Optimally Scheduling Jobs with Multiple Tasks

Ziv Scully
Guy Blelloch
Mor Harchol-Balter
Alan Scheller-Wolf

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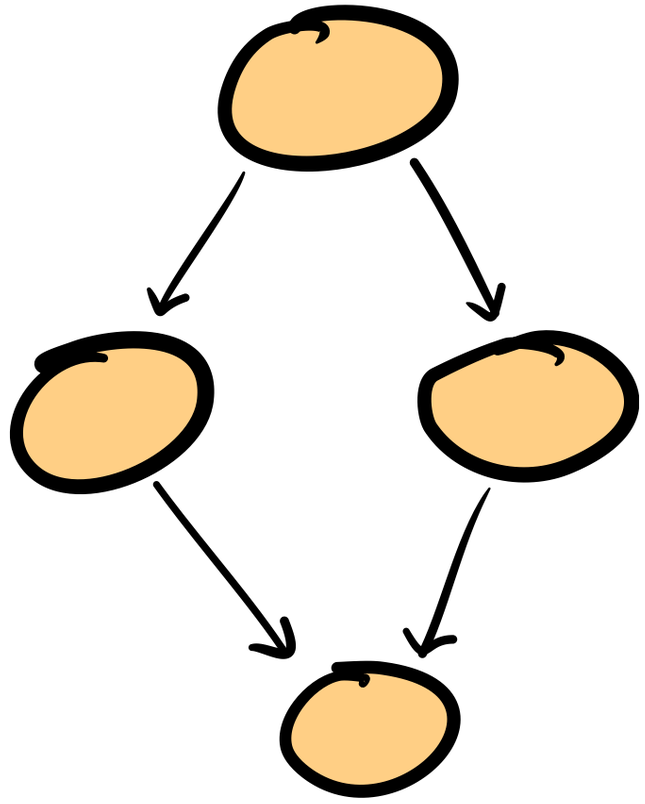
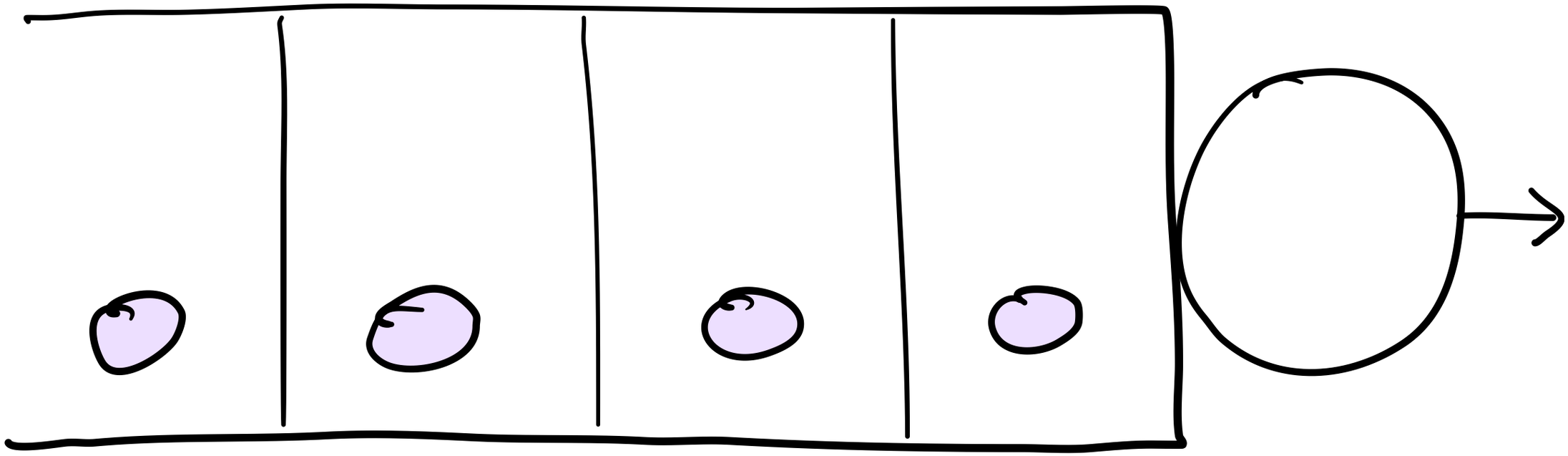


Classic: Multiclass M/G/1



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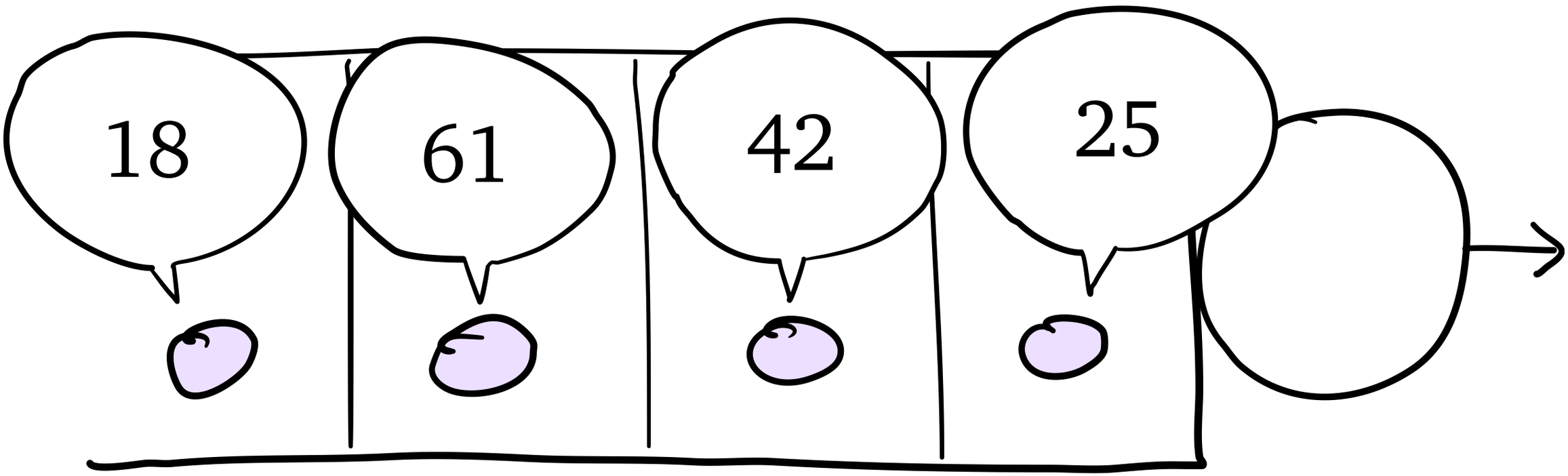

Known sizes:

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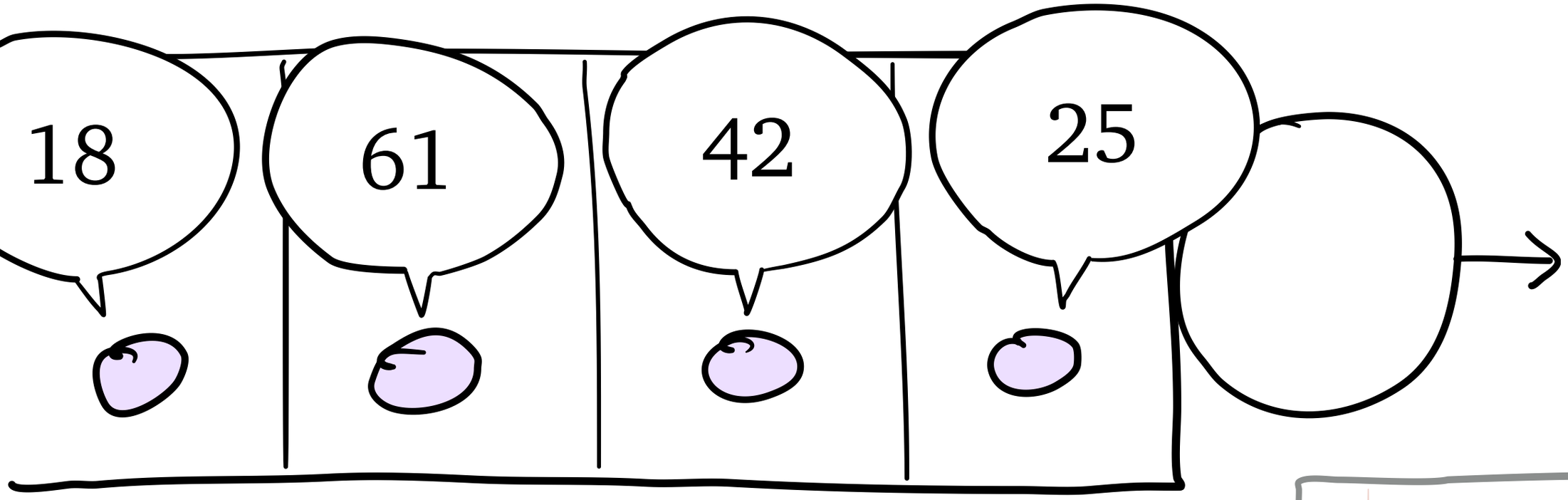
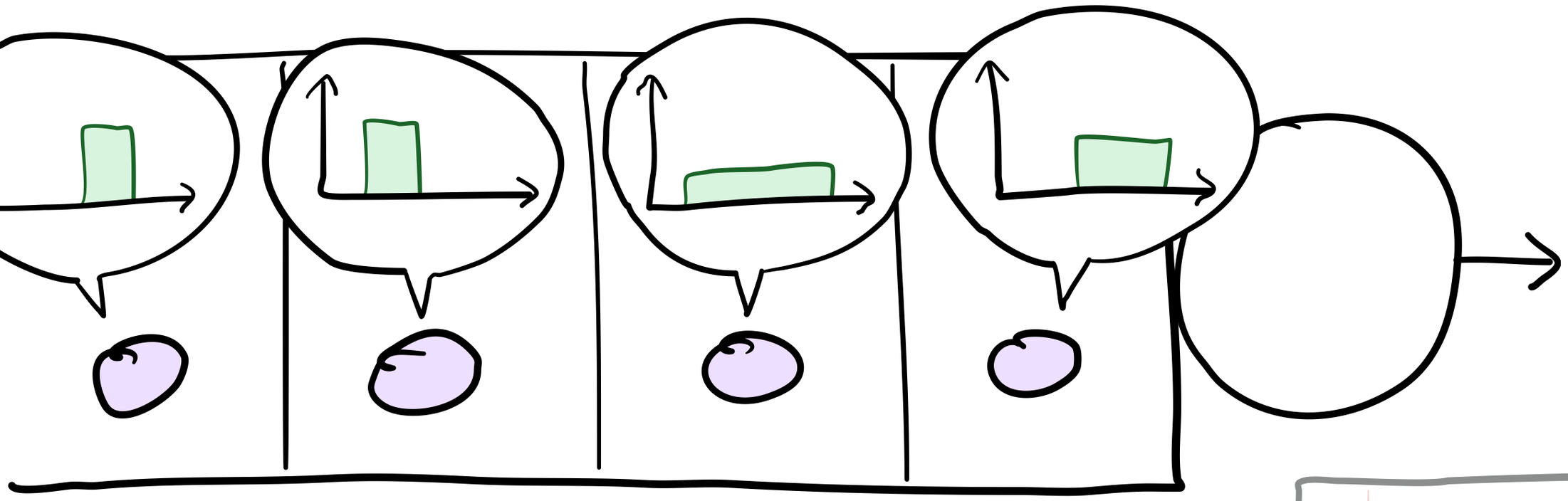


Known sizes: **SRPT**

Shortest
Remaining
Processing
Time

Classic: Multiclass M/G/1



Known sizes: **SRPT**

Uniform distributions:

Shortest
Remaining
Processing
Time

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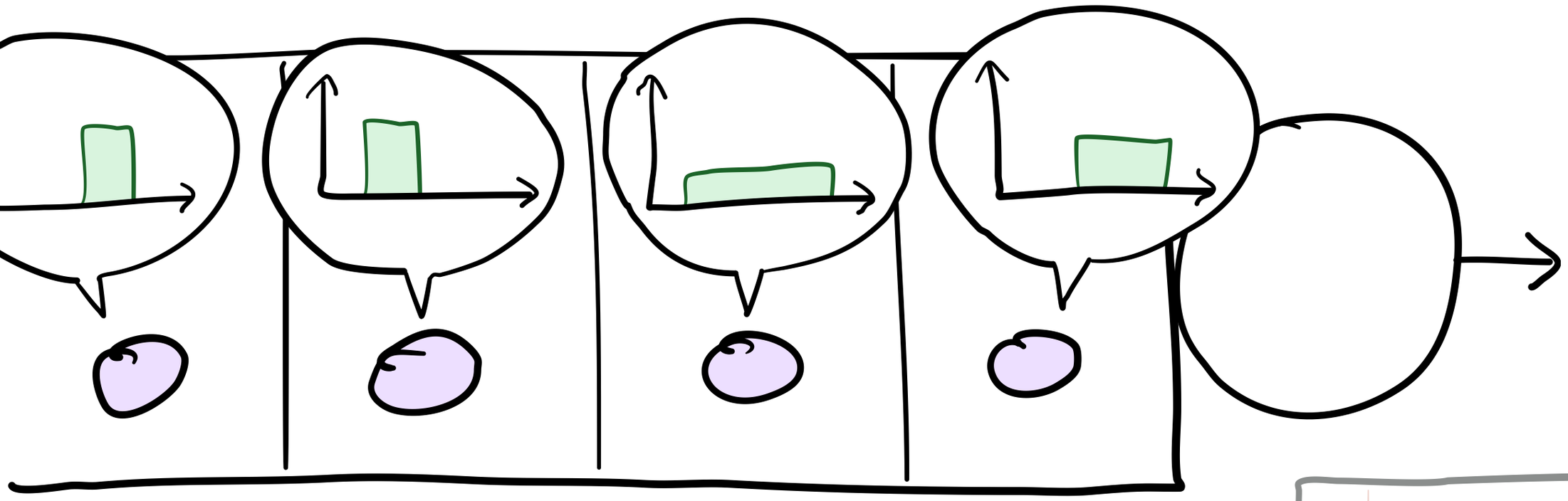
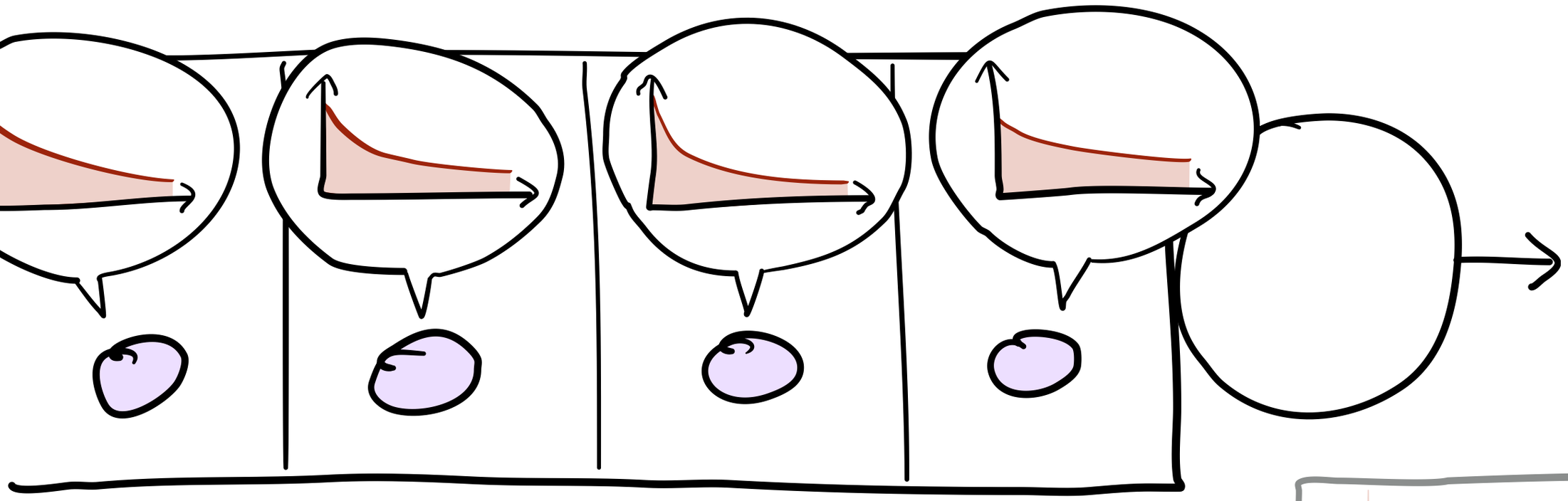


Known sizes: **SRPT**

Uniform distributions: **SERPT**

Shortest
Expected
Remaining
Processing
Time

Classic: Multiclass M/G/1



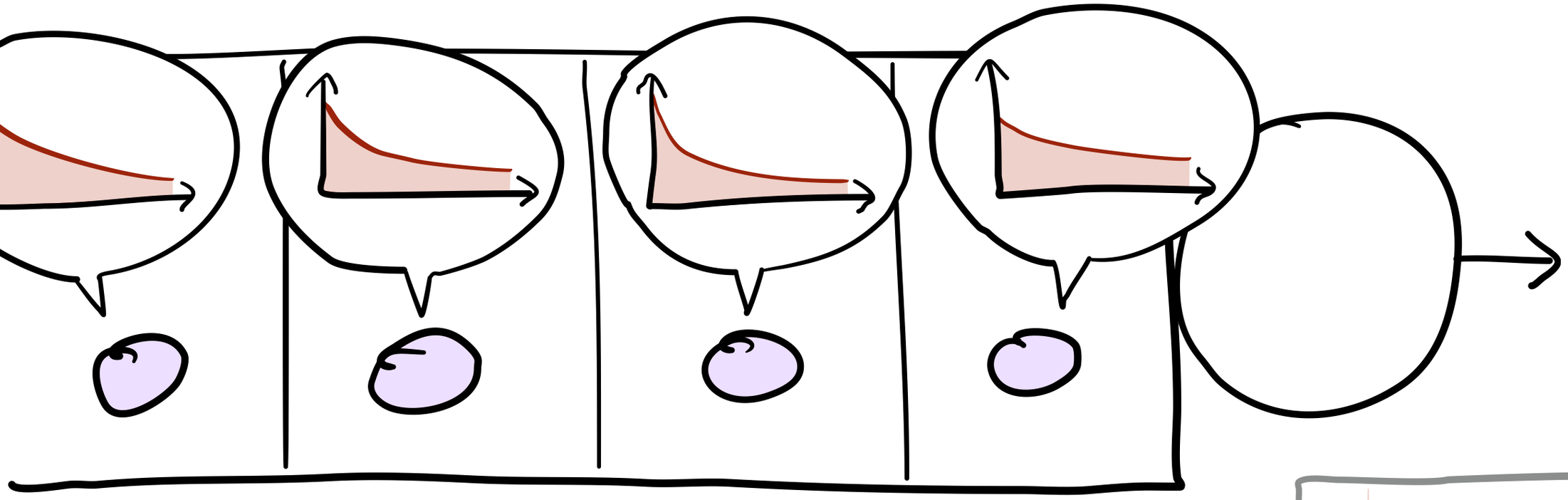
Known sizes: **SRPT**

Uniform distributions: **SERPT**

Pareto distributions:

Shortest
Expected
Remaining
Processing
Time

Classic: Multiclass M/G/1



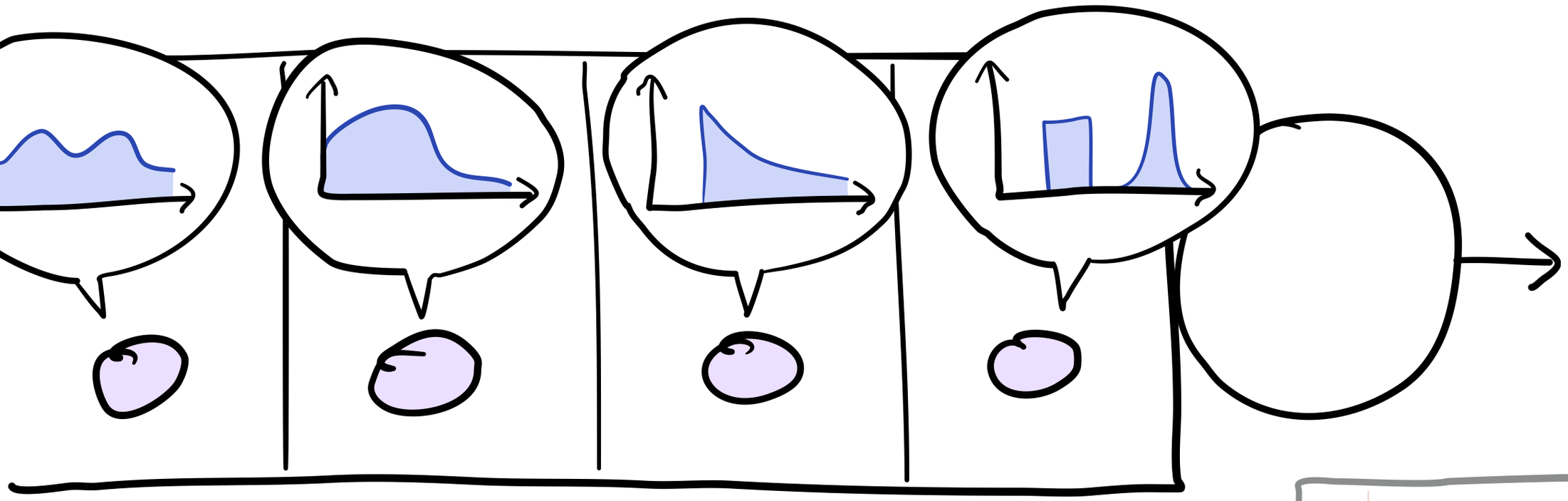
Known sizes: **SRPT**

Uniform distributions: **SERPT**

Pareto distributions: **HHR** (not SERPT!)

Highest
Hazard
Rate

Classic: Multiclass M/G/1



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Hazard
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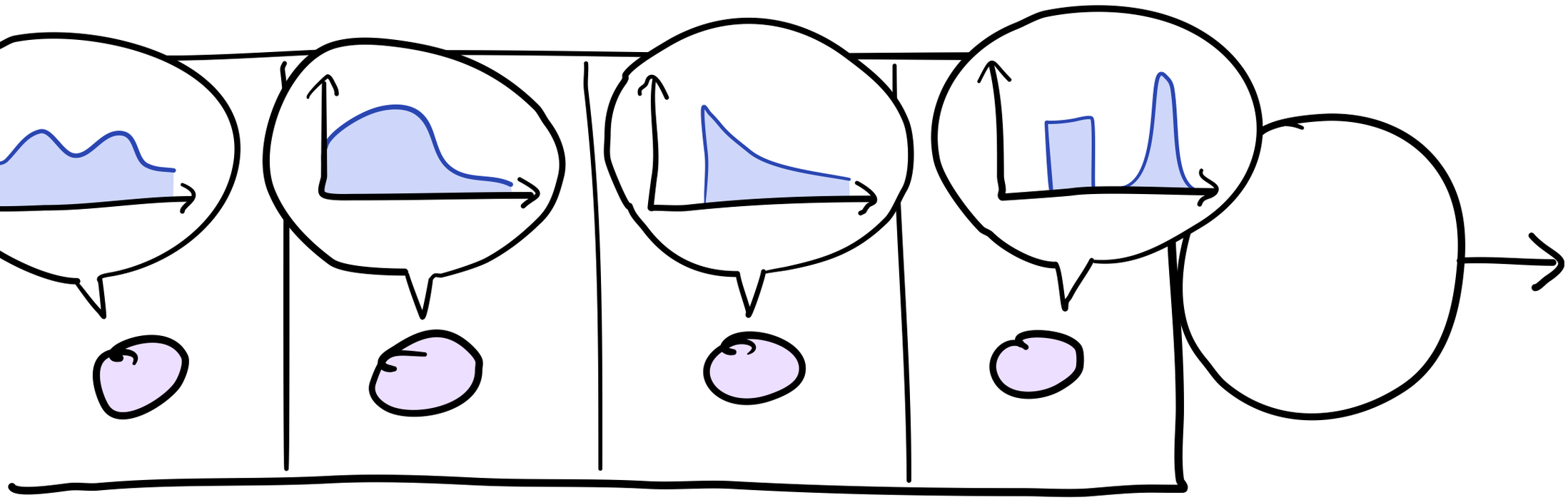


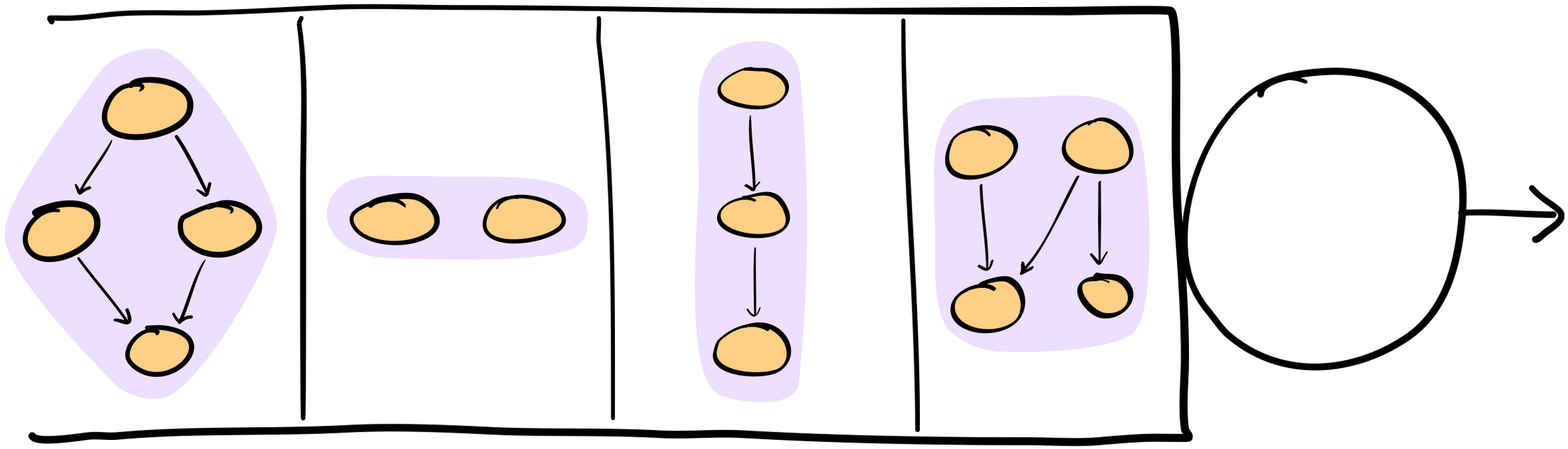
Known sizes: **SRPT**

Uniform distributions: **SERPT**

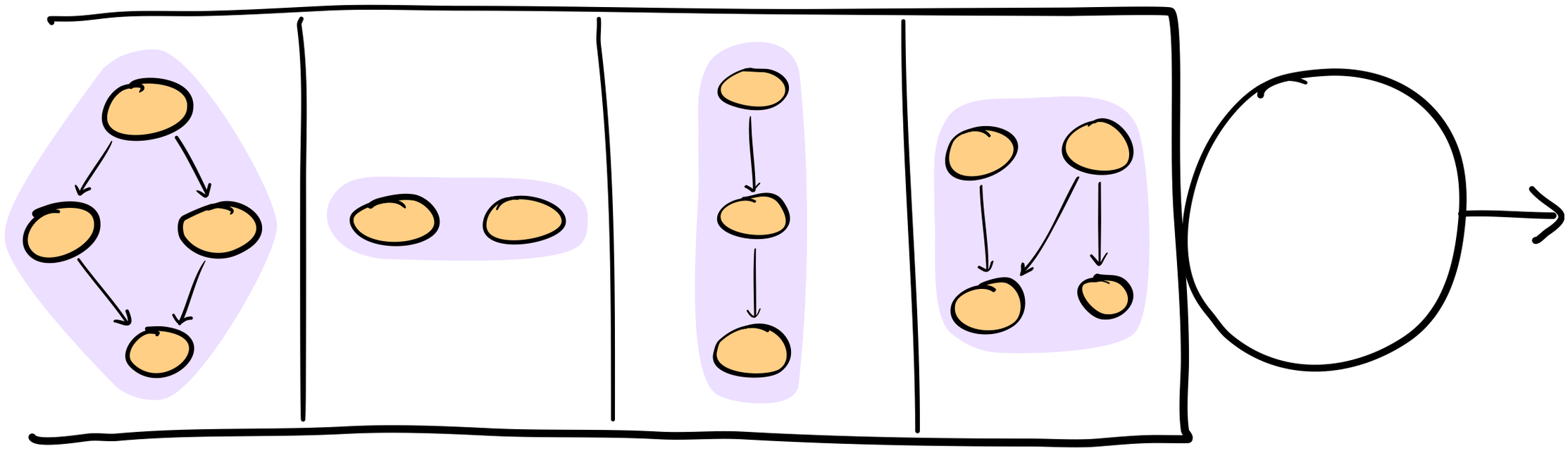
Pareto distributions: **HHR** (not SERPT!)

General distributions: **Gittins index** policy

Today: Multitask Jobs

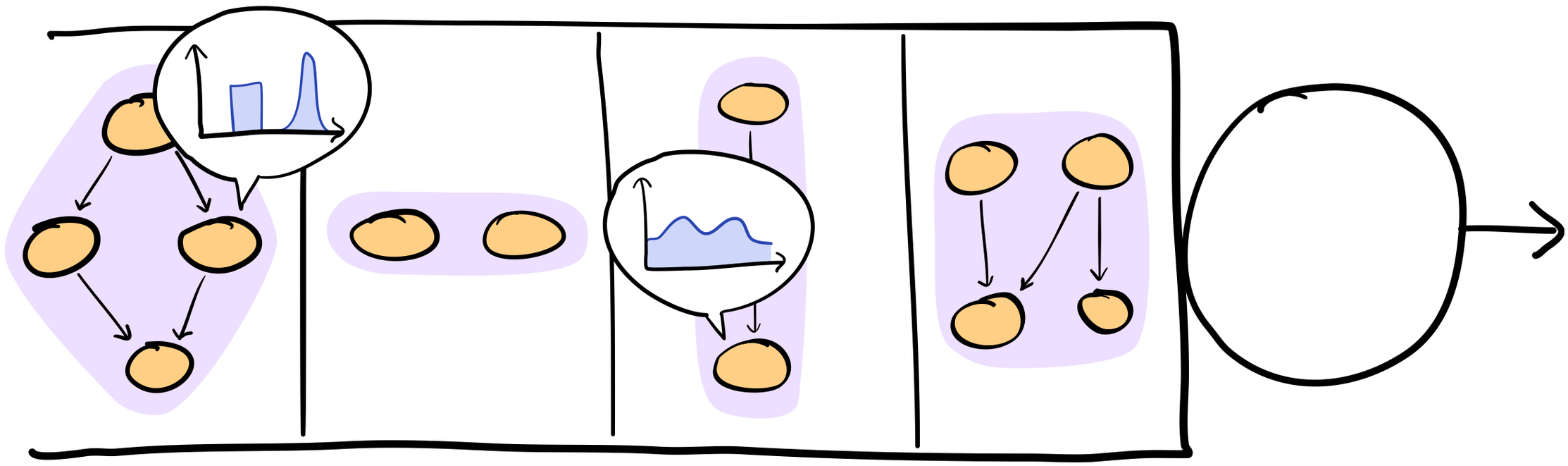


Today: Multitask Jobs



Jobs consist of multiple **tasks** in a DAG

Today: Multitask Jobs



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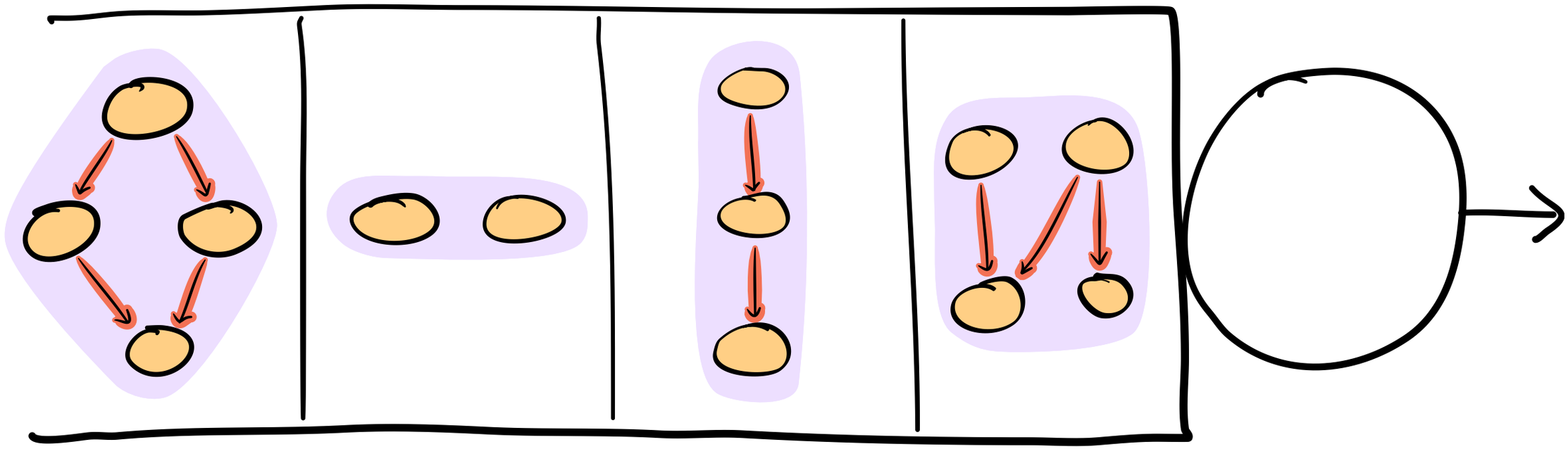

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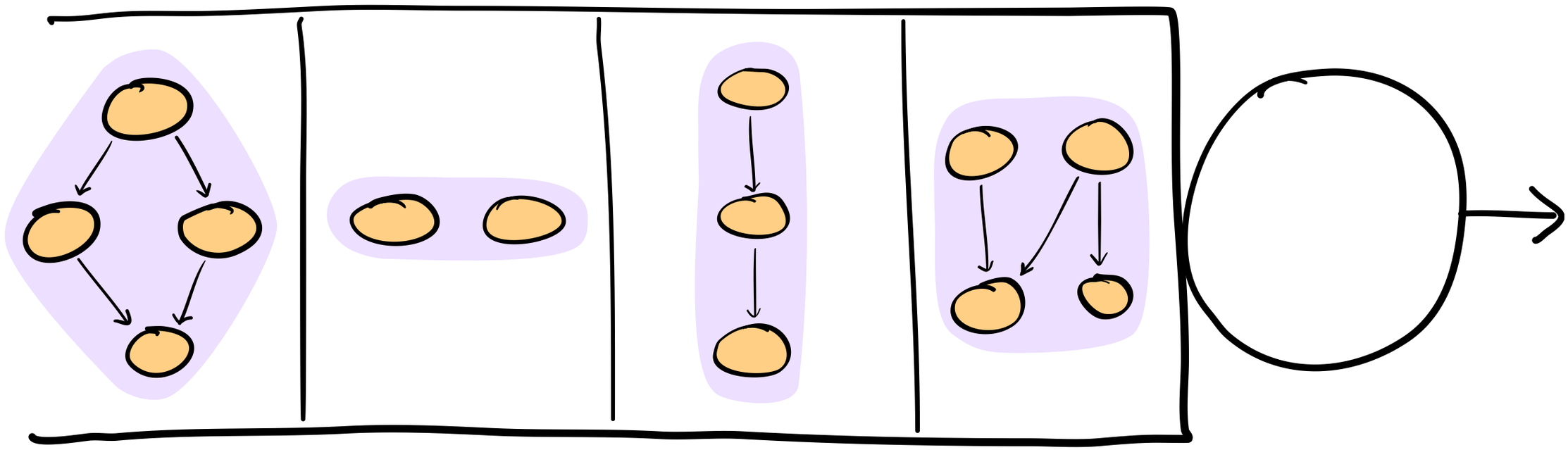
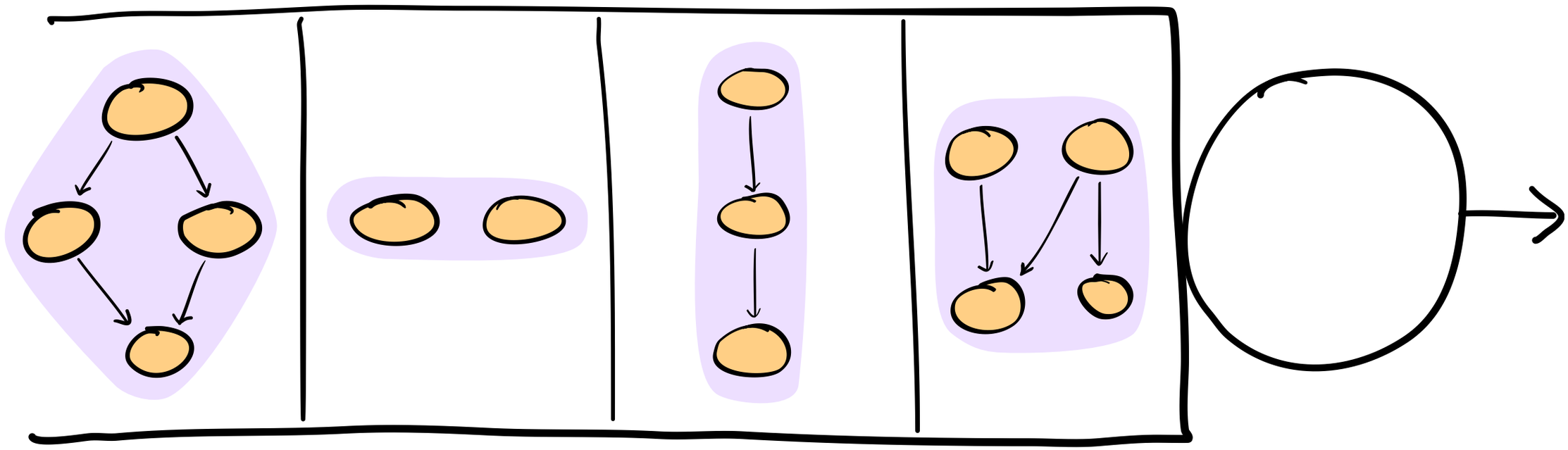


Jobs consist of multiple **tasks** in a DAG

Job not done until *all* tasks done

Today: Multitask Jobs



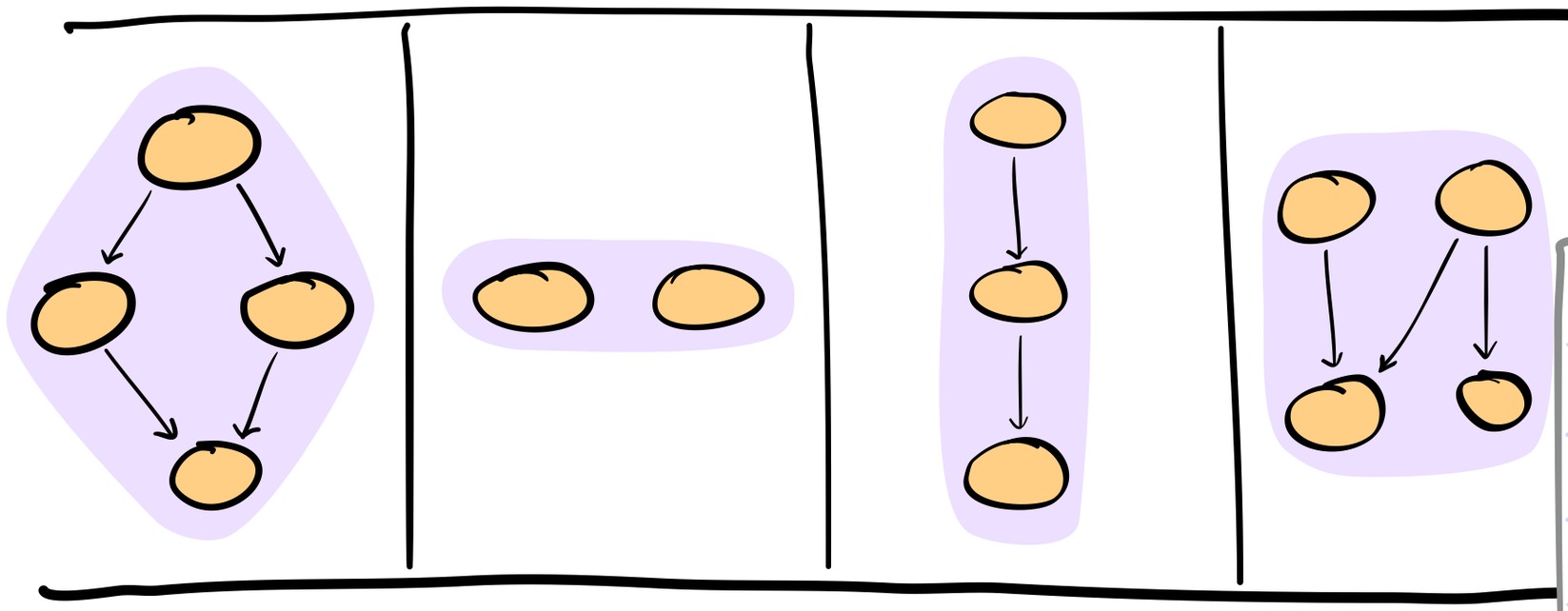
Jobs consist of multiple **tasks** in a DAG

Job not done until *all* tasks done

Goal: minimize mean response time of **jobs**

- Which **job**?
- Which **task**?

Today: Multitask Jobs



Serve tasks, not jobs
Preemption allowed
Single processor
No arrivals

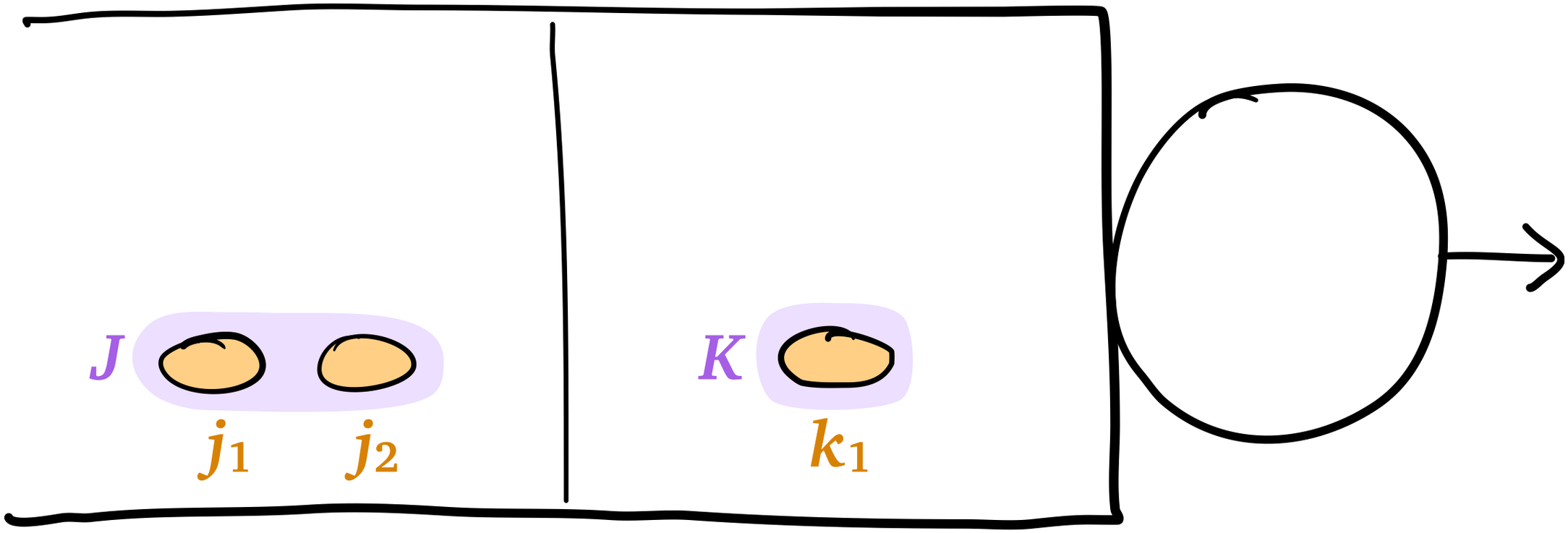
Jobs consist of multiple **tasks** in a DAG

Job not done until *all* tasks done

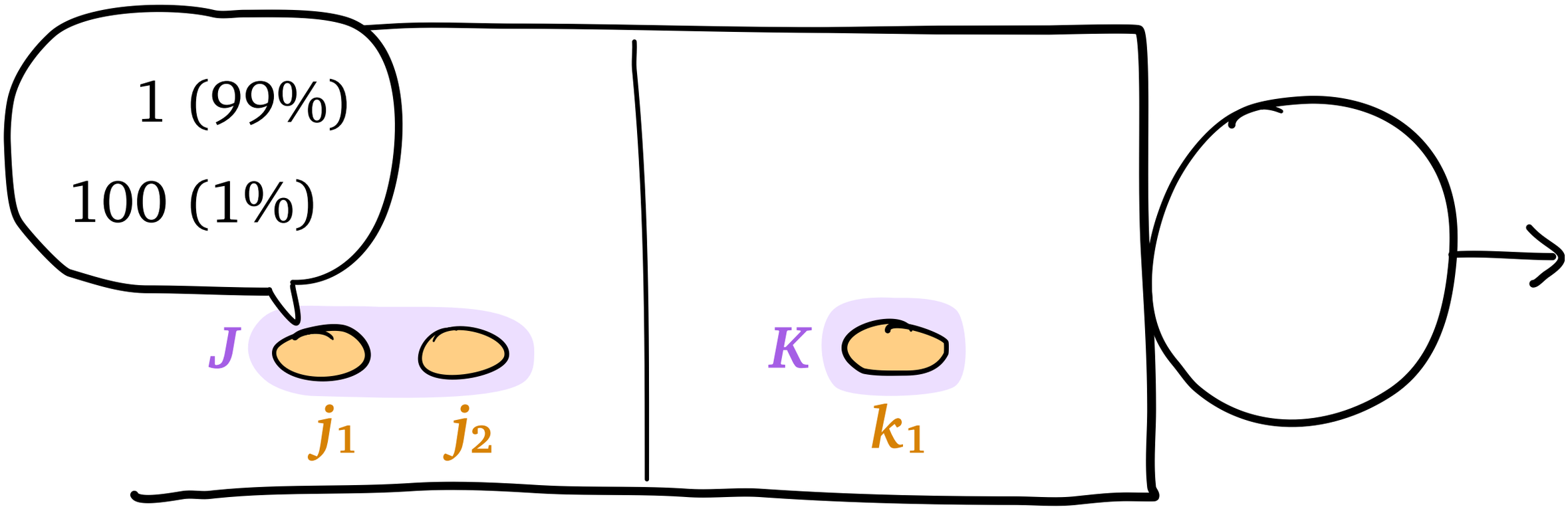
Goal: minimize mean response time of **jobs**

- Which **job**?
- Which **task**?

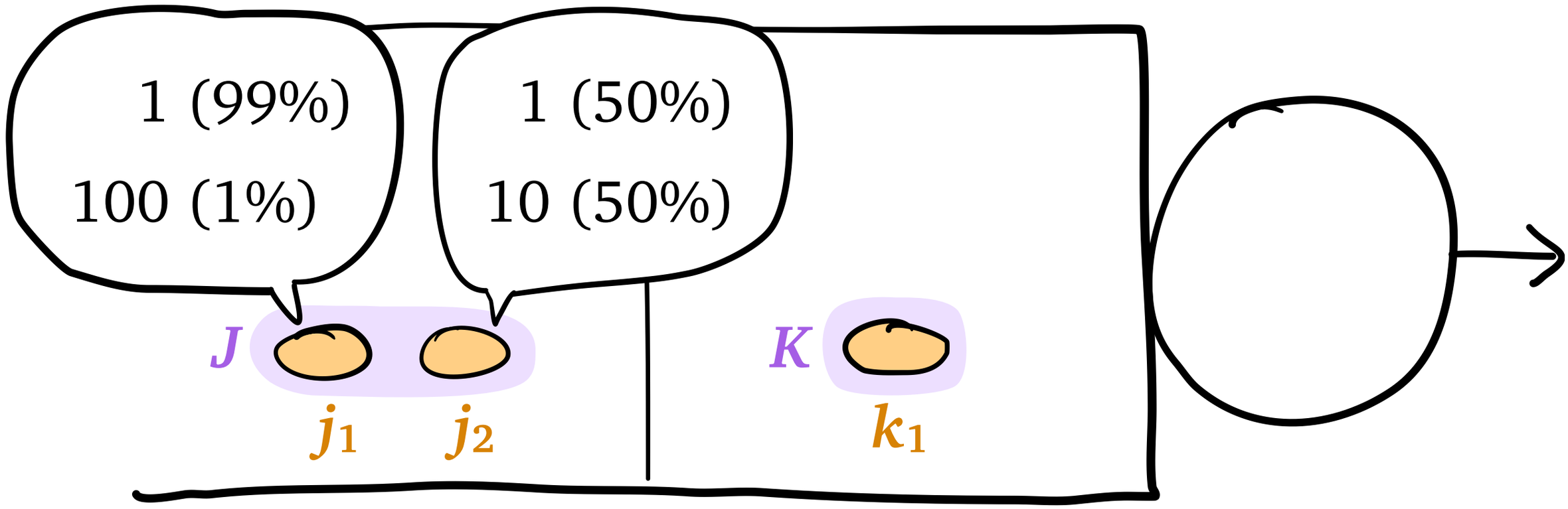
Example



Example



Example



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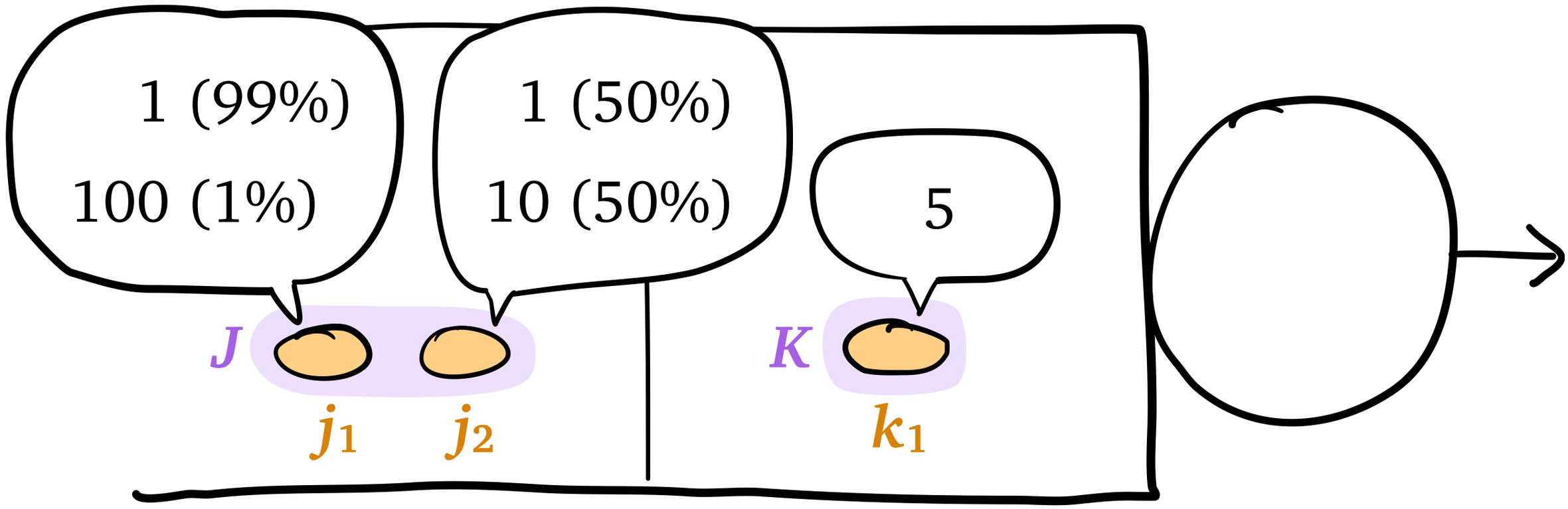
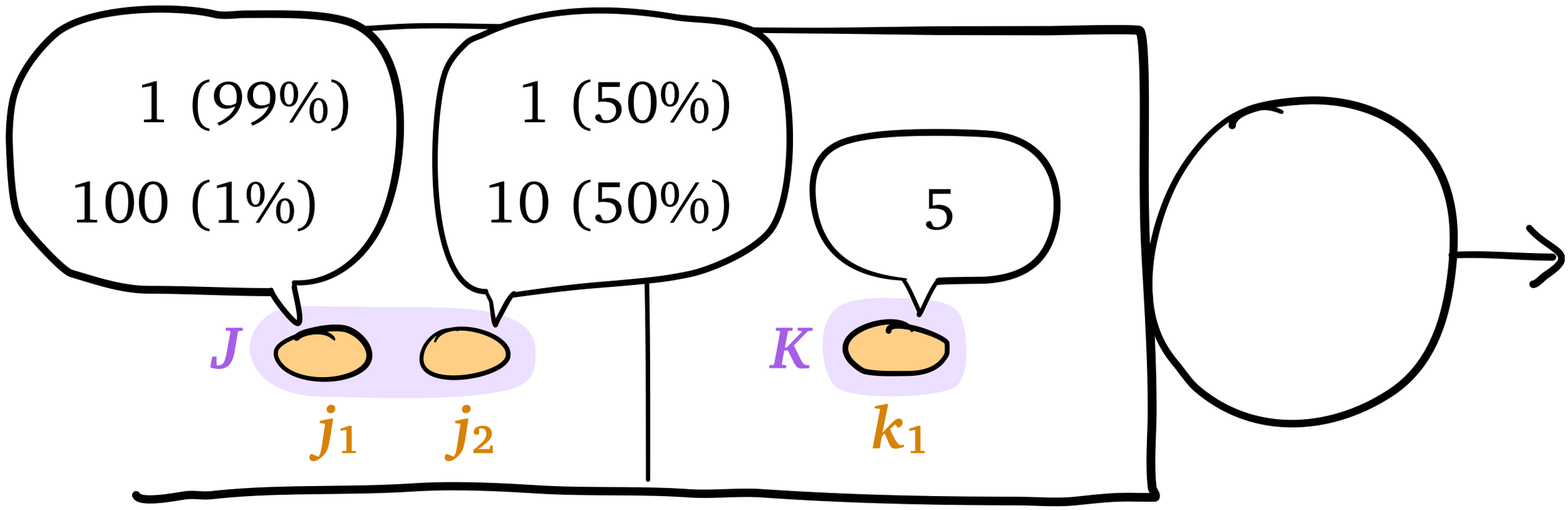
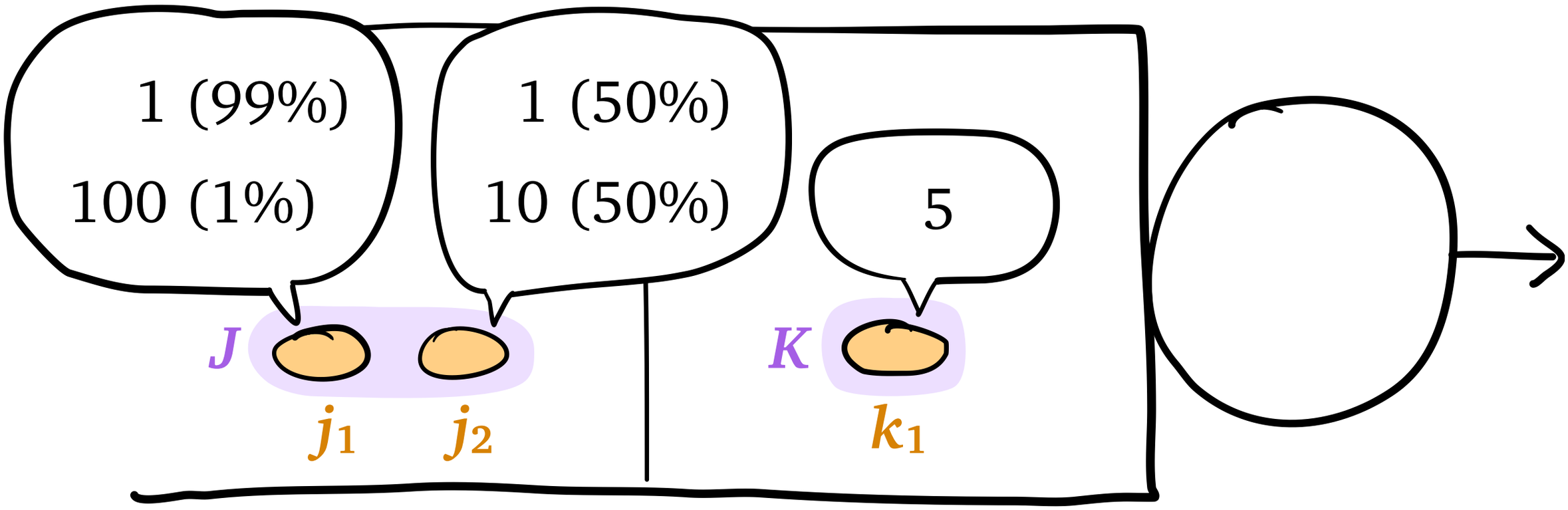


Example



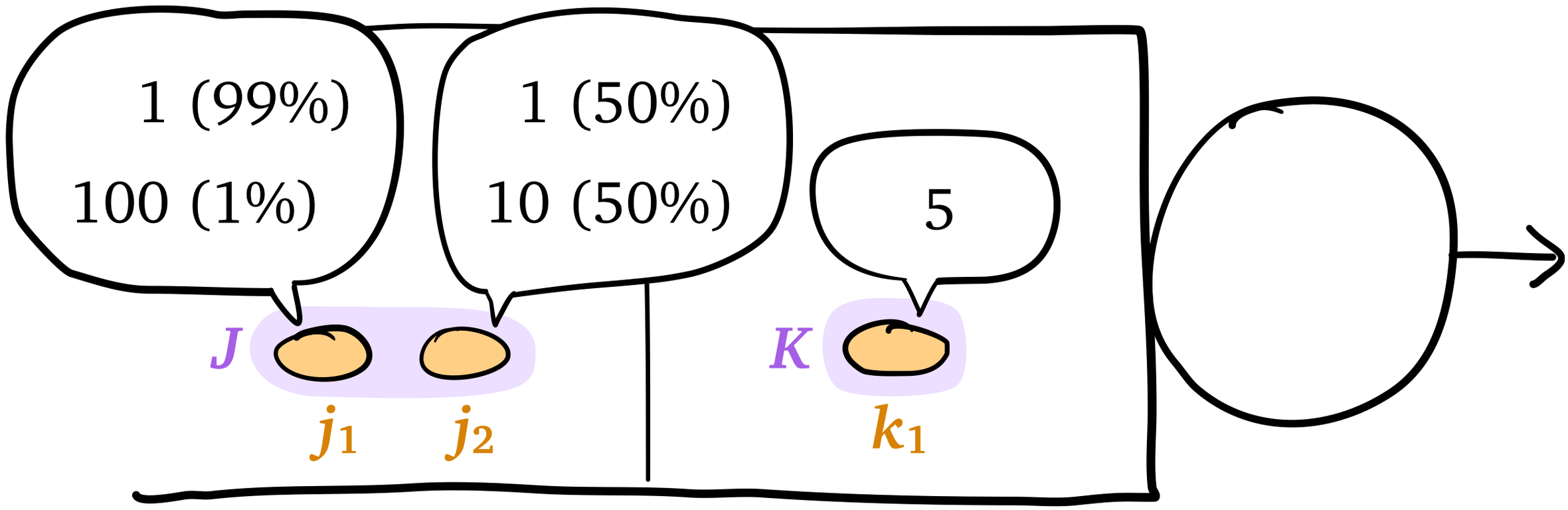
Which **job** should we run first?

Example



Which **job** should we run first? *J*

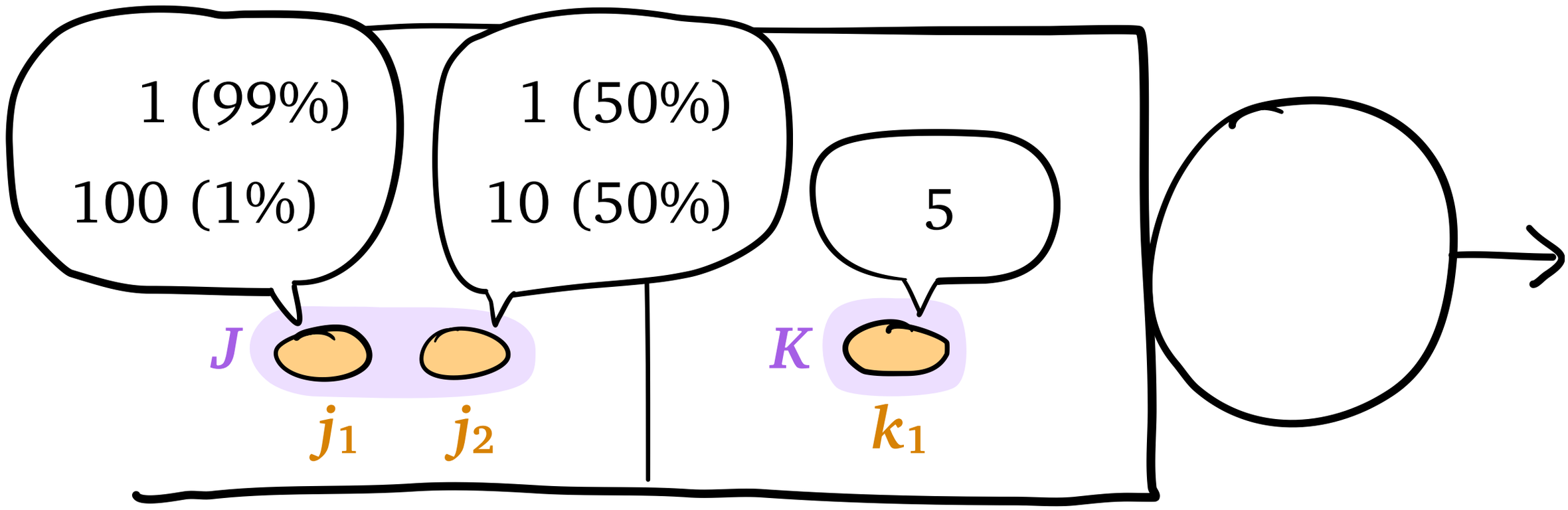
Example



Which **job** should we run first? *J*

Which **task** of *J* should we serve first?

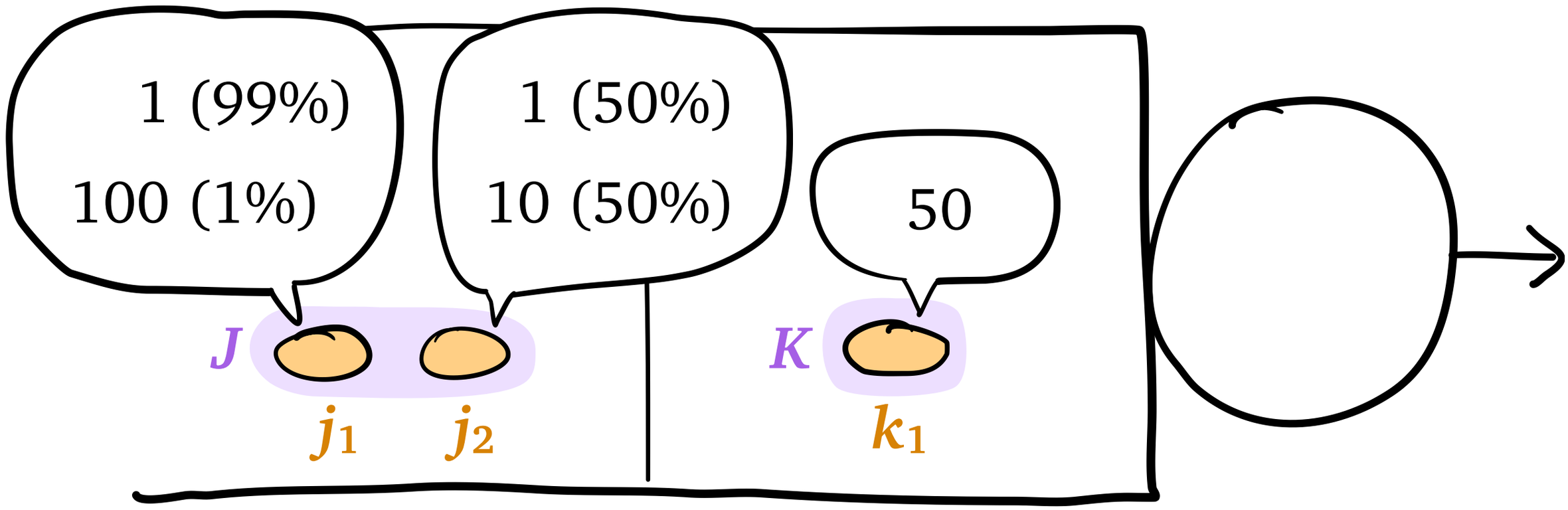
Example



Which **job** should we run first? J

Which **task** of J should we serve first? j_2

Example



Which **job** should we run first? J

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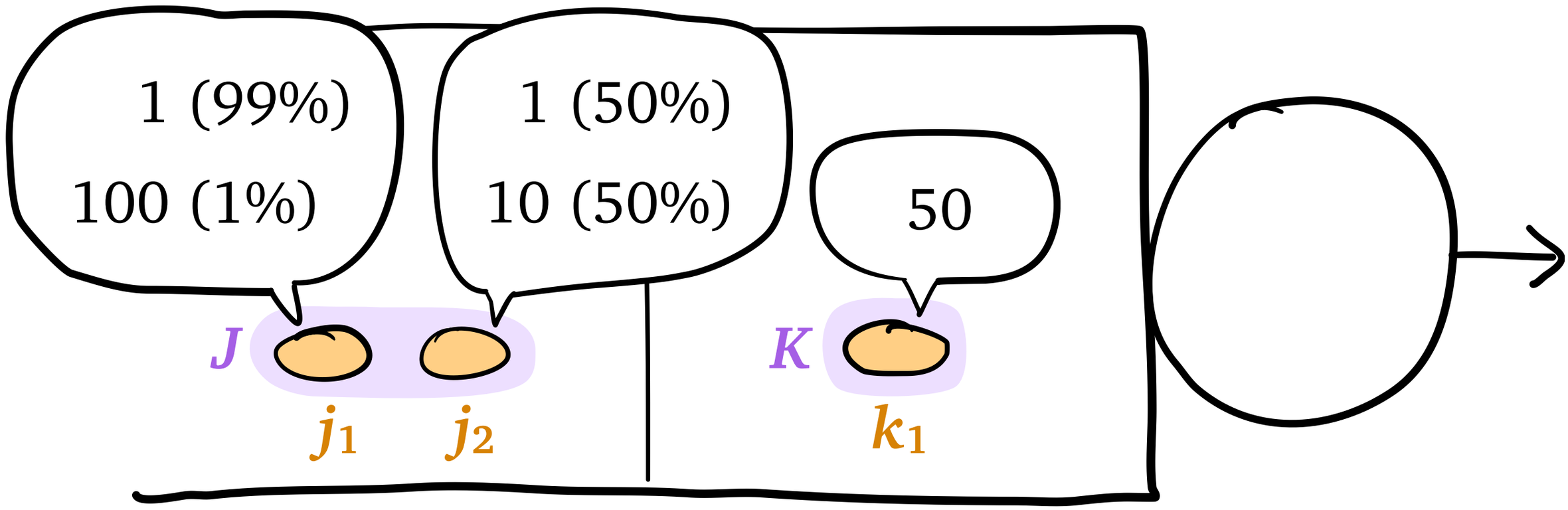
What if we increase size of K to 50?

Example

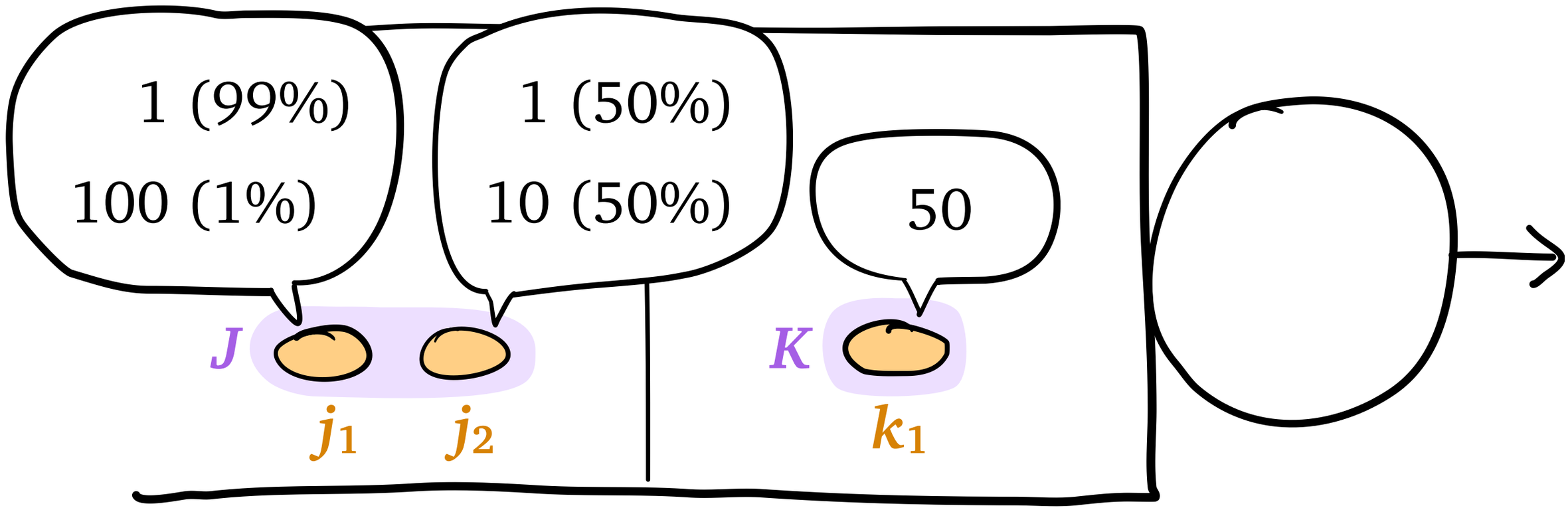


Which **job** should we run first? J

Which **task** of J should we serve first? j_2

What if we increase size of K to 50? j_1

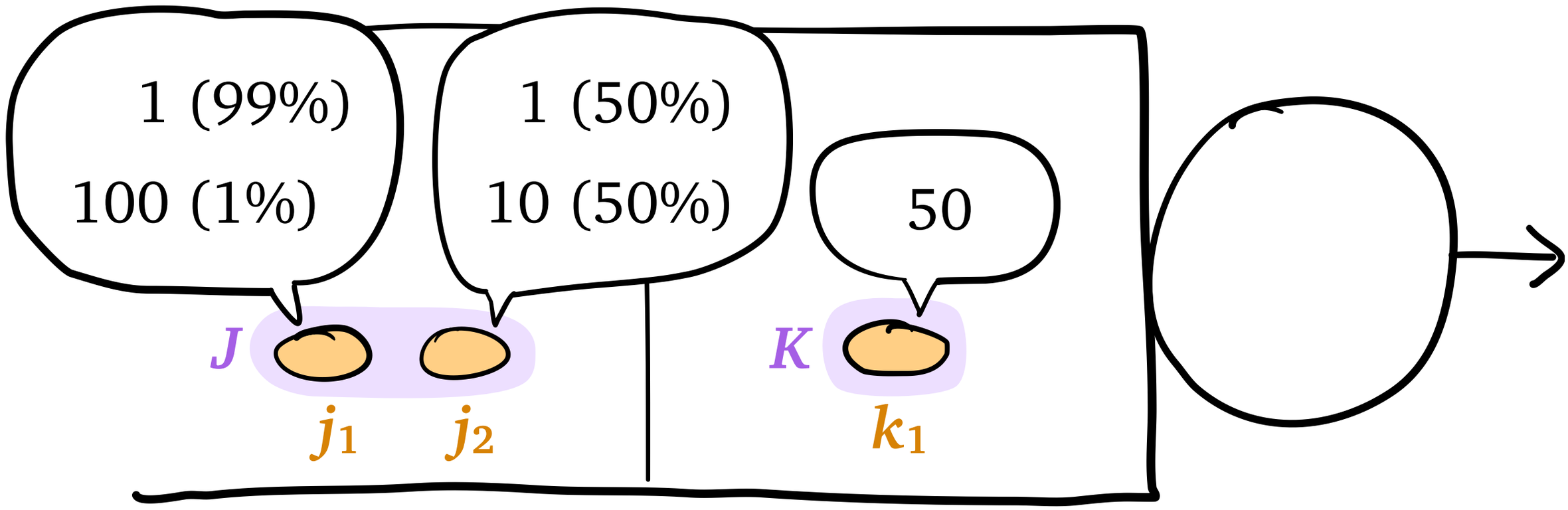
Example



Takeaways:

- Which **job**?
- Which **task**?

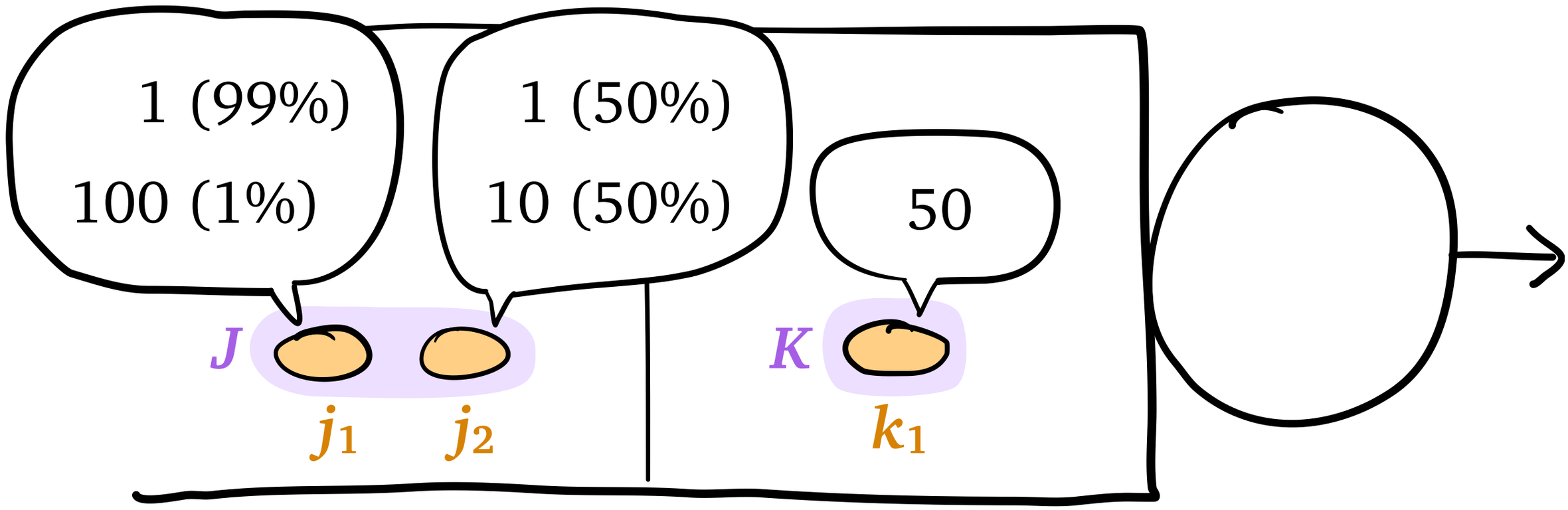
Example



Takeaways:

- Which **job**? *Not SERPT!*
- Which **task**?

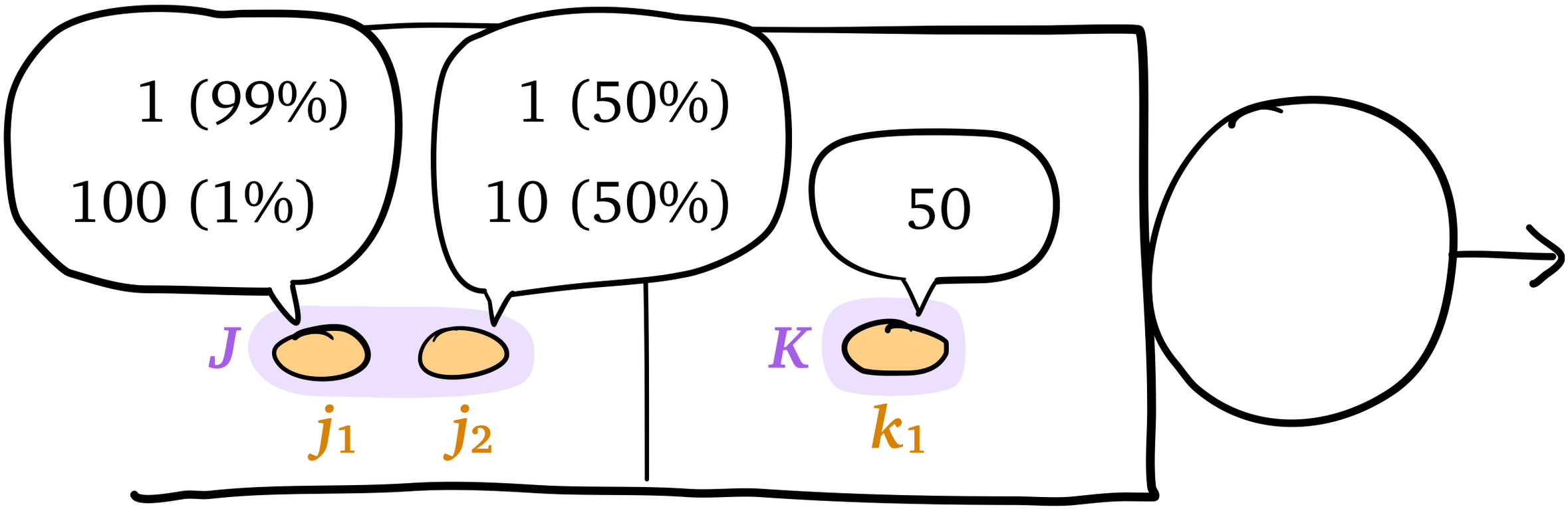
Example



Takeaways:

- Which **job**? *Not SERPT!*
- Which **task**? *"Most informative" first*

Example

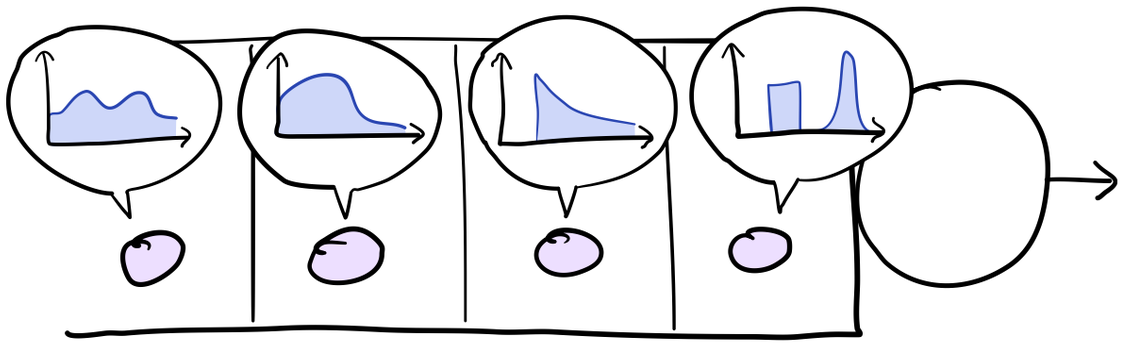


Takeaways:

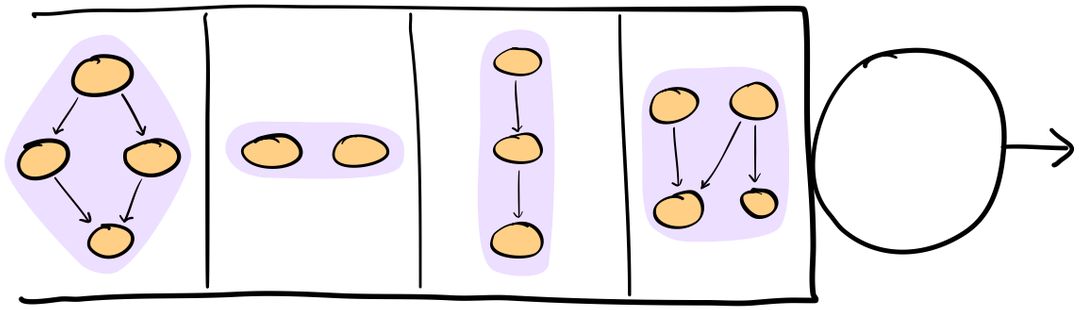
- Which **job**? *Not SERPT!*
- Which **task**? *"Most informative" first*

⚠ "Most informative" is context-dependent!

Prior Work: Single-Task



New Problem: Multitask



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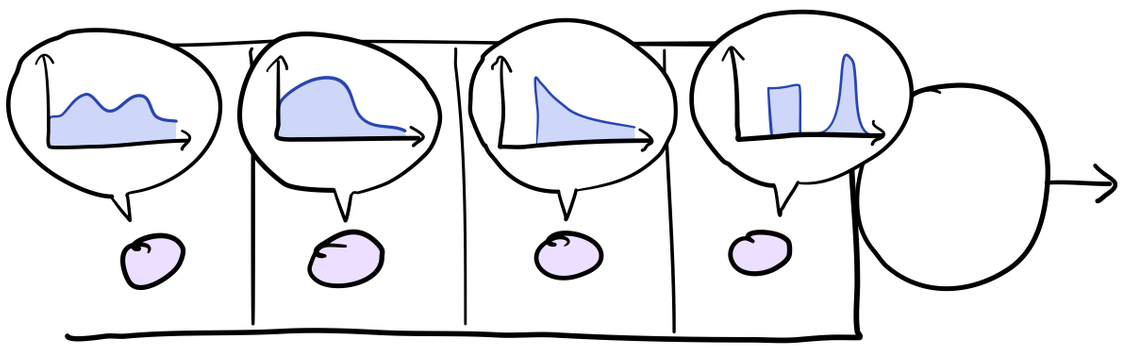

Gittins index policy optimal

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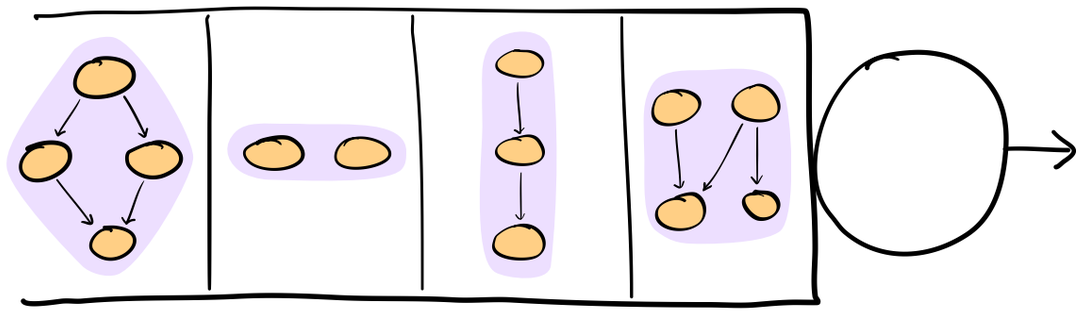

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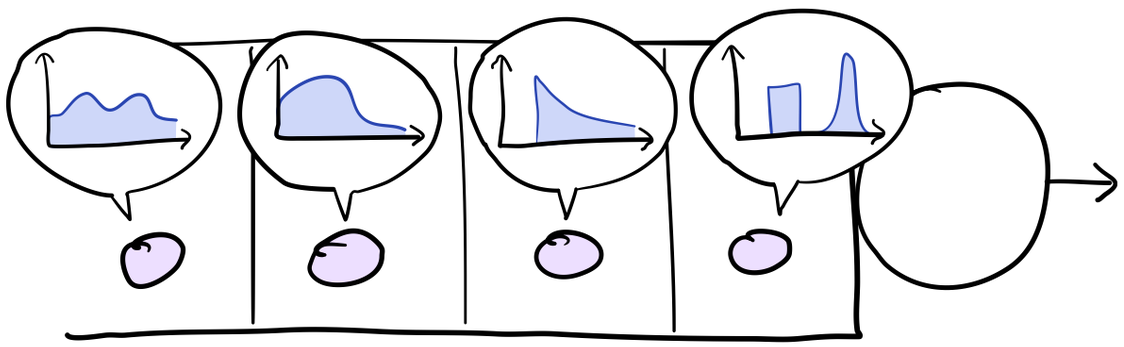

Gittins index policy optimal

Key property: analyzes each job *independently*

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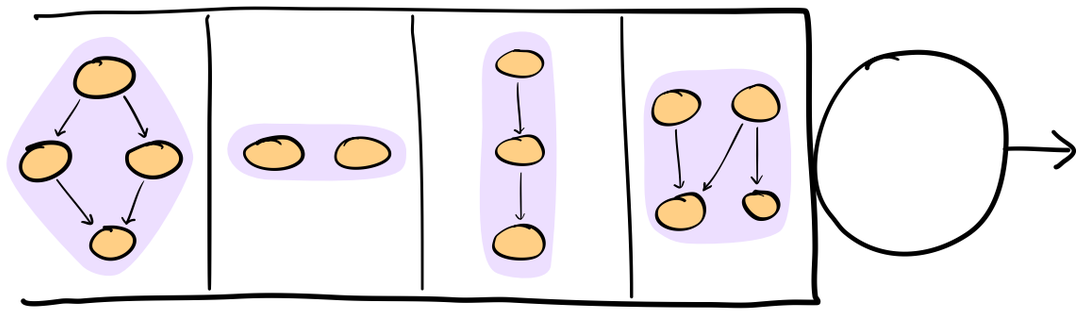

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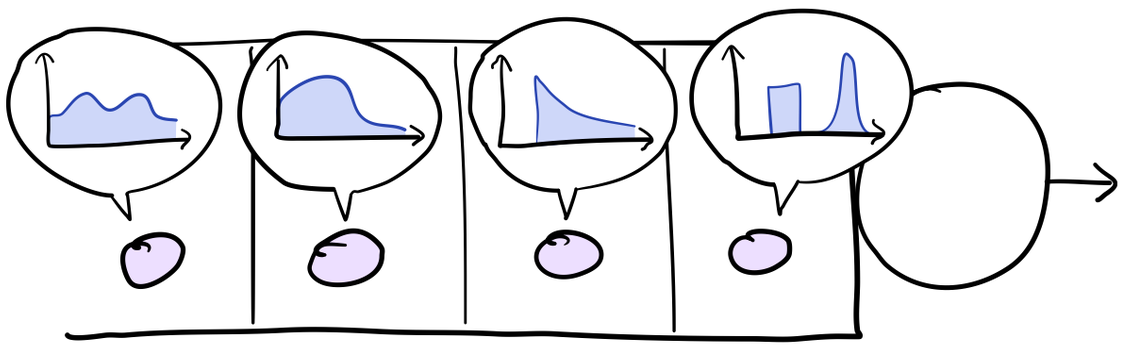

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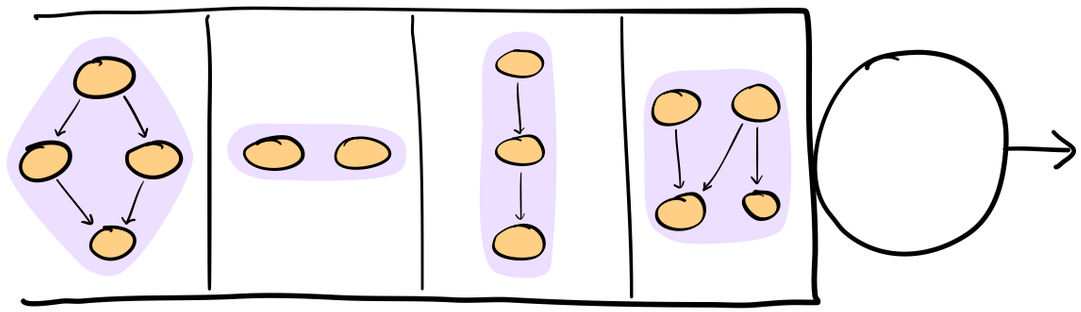

Goal:

- Which **job**?
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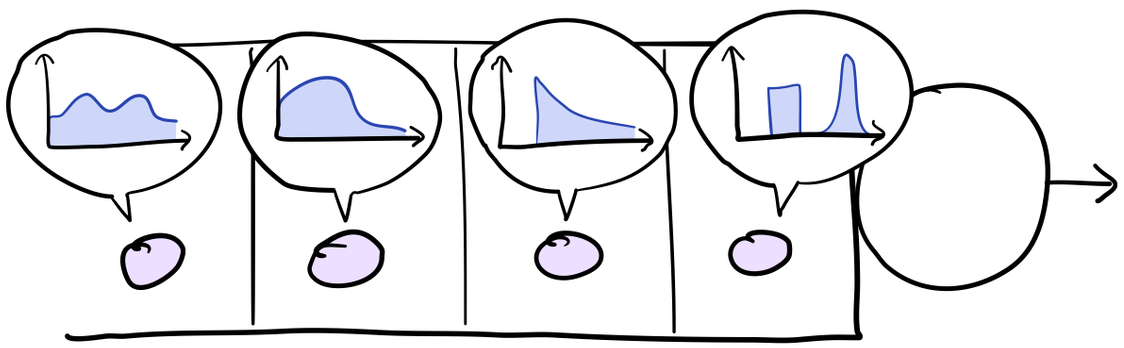

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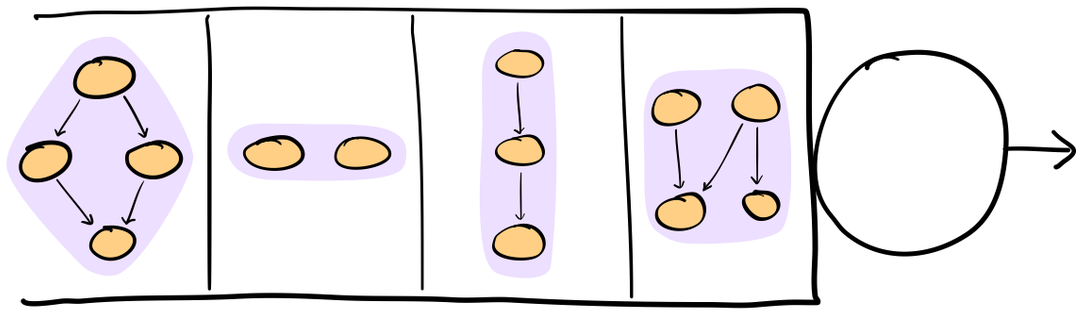

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Gittins index??

Gittins index

Gittins index

many equivalent definitions

- Efficiency function [Gittins ’79]
- Fair charge [Weber ‘92]
- Retirement option [Whittle ‘80]
- Restart-in-state [Katehakis and Veinott ‘87]

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- **Single-job profit** (this talk)

Gittins index

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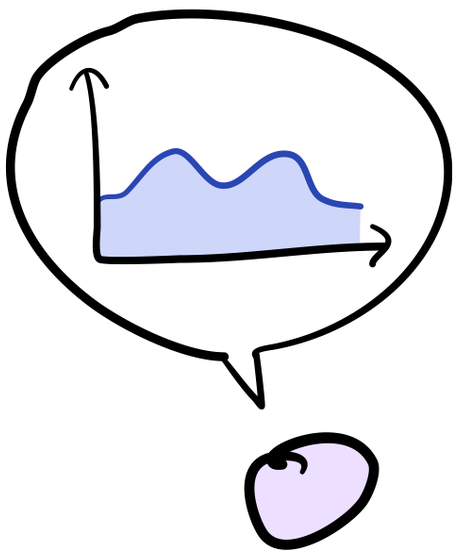
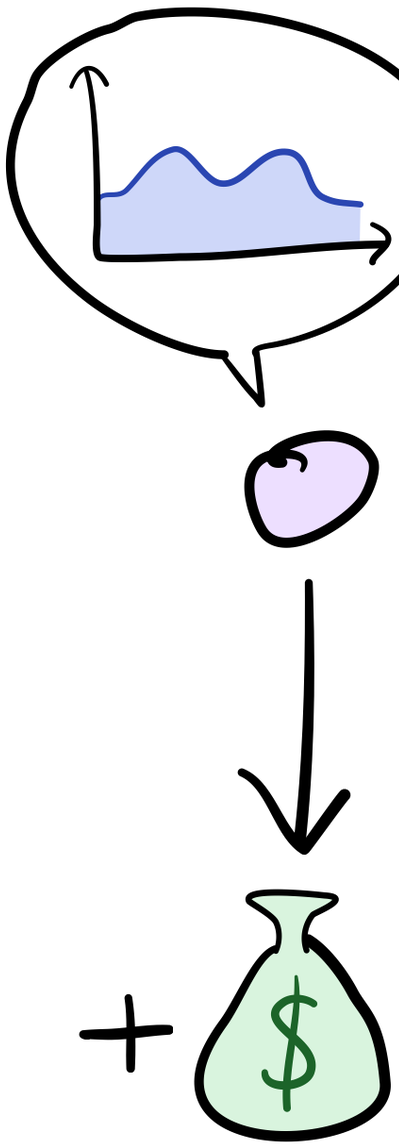
- Efficiency function [Gittins ’79]
- Fair charge [Weber ‘92]
- Retirement option [Whittle ‘80]
- Restart-in-state [Katehakis and Veinott ‘87]
- **Single-job profit** (this talk)
  - Natural definition for multitask jobs

Single-Job Profit



Game with a **job** and potential **reward**

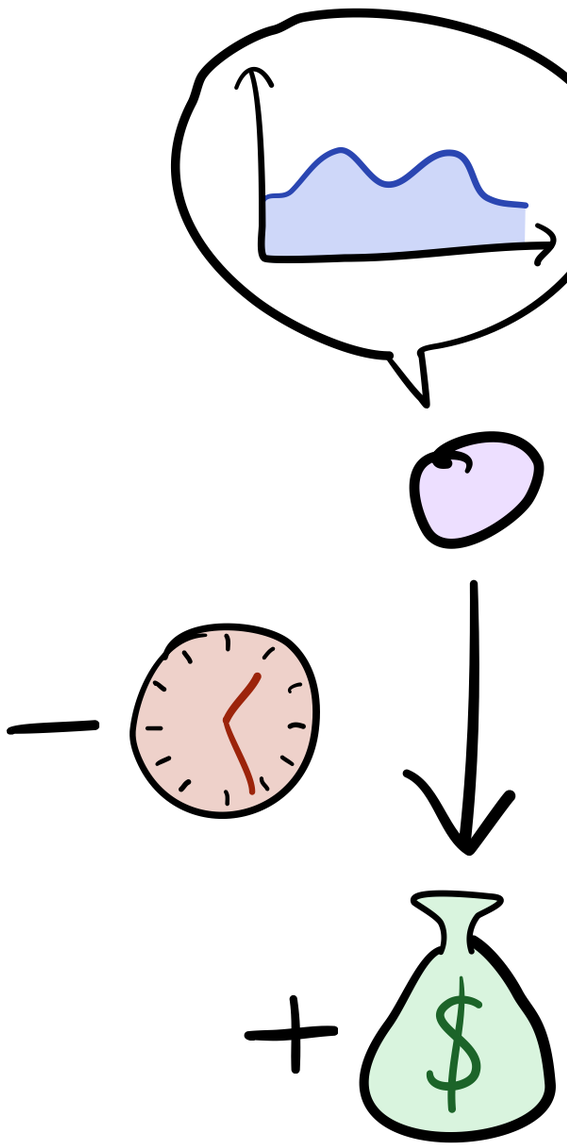
Single-Job Profit



Game with a **job** and potential **reward**

Run **job** as long as we like, get **reward** if we complete it

Single-Job Profit



Game with a **job** and potential **reward**

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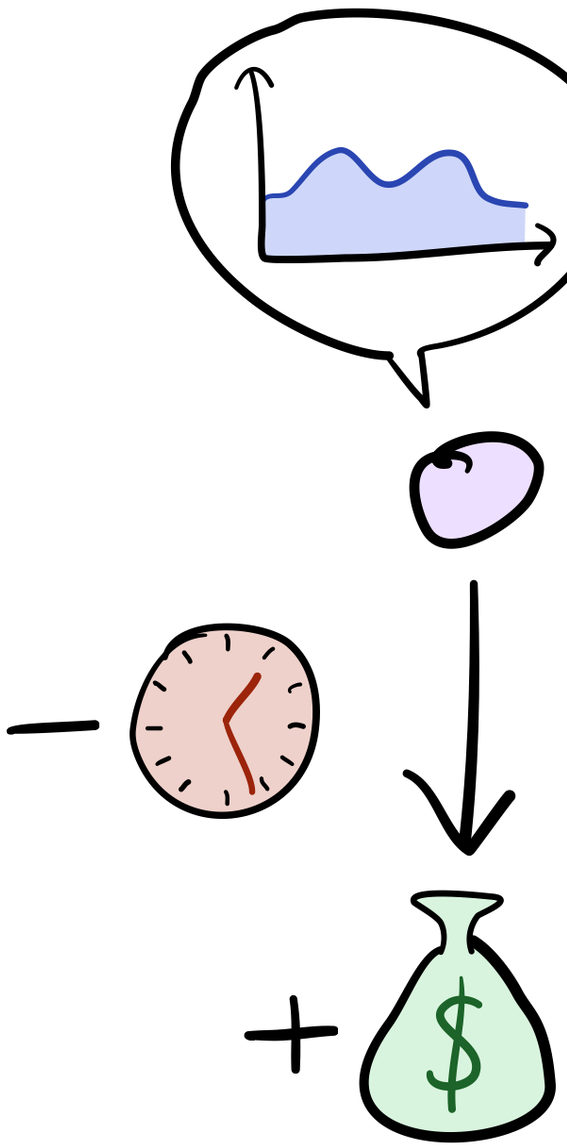
Pay for **time** spent running job

Single-Job Profit



Game with a **job** and potential **reward**

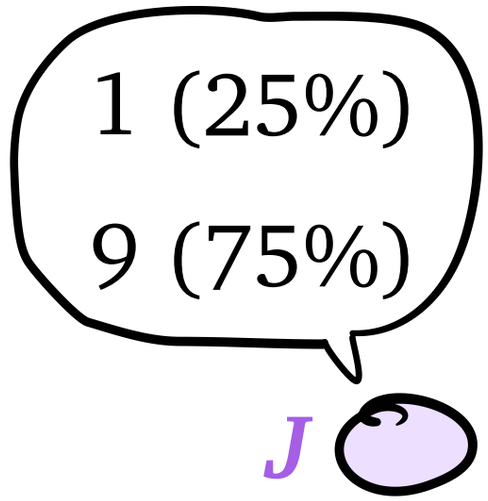
Run **job** as long as we like, get **reward** if we complete it

Pay for **time** spent running job

Goal: maximize *profit*,

E[**reward** recieved − **time** spent]

Example



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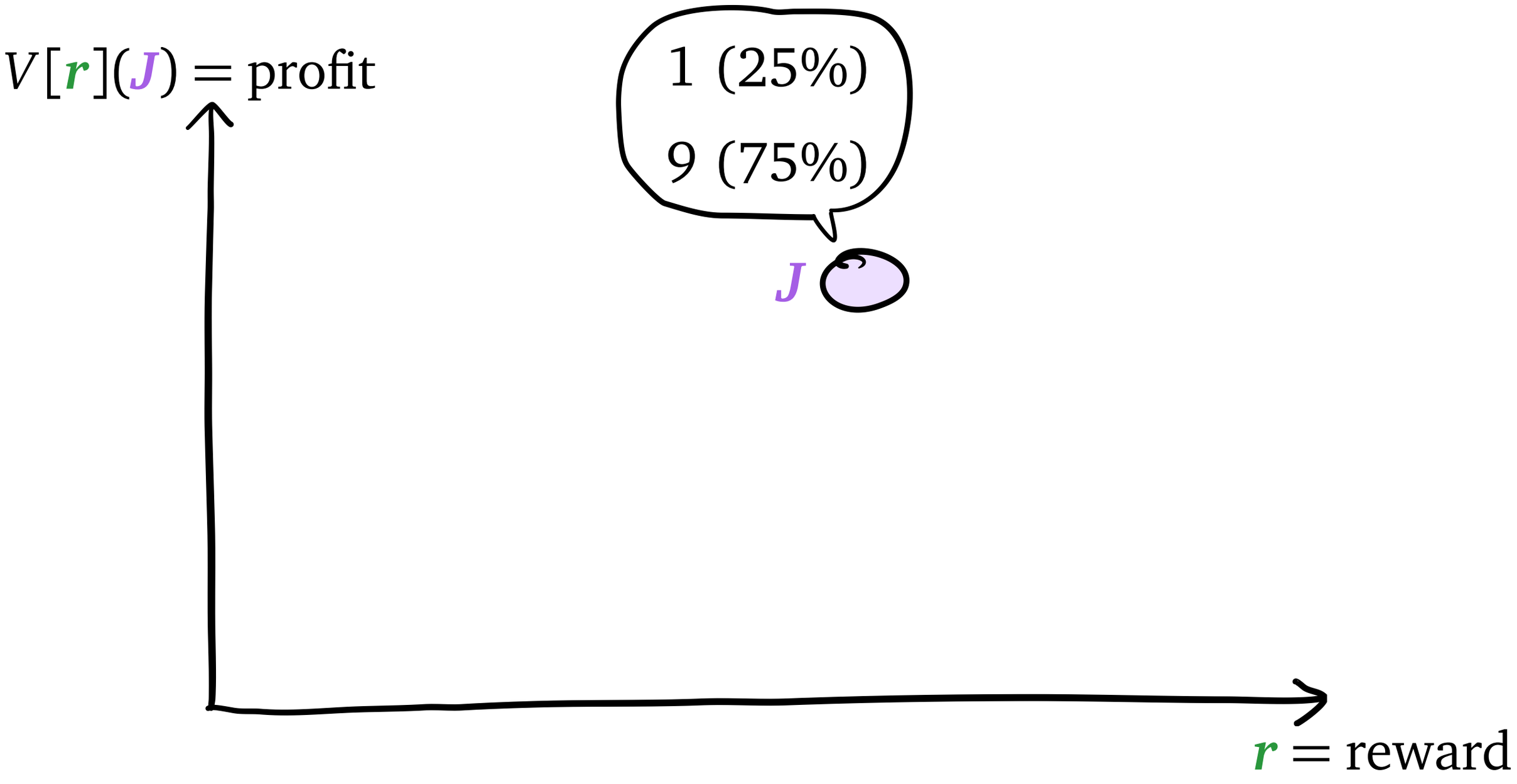

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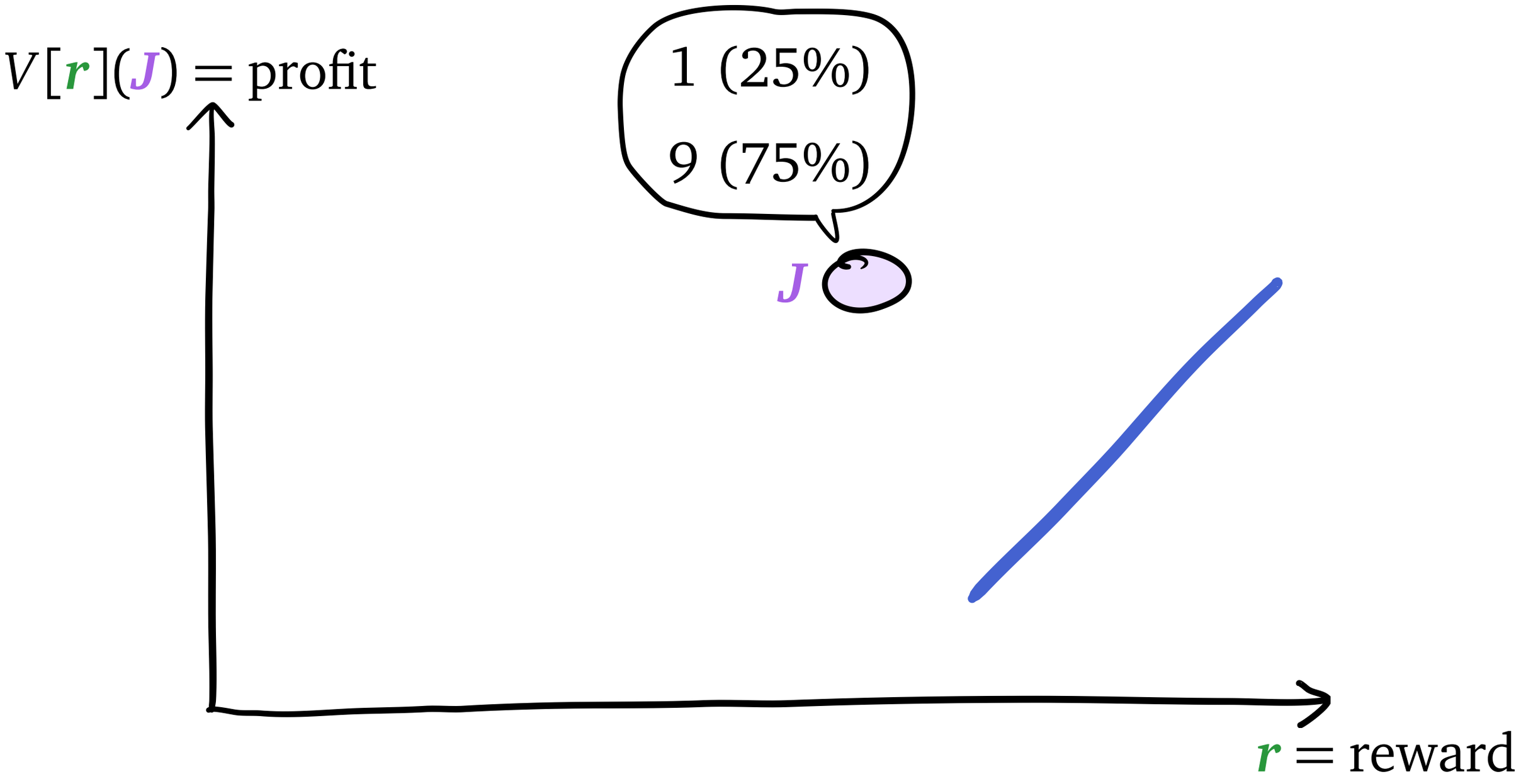

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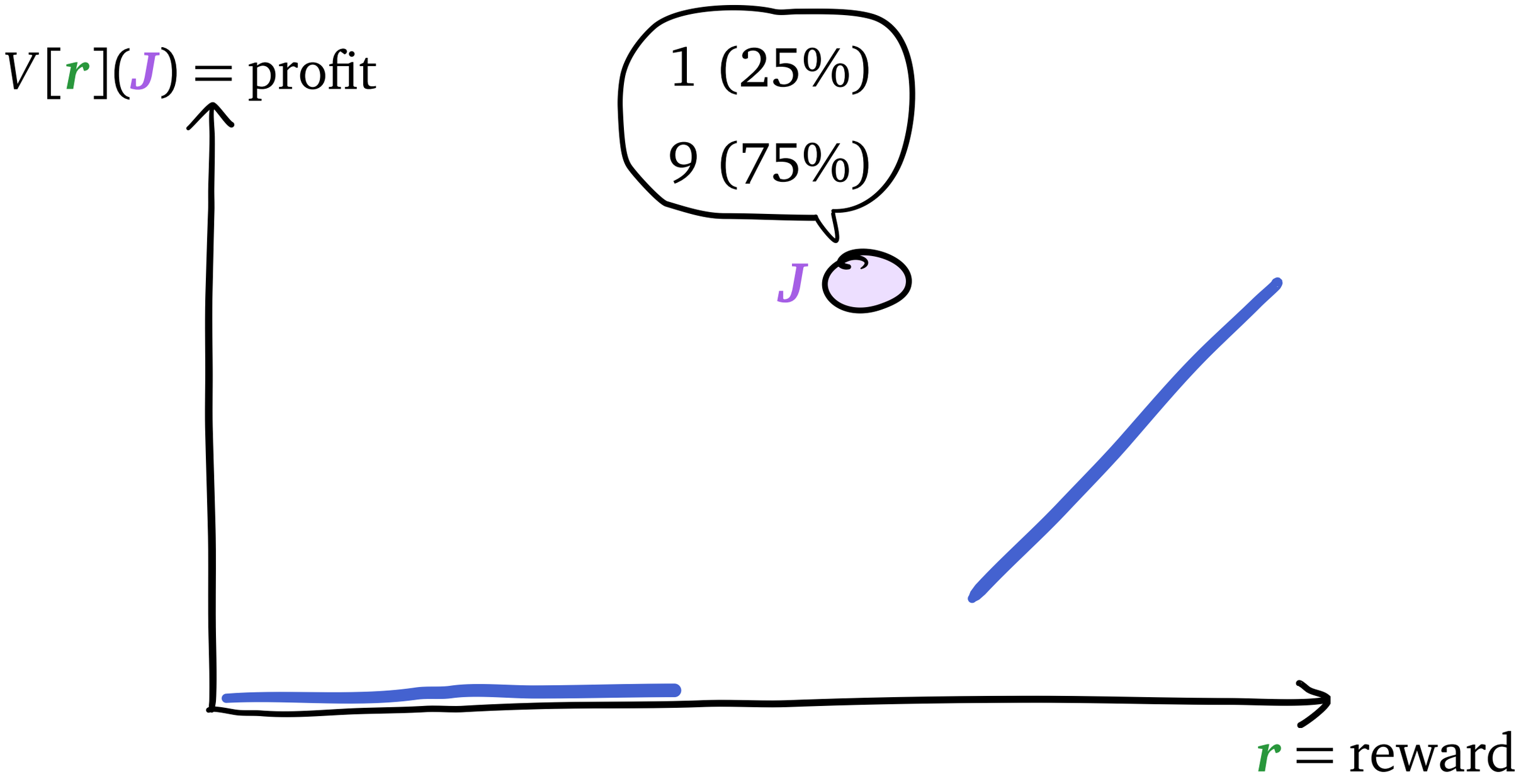

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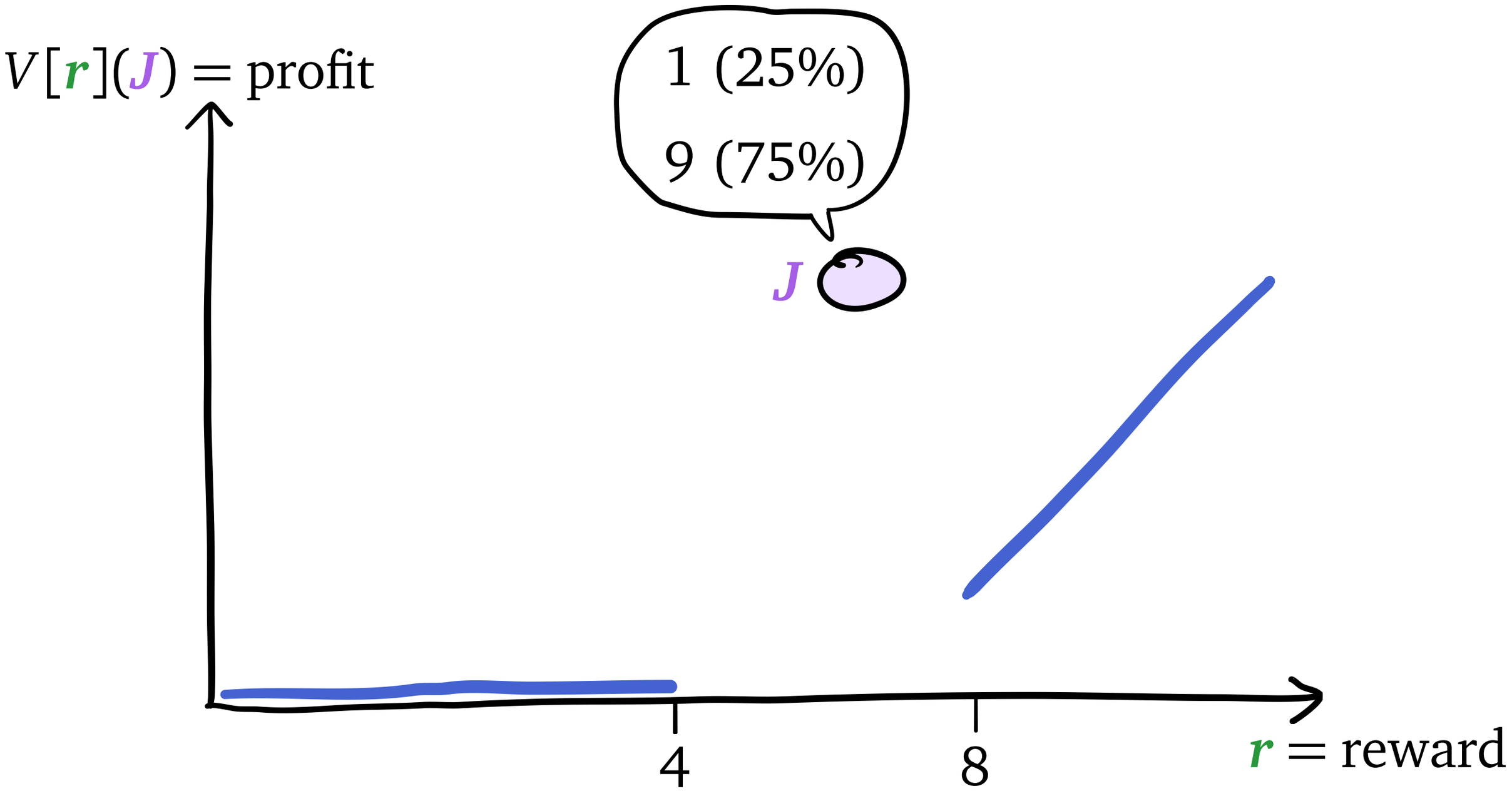
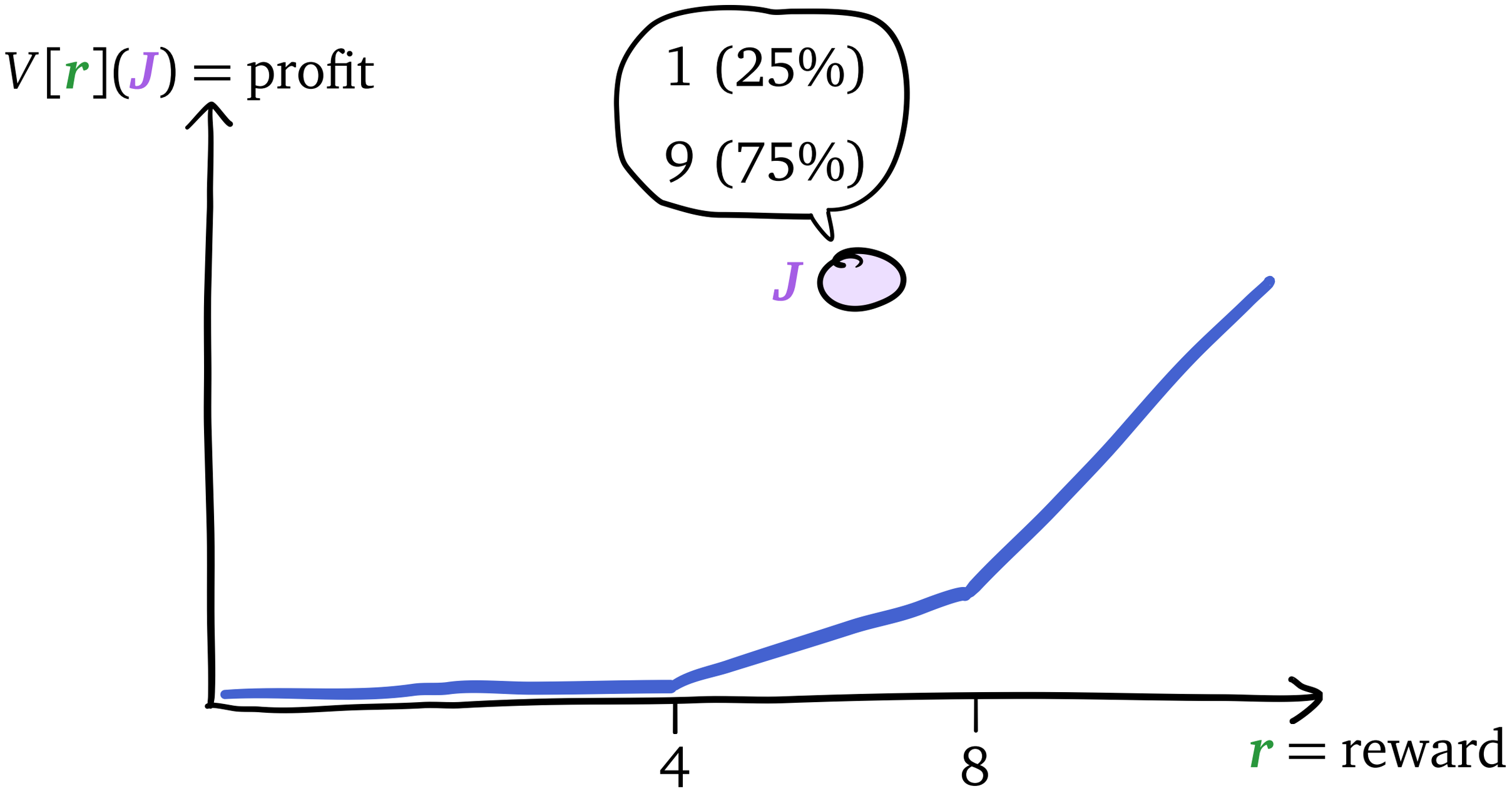


Example



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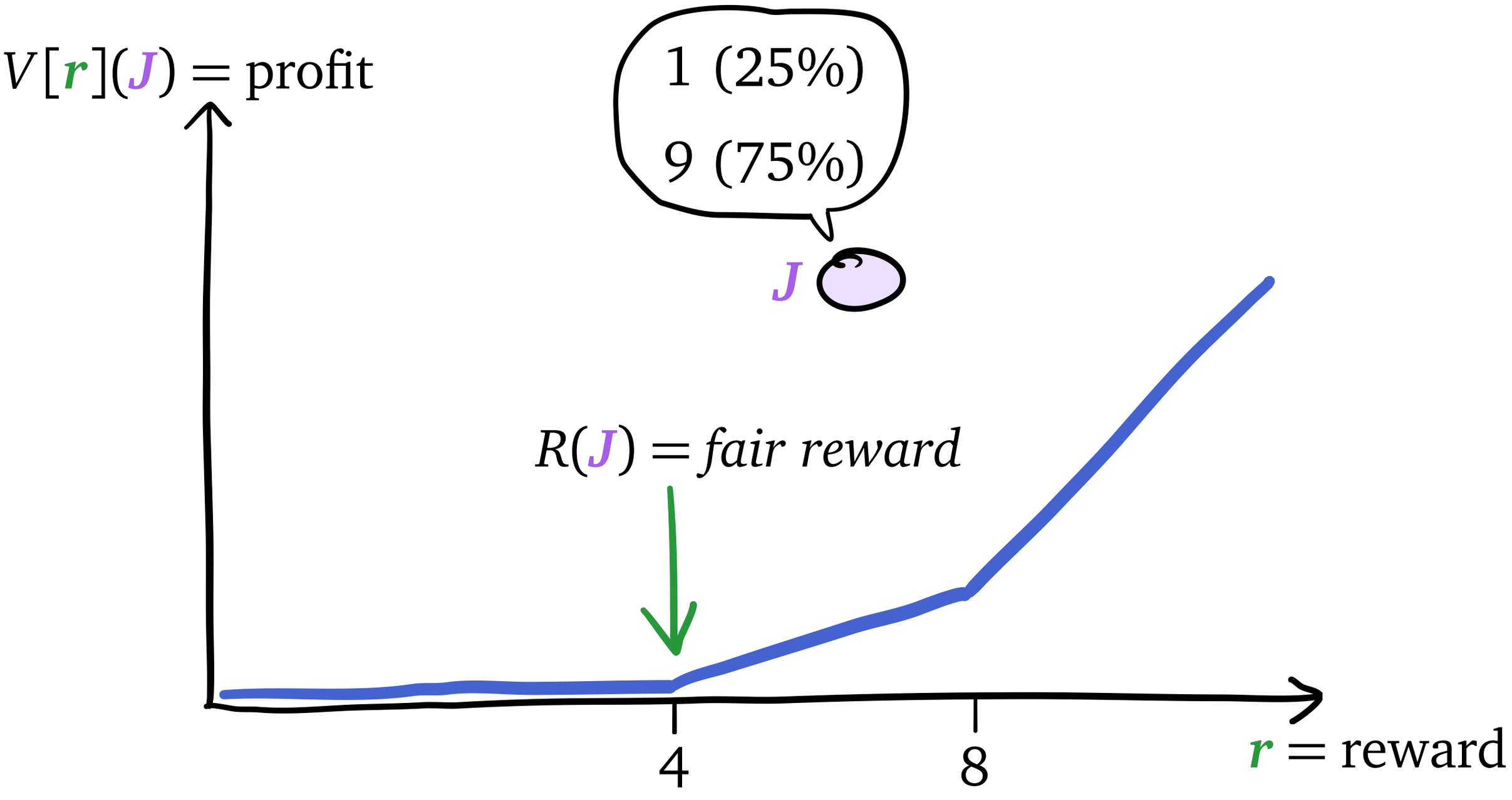
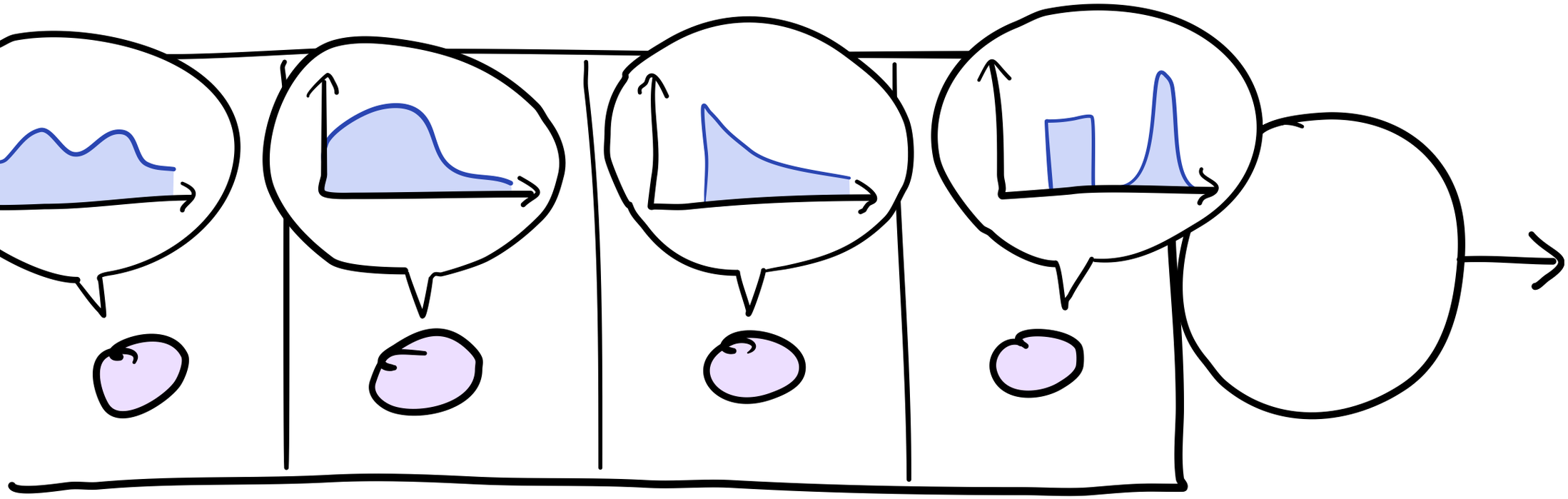


Fair Reward in Scheduling



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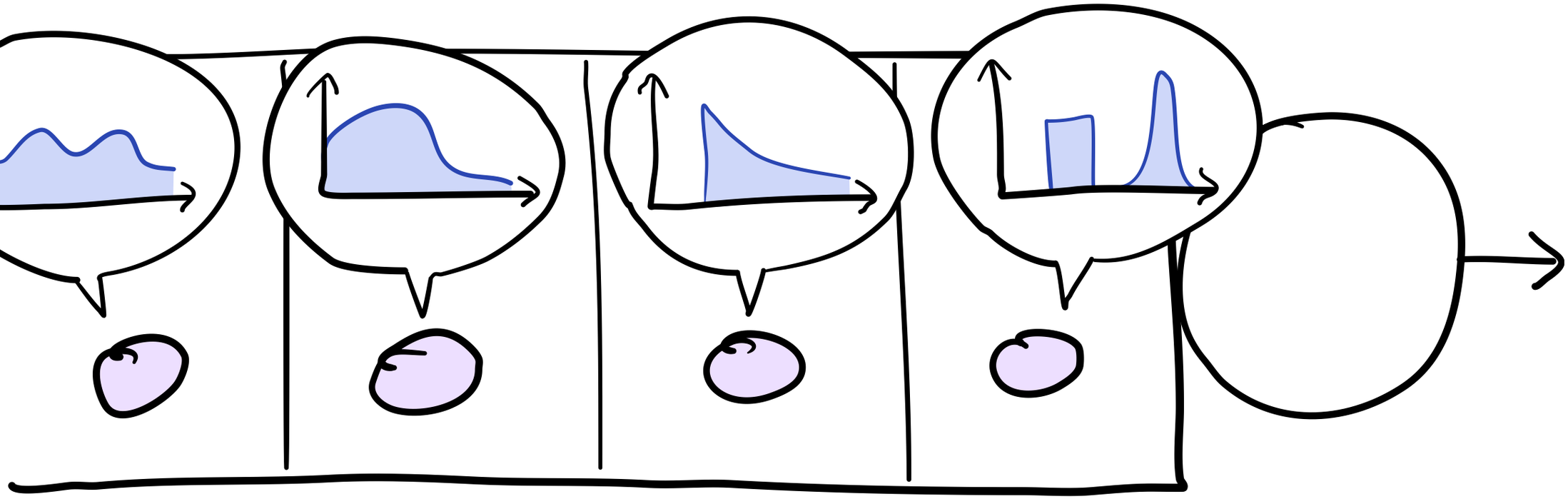
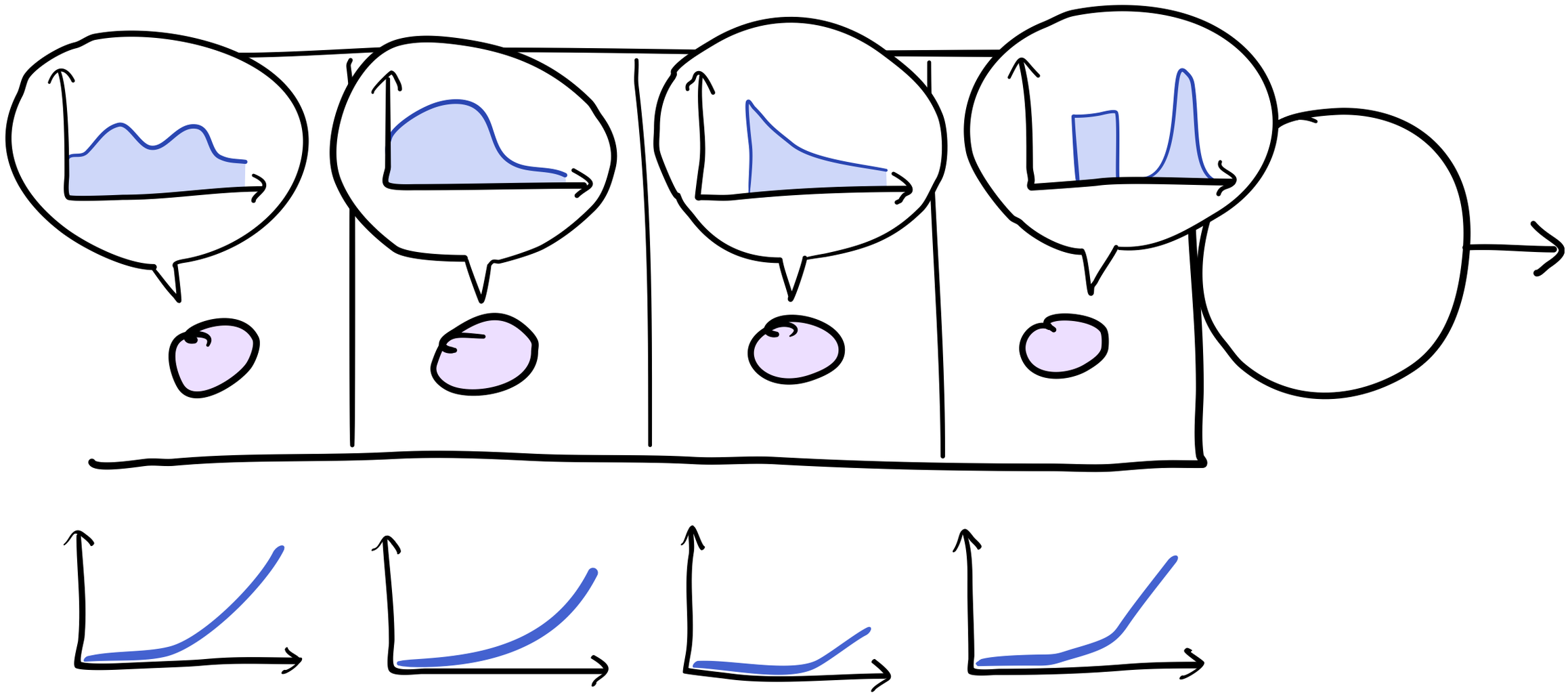


Theorem: optimal to serve
job of *minimum fair reward*

Fair Reward in Scheduling



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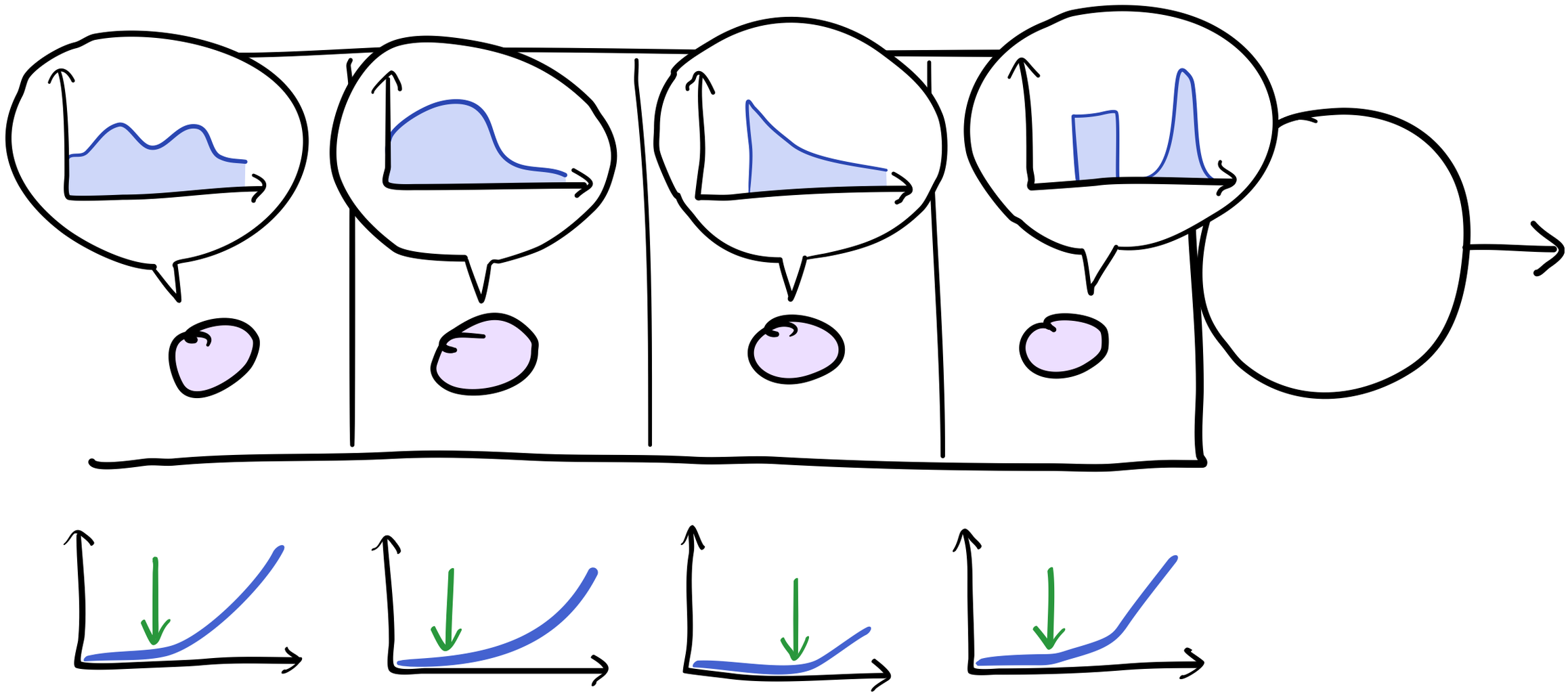

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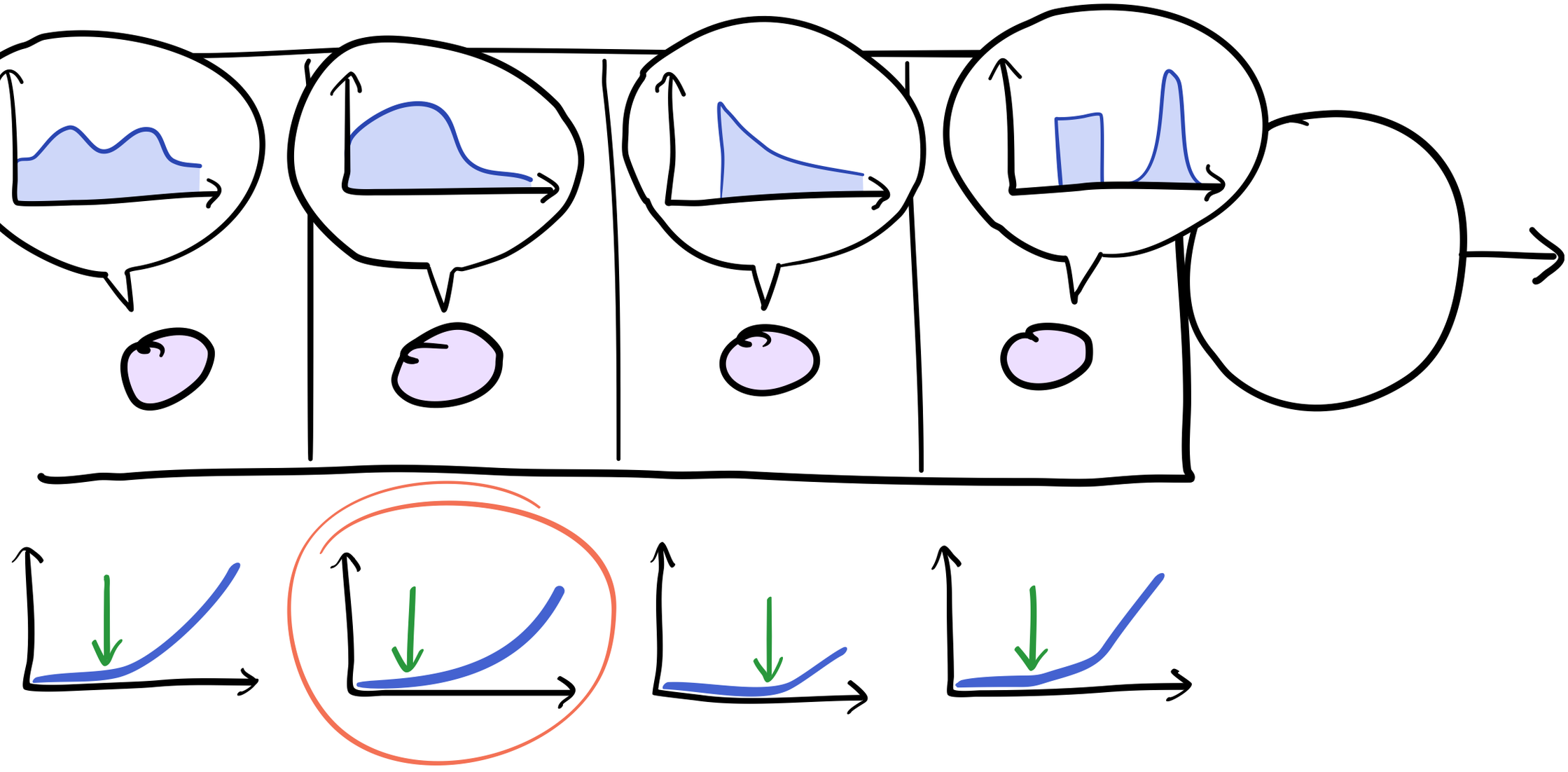

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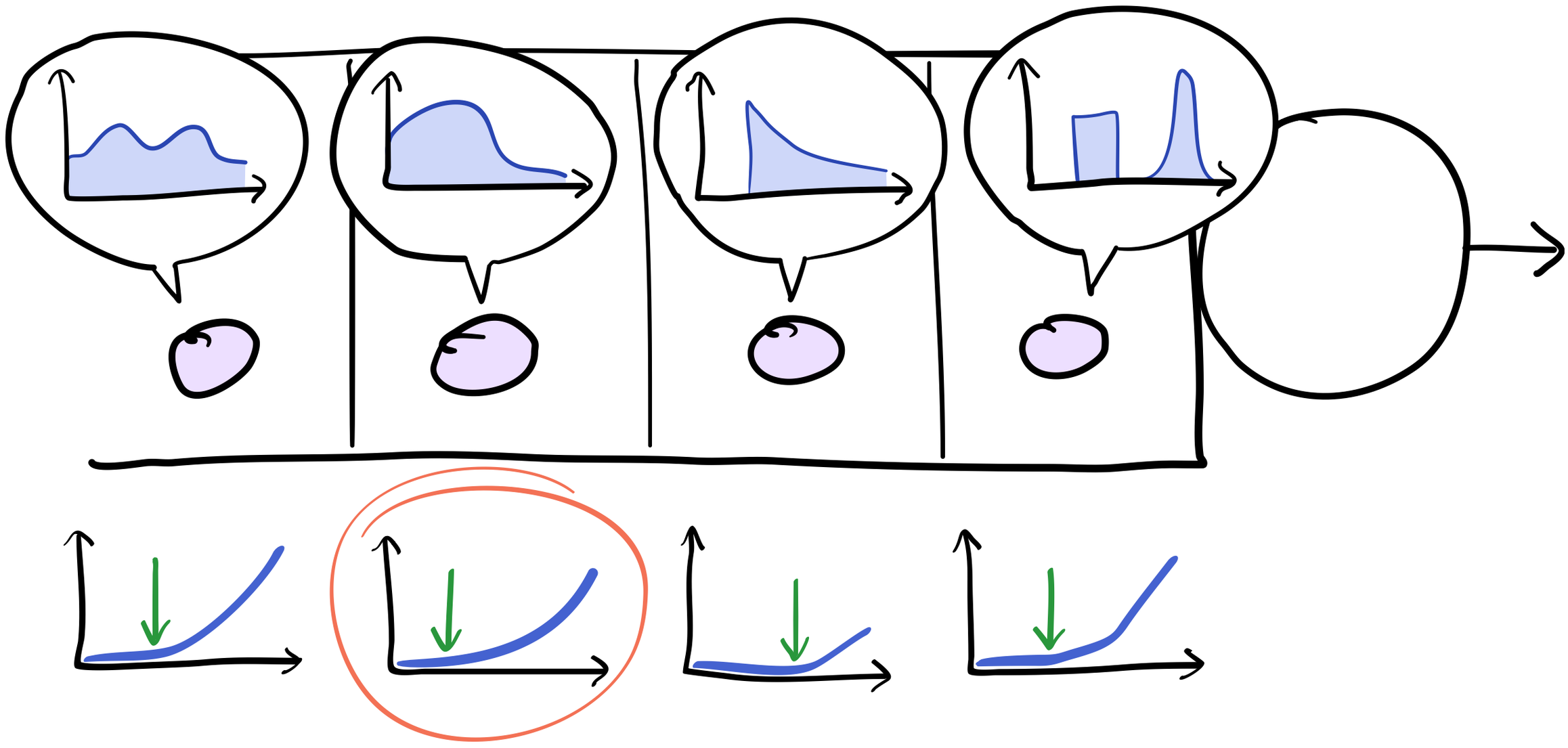
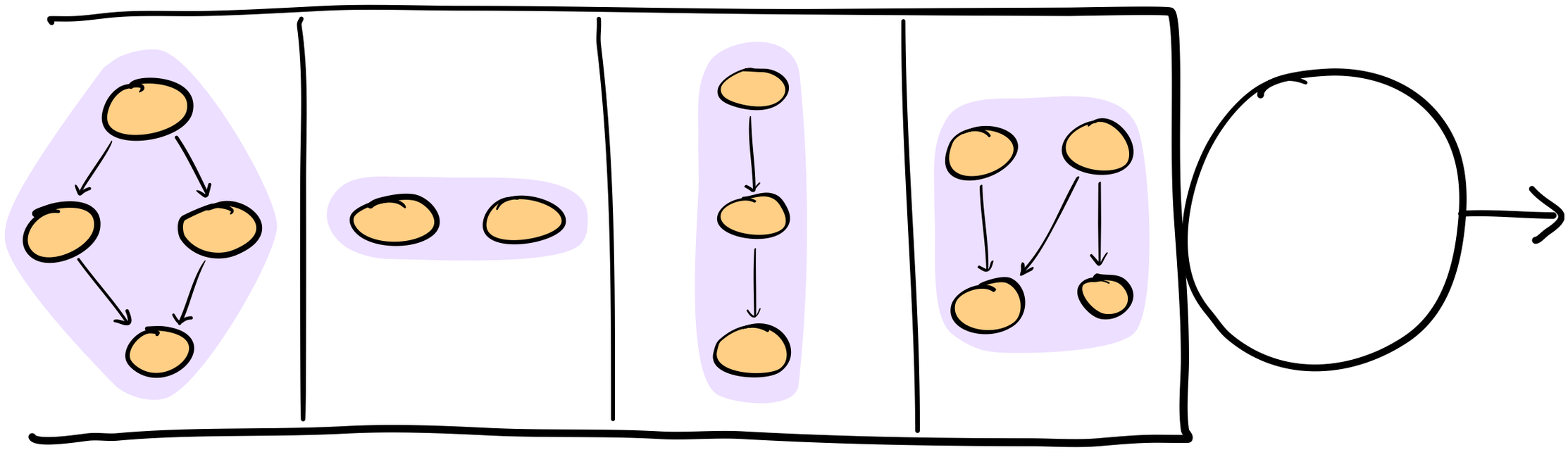


Theorem: optimal to serve job of *minimum fair reward*

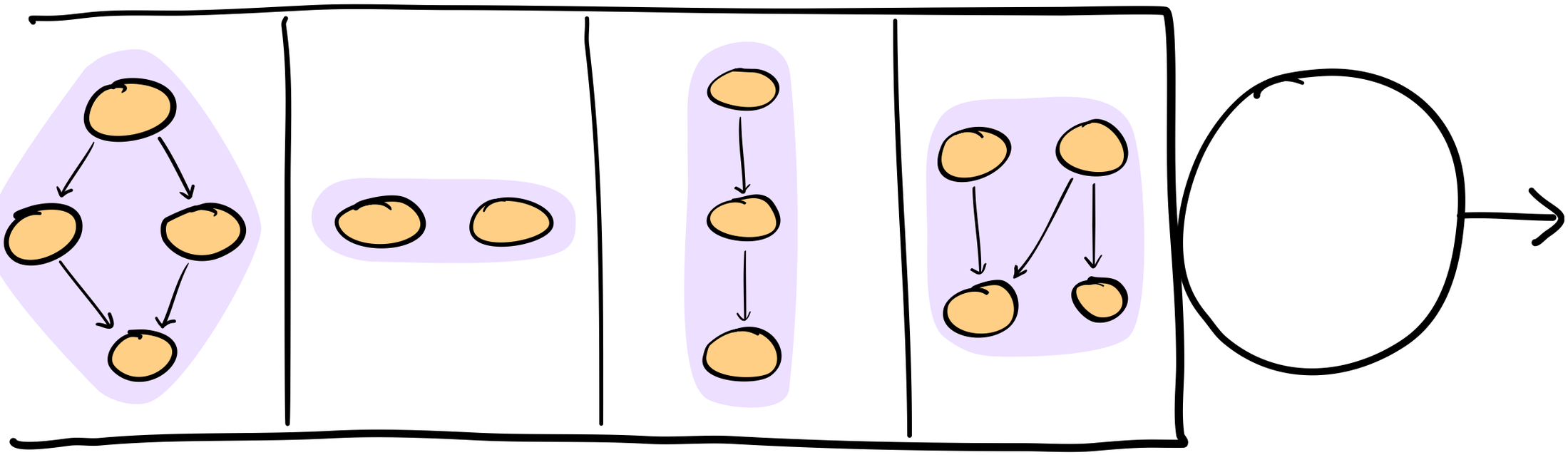
$$G(J) = \frac{1}{R(J)}$$

Gittins index

Generalizing to Multitask

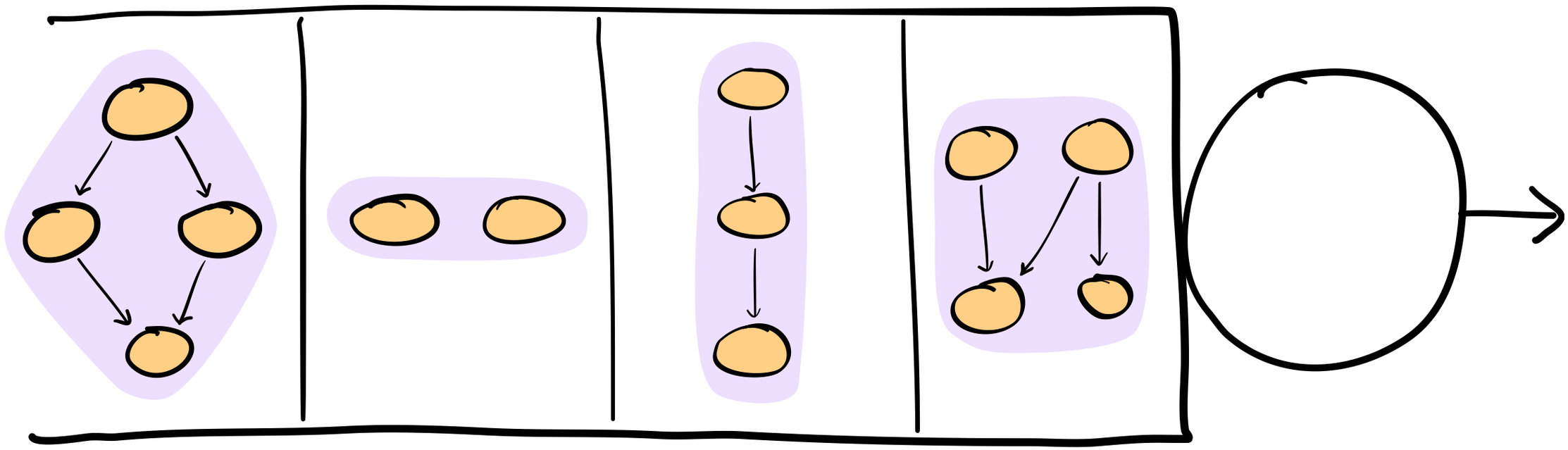


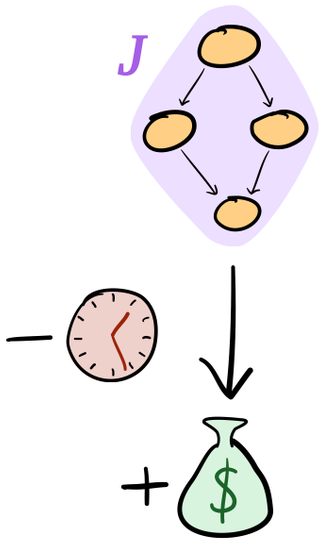
Generalizing to Multitask



Single-job profit easily generalizes

Generalizing to Multitask





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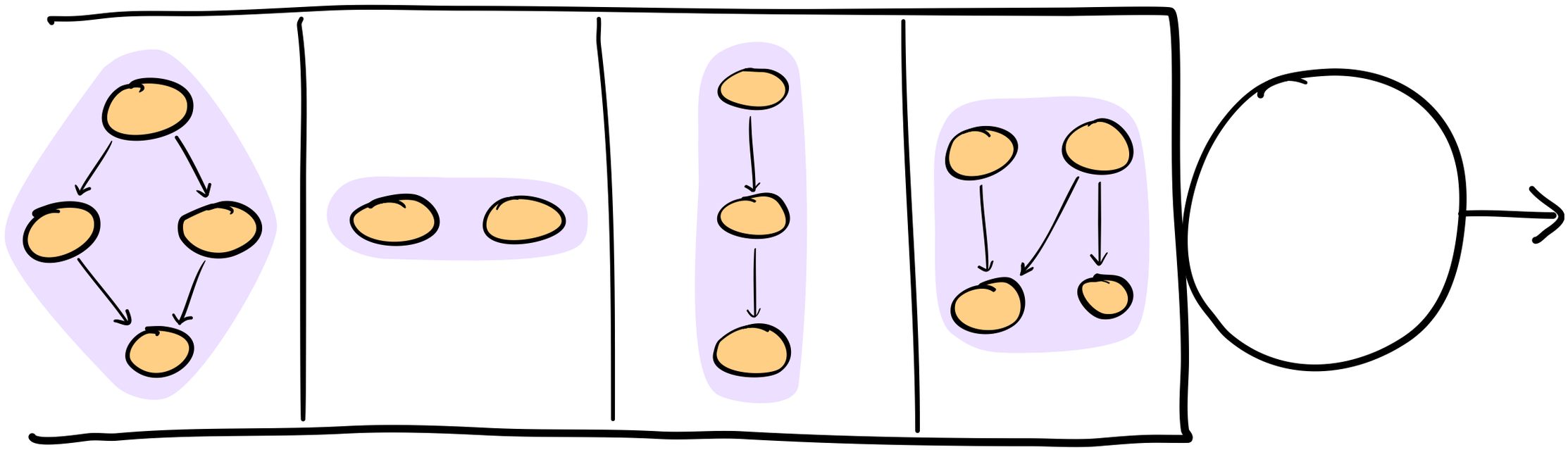

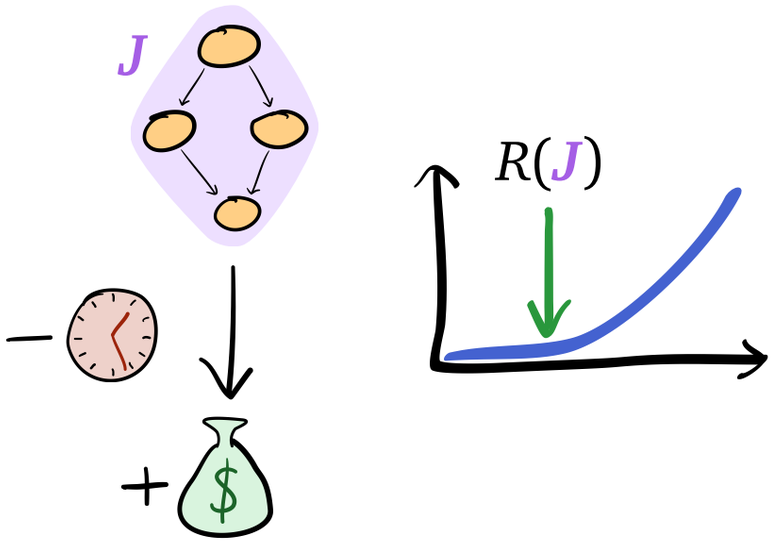

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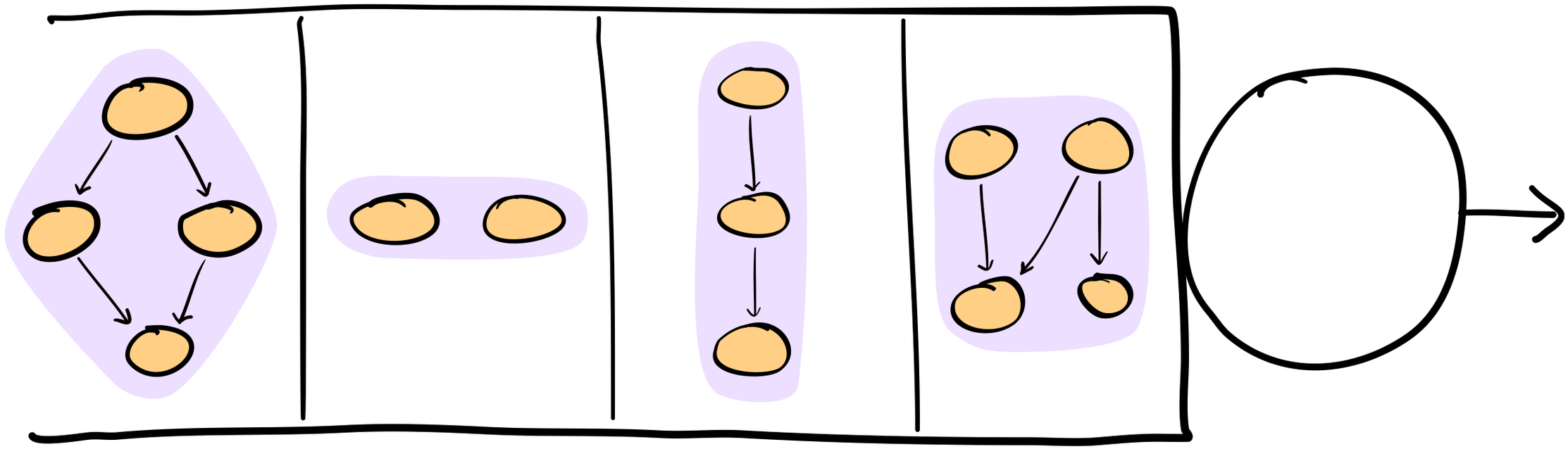
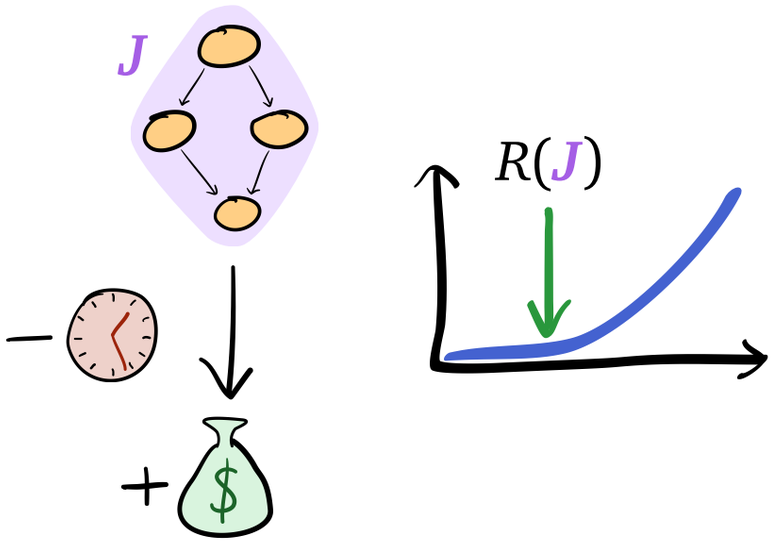
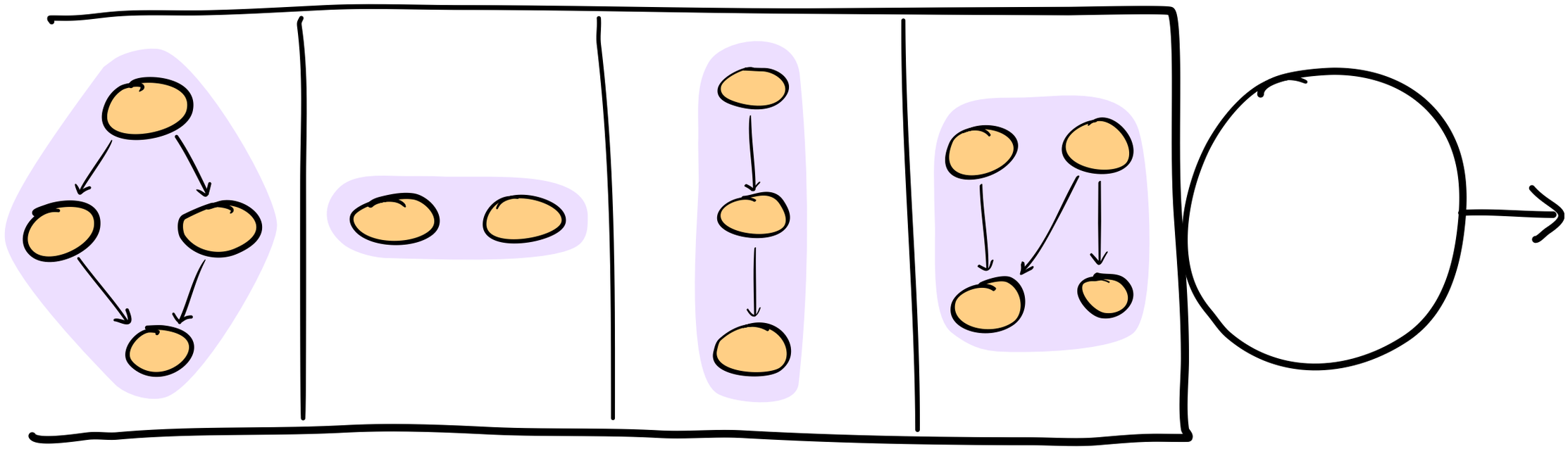


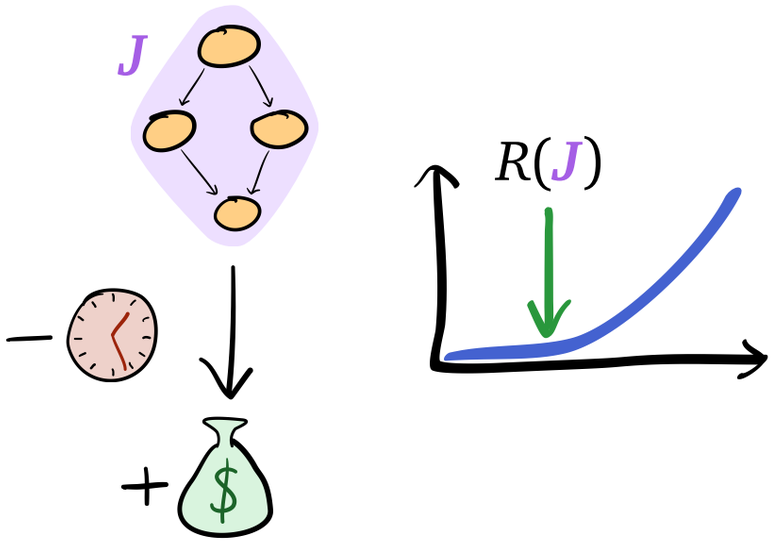


Single-job profit easily generalizes

⚠ **Obstacles**:

Generalizing to Multitask





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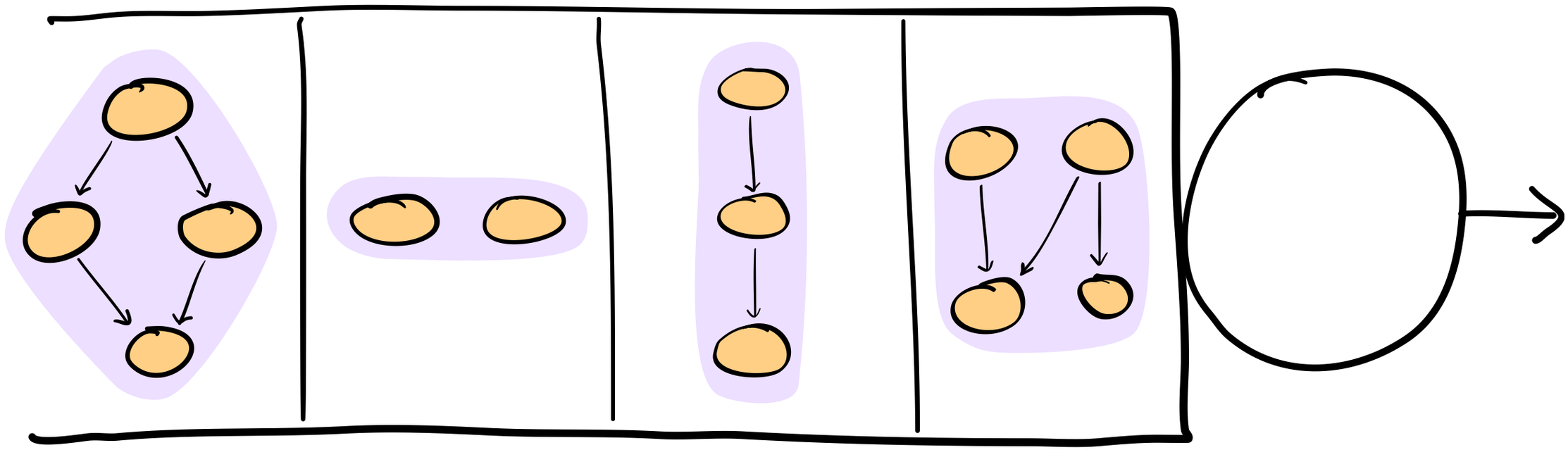
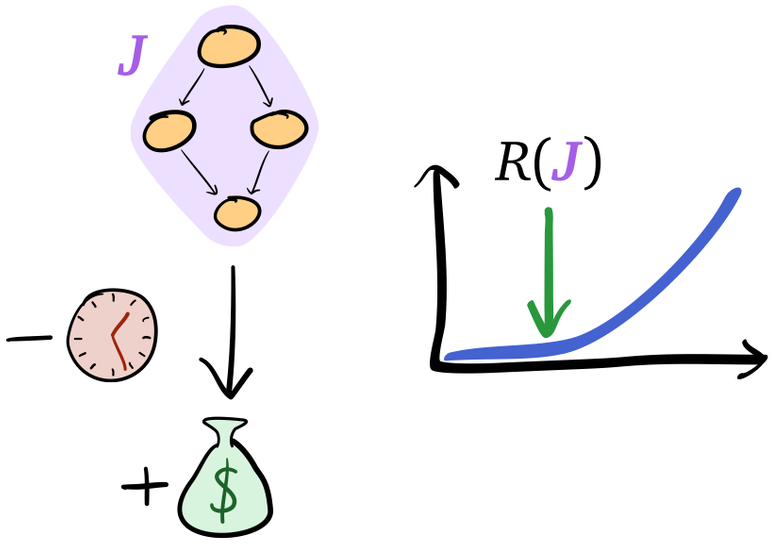
Obstacles:

- Hard to compute profit

Generalizing to Multitask



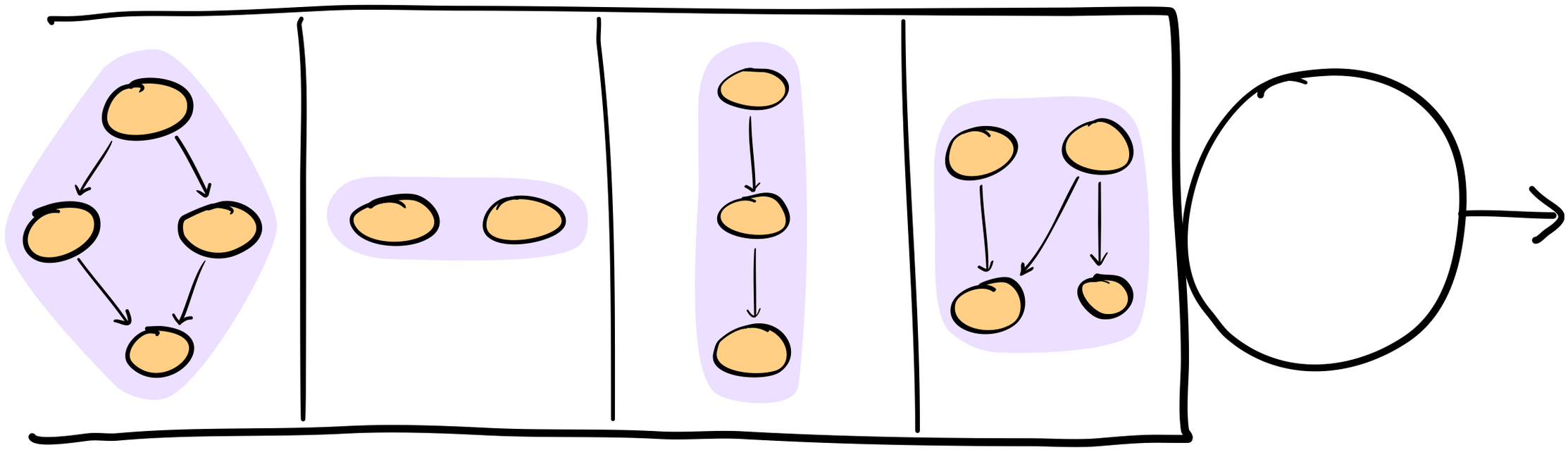


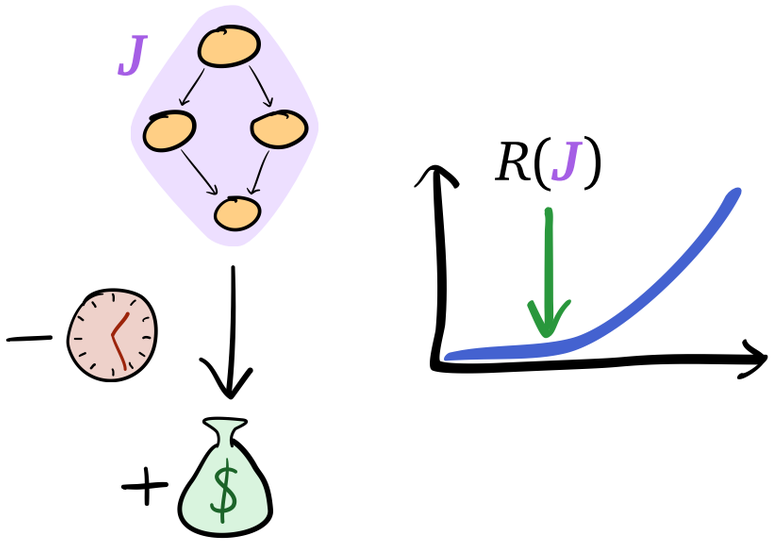
Single-job profit easily generalizes

Obstacles:

- Hard to compute profit
- Fair reward policy not always optimal

Generalizing to Multitask





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this talk

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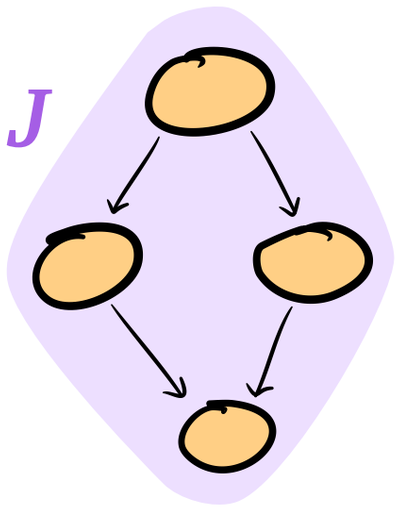
Splitting Big Jobs



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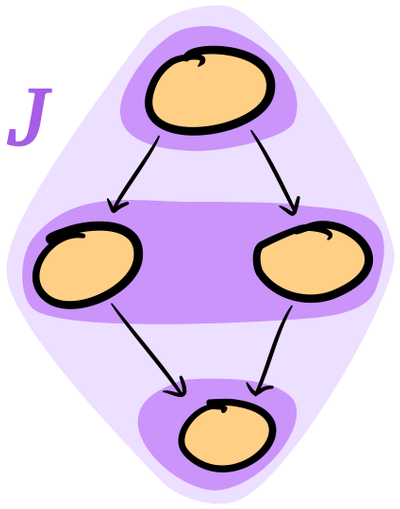

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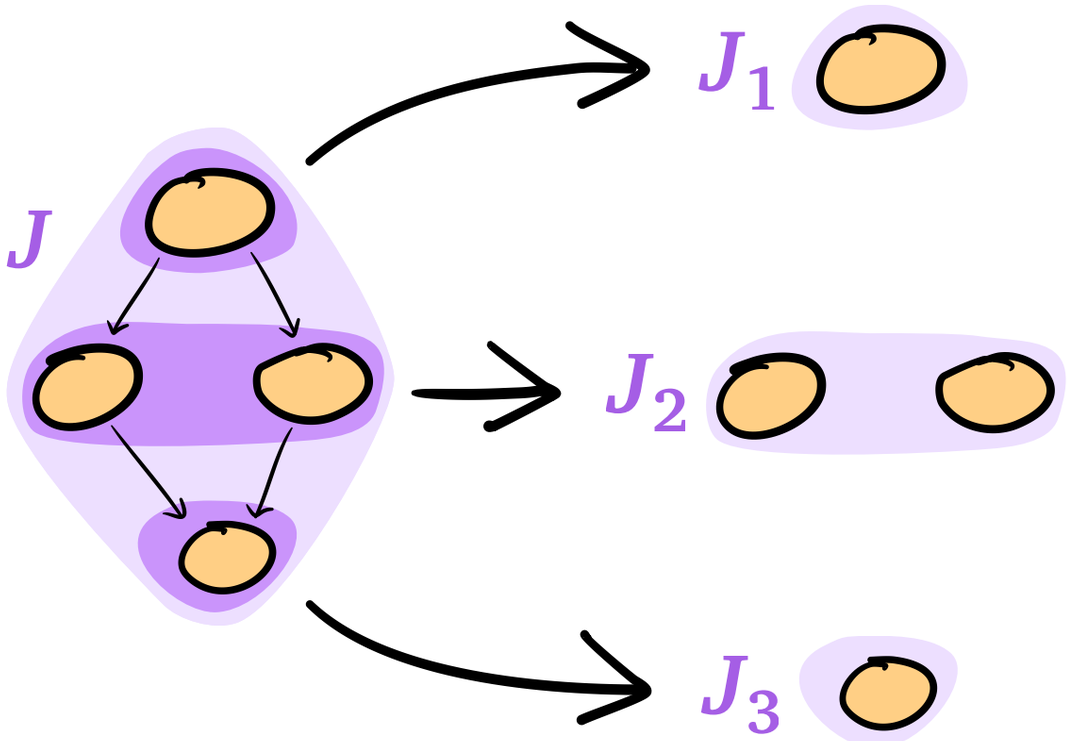

phases

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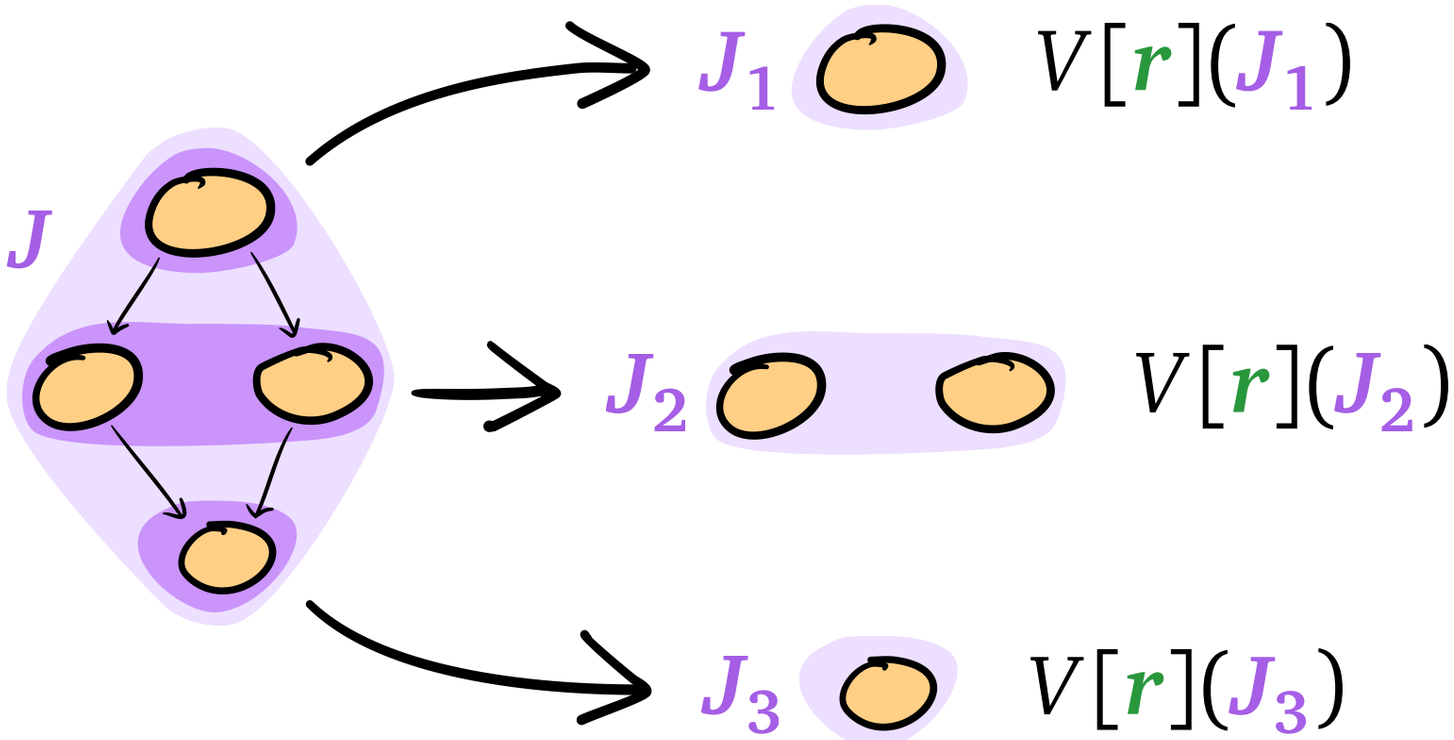

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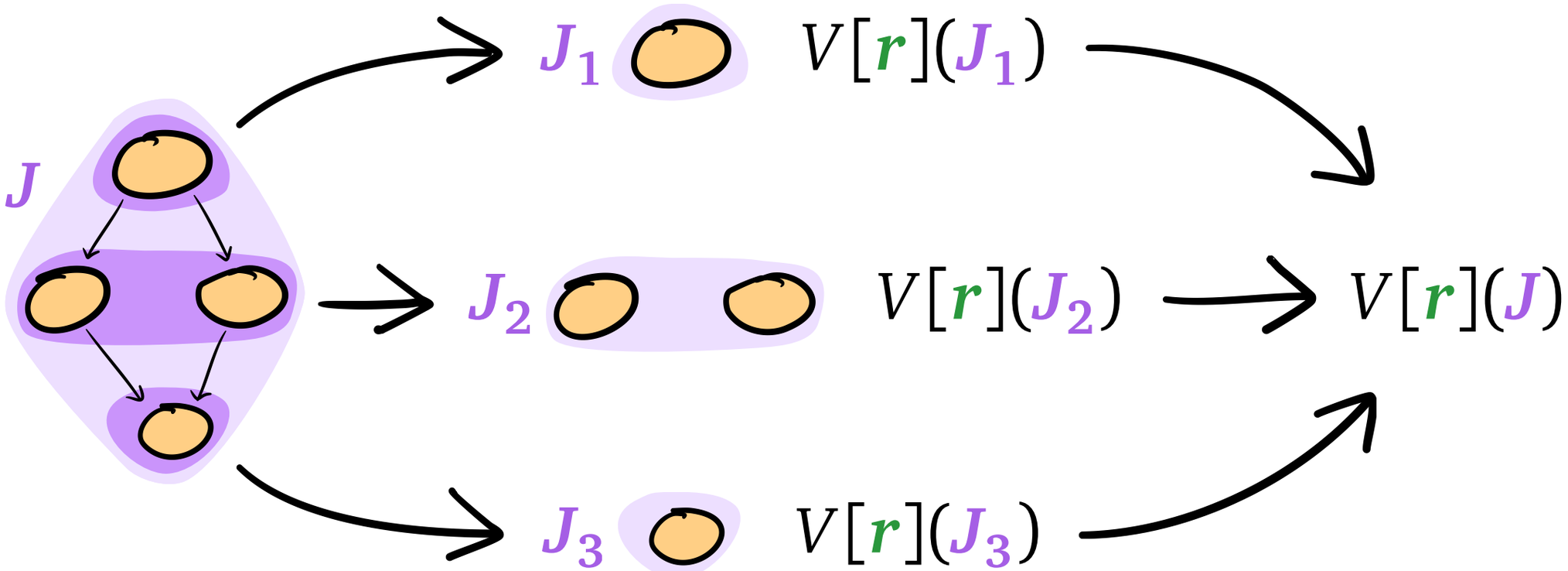
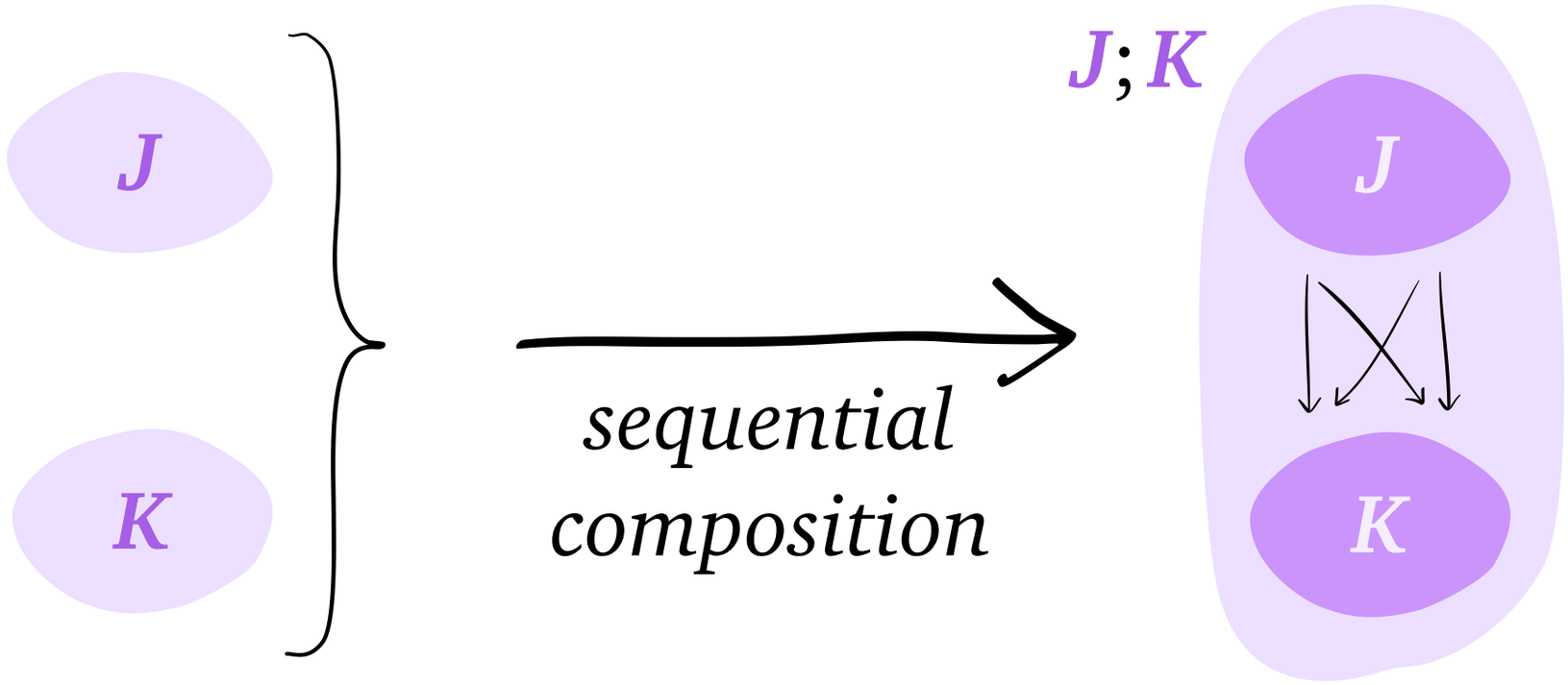


Composition Law



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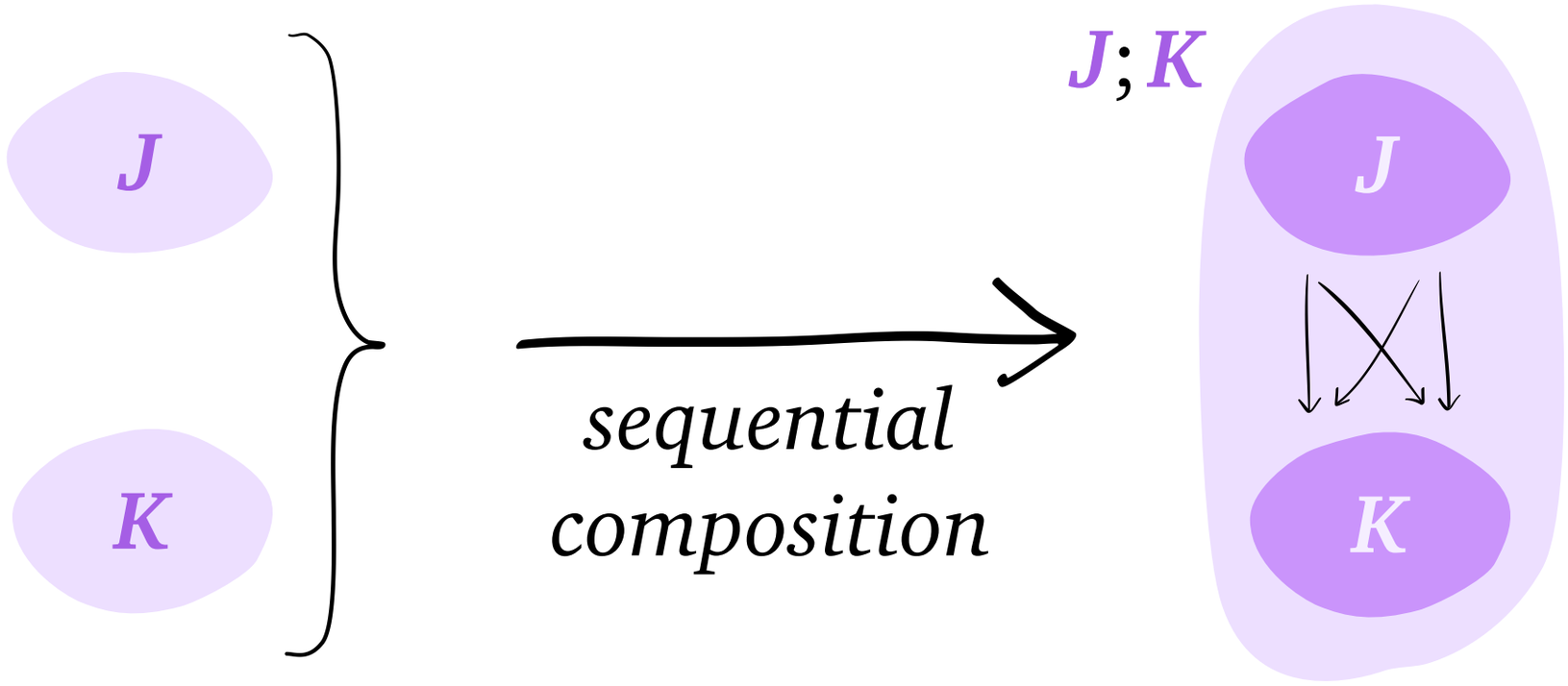

Theorem (Composition Law):

$$V[\boldsymbol{r}](\boldsymbol{J};\boldsymbol{K}) = V[V[\boldsymbol{r}](\boldsymbol{K})](\boldsymbol{J})$$

Proof Idea

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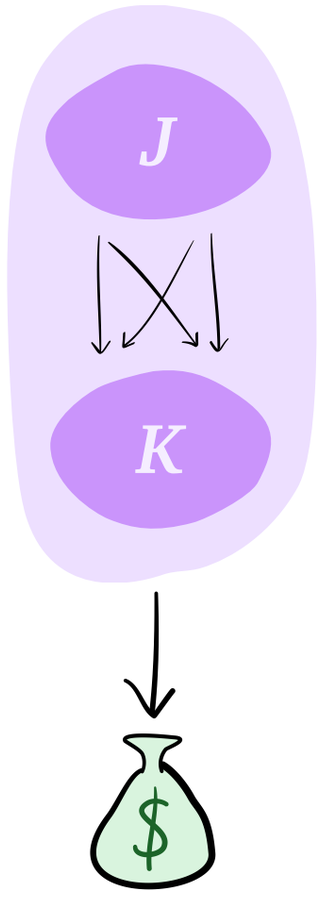

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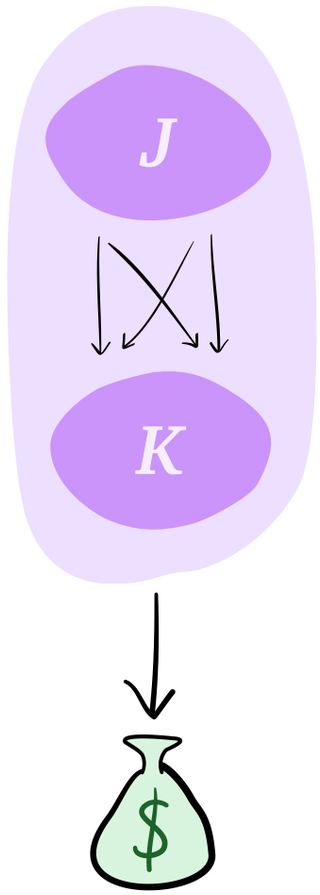

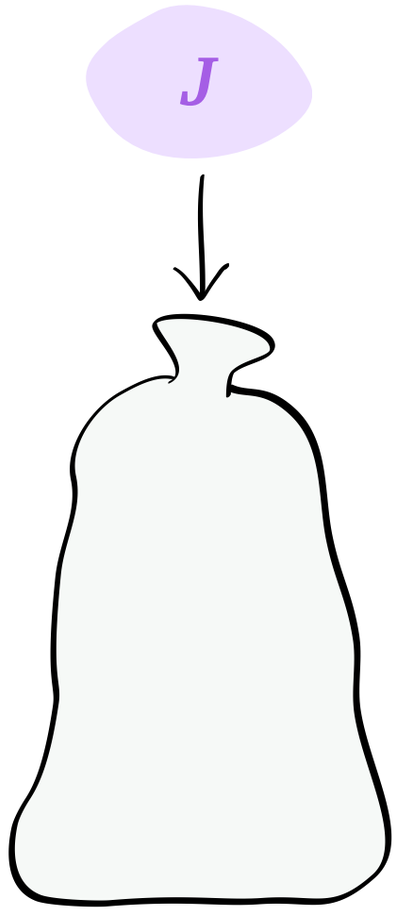

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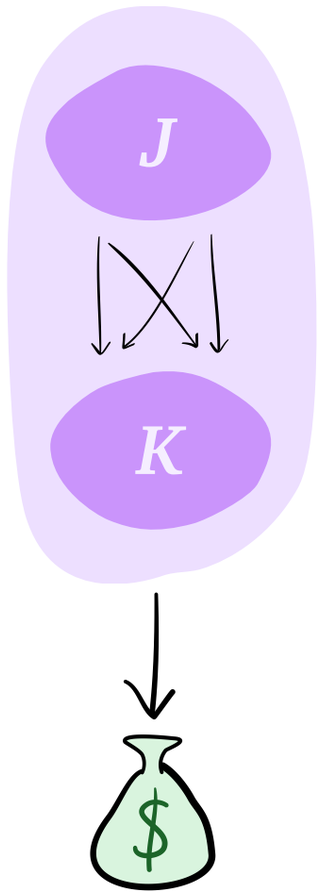

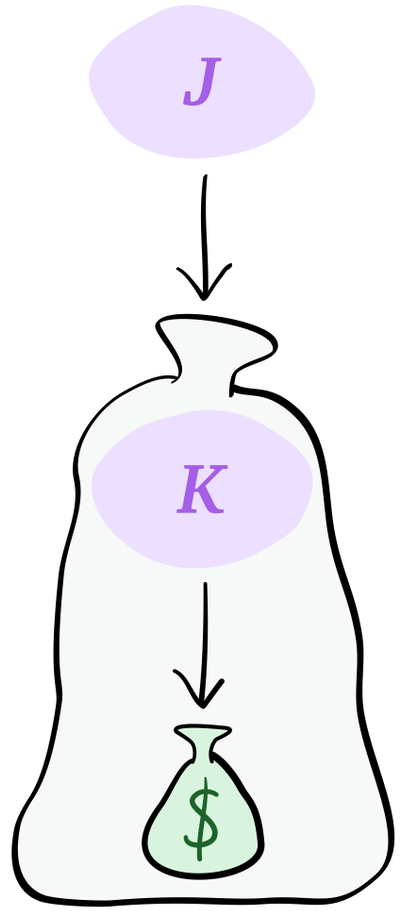

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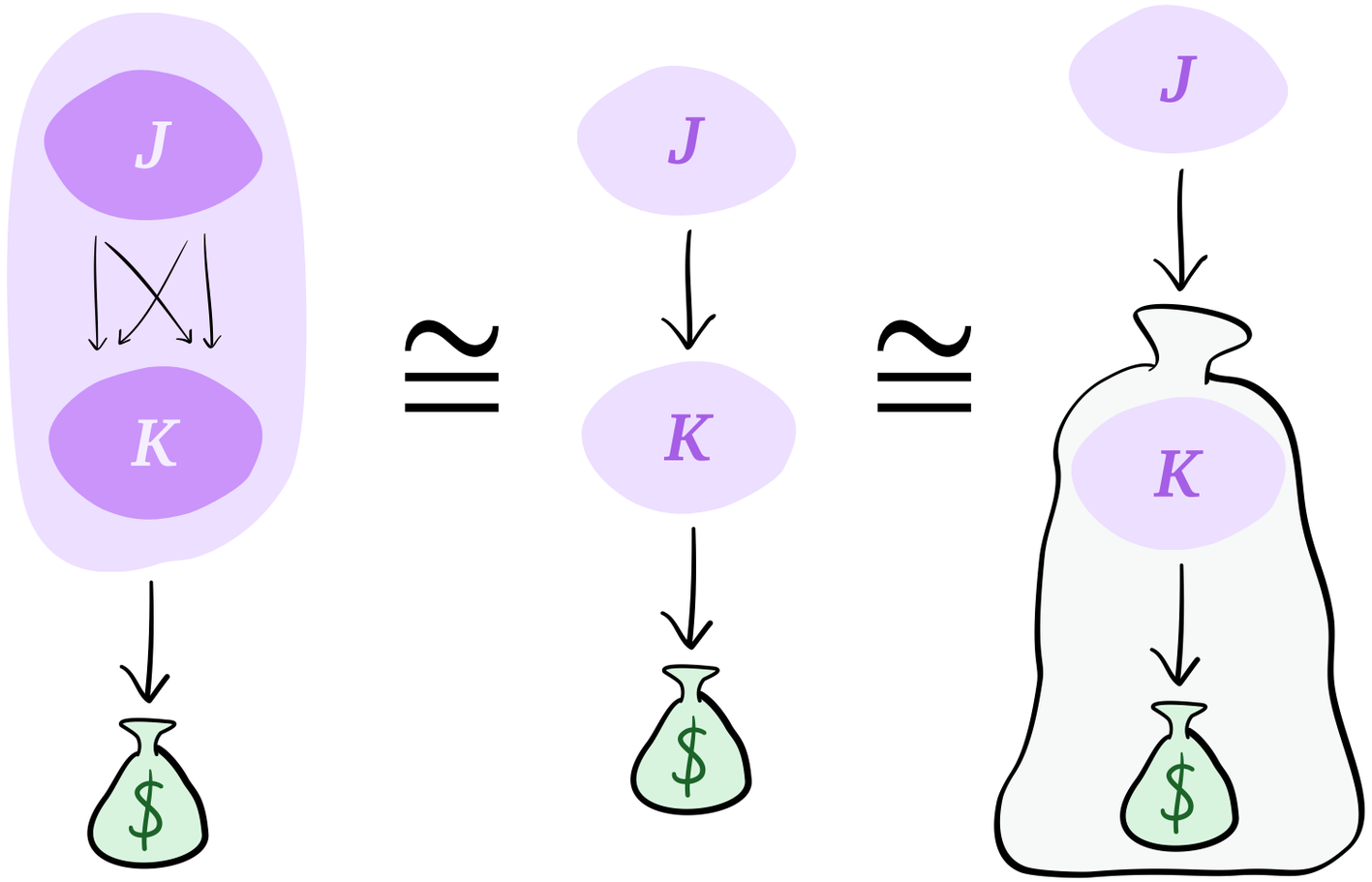
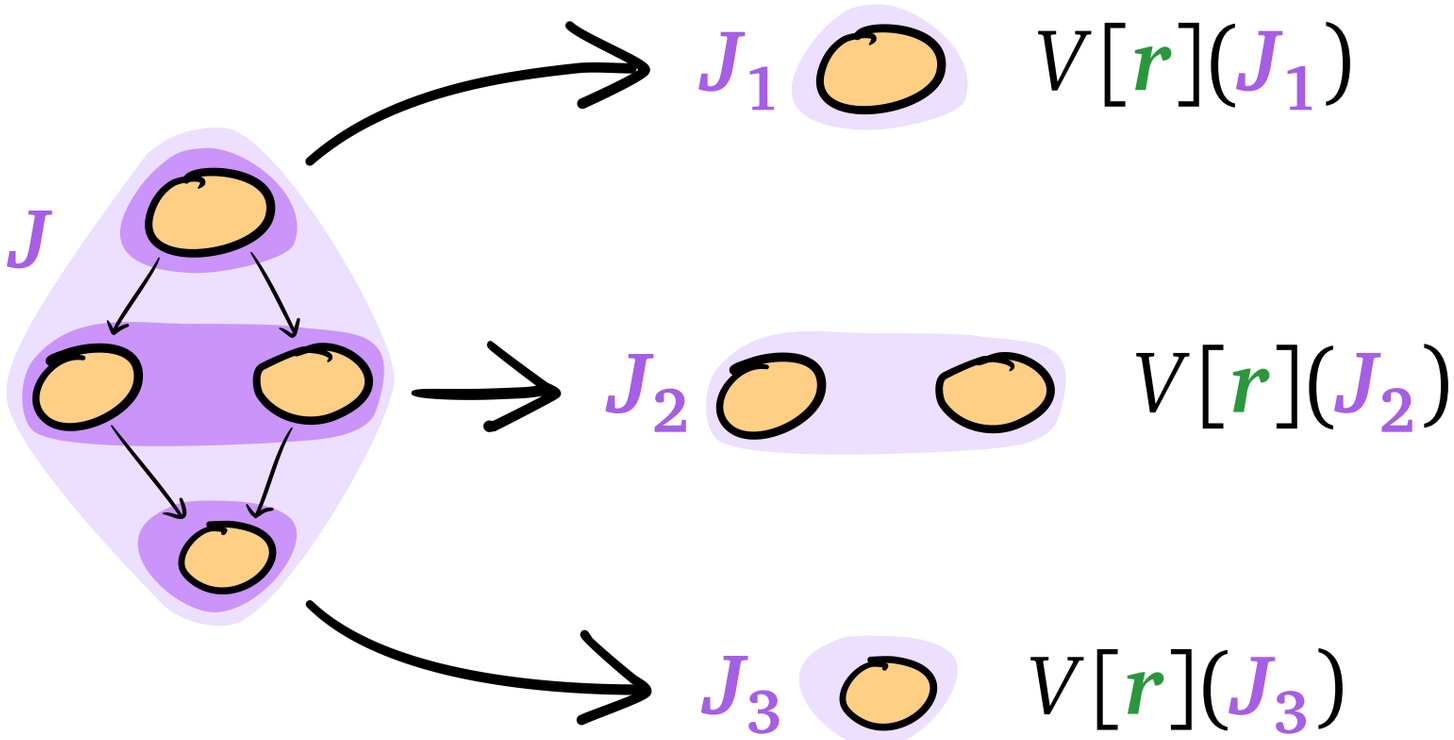


Applying Composition



phases

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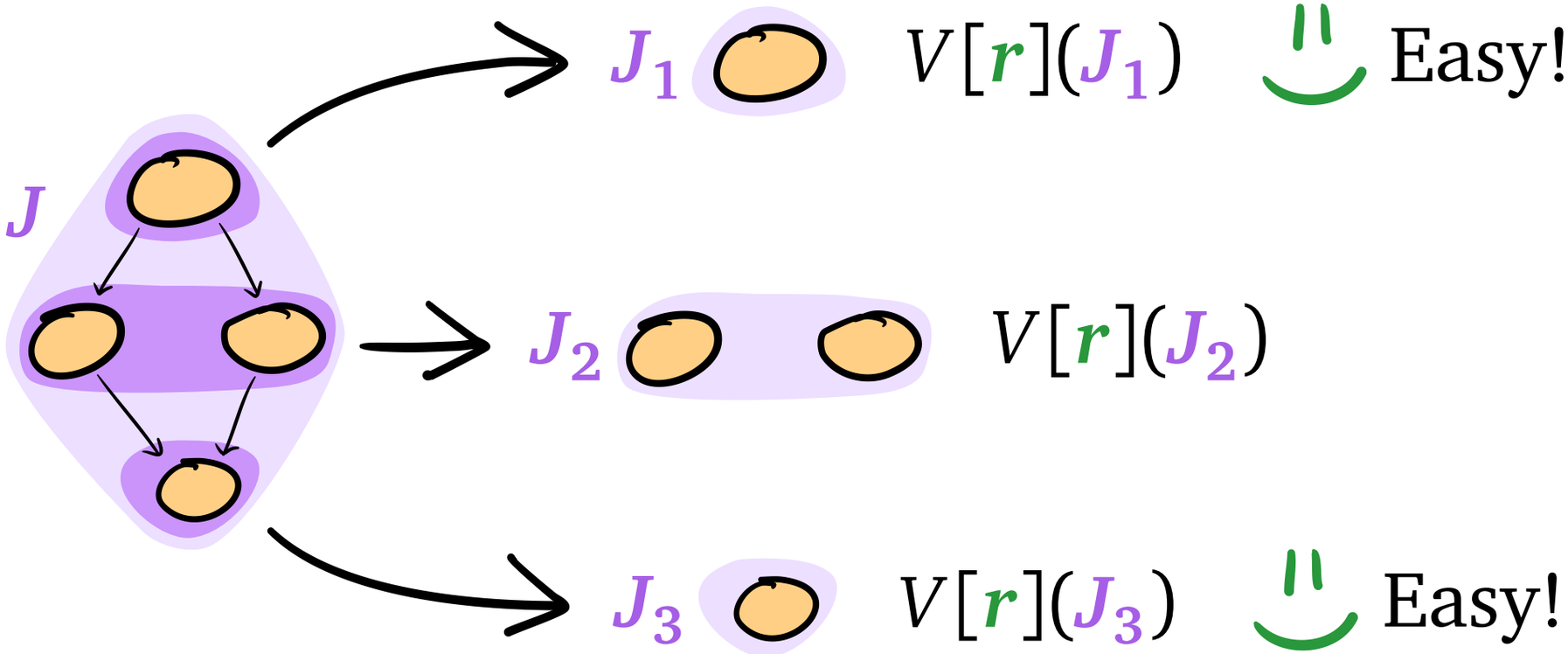

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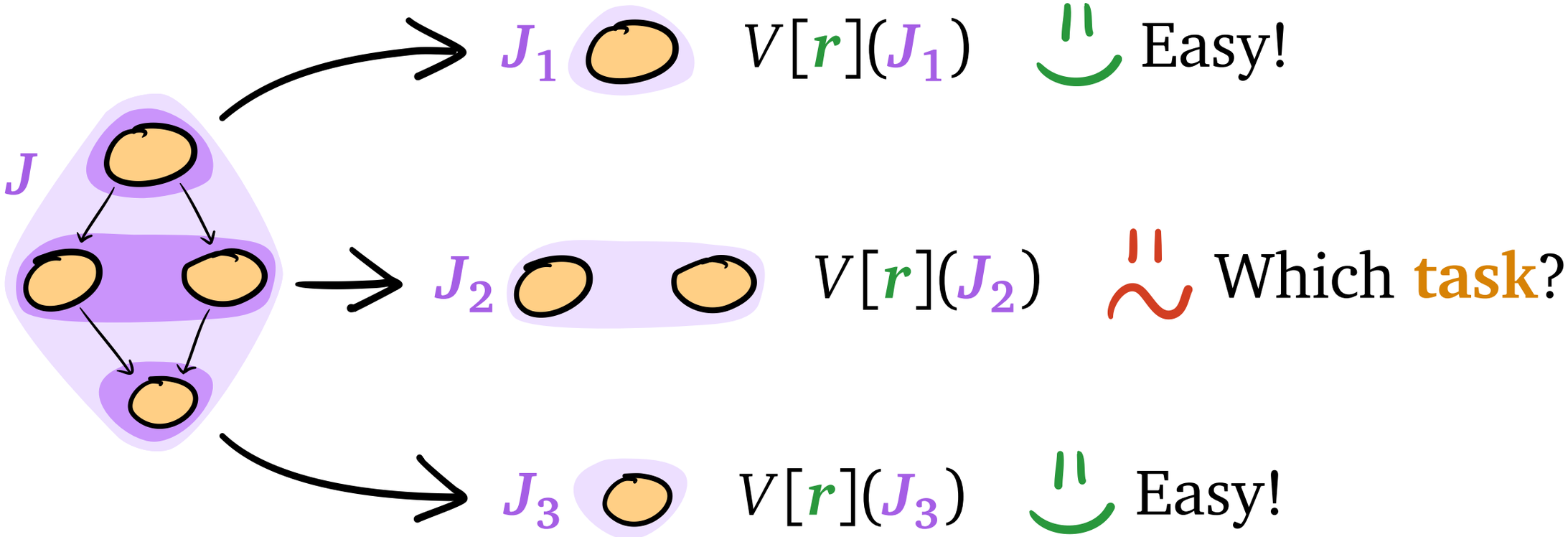
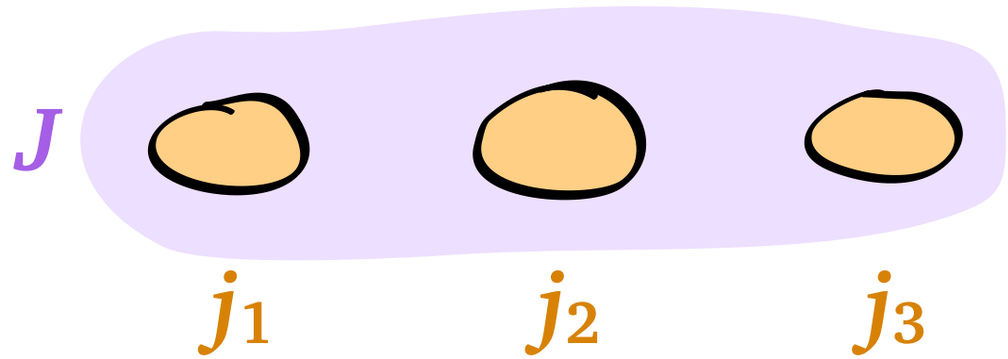


Autopiloting Law



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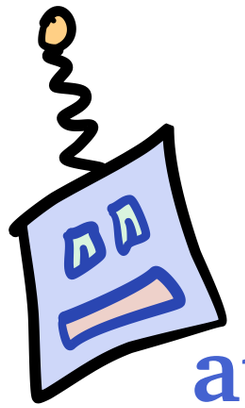
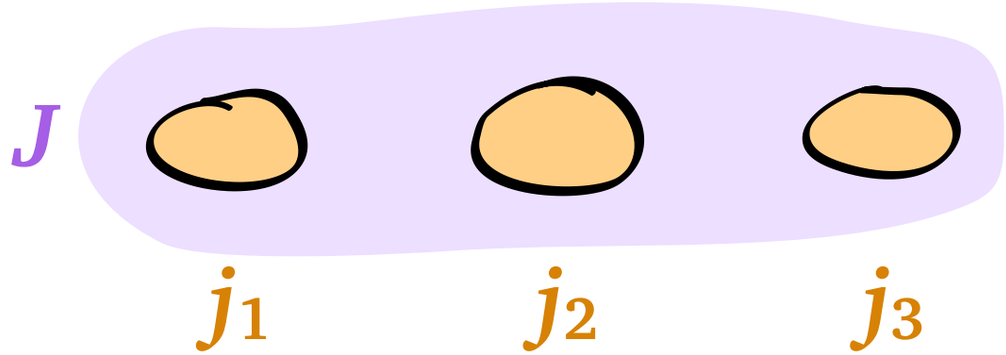


autopilot



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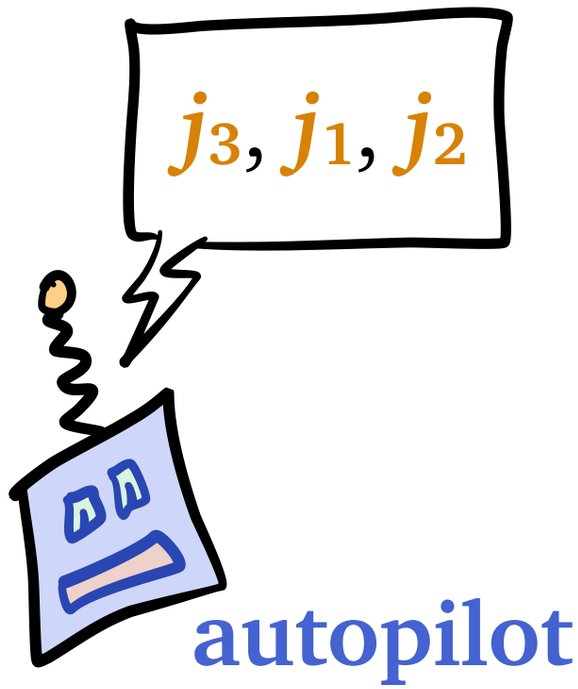

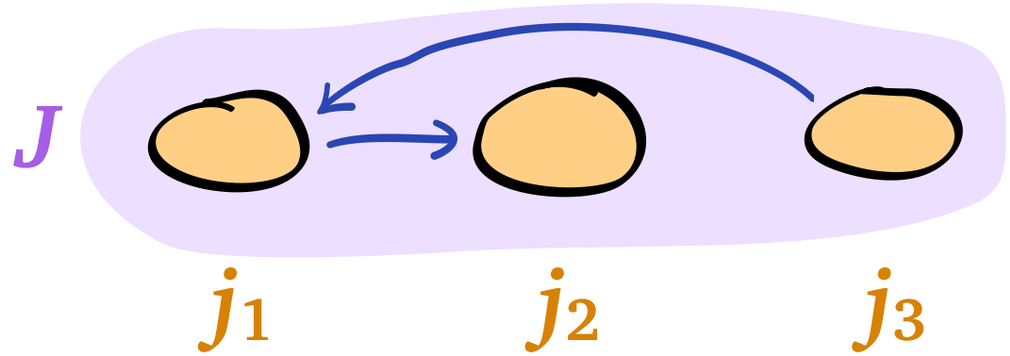

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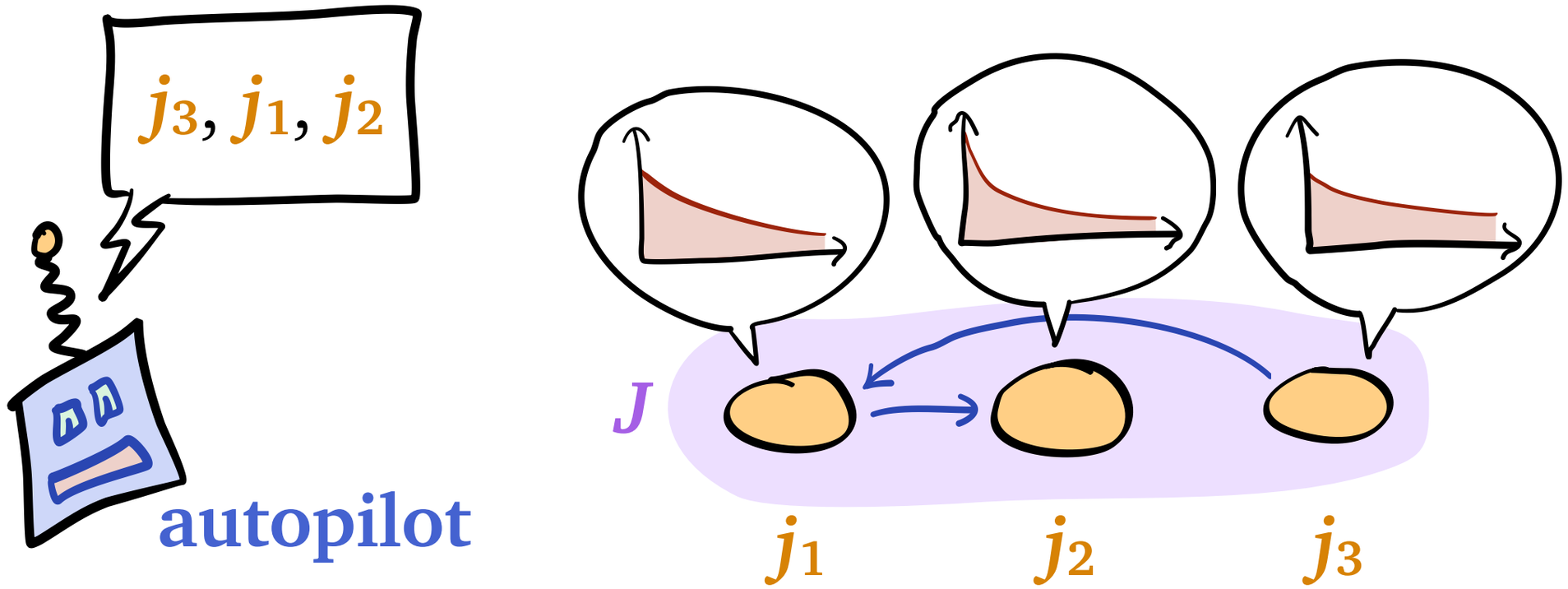
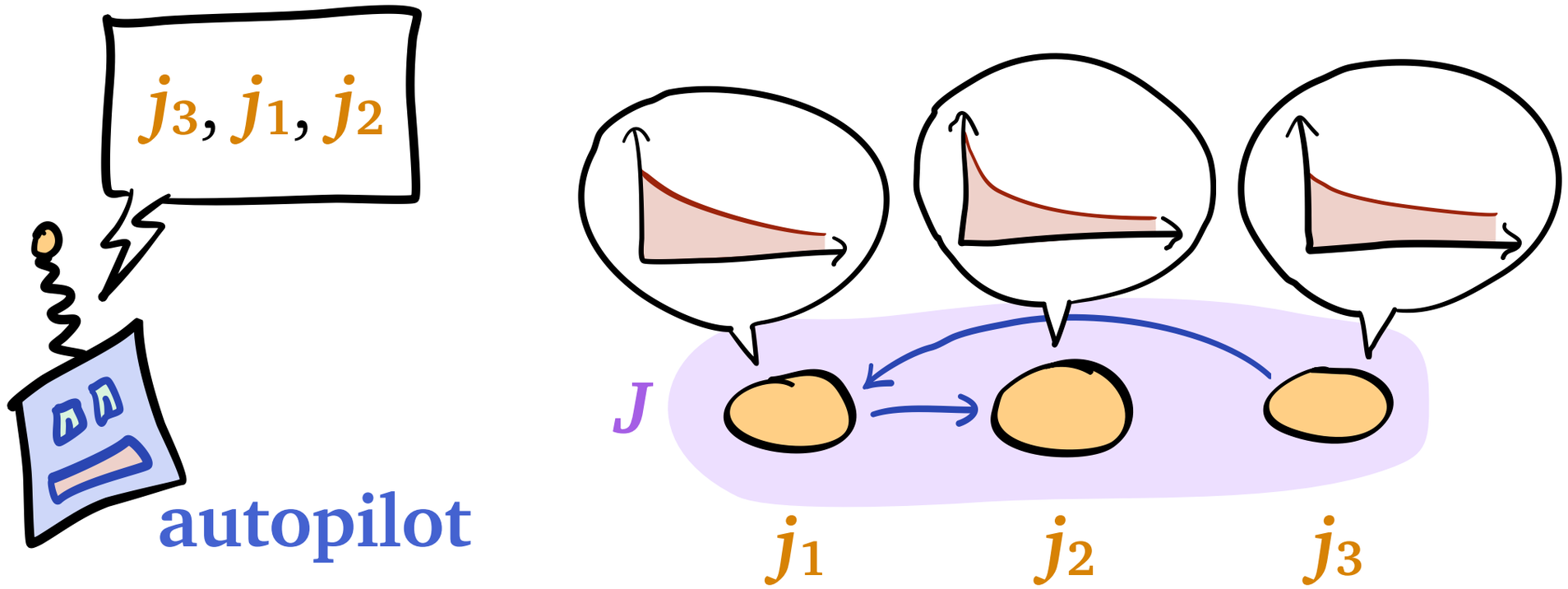


Theorem (Autopiloting Law): any fully parallel job with **Pareto** tasks of same α has an **autopilot**

Autopiloting Law



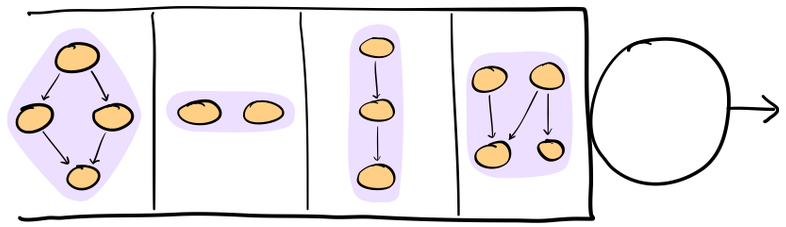
Theorem (Autopiloting Law): any fully parallel job with **Pareto** tasks of same α has an **autopilot**

- serves task that is *longest in expectation*

Summary

Summary

Goal: multitask scheduling



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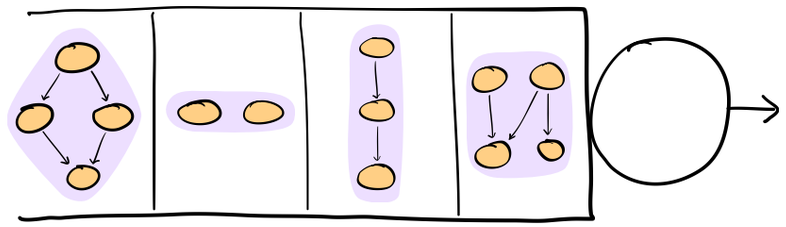
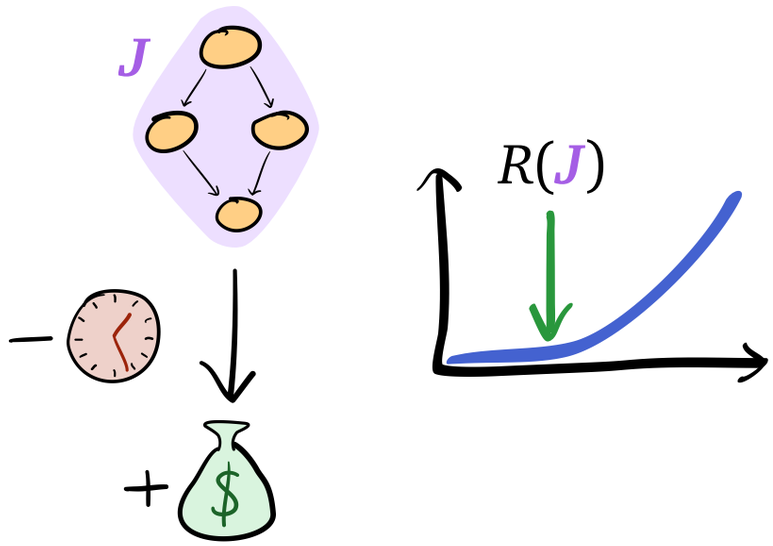


Approach: single-job profit



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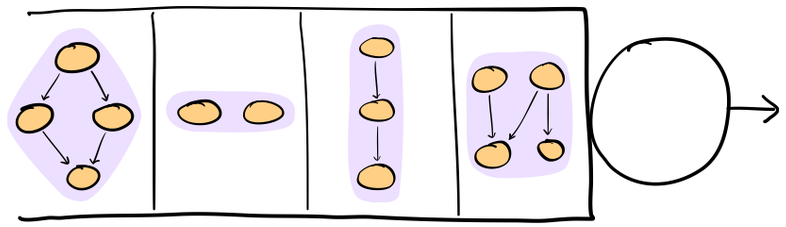
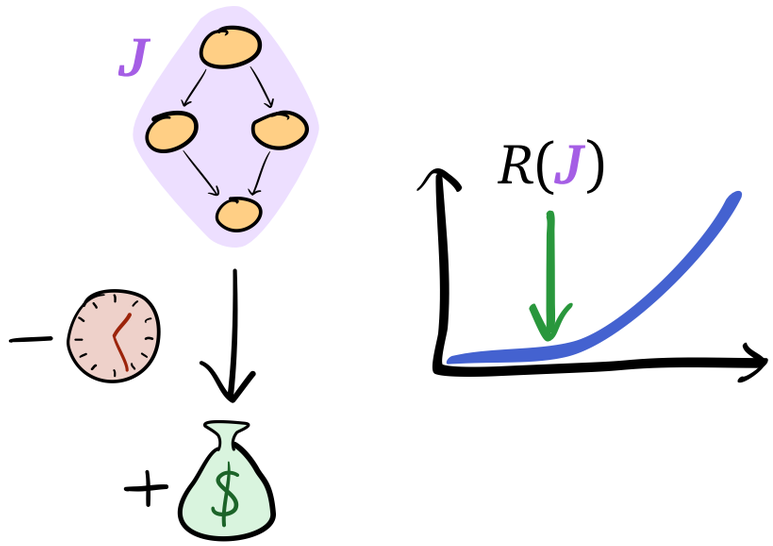


Obstacle: computation

Approach: single-job profit



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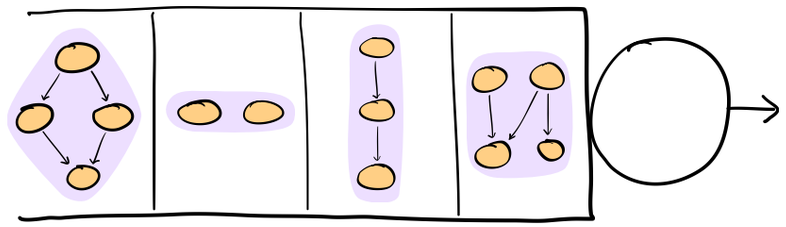

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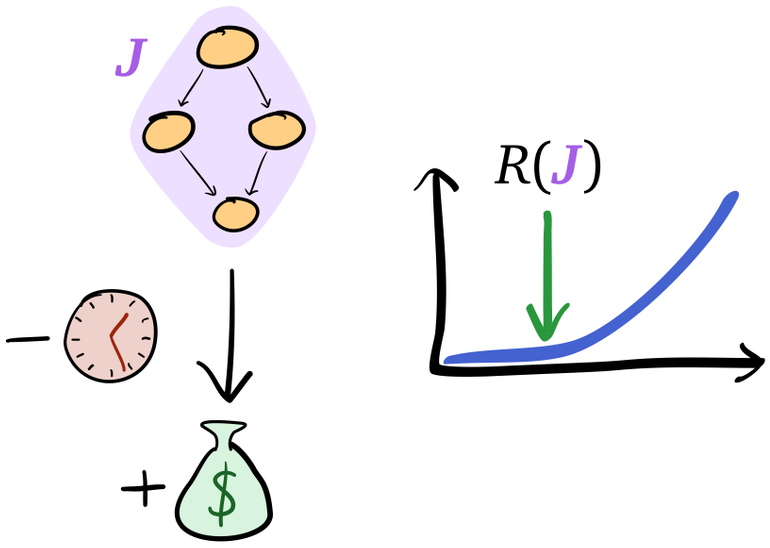
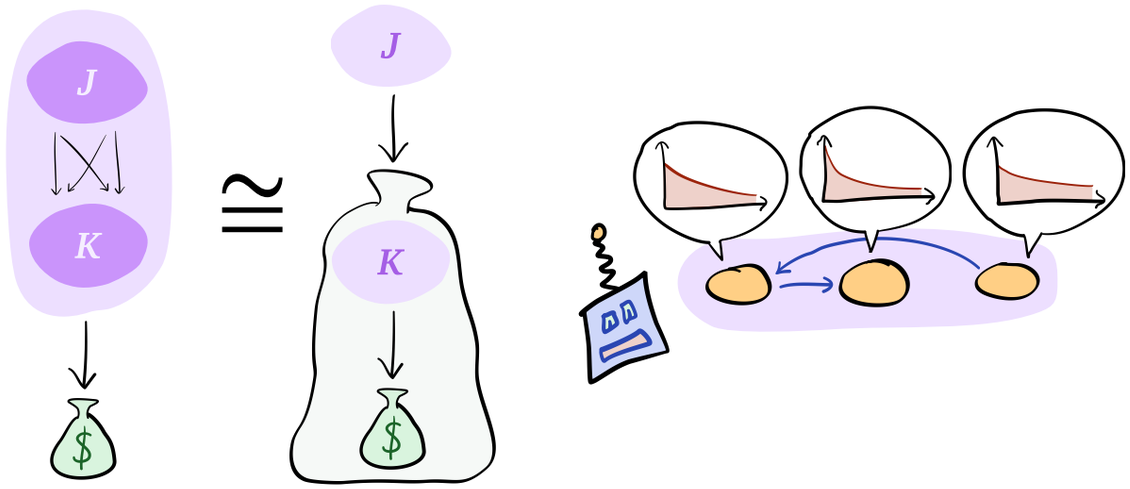


Obstacle: computation

Solutions: composition law
autopiloting law



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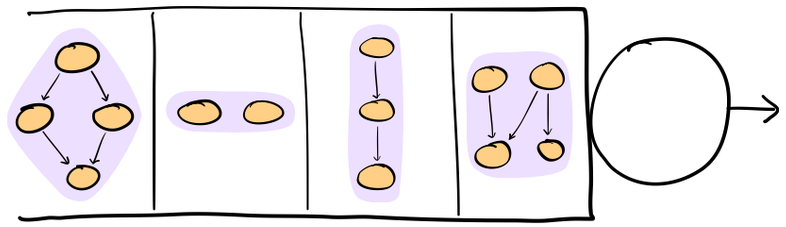
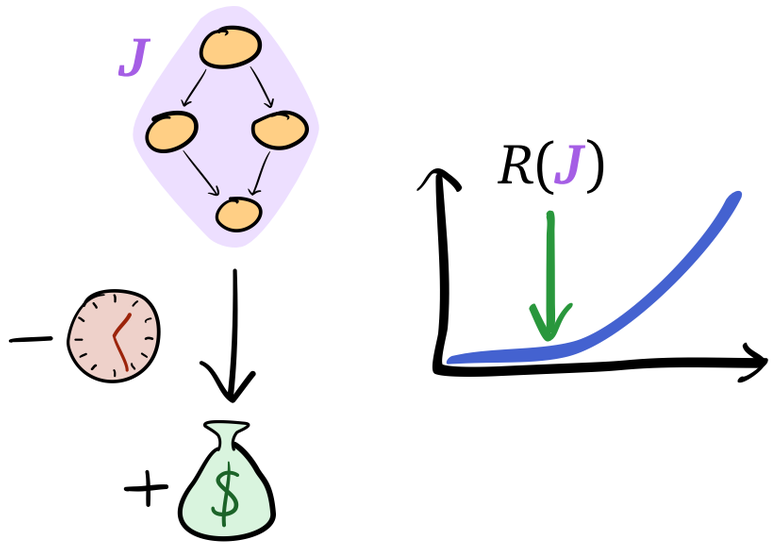
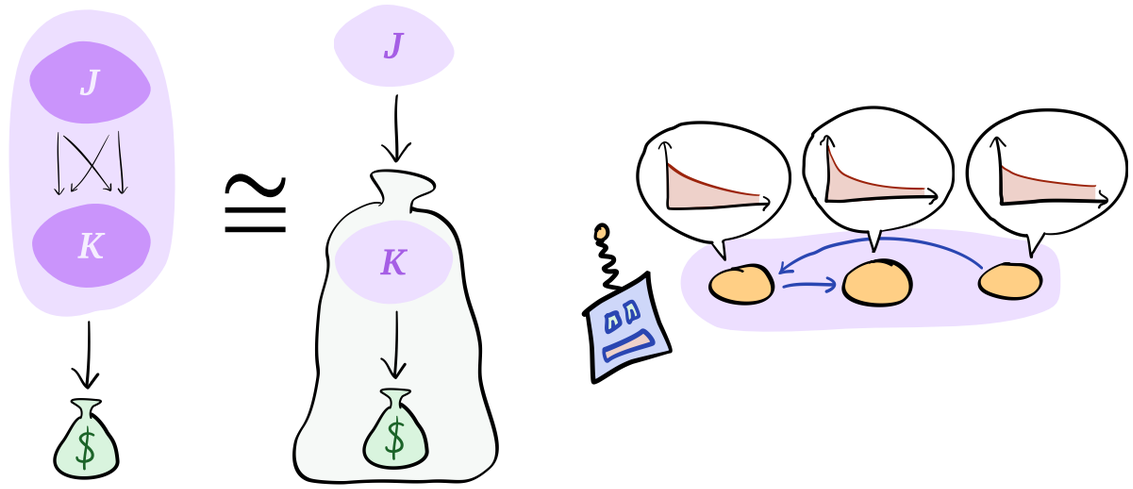


Approach: single-job profit



Obstacle: computation

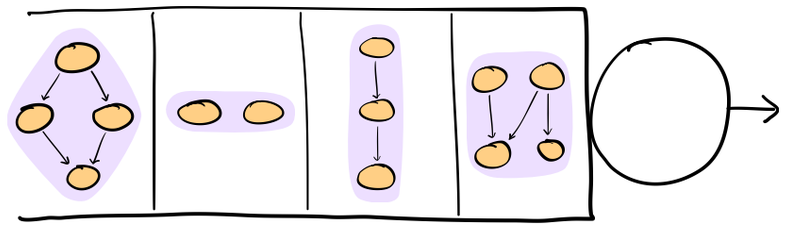
Solutions: composition law
autopiloting law



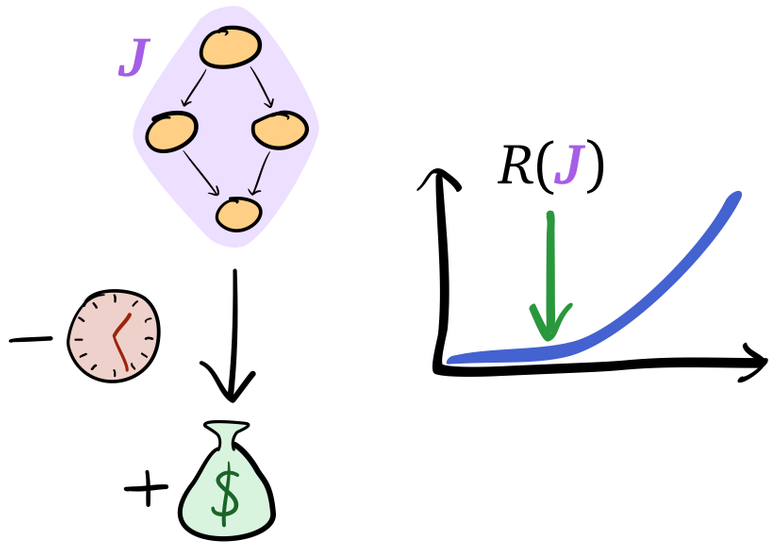
Obstacle: proving optimality

Summary

Goal: multitask scheduling

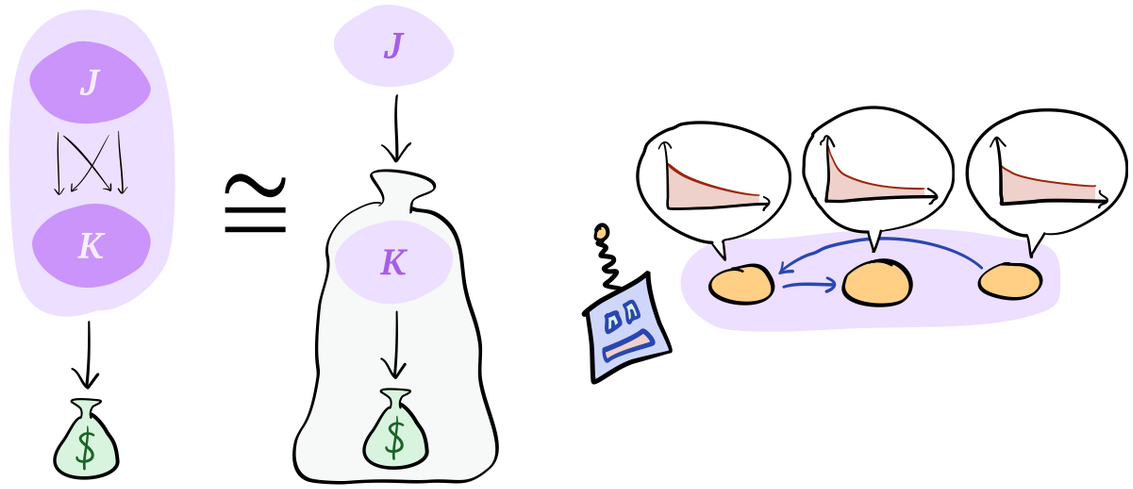


Approach: single-job profit



Obstacle: computation

Solutions: composition law
autopiloting law



Obstacle: proving optimality

Solution: autopiloting law