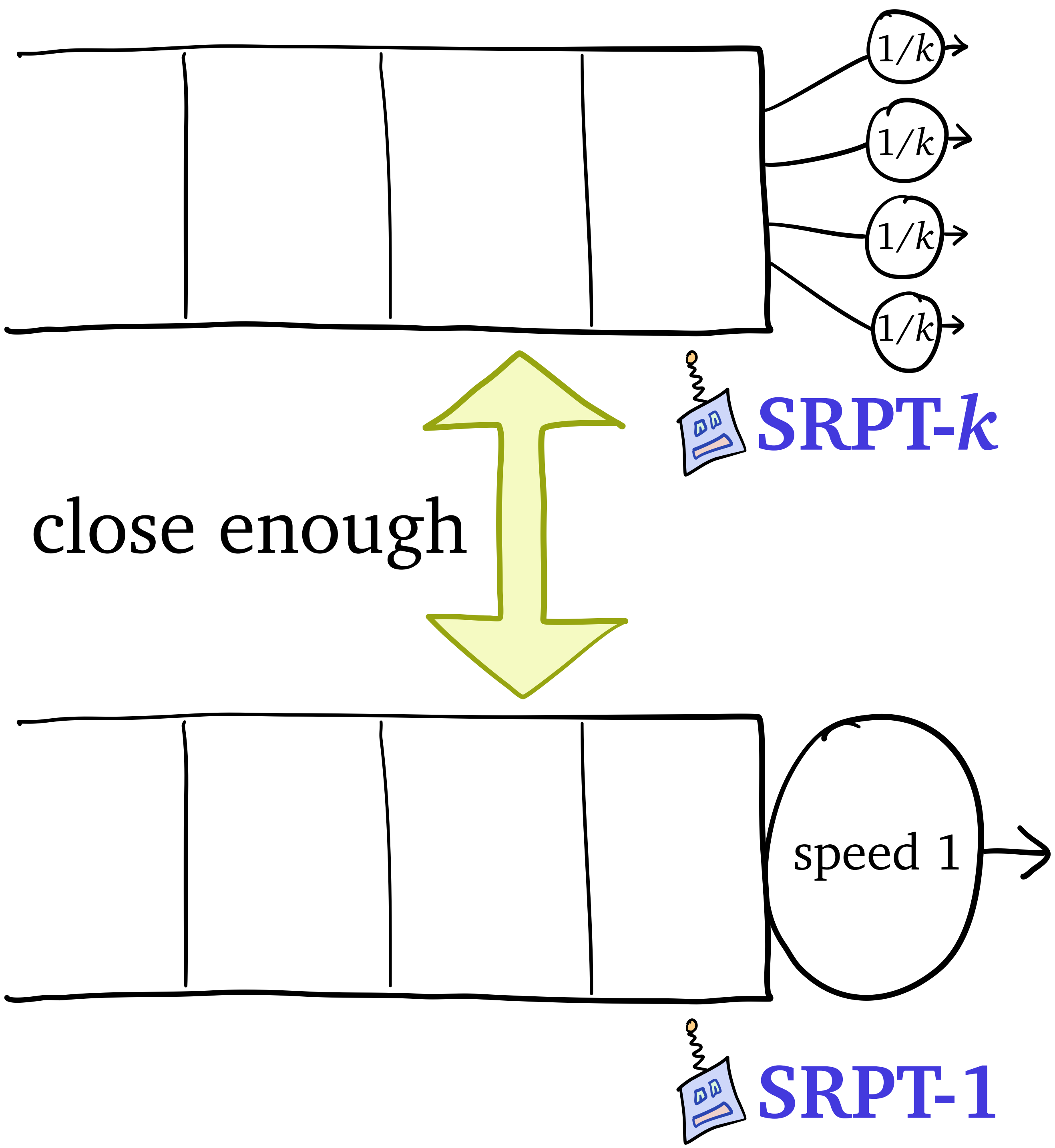
$$\mathbf{E}[T_{\text{SRPT-1}}] = \omega\left(\log \frac{1}{1-\rho}\right)$$

Theorem: for any fixed k,

$$\lim_{\rho \to 1} \frac{\mathbf{E}[T_{\text{SRPT-}k}]}{\mathbf{E}[T_{\text{SRPT-1}}]} = 1$$

SRPT-k Proof Sketch



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(Lin, Wierman, & Zwart, 2011)

Step 2:

analyze heavy-traffic **SRPT-1**

$$\mathbf{E}[T_{\text{SRPT-1}}] = \omega\left(\log \frac{1}{1-\rho}\right)$$

needs X to be "finite variance"

Theorem: for any fixed k,

$$\lim_{\rho \to 1} \frac{\mathbf{E}[T_{\text{SRPT-}k}]}{\mathbf{E}[T_{\text{SRPT-1}}]} = 1$$

Step 1: link **Gittins-*k*** to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

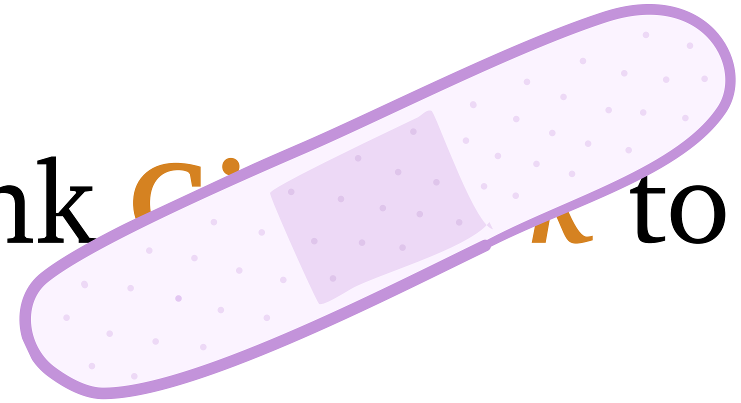
Step 1: link **Gittins-*k*** to **Gittins-1**

✓ **Step 2:** analyze heavy-traffic **Gittins-1**

Step 1: link **Gittins-*k*** to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

Step 1: link **Gi[illegible]k** to **Gittins-1**



Step 2: analyze heavy-traffic **Gittins-1**

Step 1: link M-Gittins-k to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

new policy!

M-Gittins-k

Step 1: link G…k to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

new policy!

Step 1: link **M-Gittins-k** to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

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Step 1: link M-Gittins-k to **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

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new policy!

Step 1: link **M-Gittins-*k*** to **Gittins-1**

- **Gittins-*k*** vs. **Gittins-1**
- What **M-Gittins** is
- **M-Gittins-*k*** vs. **Gittins-1**

Step 2: analyze heavy-traffic **Gittins-1**

Theorem: for any fixed k,

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Step 1: link M-Gittins-k to **Gittins-1**

- **Gittins-k** vs. **Gittins-1**
- What **M-Gittins** is
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Step 2: analyze heavy-traffic **Gittins-1**

see paper

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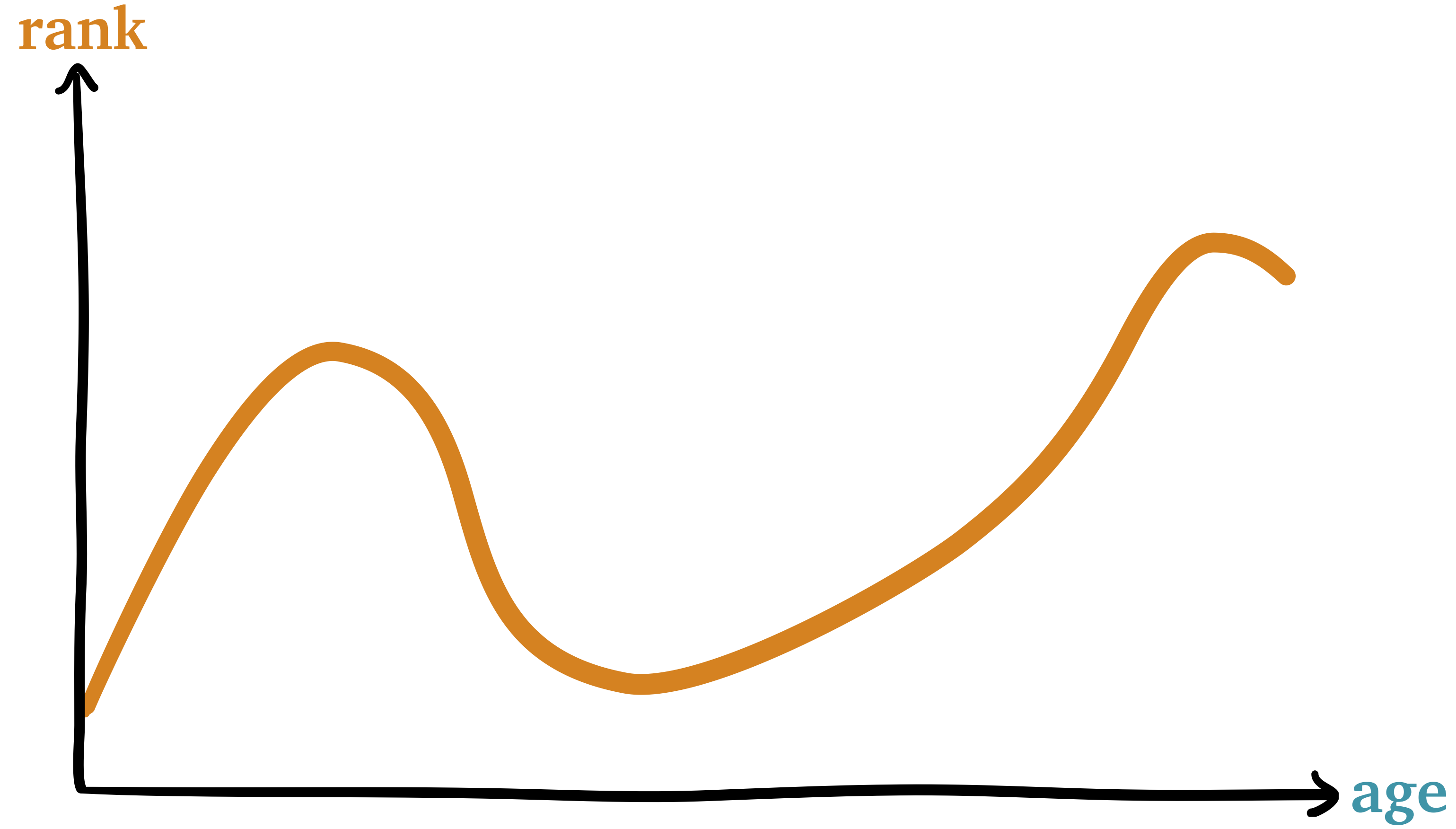
Analyzing **Gittins-1**



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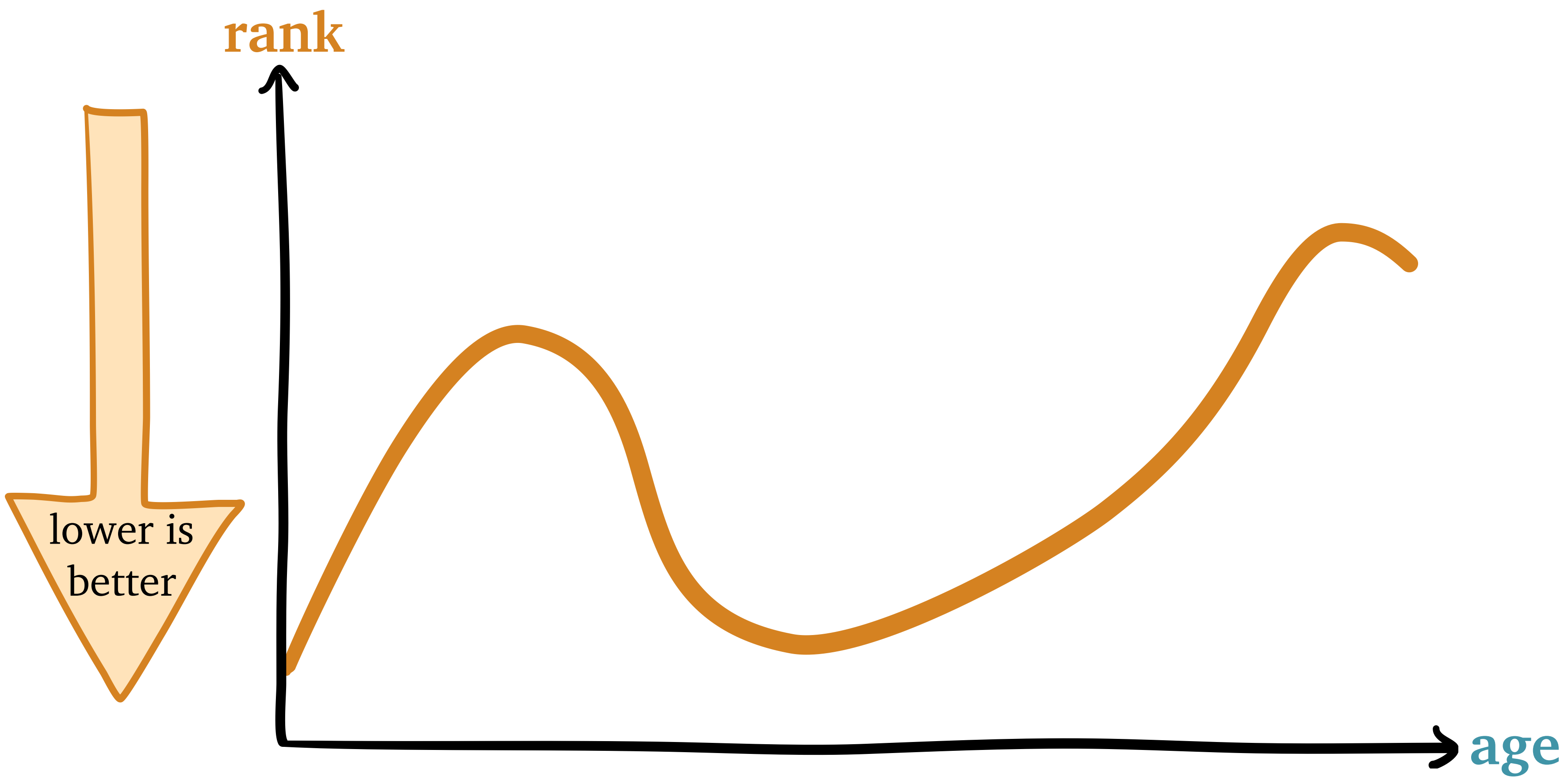
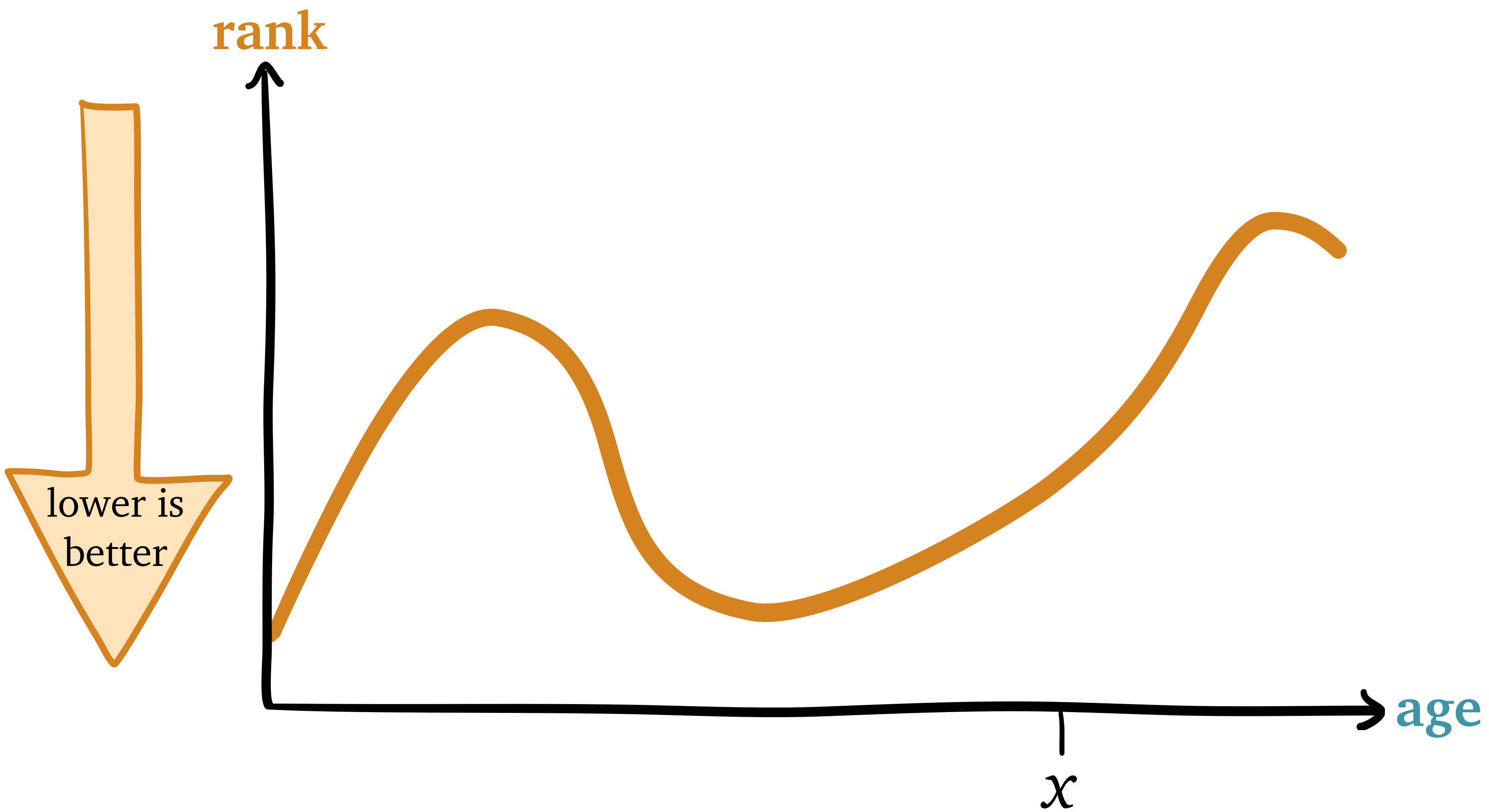


Analyzing **Gittins-1**

Suppose I'm a job of size x



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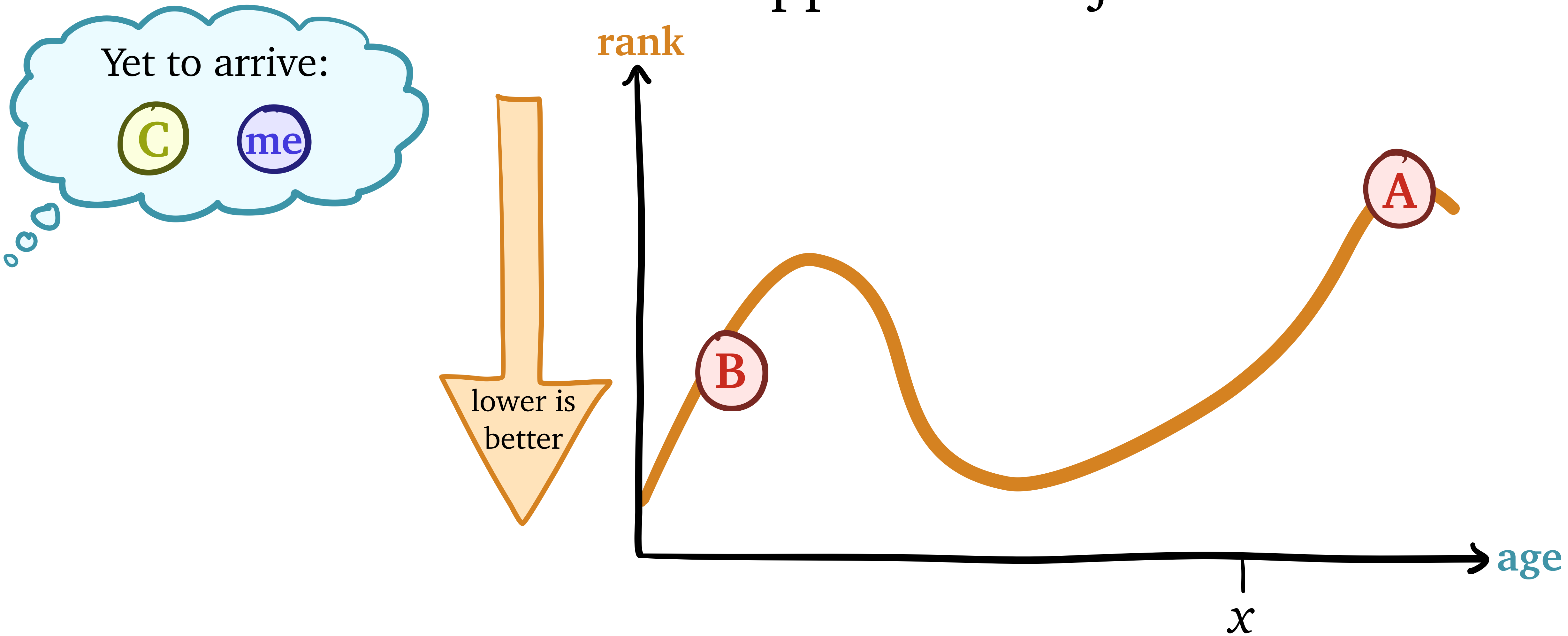

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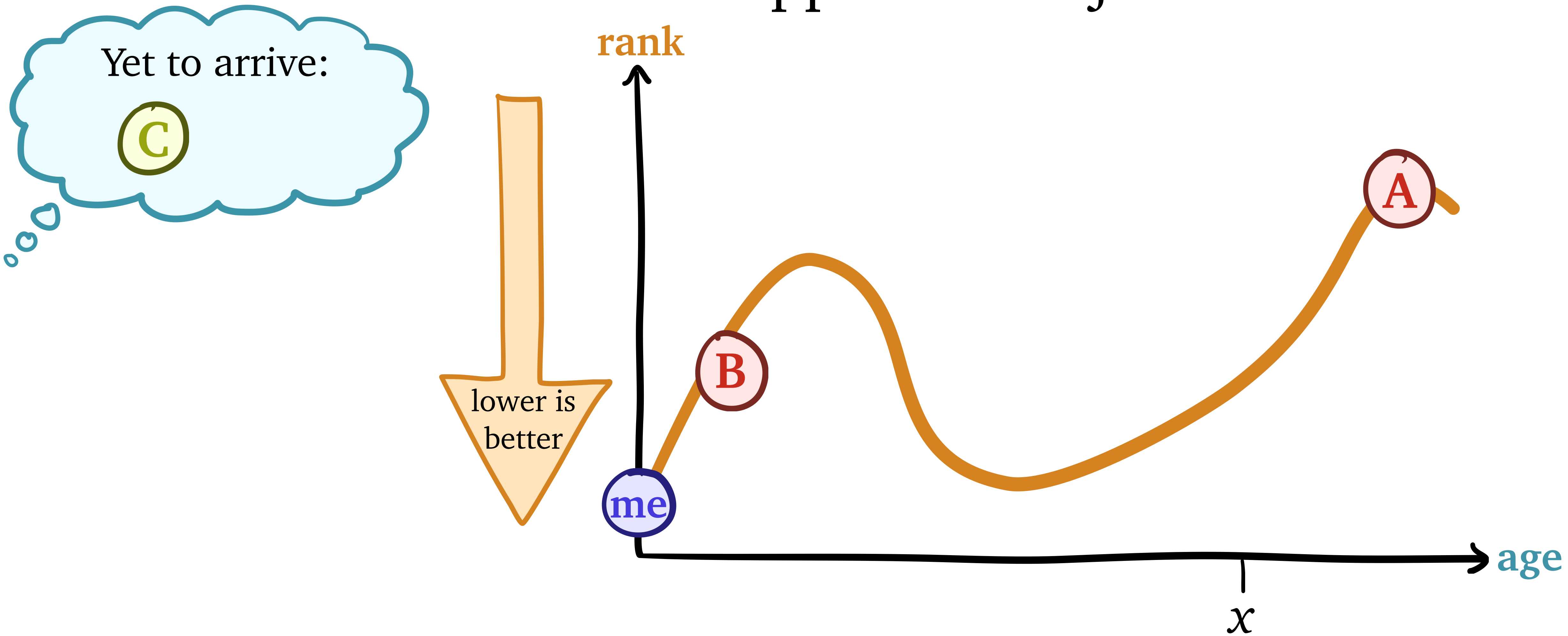

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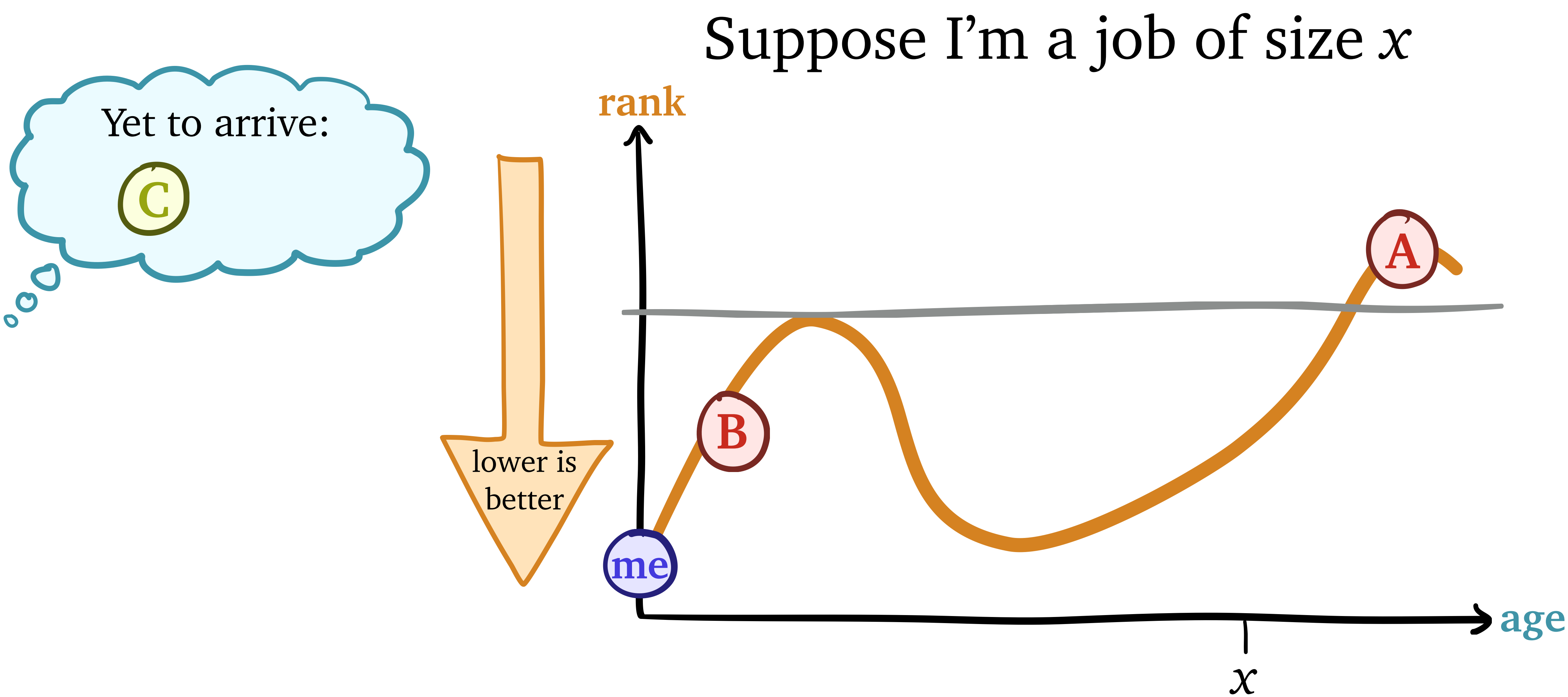

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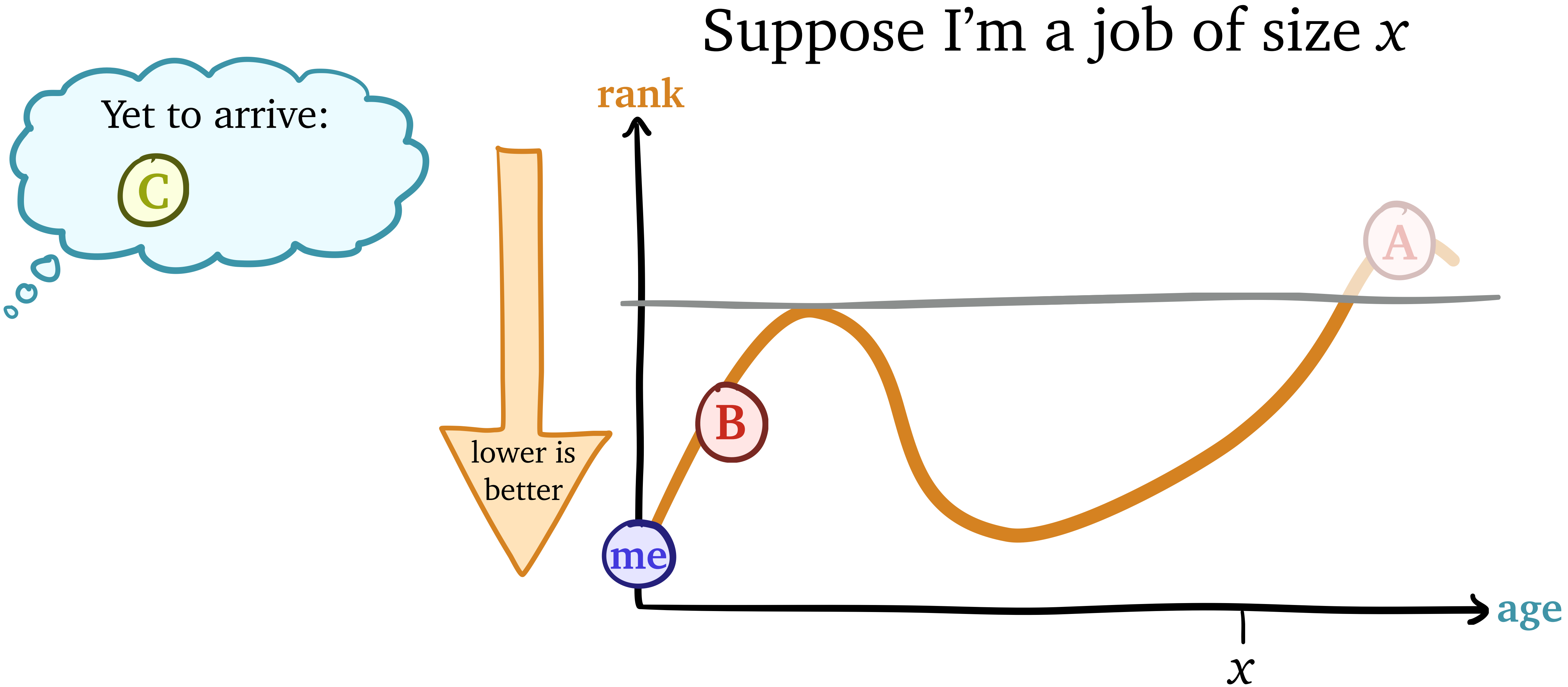

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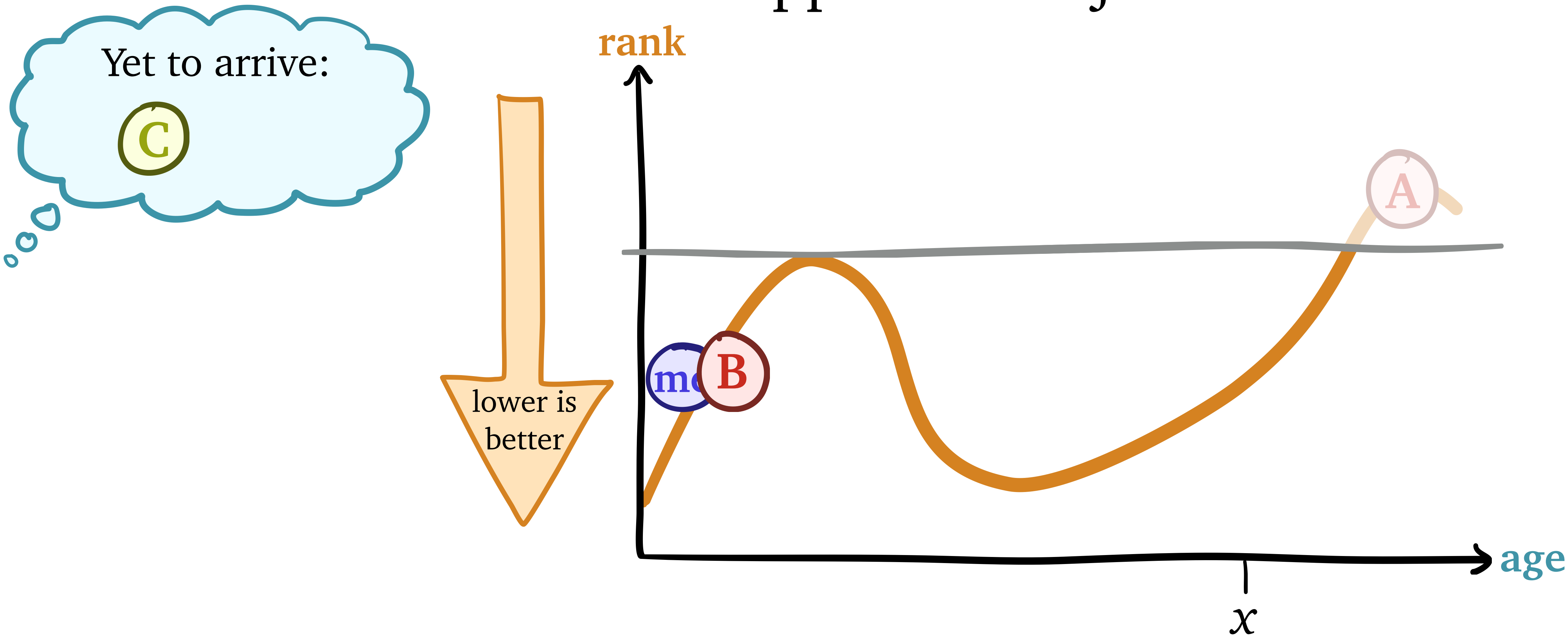

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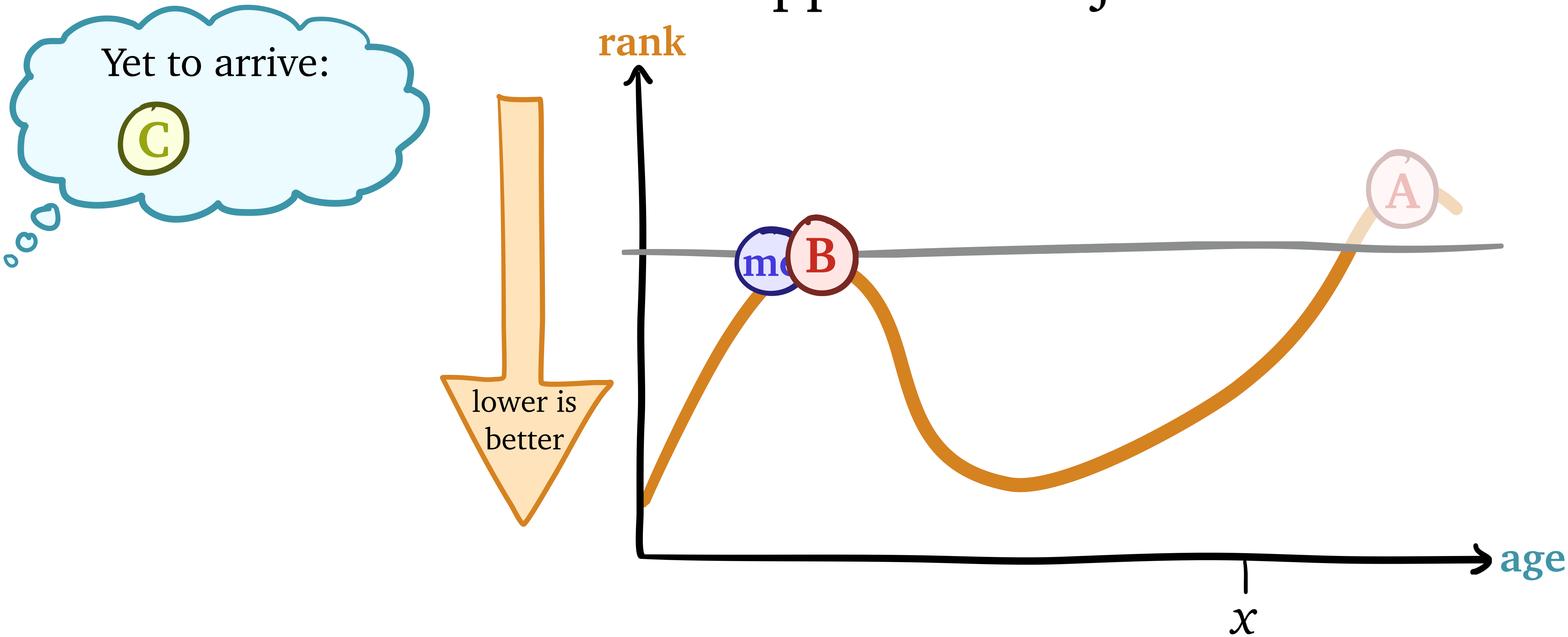

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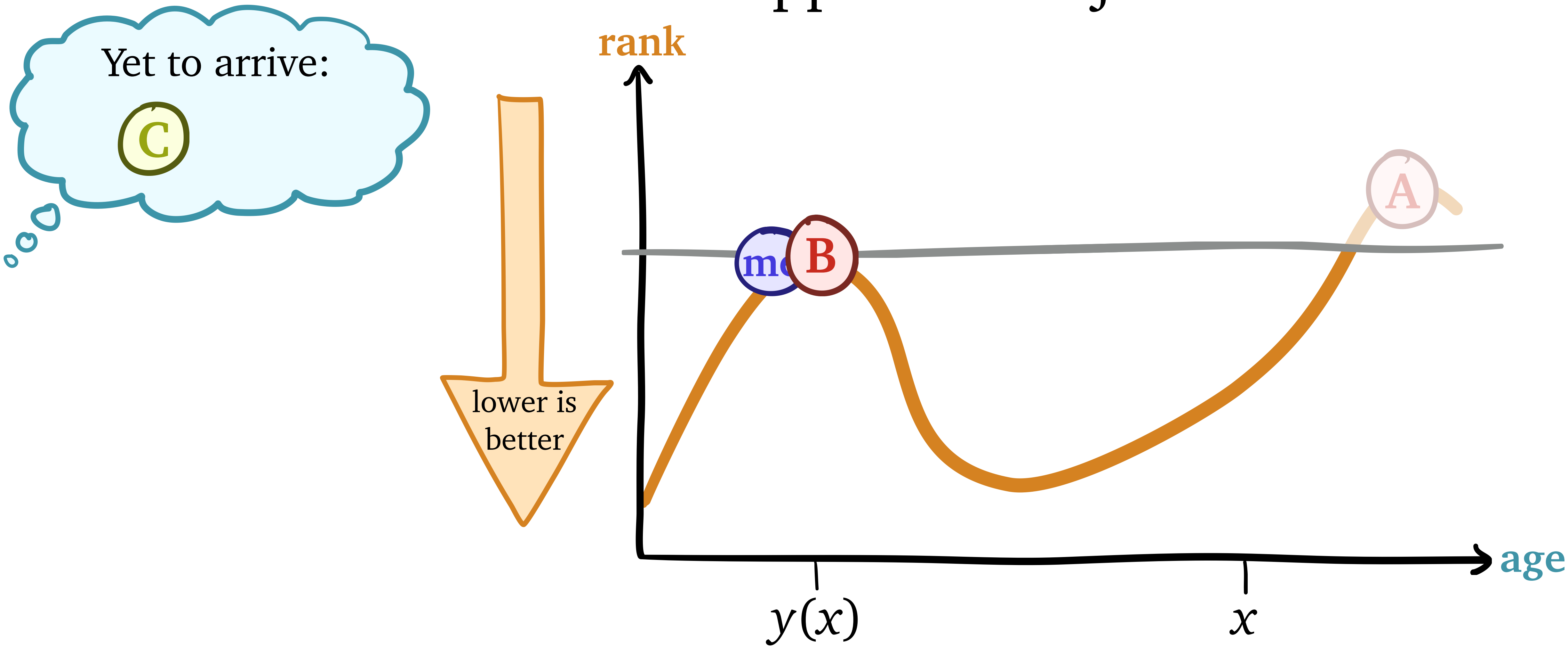

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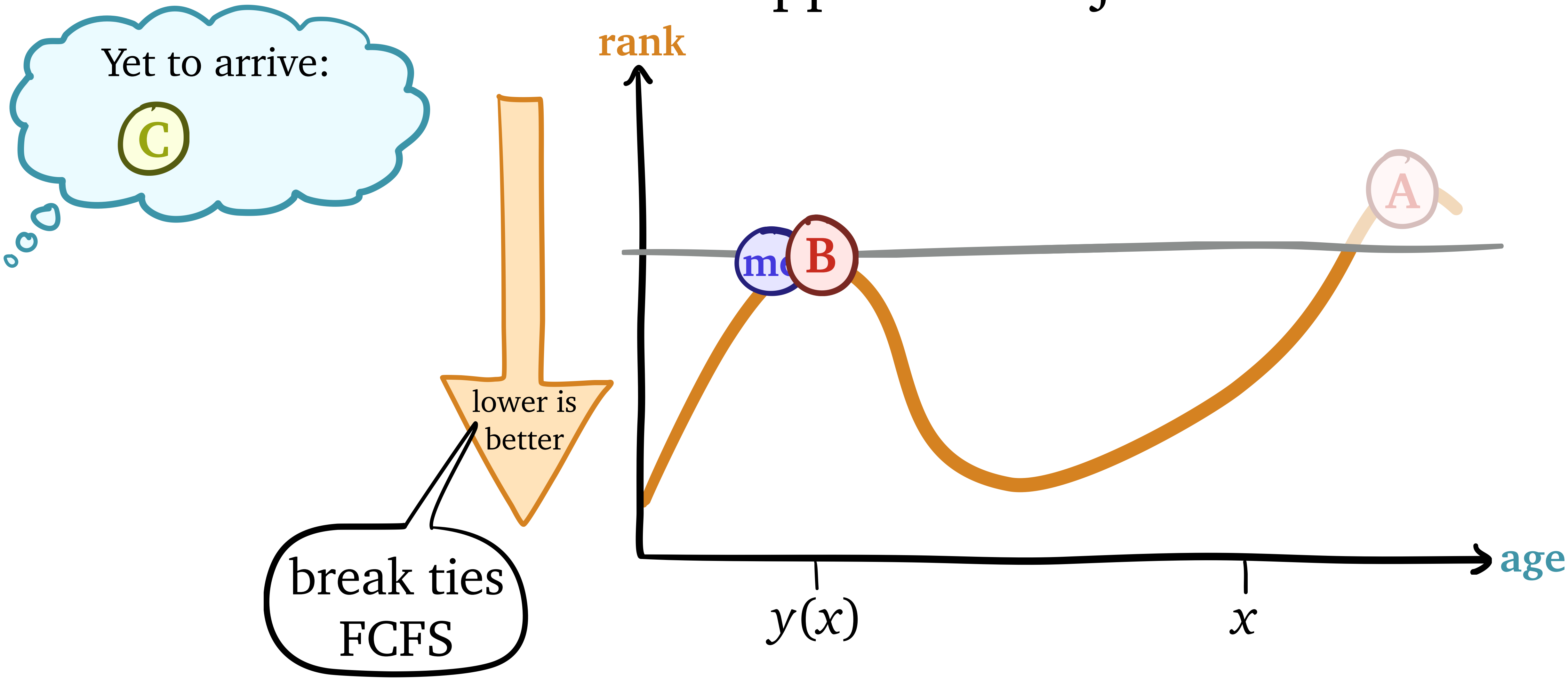

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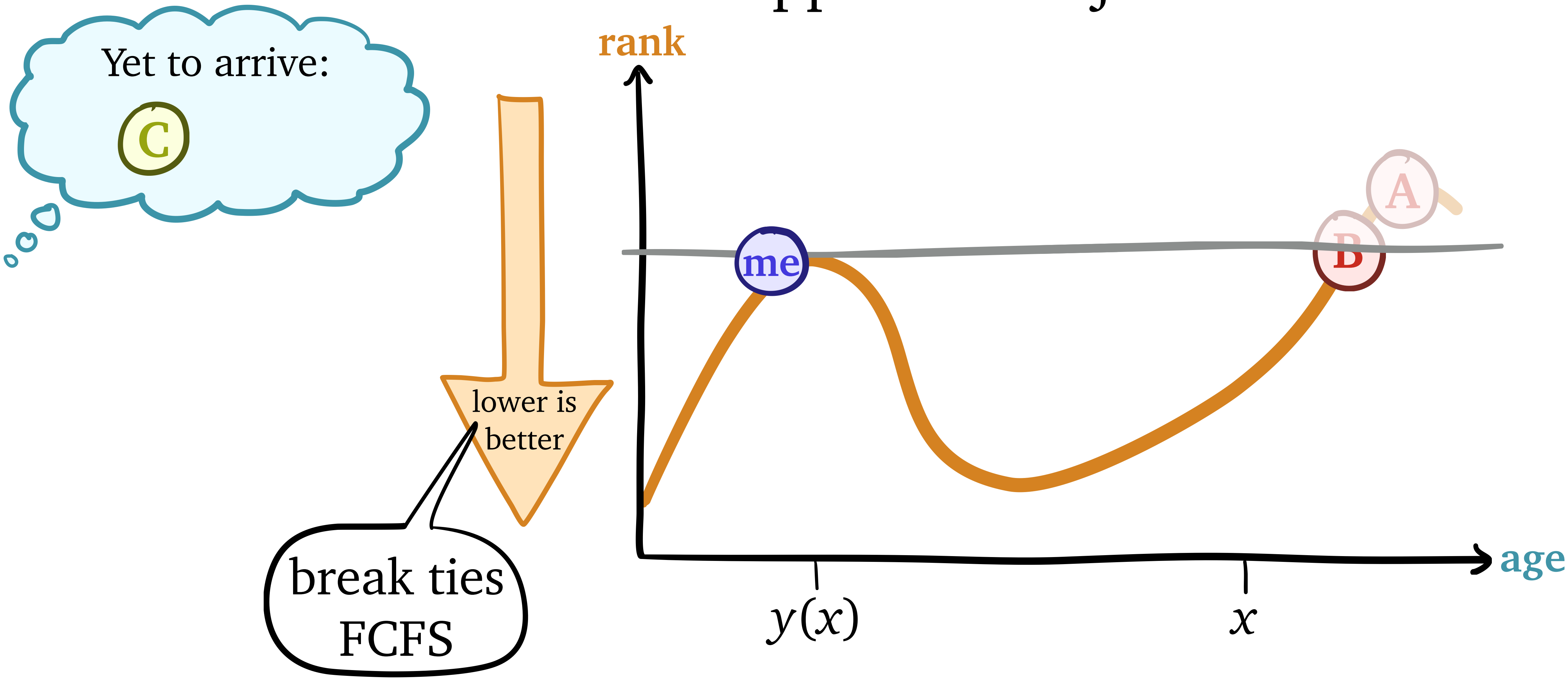

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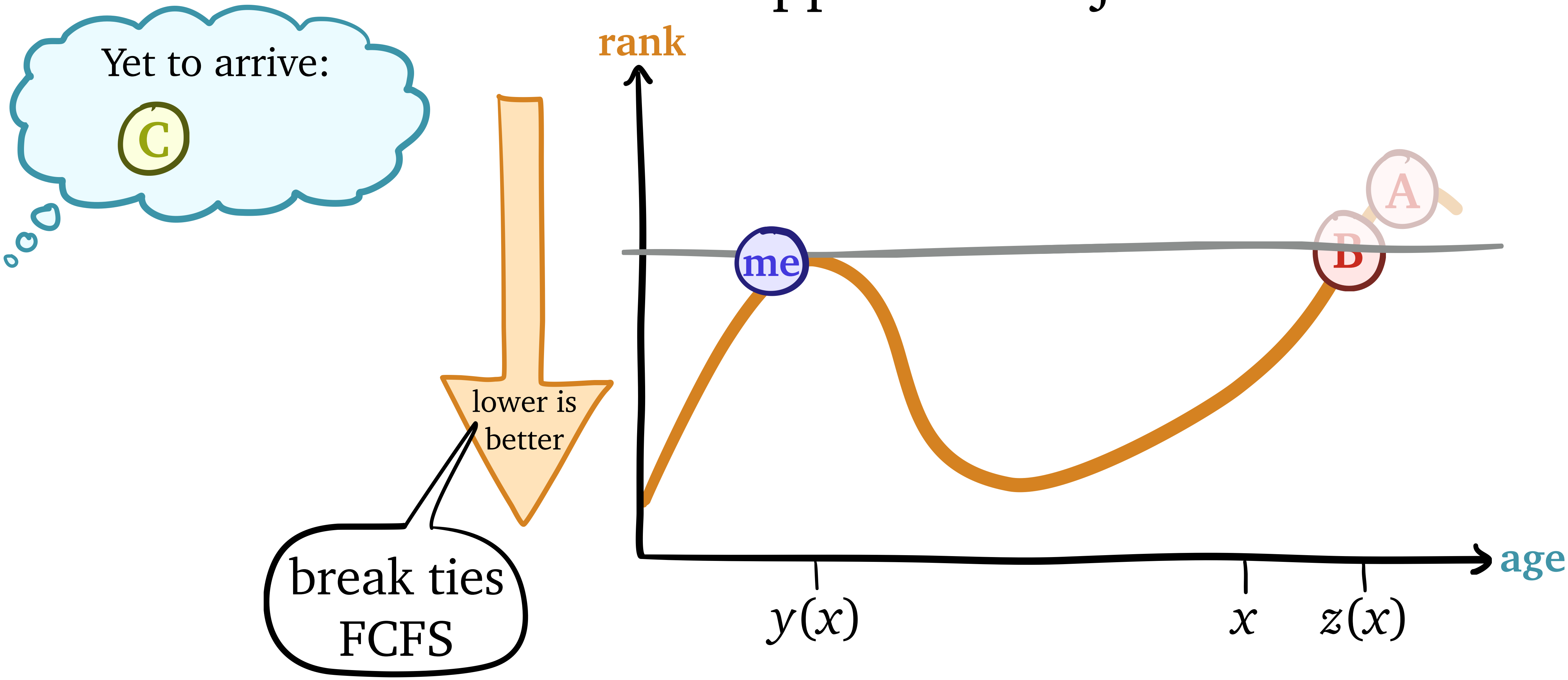

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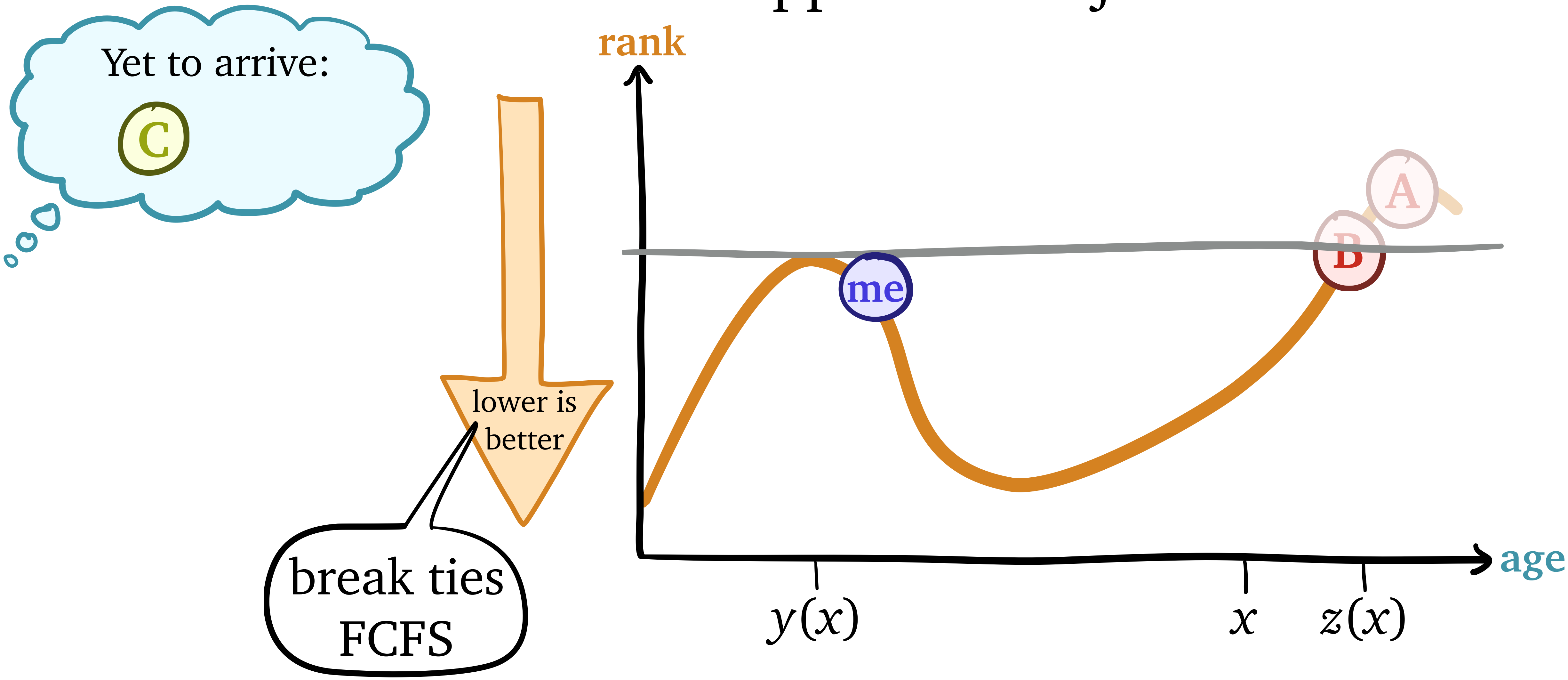

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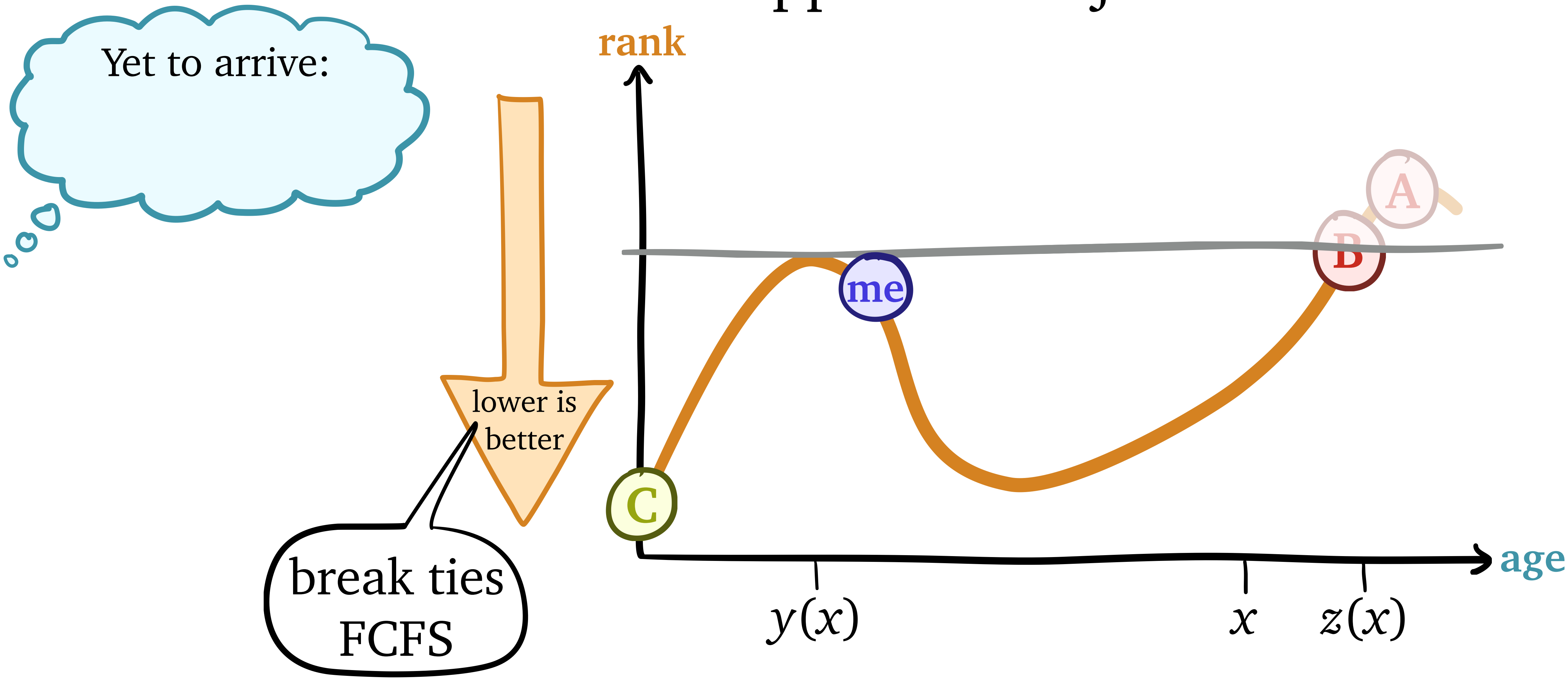

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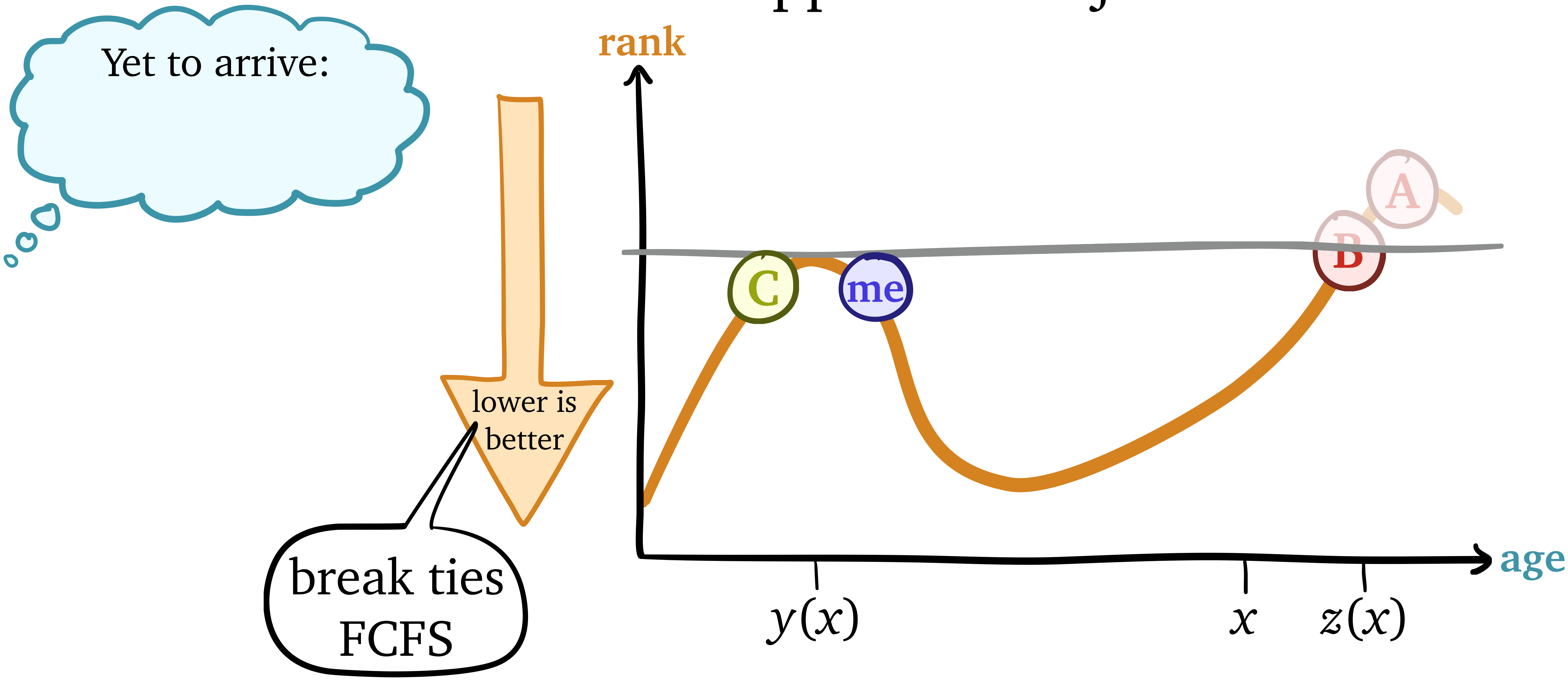

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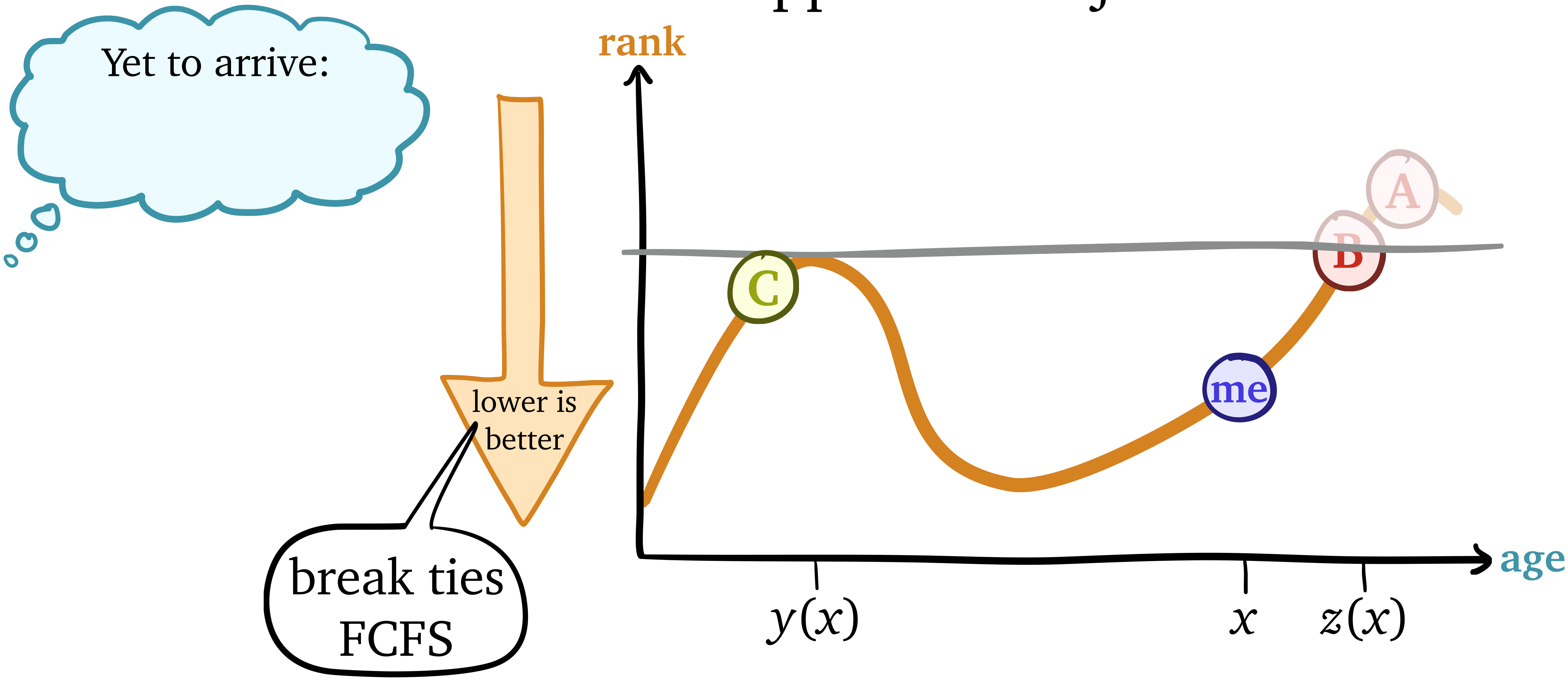

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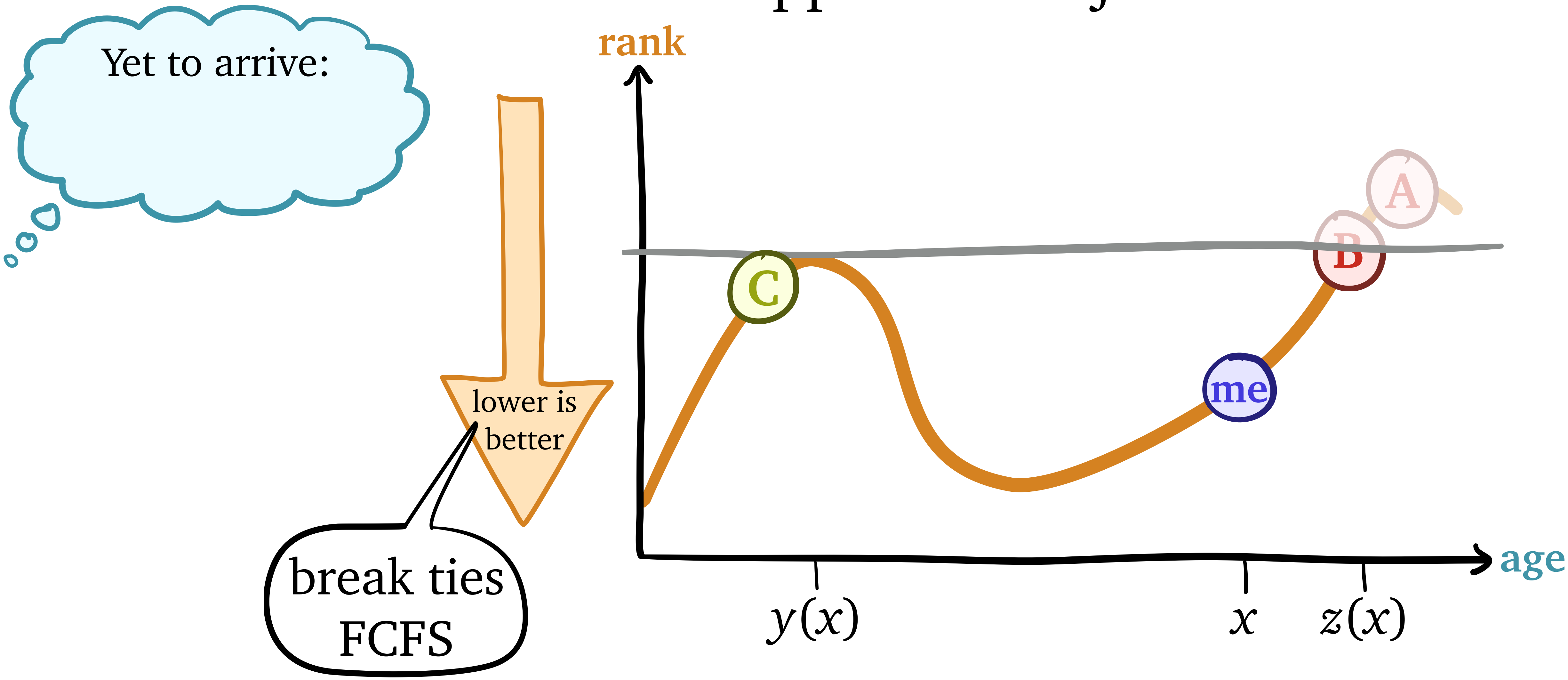
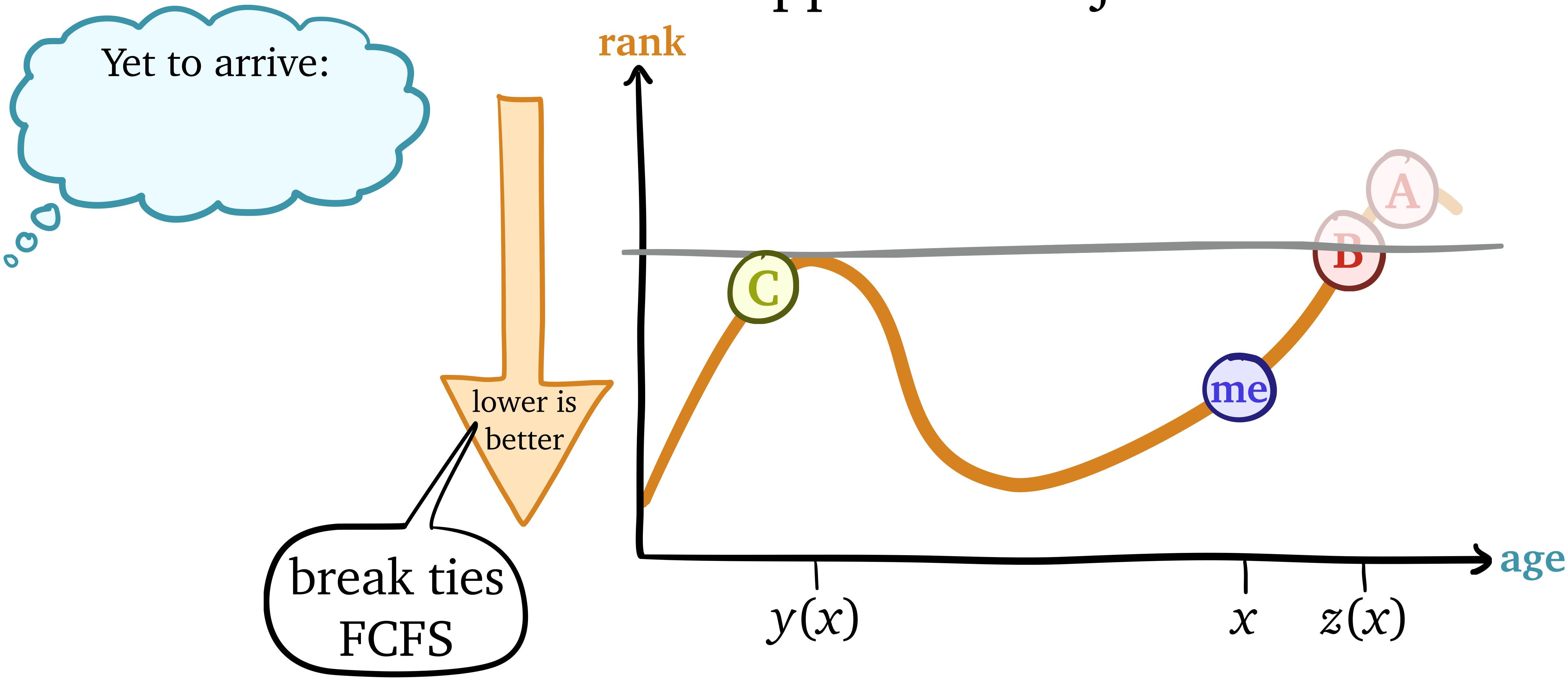


I can ignore { *old* jobs (**A** & **B**) / *new* jobs (**C**) } after age { ??? / ??? }

Analyzing **Gittins-1**

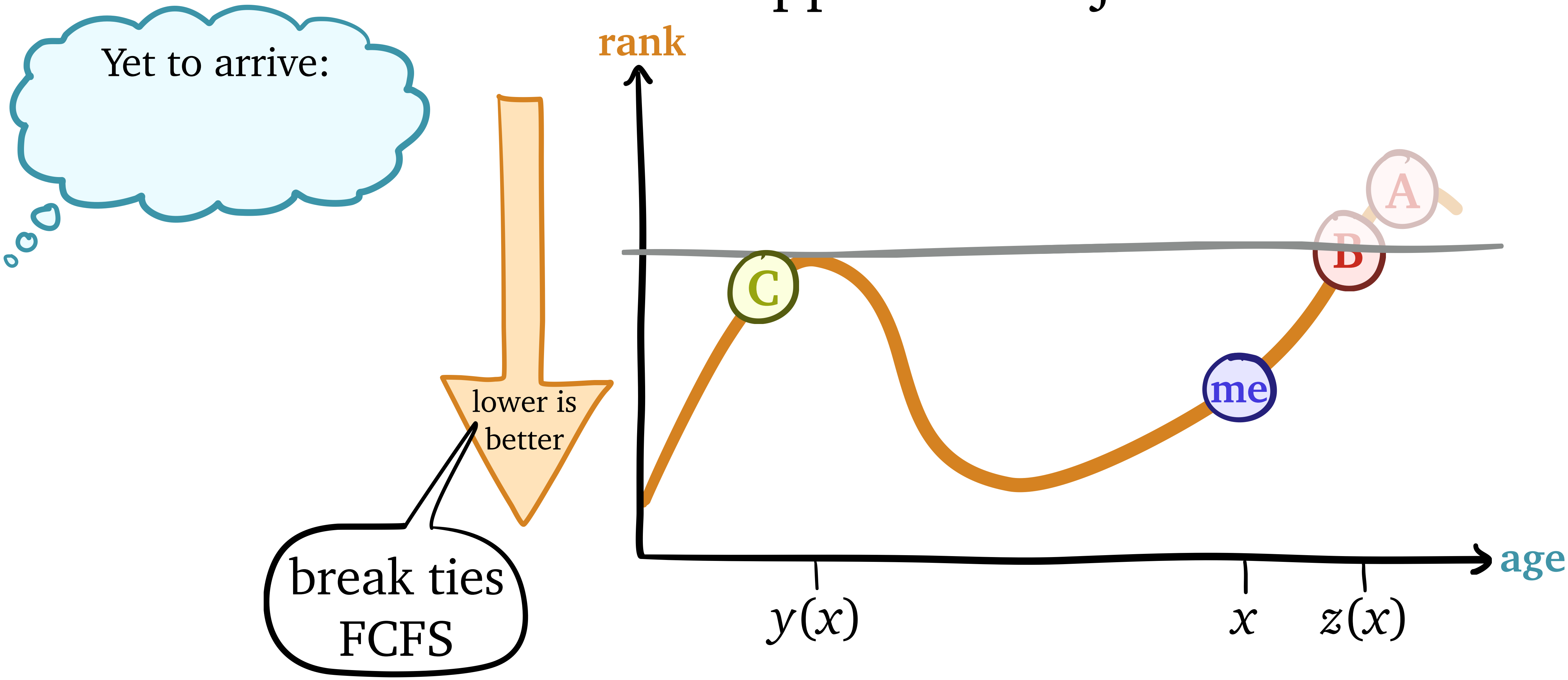
Suppose I'm a job of size x



I can ignore $\left\{\begin{array}{l} \textit{old} \text{ jobs (A \& B)} \\ \textit{new} \text{ jobs (C)} \end{array}\right\}$ after age $\left\{\begin{array}{l} z(x) \\ ??? \end{array}\right\}$

Analyzing **Gittins-1**

Suppose I'm a job of size x

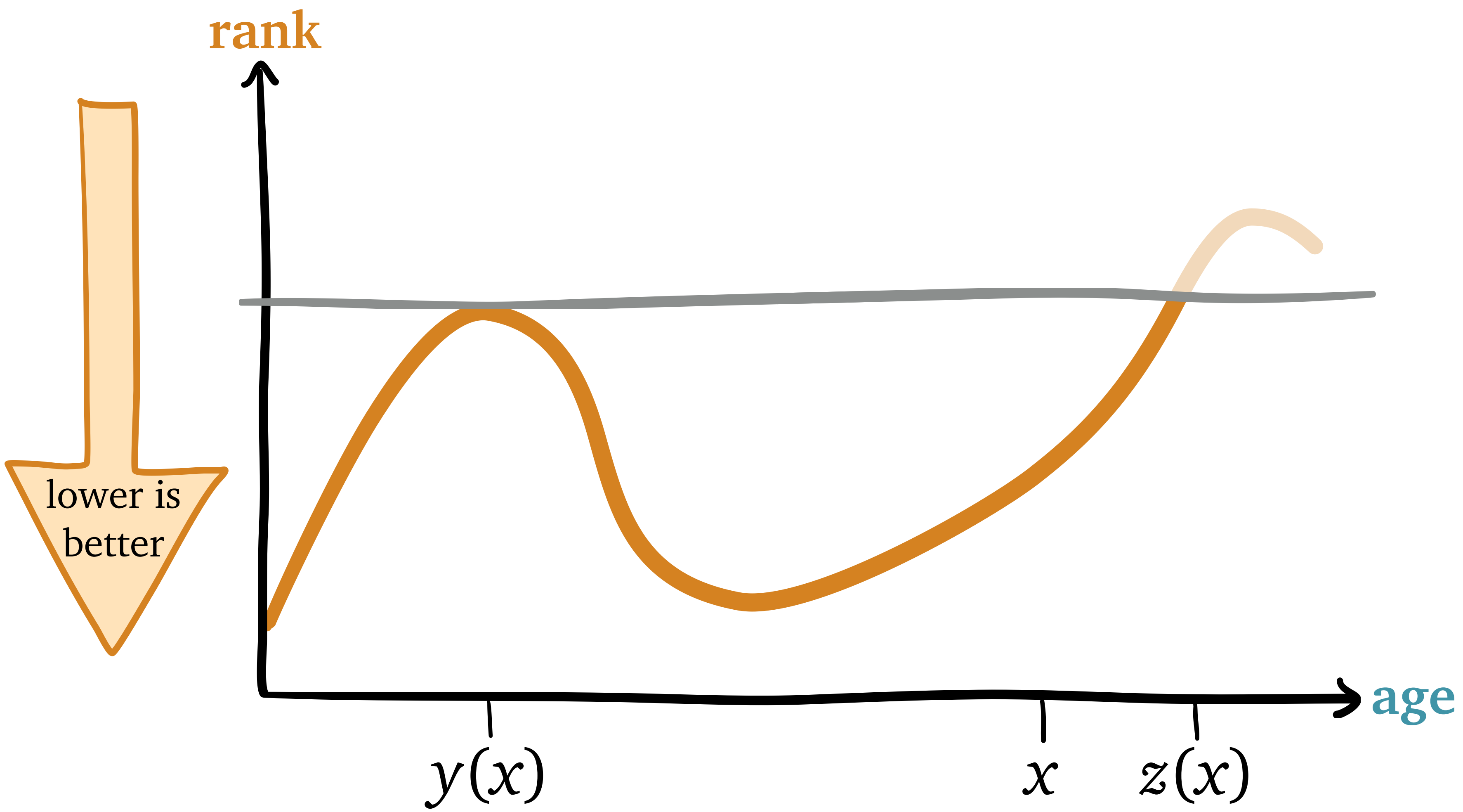


I can ignore { *old* jobs (**A** & **B**) / *new* jobs (**C**) } after age { $z(x)$ / $y(x)$ }

Question:
What goes wrong for **Gittins-*k***?

Analyzing **Gittins-*k***

Suppose I'm a job of size x



Analyzing **Gittins-*k***

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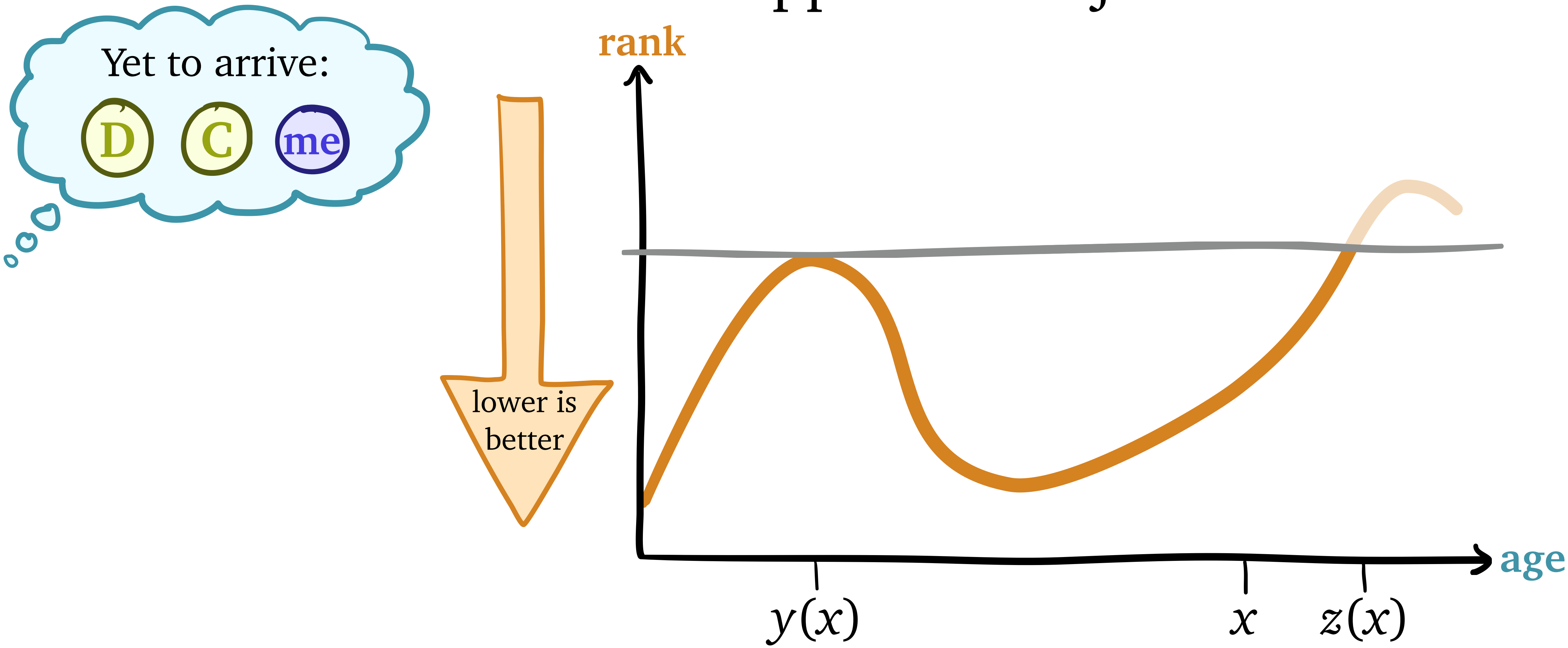
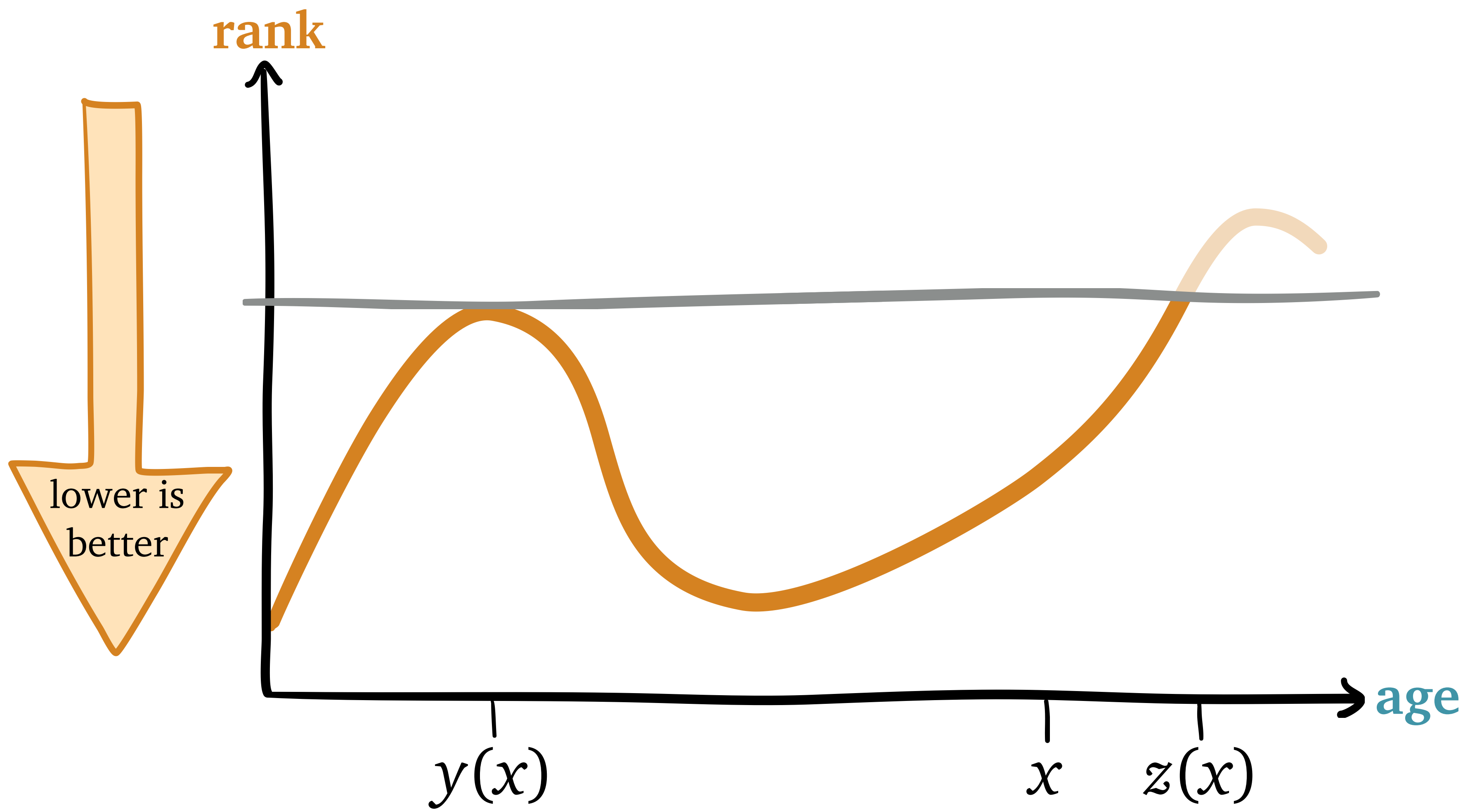


I can ignore $\left\{ \begin{array}{c} \textit{old} \text{ jobs} \\ \textit{new} \text{ jobs (C \& D)} \end{array} \right\}$ after age $\left\{ \begin{array}{c} ??? \\ ??? \end{array} \right\}$

Analyzing **Gittins-k**

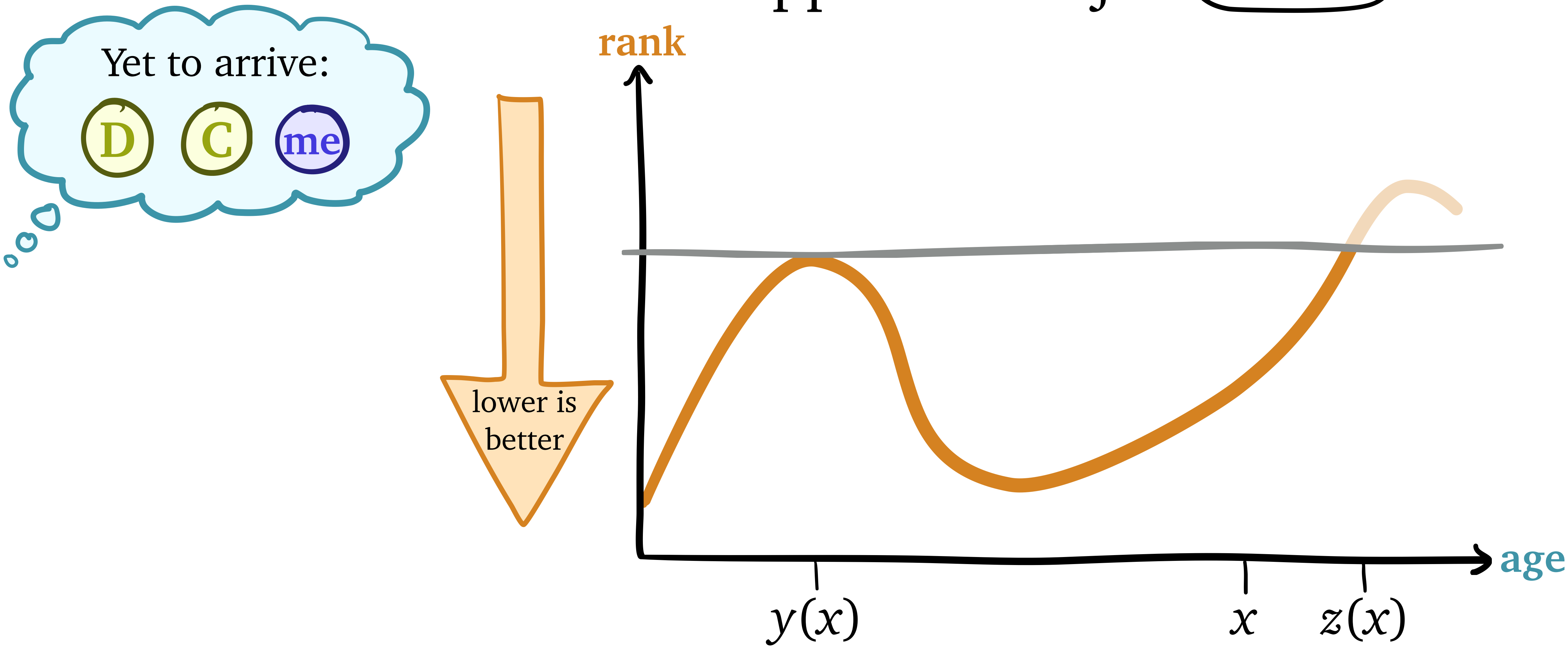
Suppose I'm a job x ($k = 2$)



I can ignore { *old* jobs / *new* jobs (C & D) } after age { ??? / ??? }

Analyzing **Gittins-k**

Suppose I'm a job $k = 2$ x

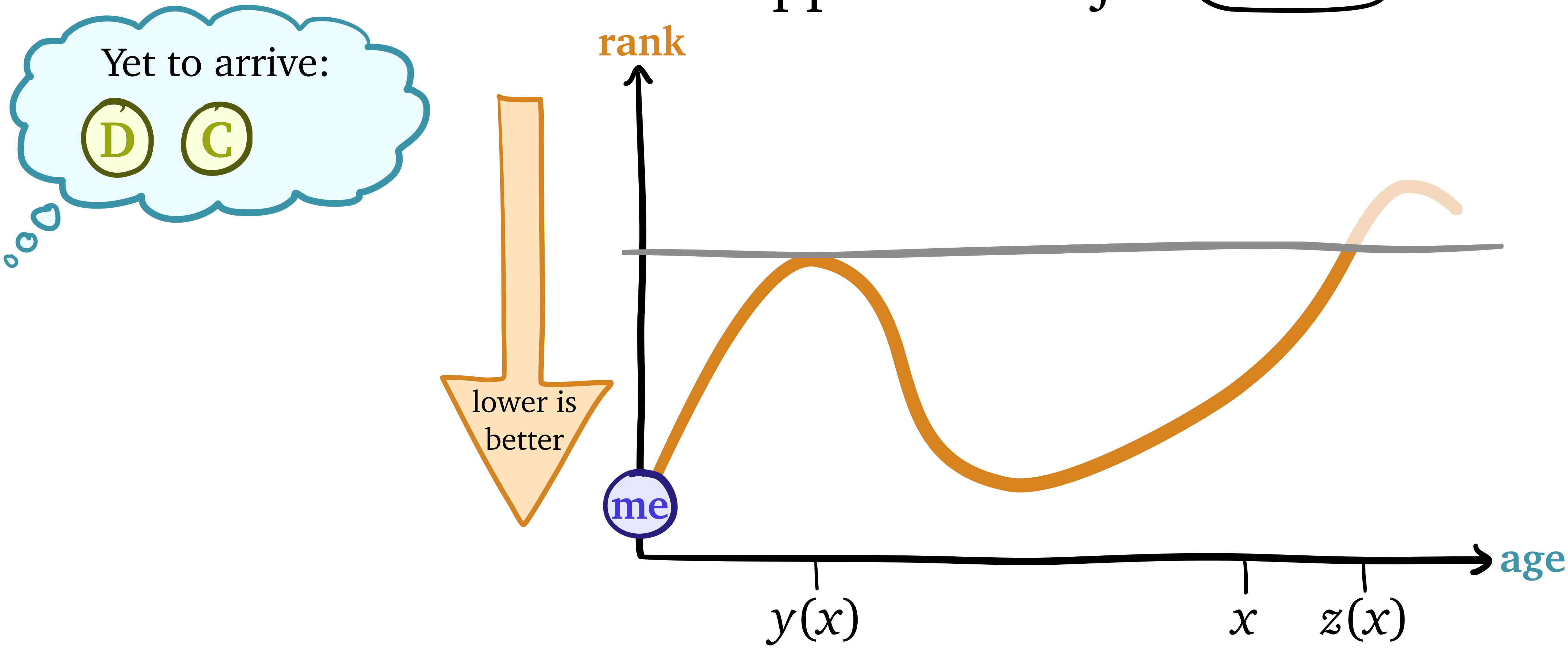


I can ignore { *old* jobs / *new* jobs (C & D) } after age { $z(x)$ / ??? }

Analyzing **Gittins-k**

Suppose I'm a job x

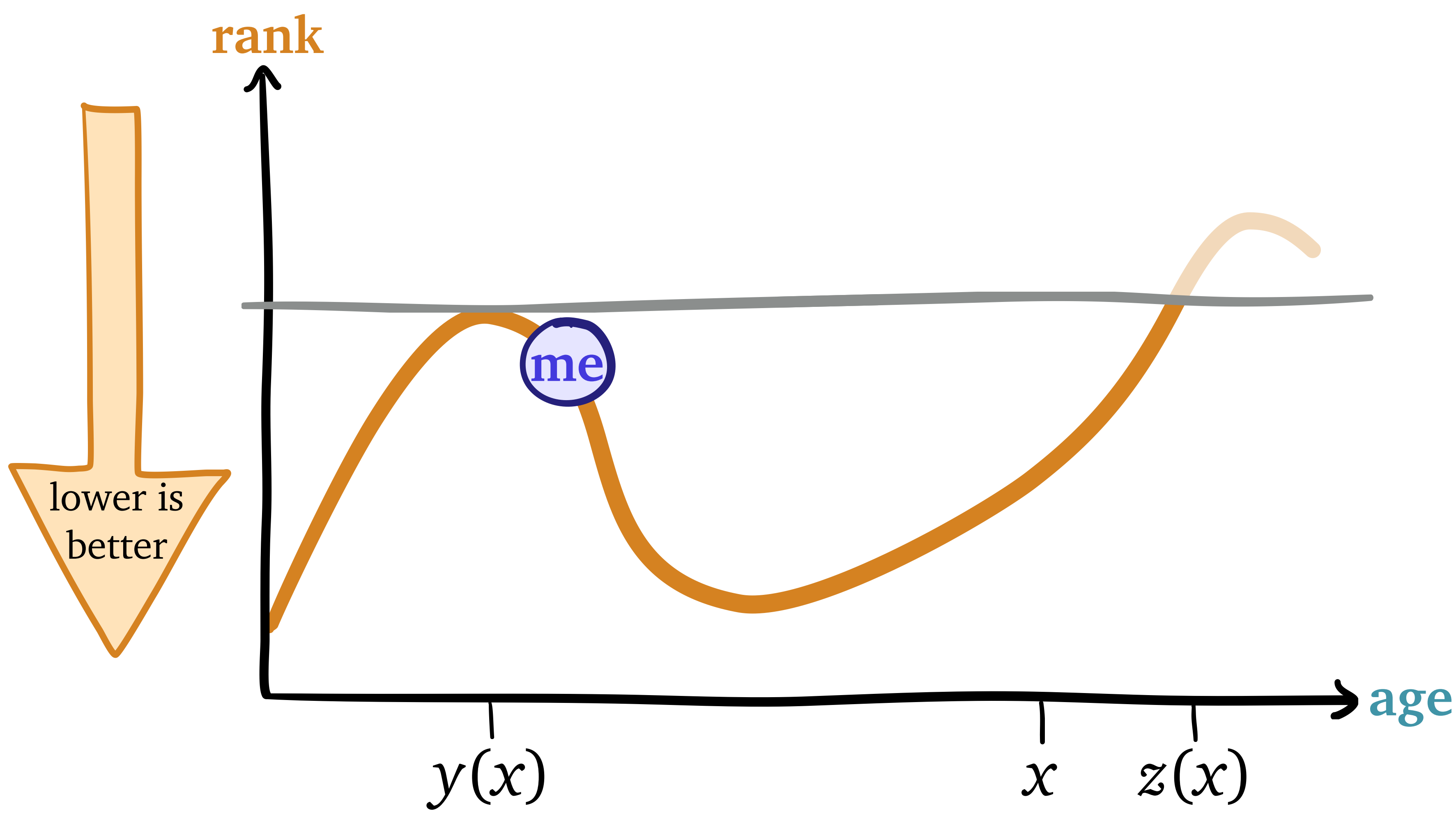
$k = 2$



I can ignore { *old* jobs / *new* jobs (C & D) } after age { $z(x)$ / ??? }

Analyzing **Gittins-*k***

Suppose I'm a job ($k = 2$) x

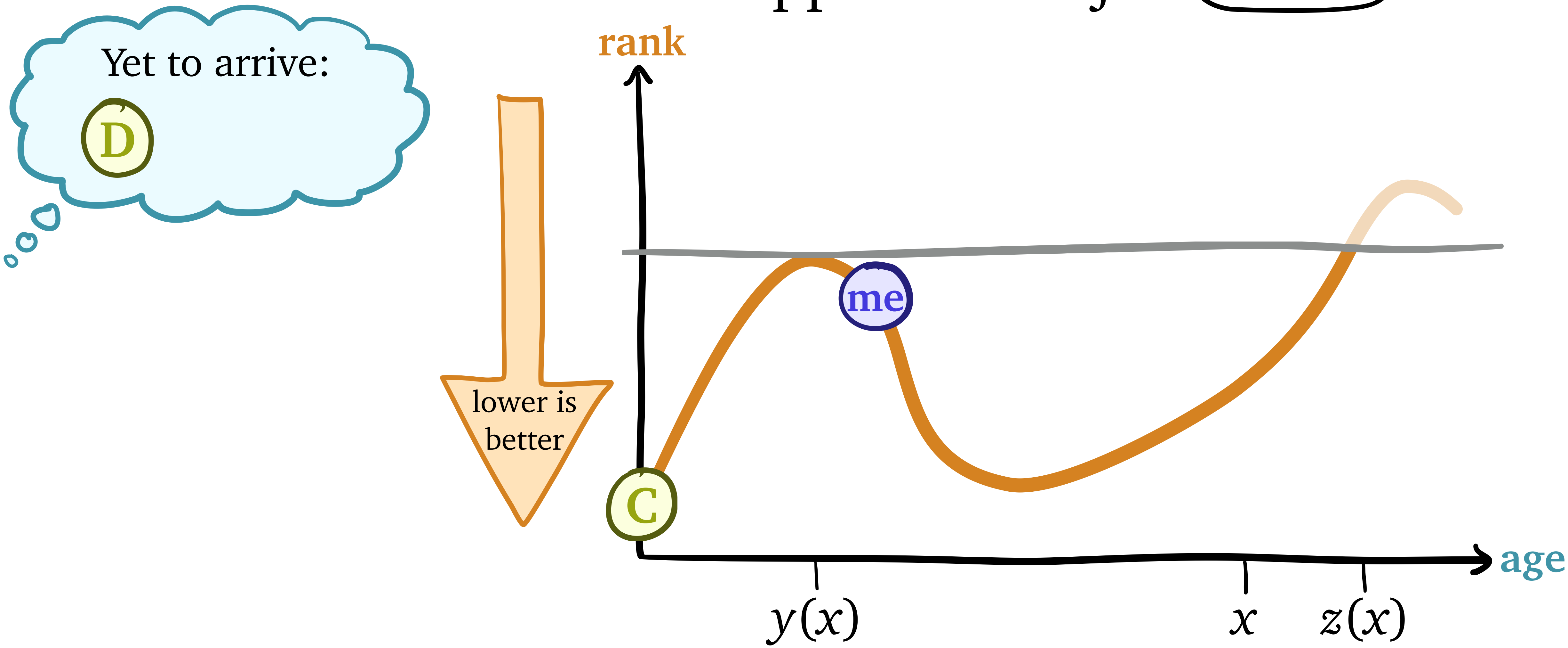


I can ignore { *old* jobs / *new* jobs (**C** & **D**) } after age { $z(x)$ / **???** }

Analyzing **Gittins-*k***

$k = 2$

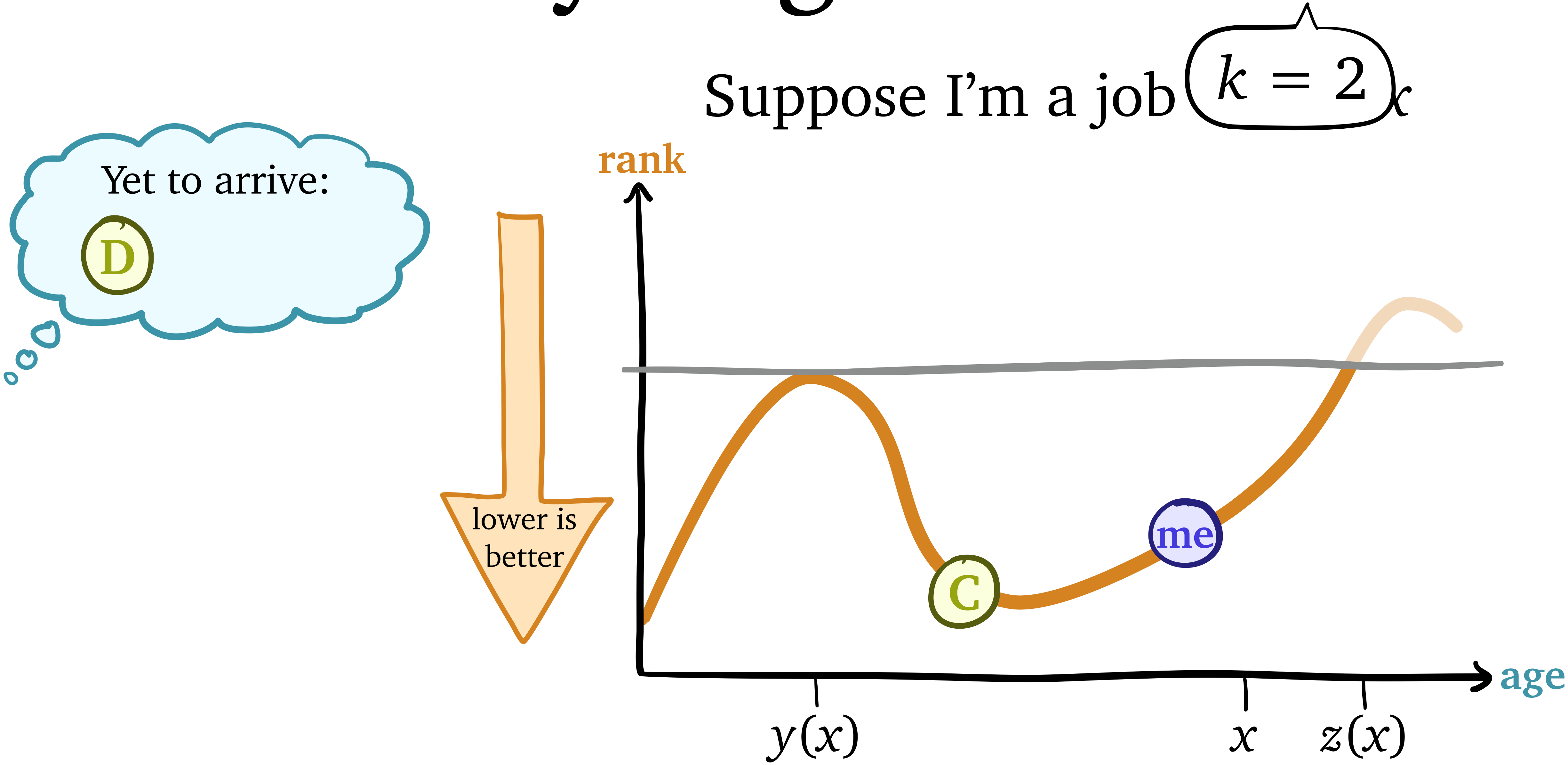
Suppose I’m a job x



I can ignore $\left\{ \begin{array}{c} \textit{old} \text{ jobs} \\ \textit{new} \text{ jobs (C \& D)} \end{array} \right\}$ after age $\left\{ \begin{array}{c} z(x) \\ ??? \end{array} \right\}$

Analyzing **Gittins-*k***

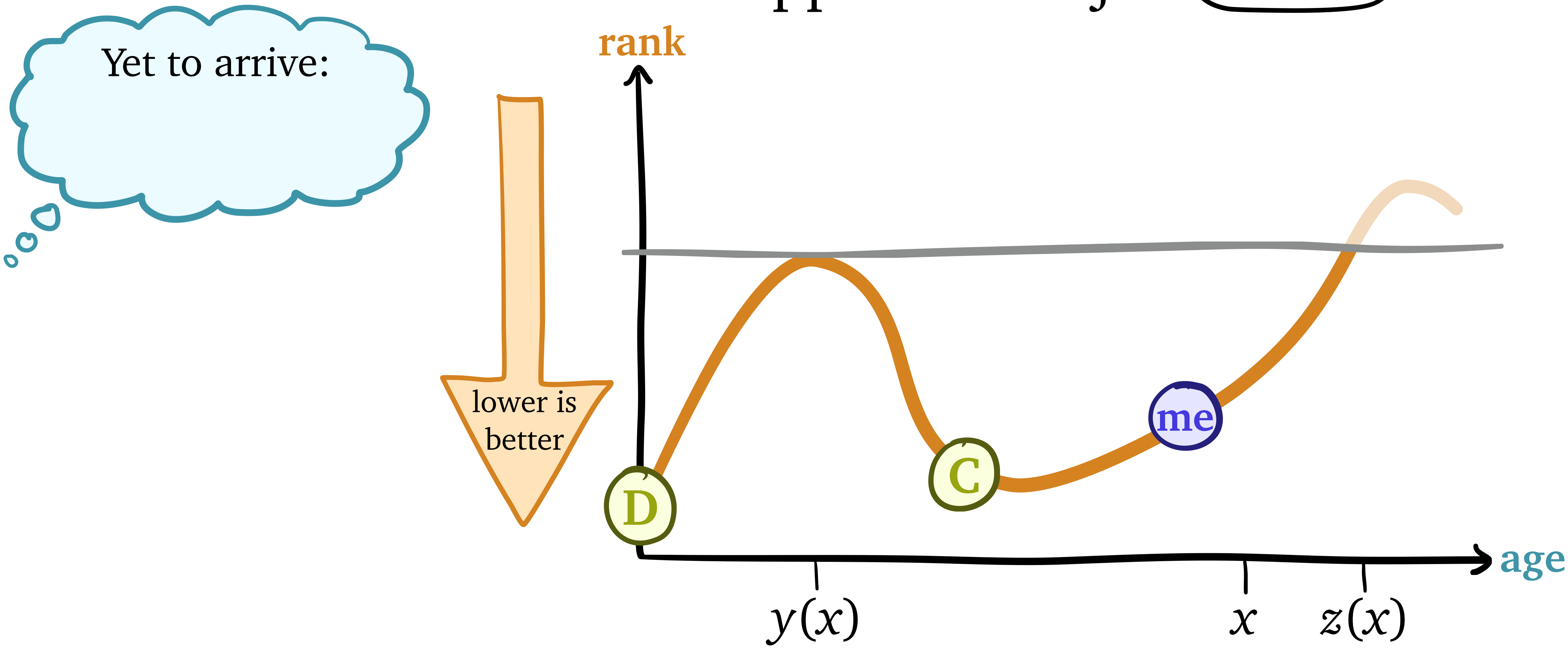
Suppose I'm a job x



I can ignore { *old* jobs / *new* jobs (C & D) } after age { $z(x)$ / ??? }

Analyzing **Gittins-k**

Suppose I'm a job $k = 2$ x



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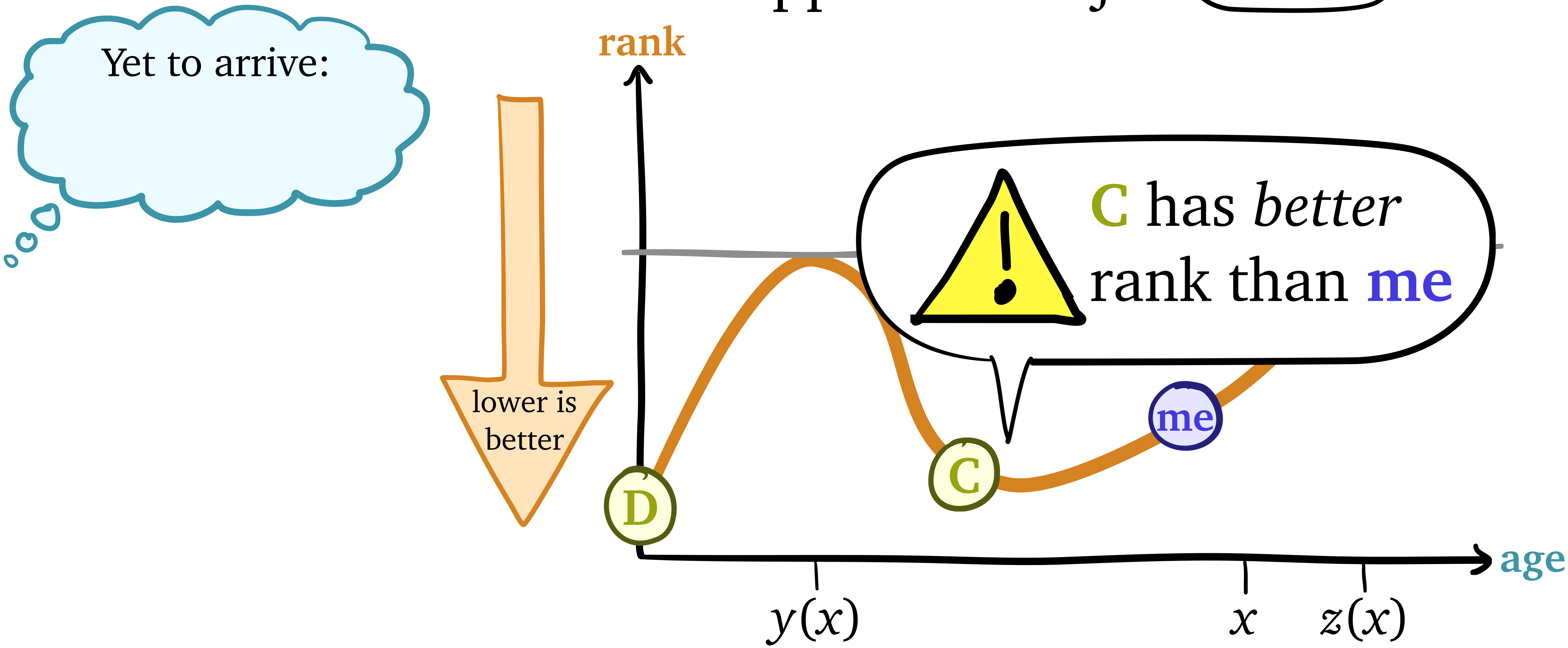

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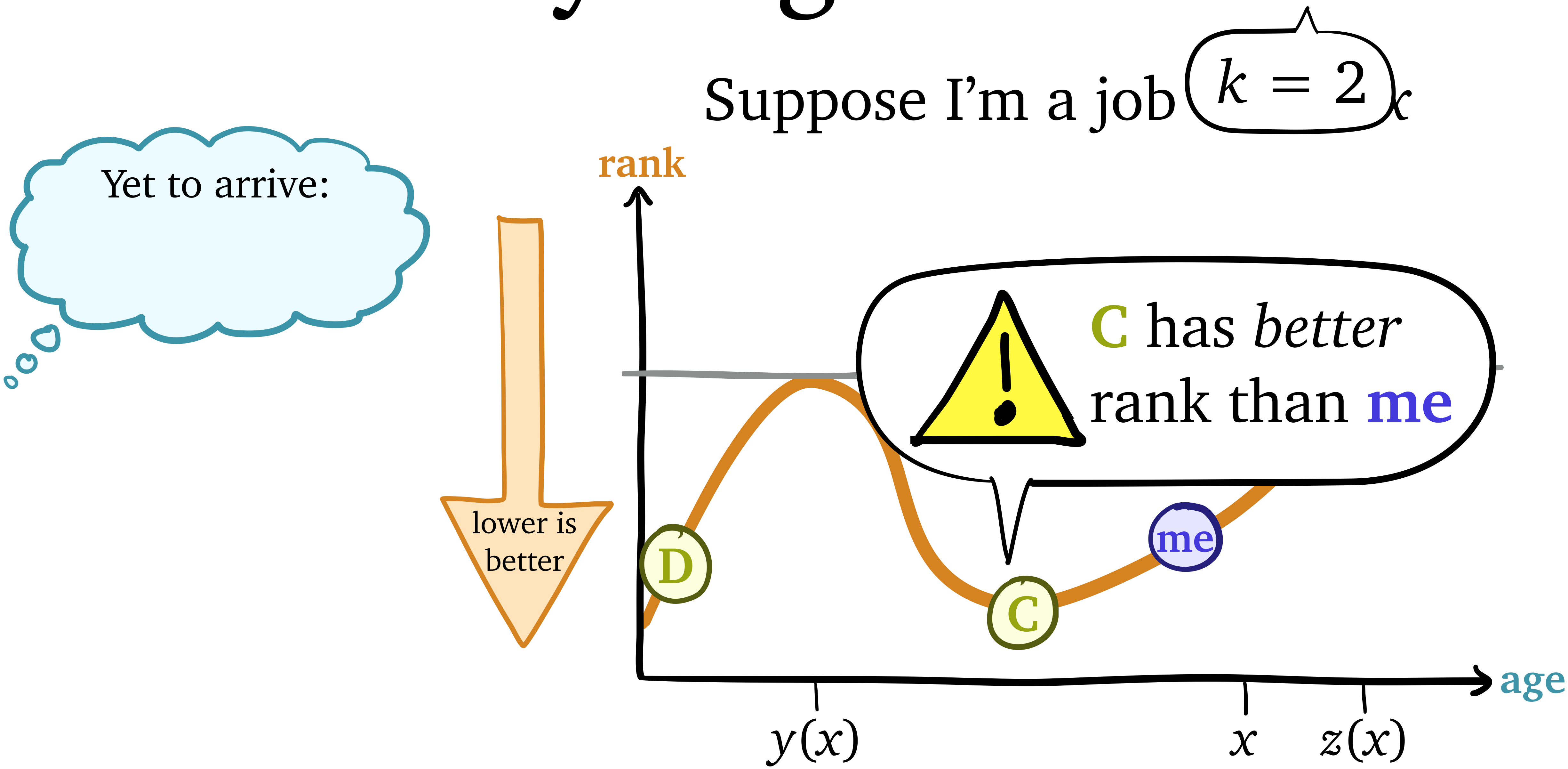

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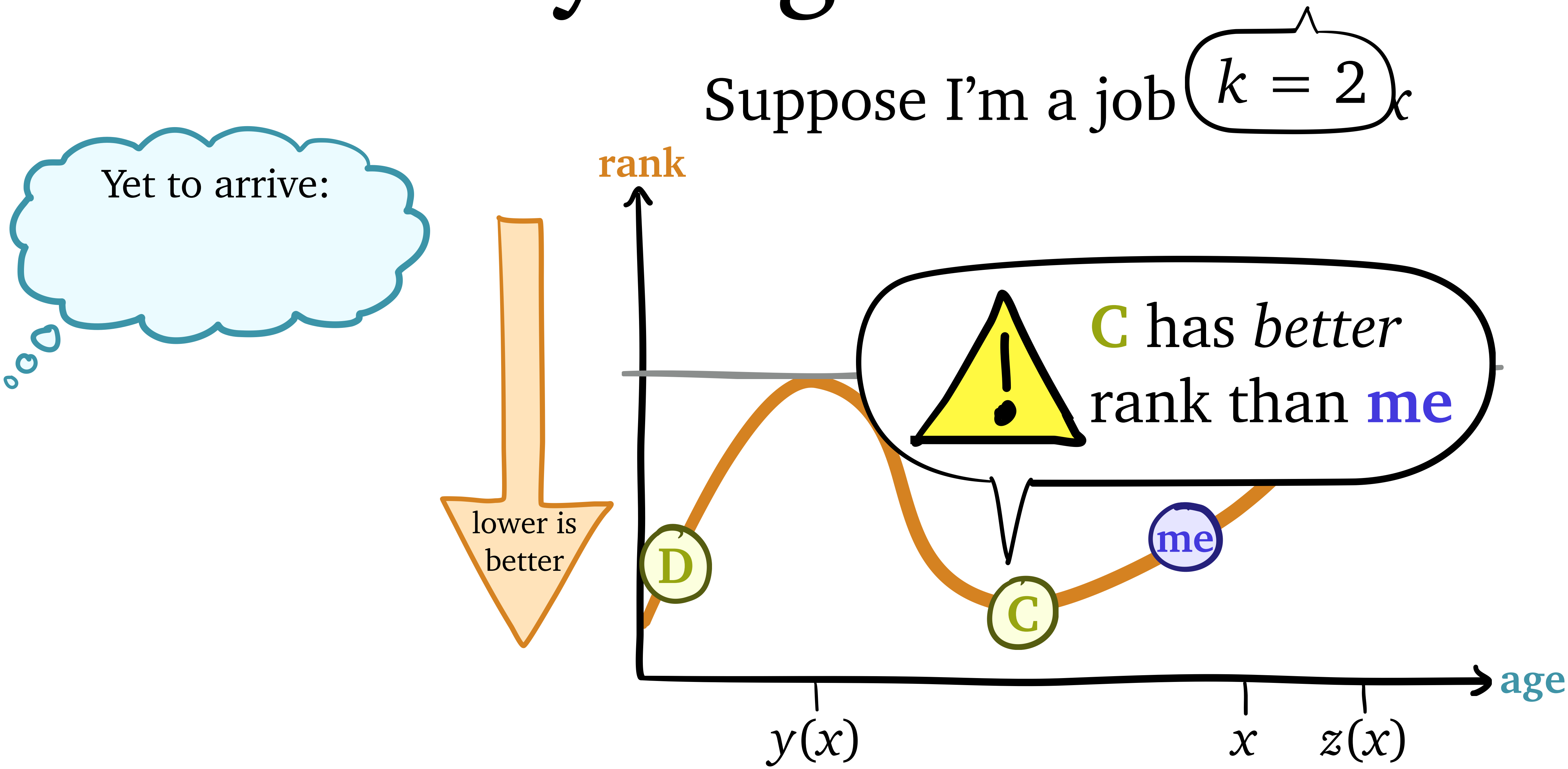

I can ignore $\left\{\begin{array}{c}\textit{old}\text{ jobs} \\ \textit{new}\text{ jobs (C \& D)}\end{array}\right\}$ after age $\left\{\begin{array}{c} z(x) \\ x \end{array}\right\}$

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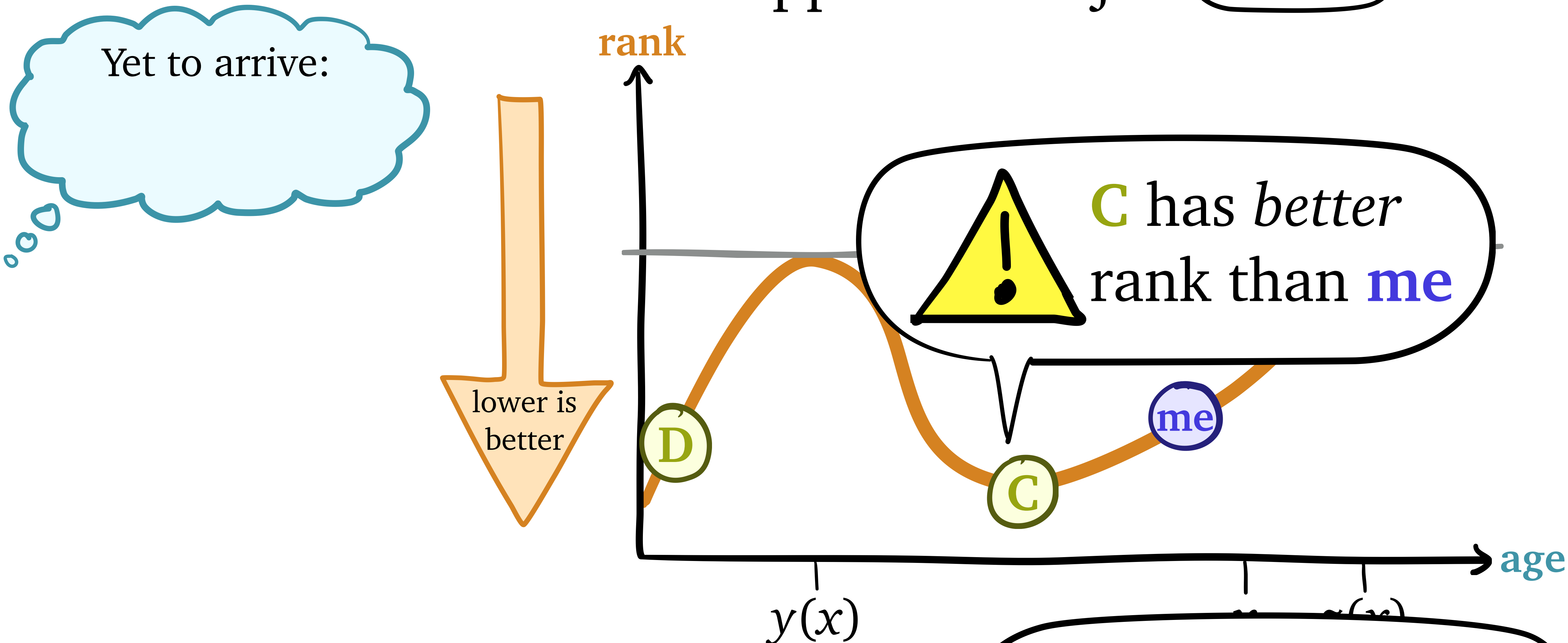

Was $y(x)$ for **Gittins-1**

I can ignore { *old* jobs / *new* jobs (C & D) } after age { x }

New Policy: **M-Gittins**

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monotonic

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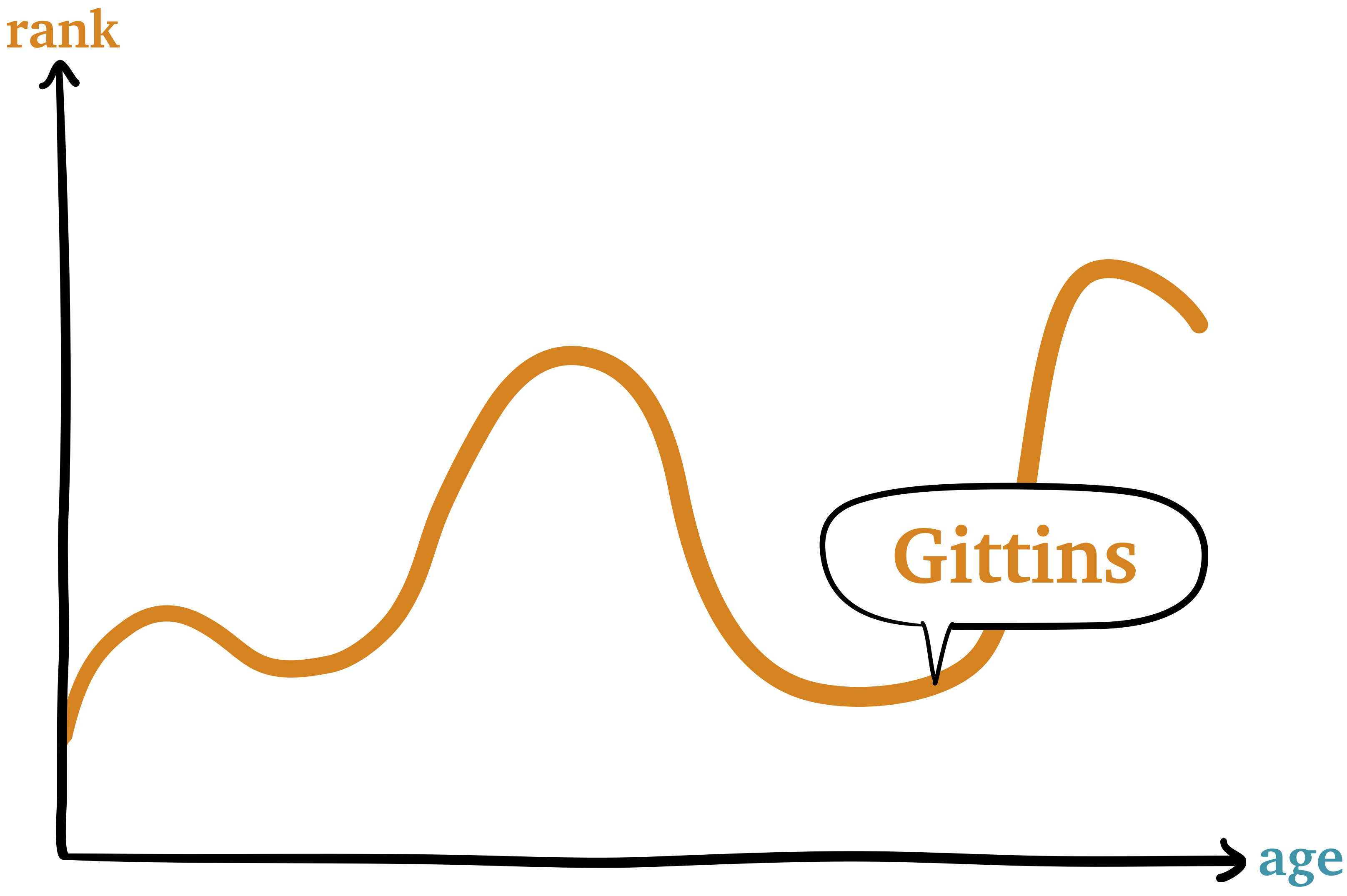

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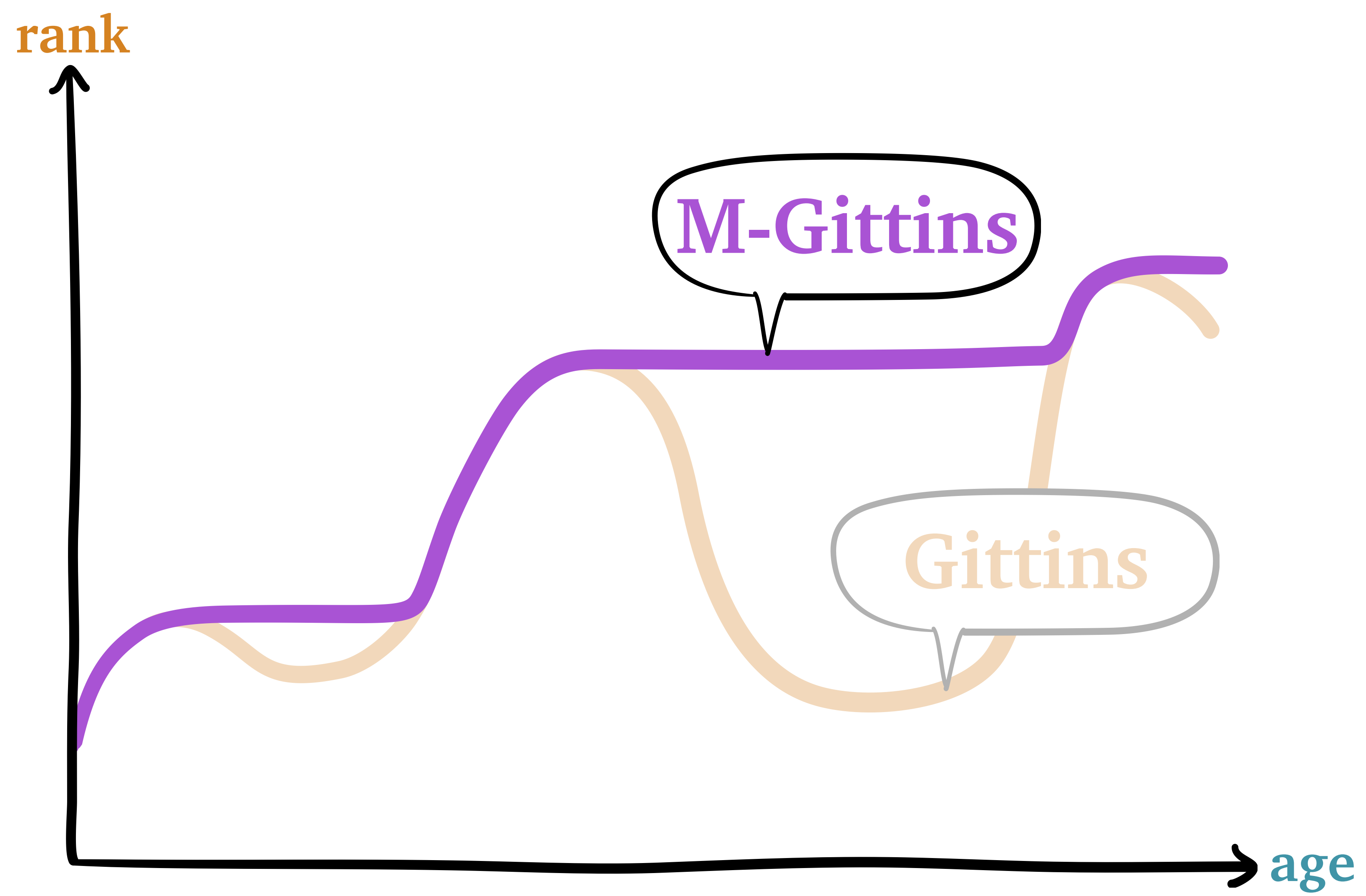
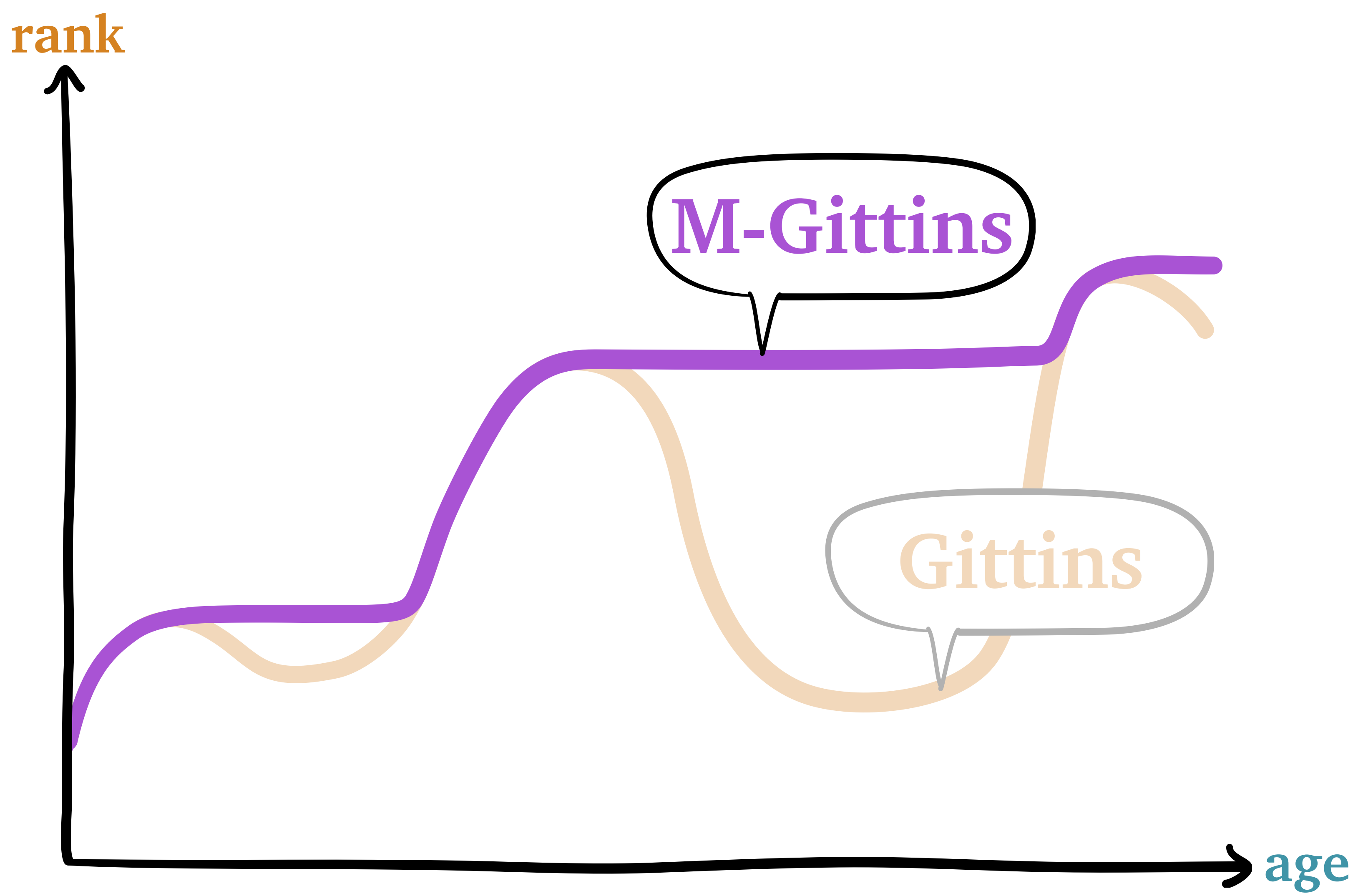


New Policy: M-Gittins

$$r_{\text{M-Gittins}}(a) = \max_{0 \le b \le a} r_{\text{Gittins}}(b)$$



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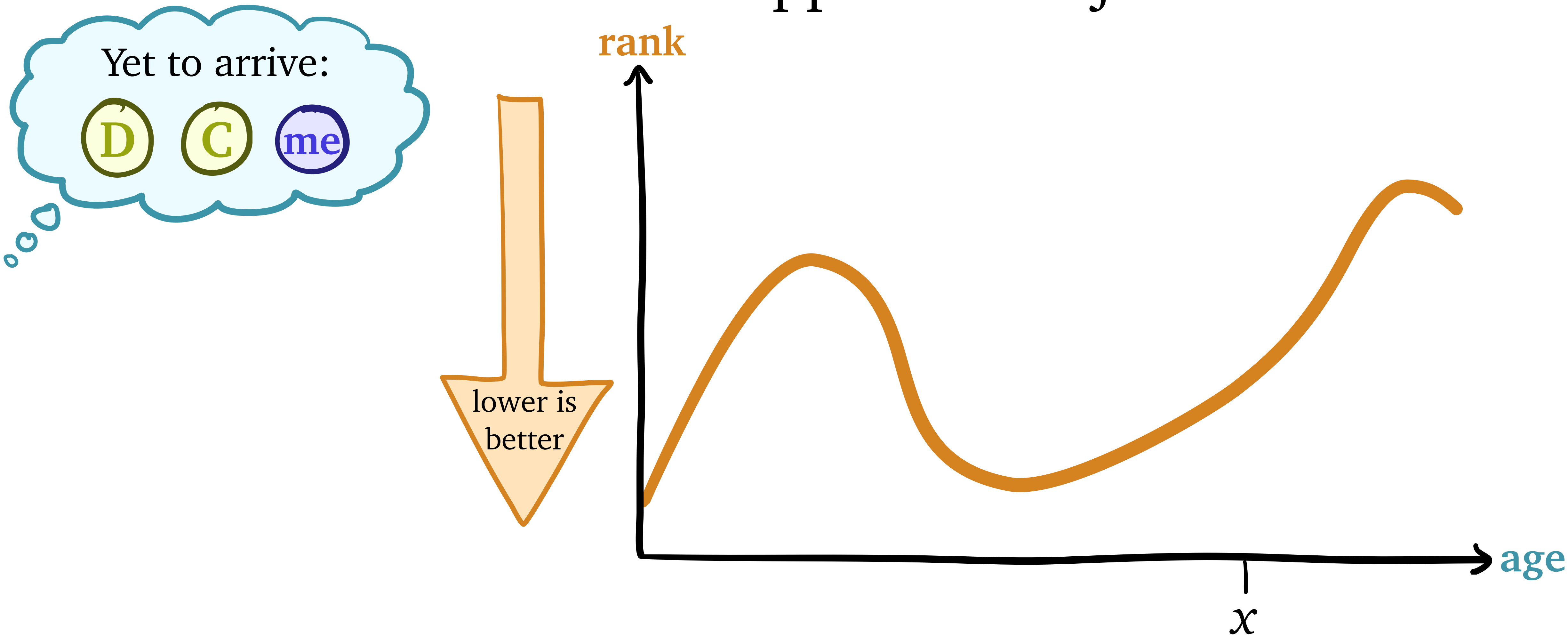
Suppose I'm a job of size x



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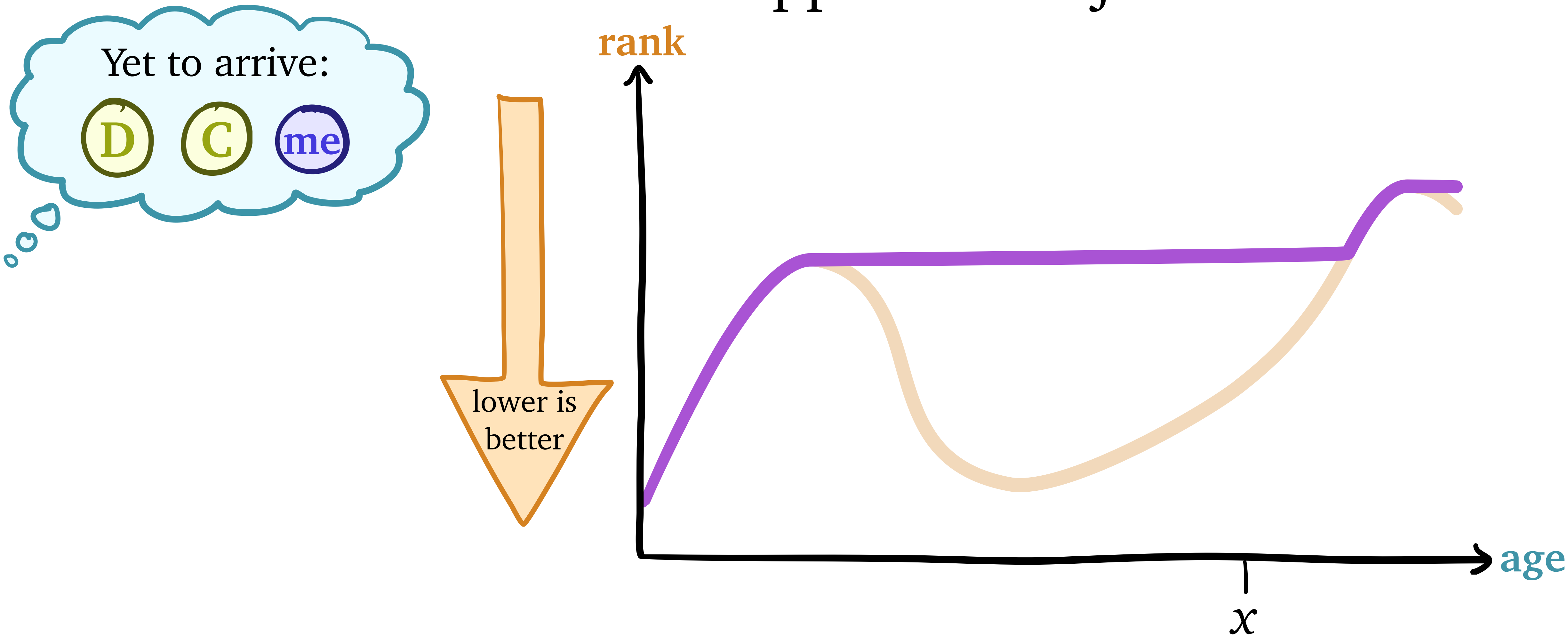

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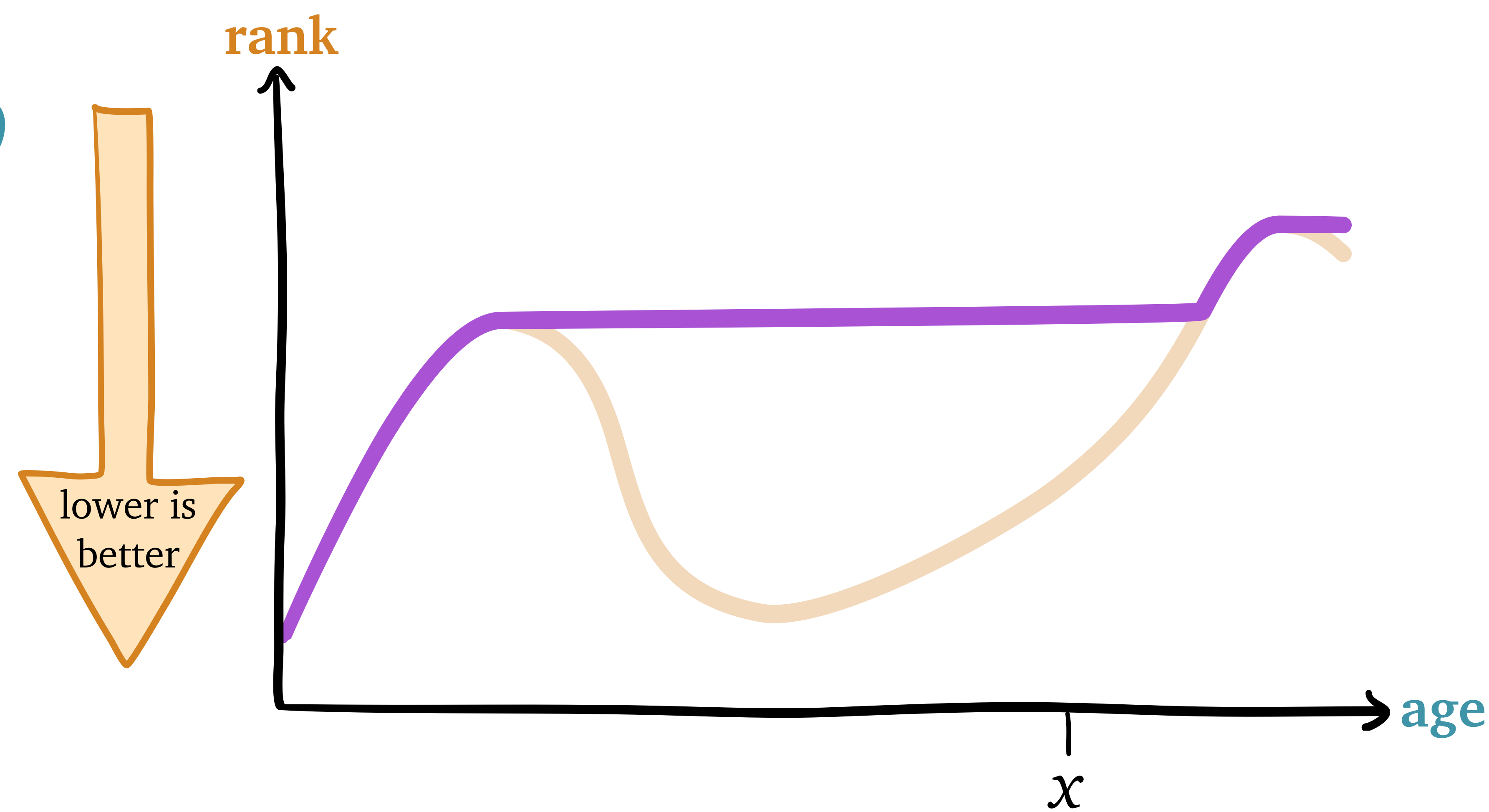

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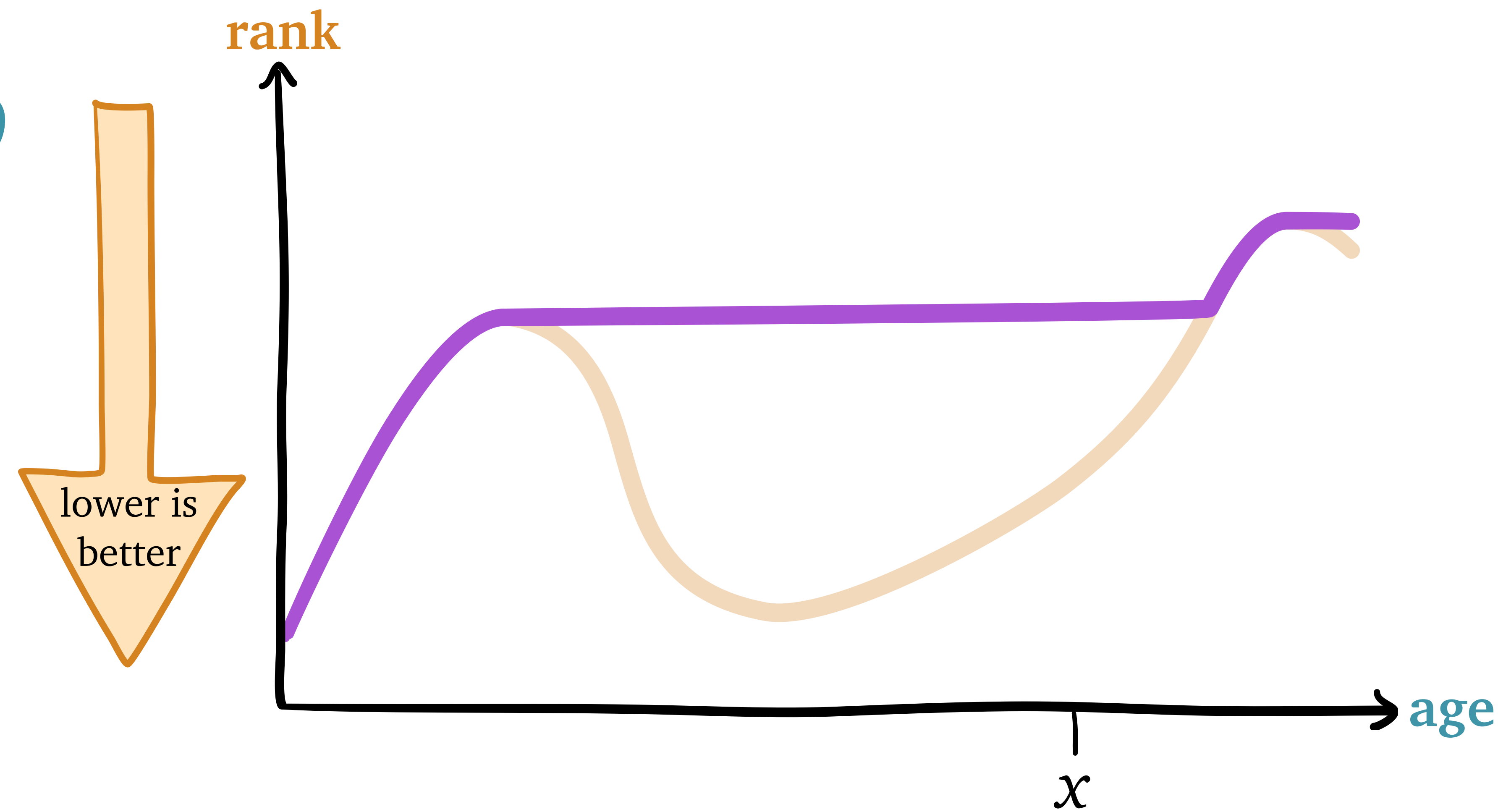

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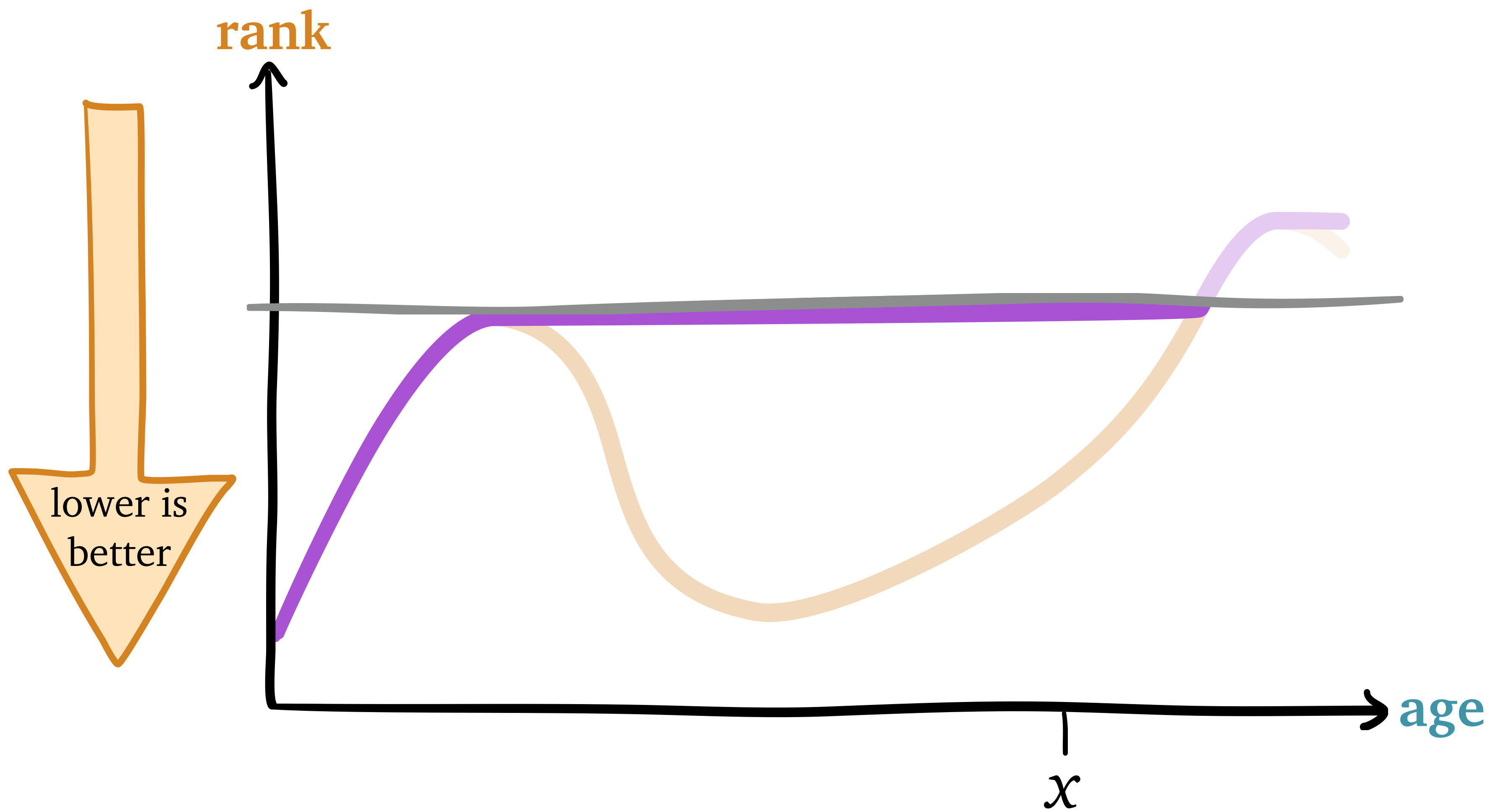

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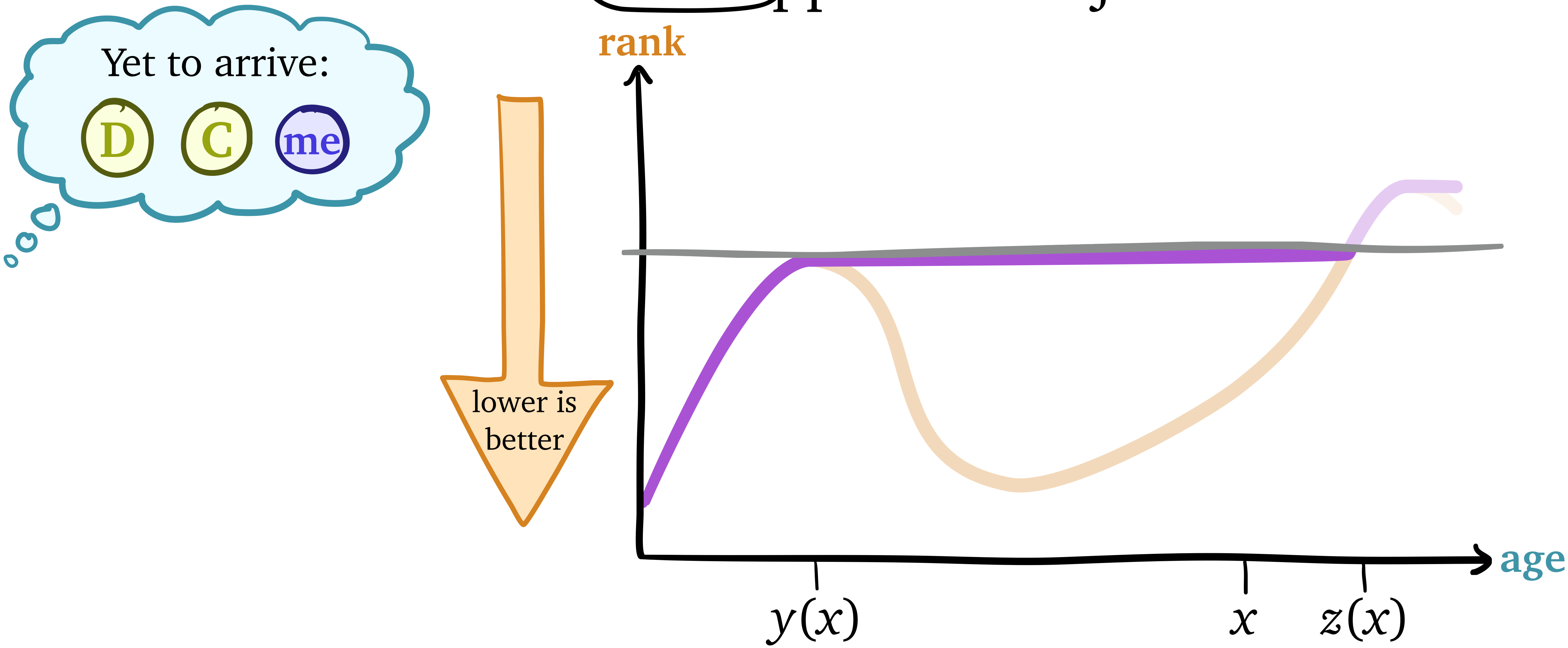

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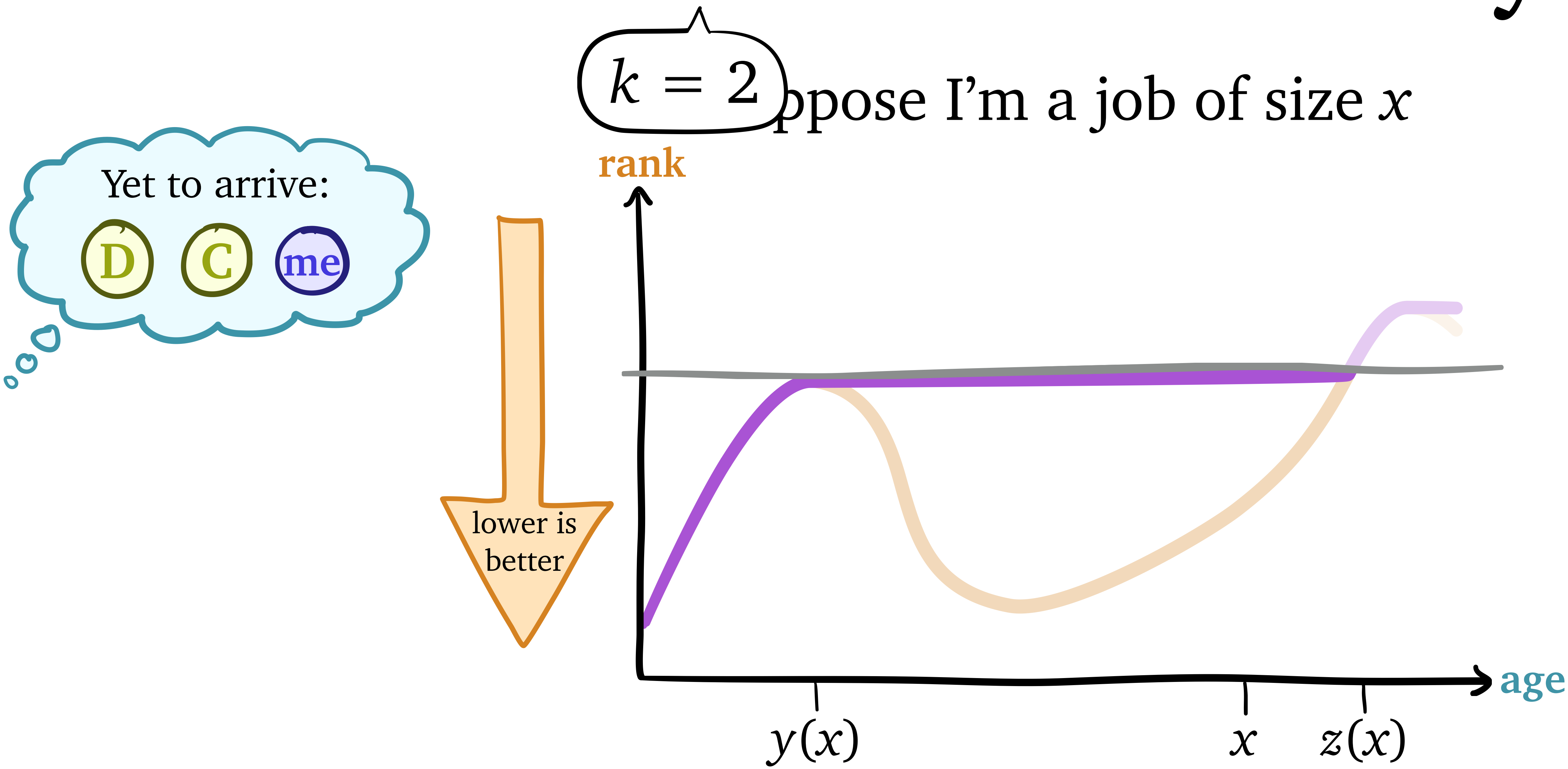

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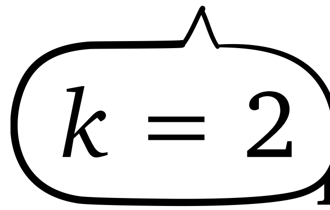

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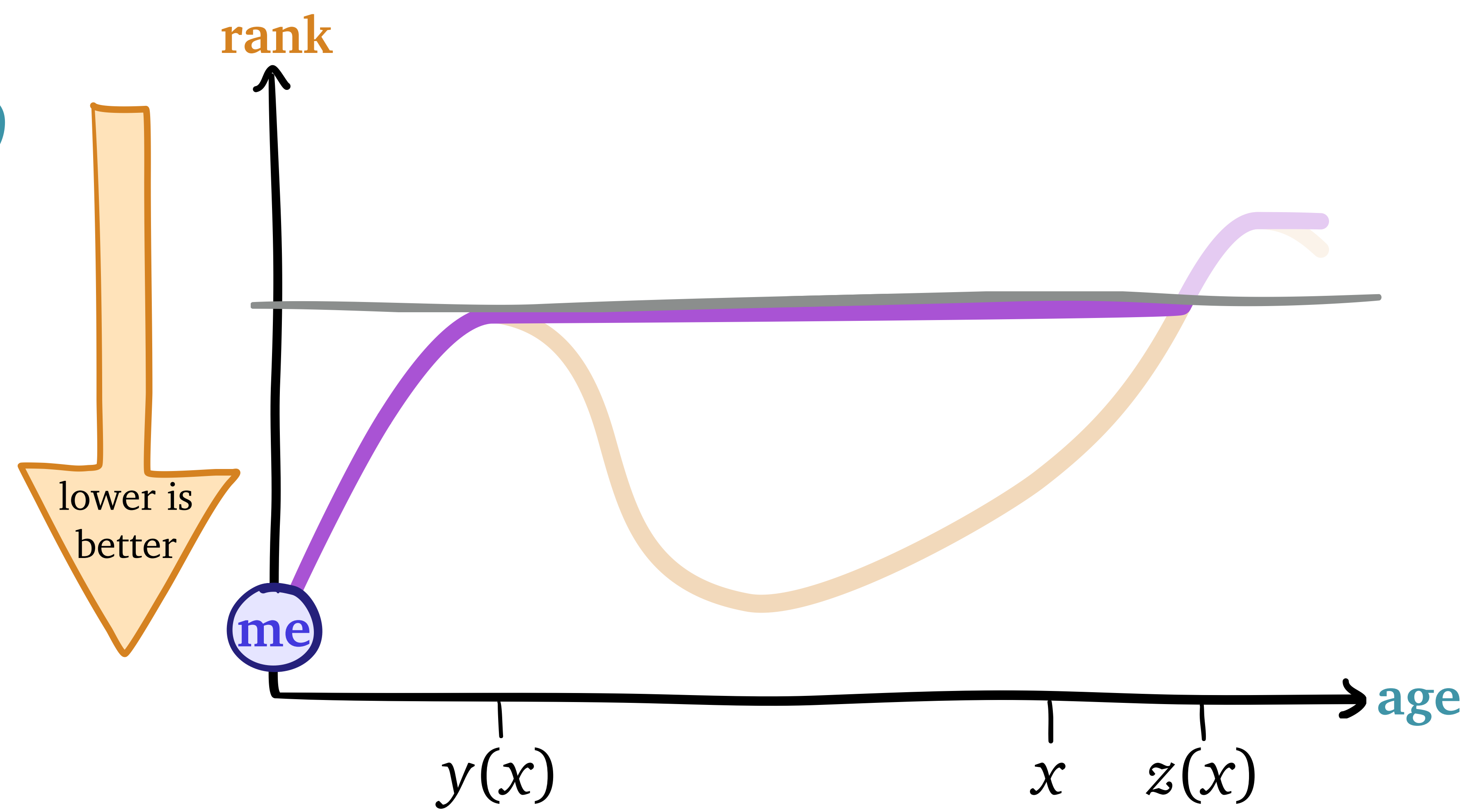

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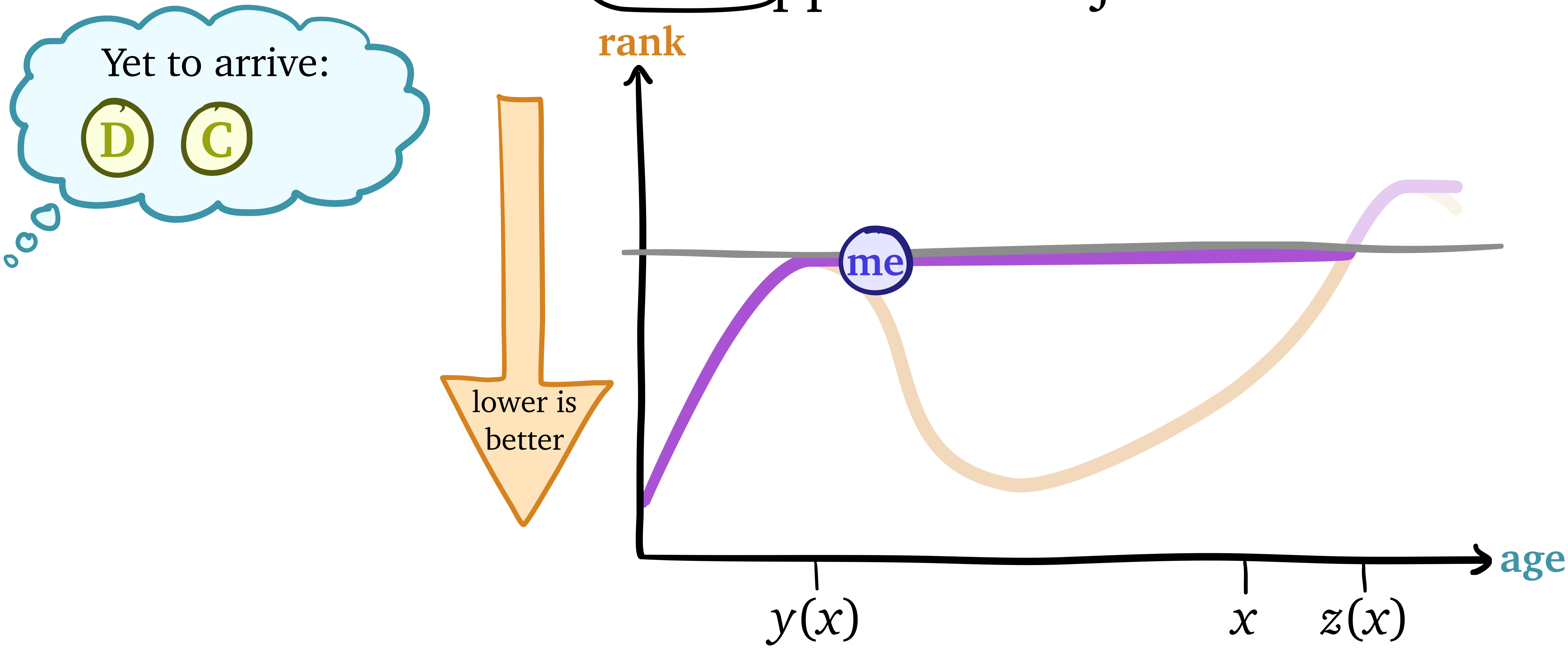

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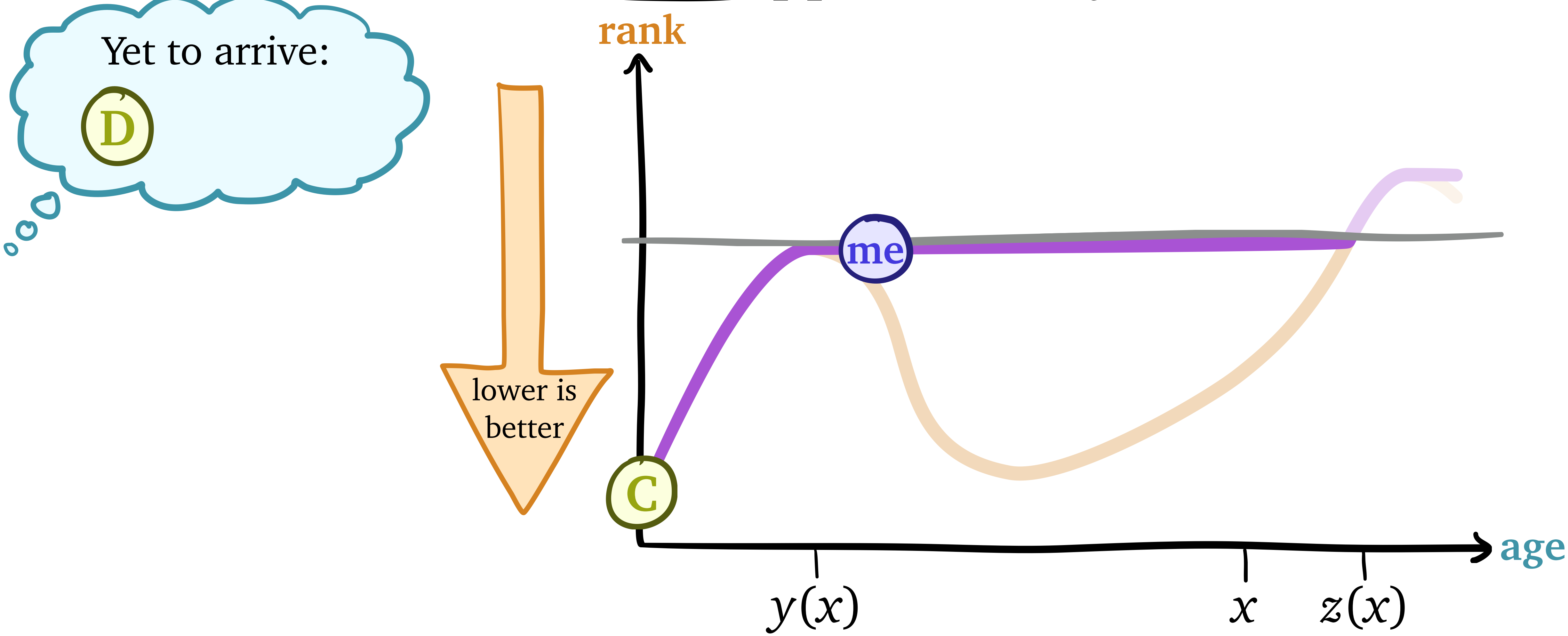

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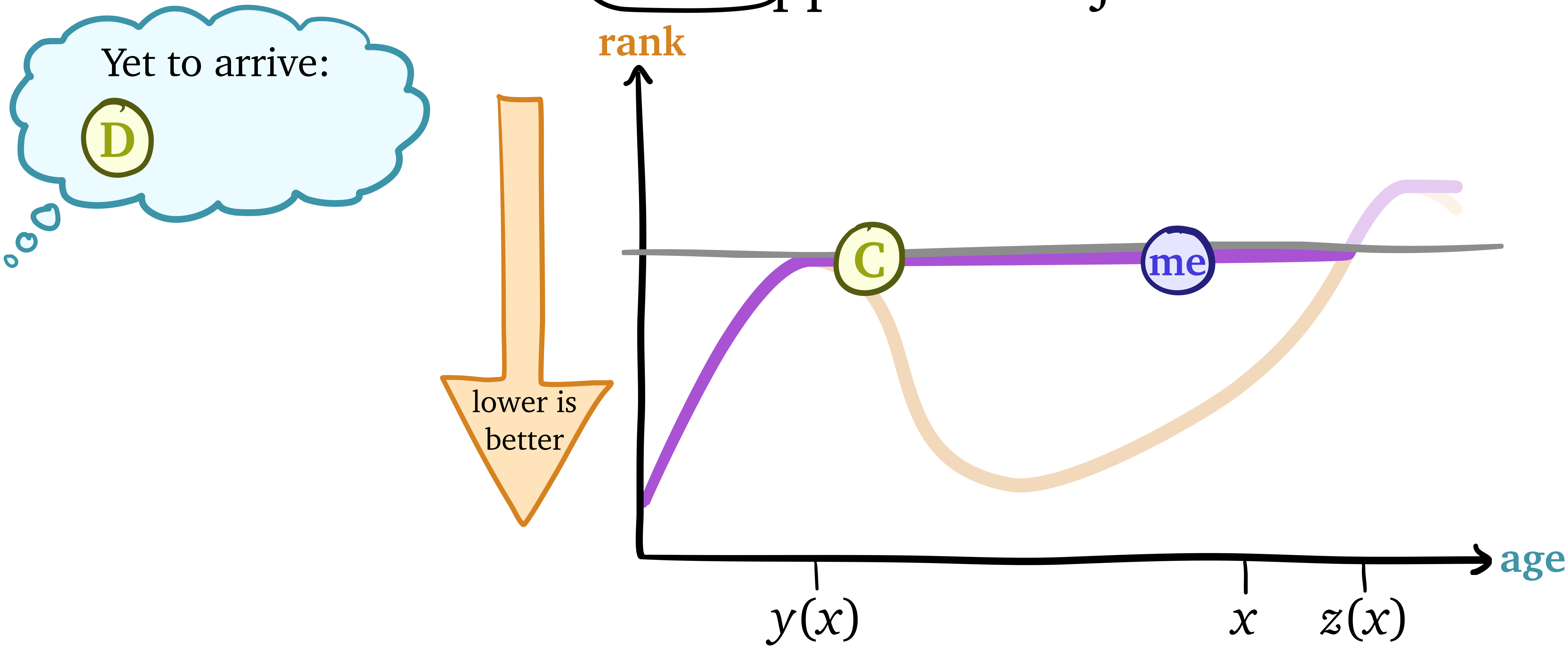

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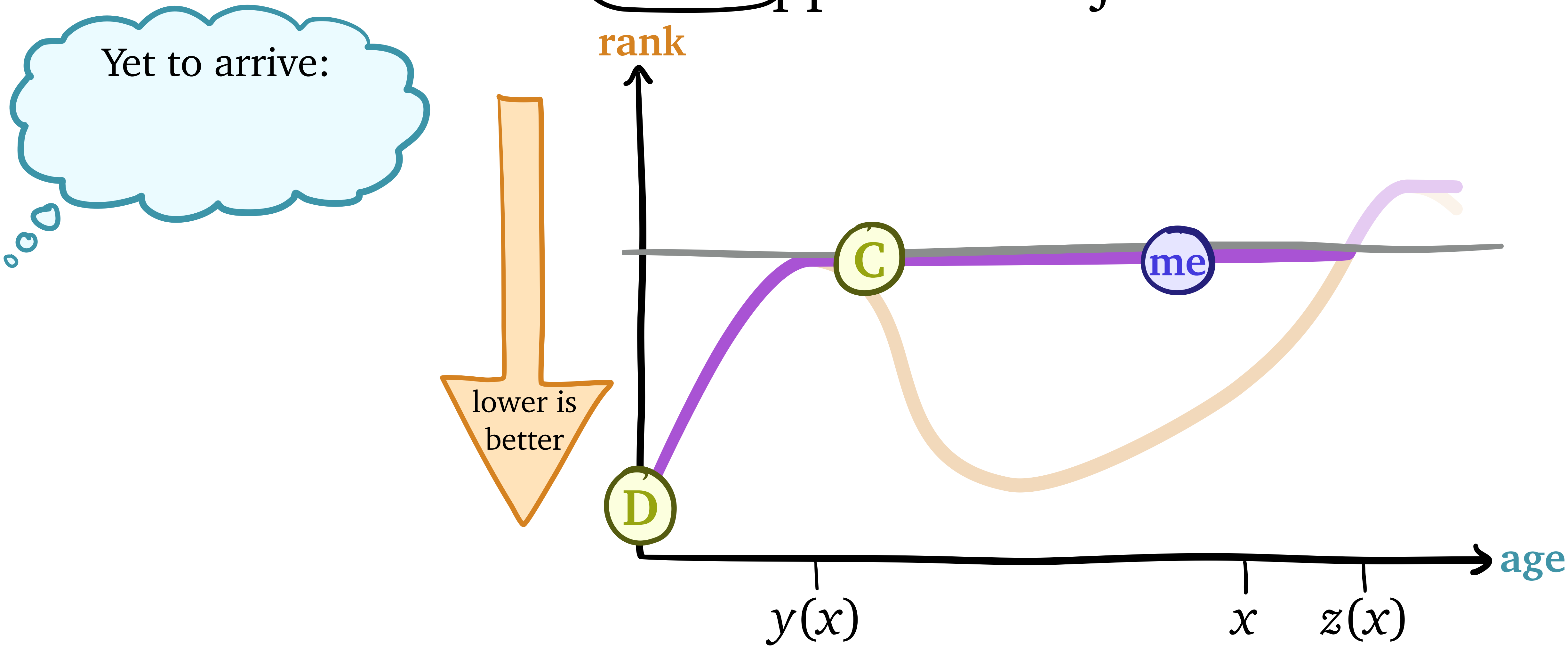

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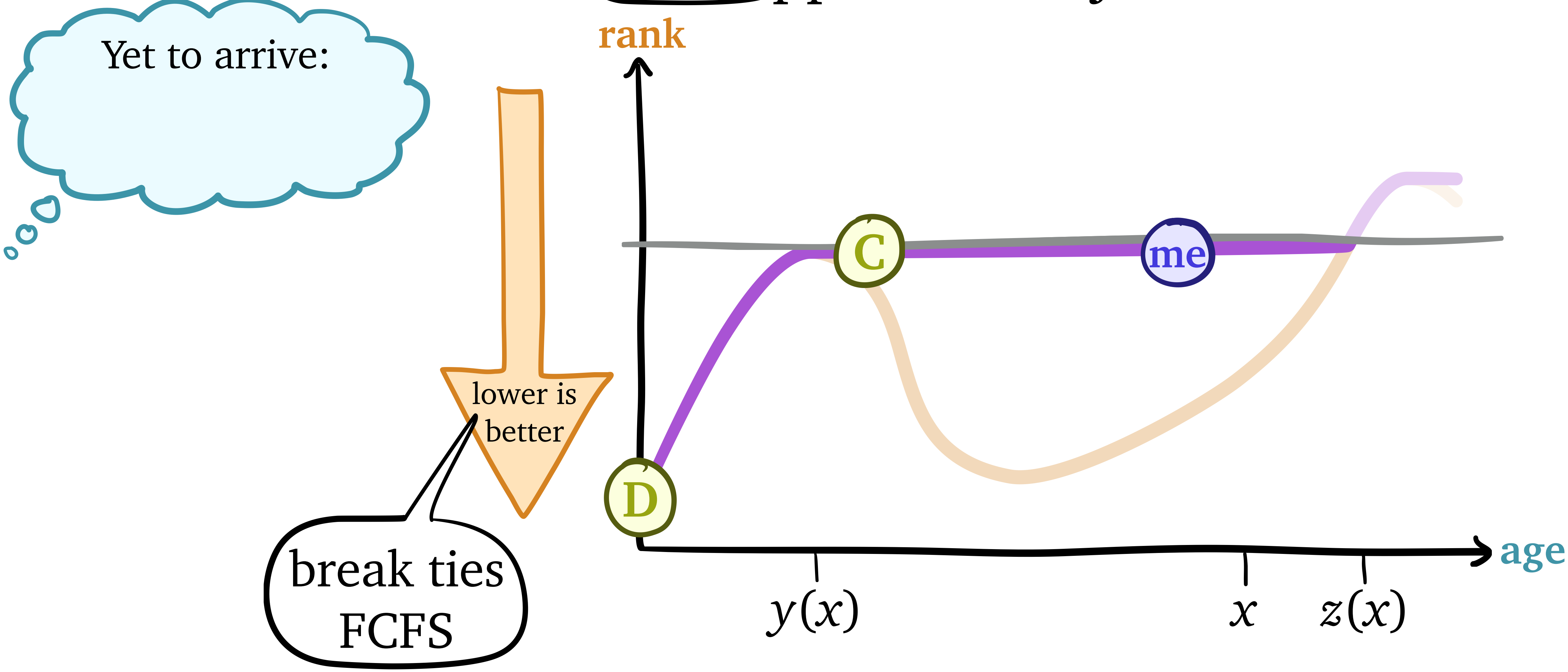

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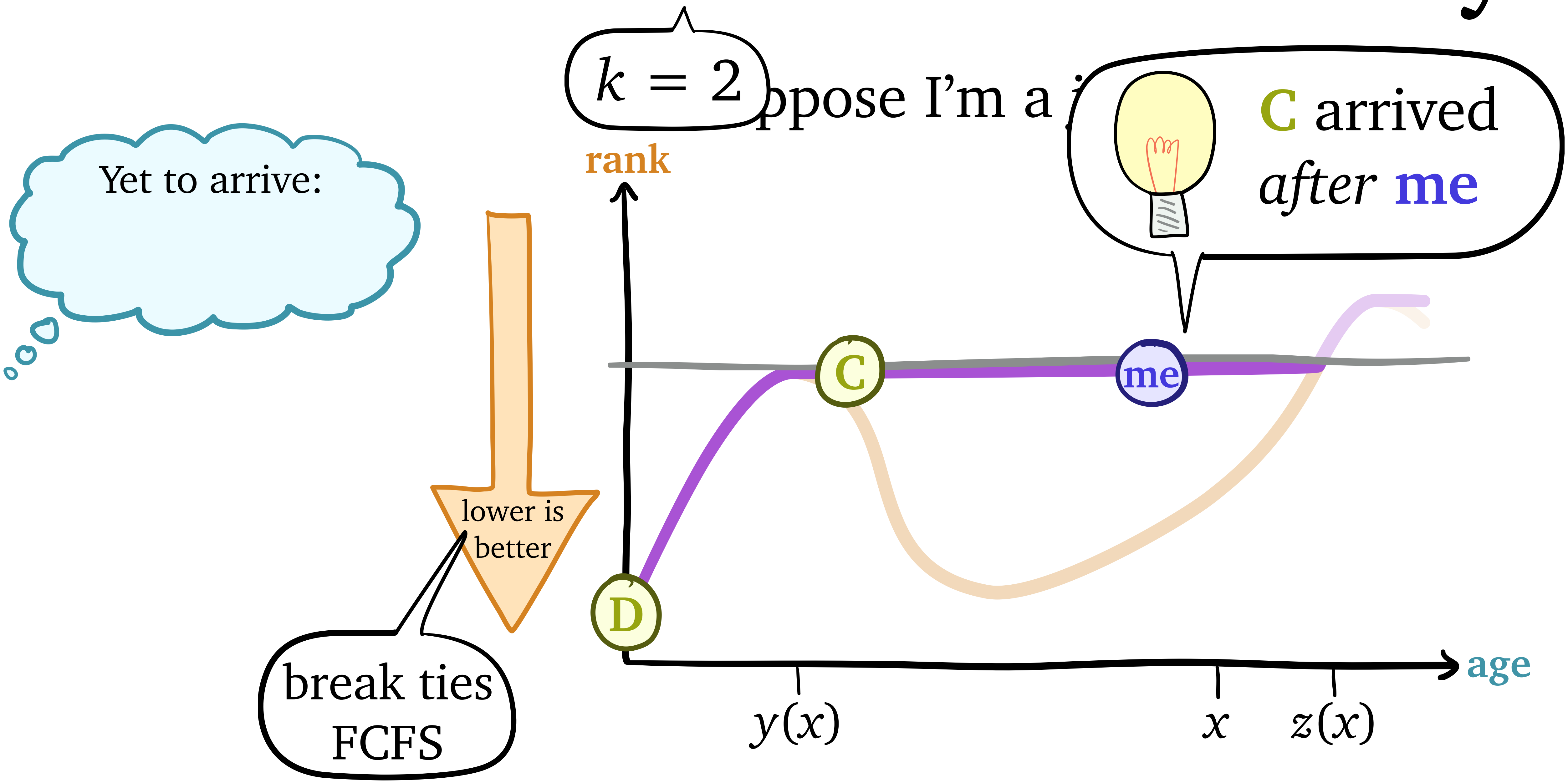

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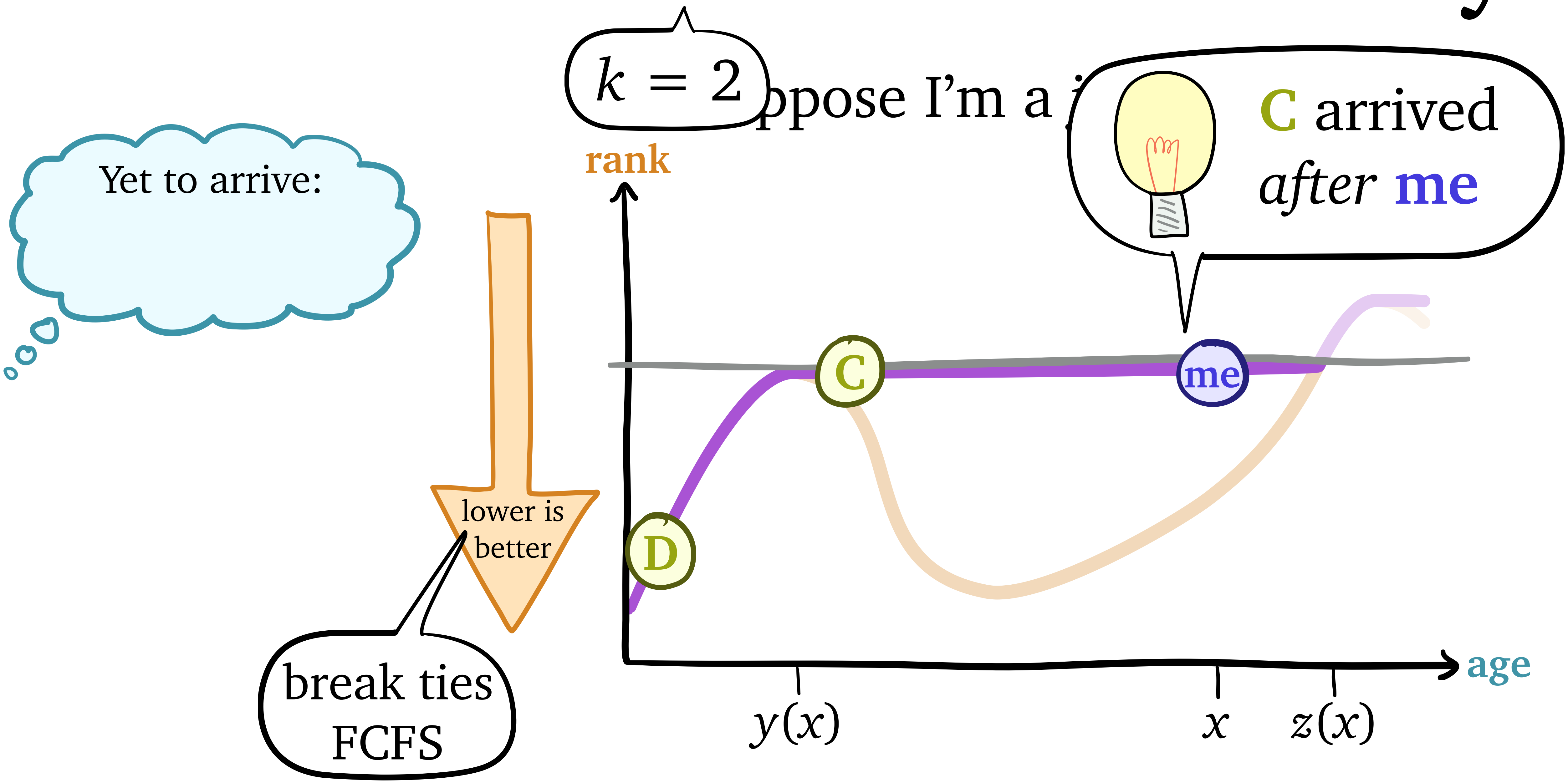

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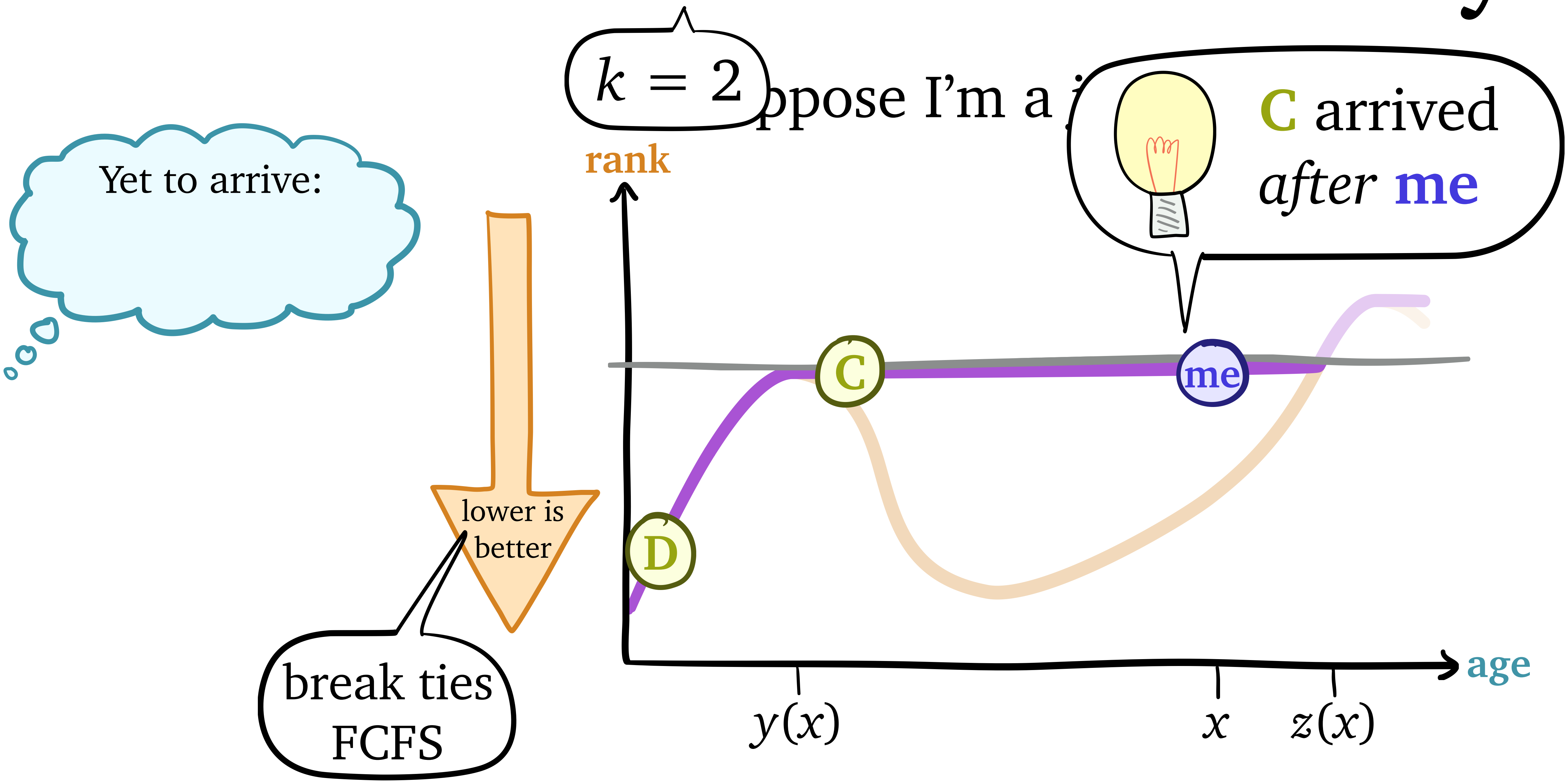

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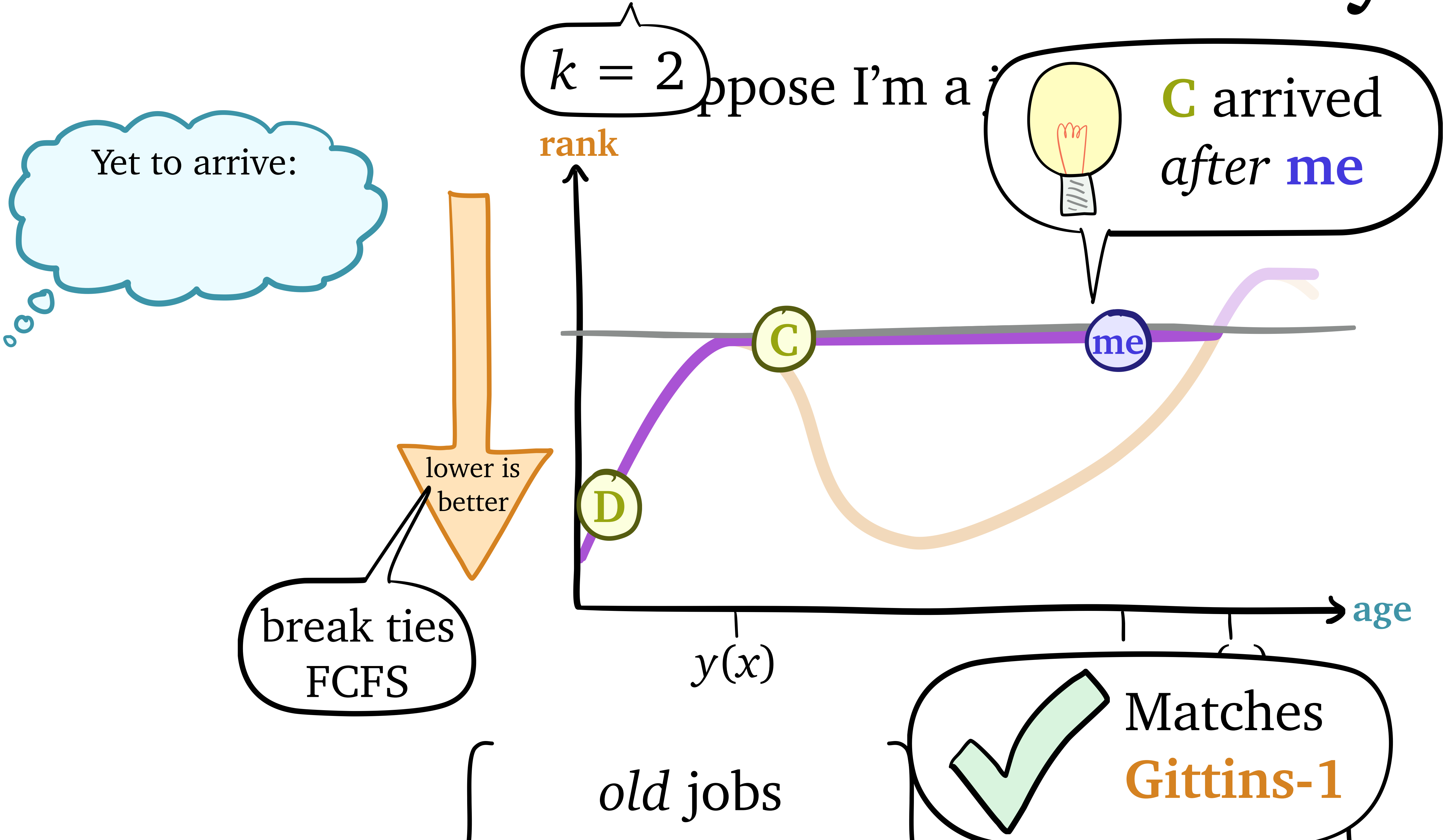

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Main Results

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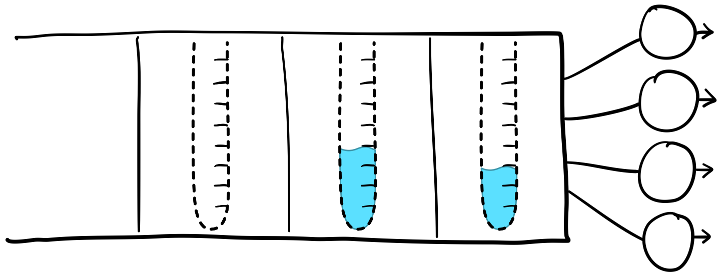
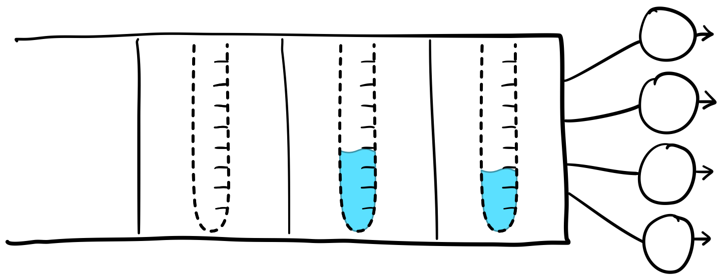


Goal: minimize $\mathbf{E}[T]$ in M/G/k with unknown job sizes



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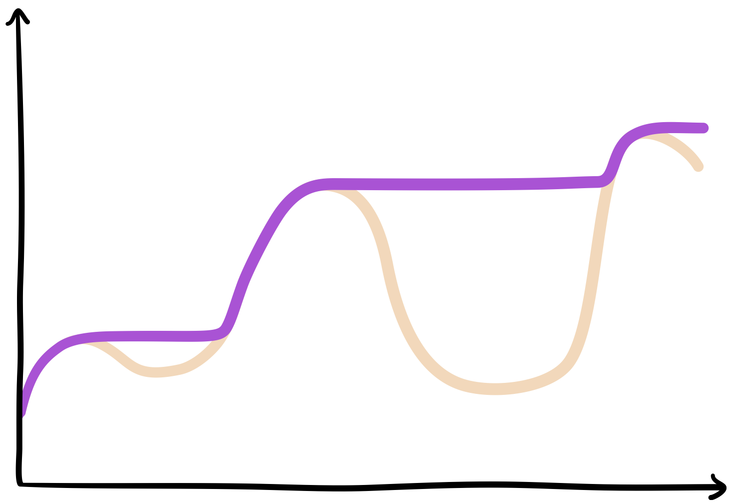

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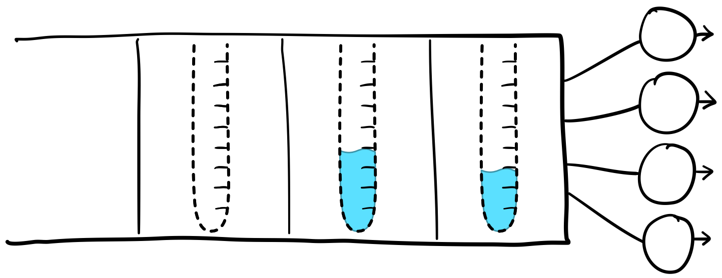

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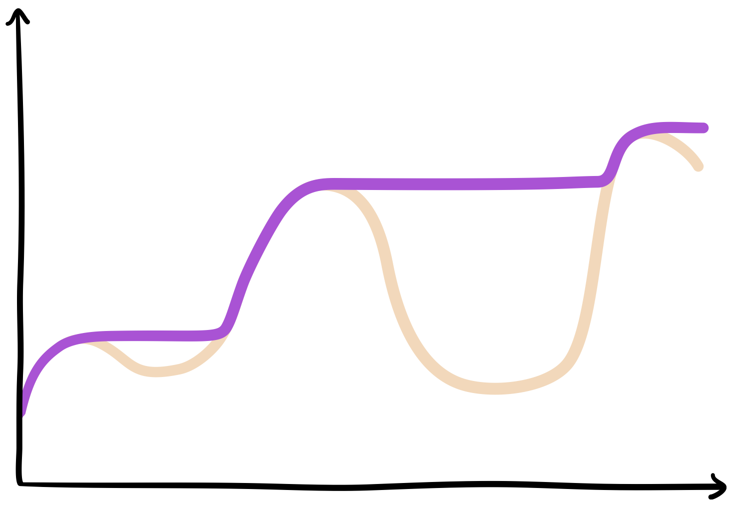

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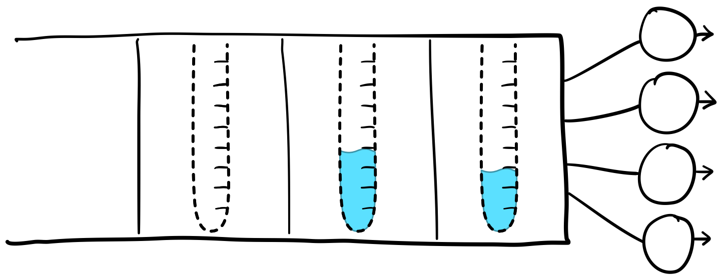

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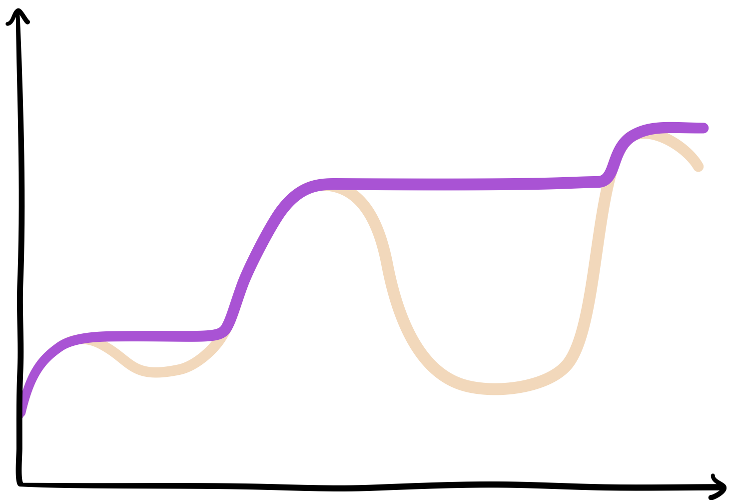

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Get in touch: `zscully@cs.cmu.edu`

Bonus Slides

Heavy-Traffic Optimality

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exponential, log-normal, Weibull…

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and if $X \in \mathrm{OR}(-\infty,-2) \cup \mathrm{MDA}(\Lambda) \cup \mathrm{ENBUE}$, then

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}