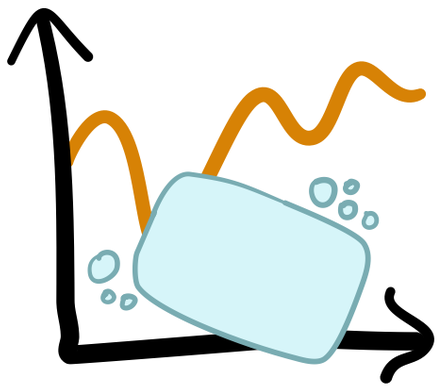
SOAP:

One Clean Analysis of All Age-Based Scheduling Policies

Ziv Scully
Mor Harchol-Balter
Alan Scheller-Wolf

Carnegie Mellon University



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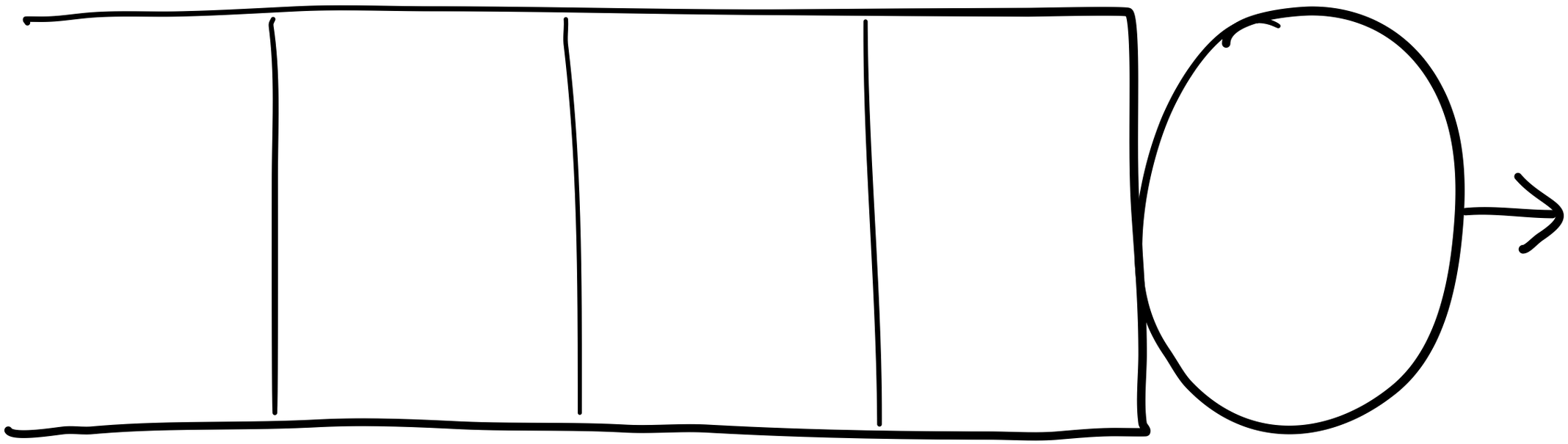

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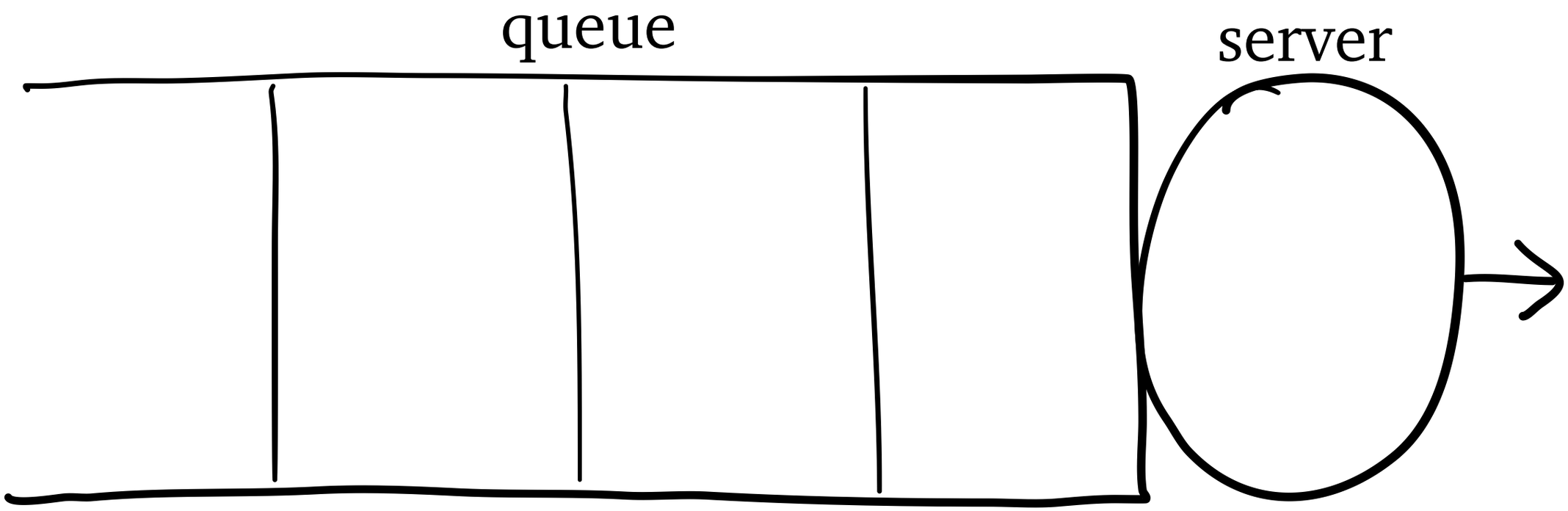

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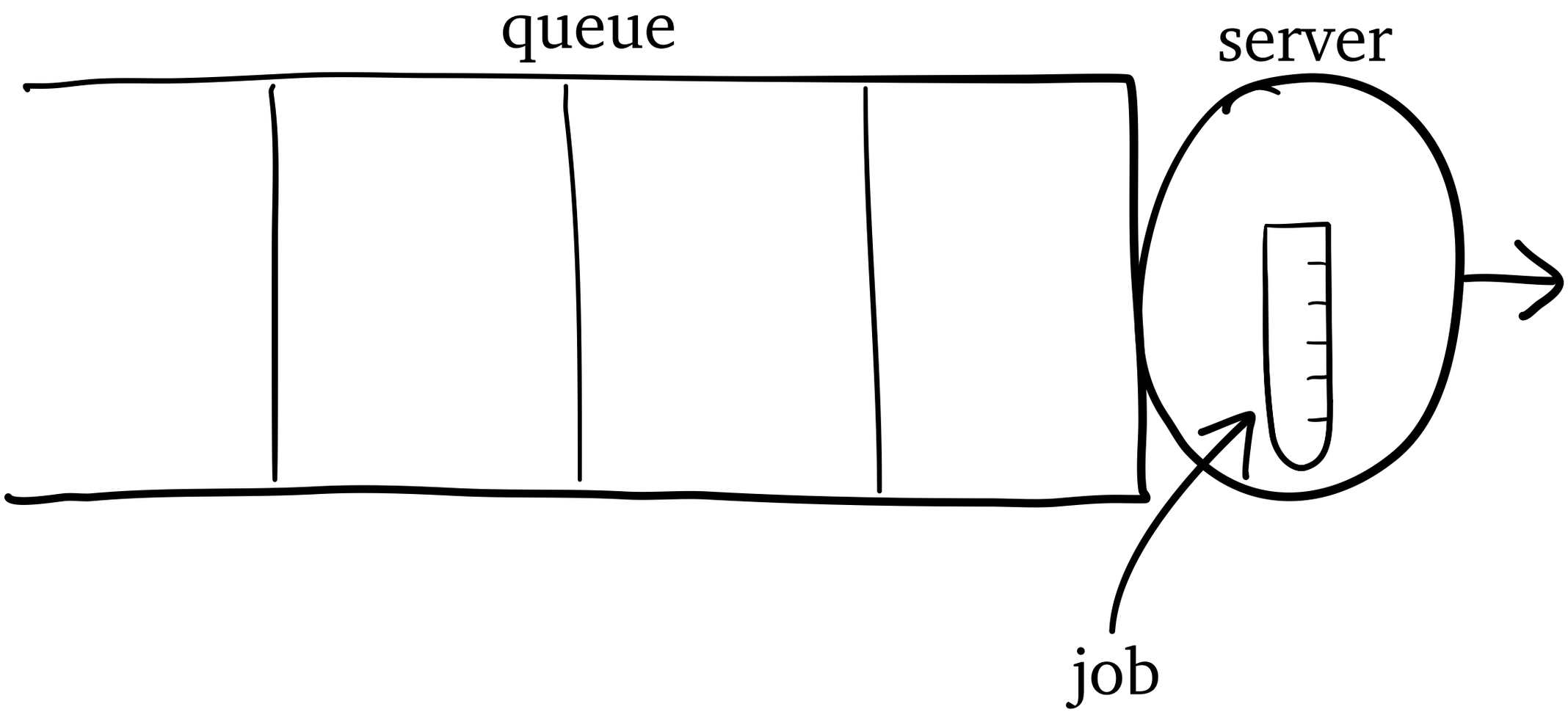

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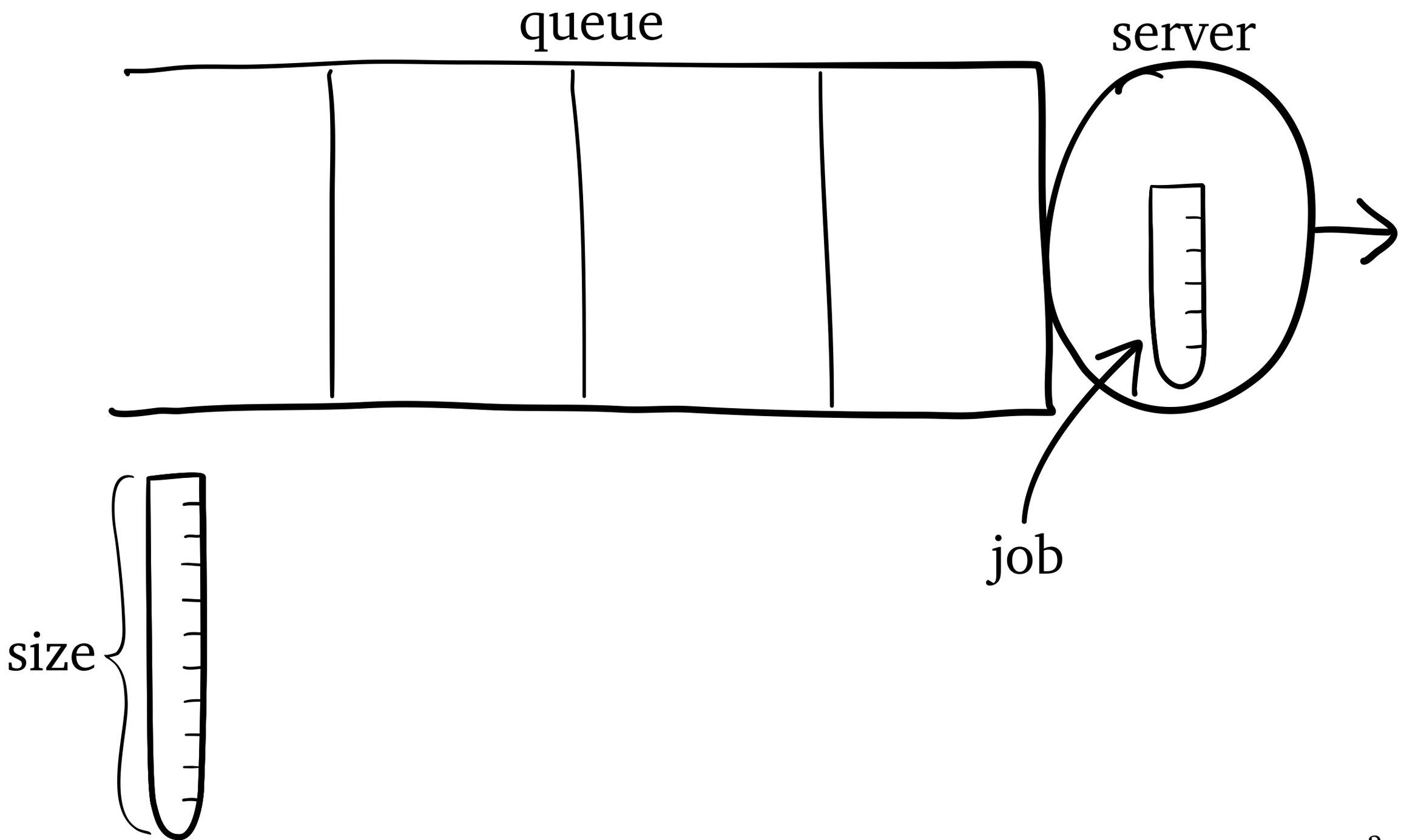

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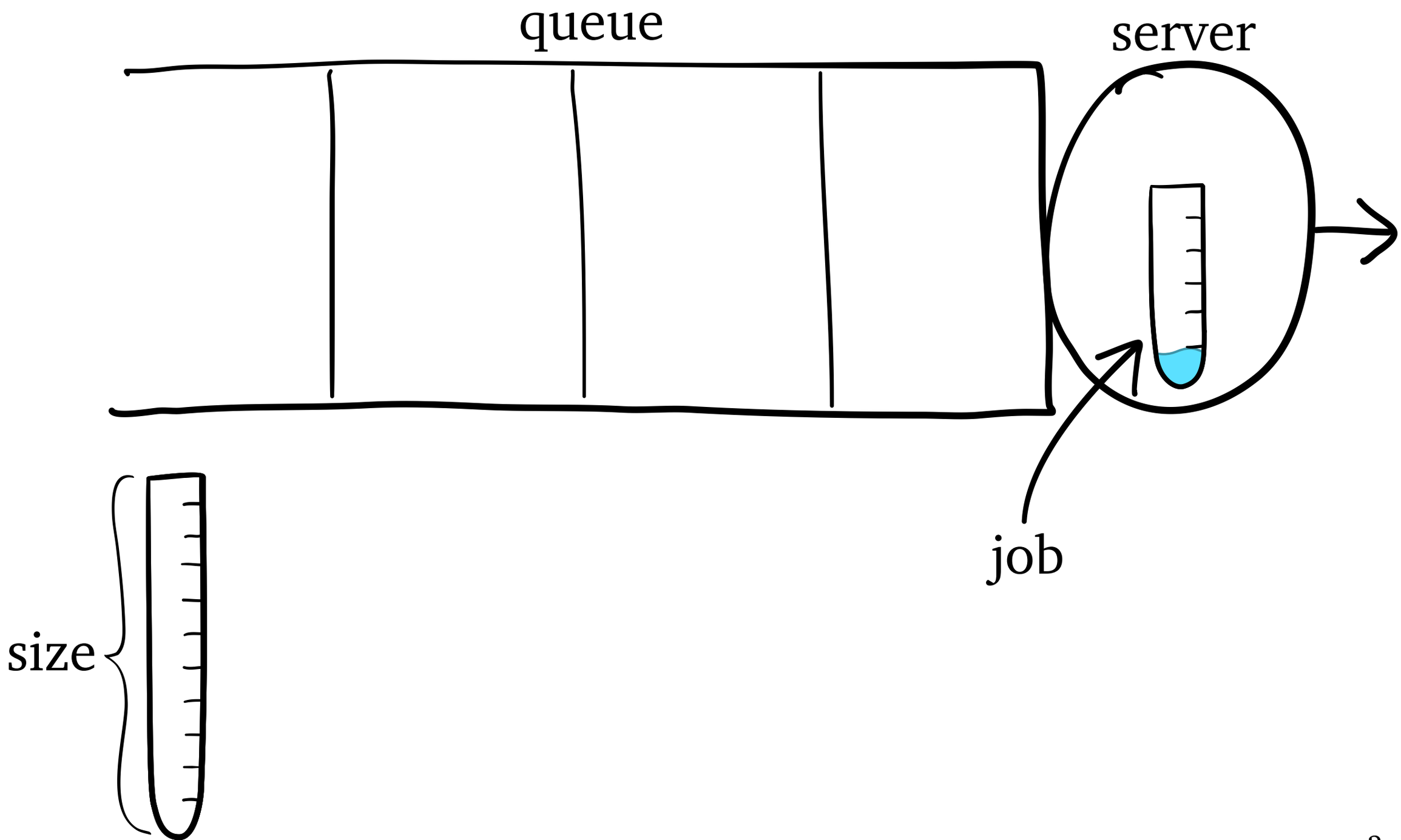

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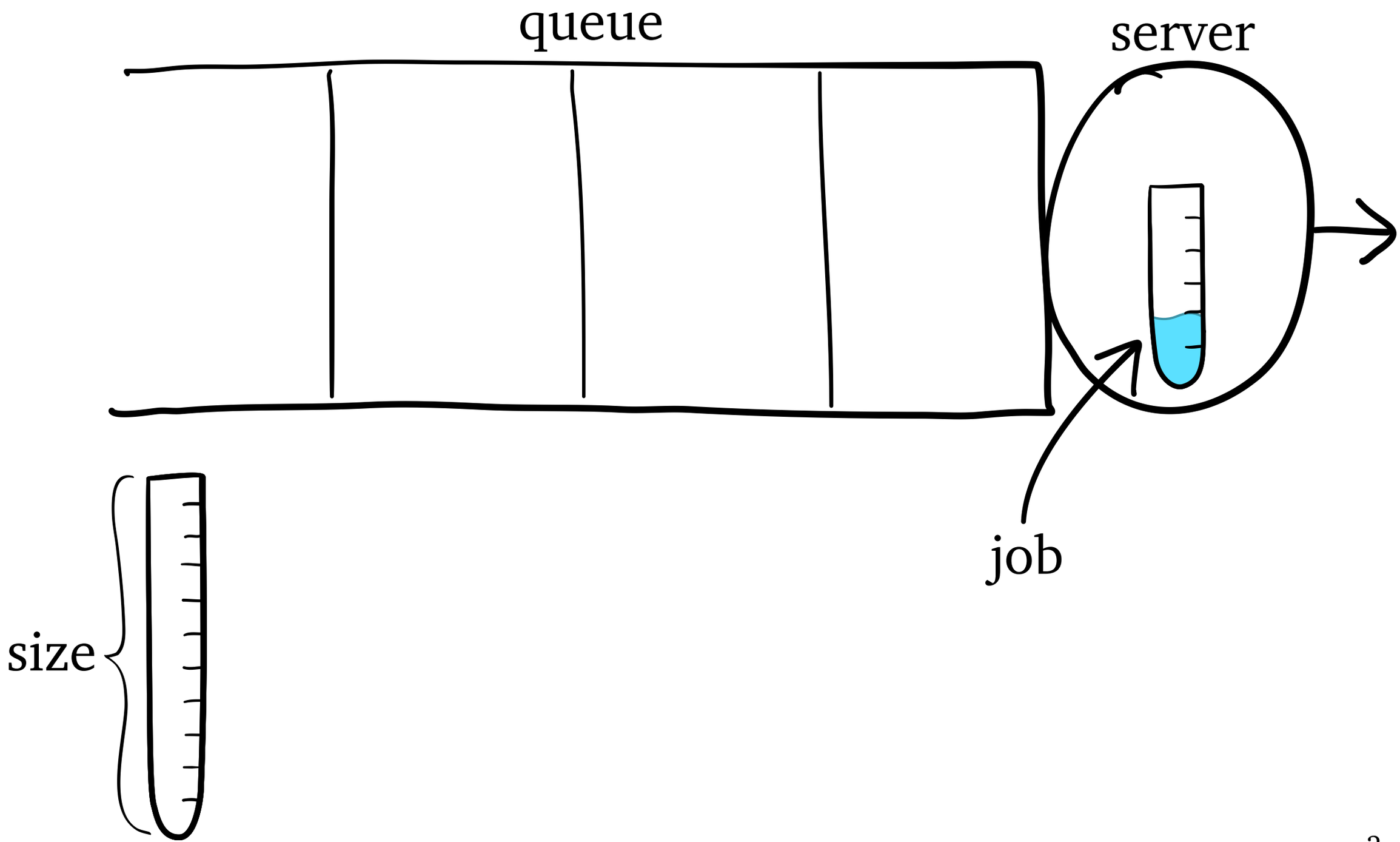

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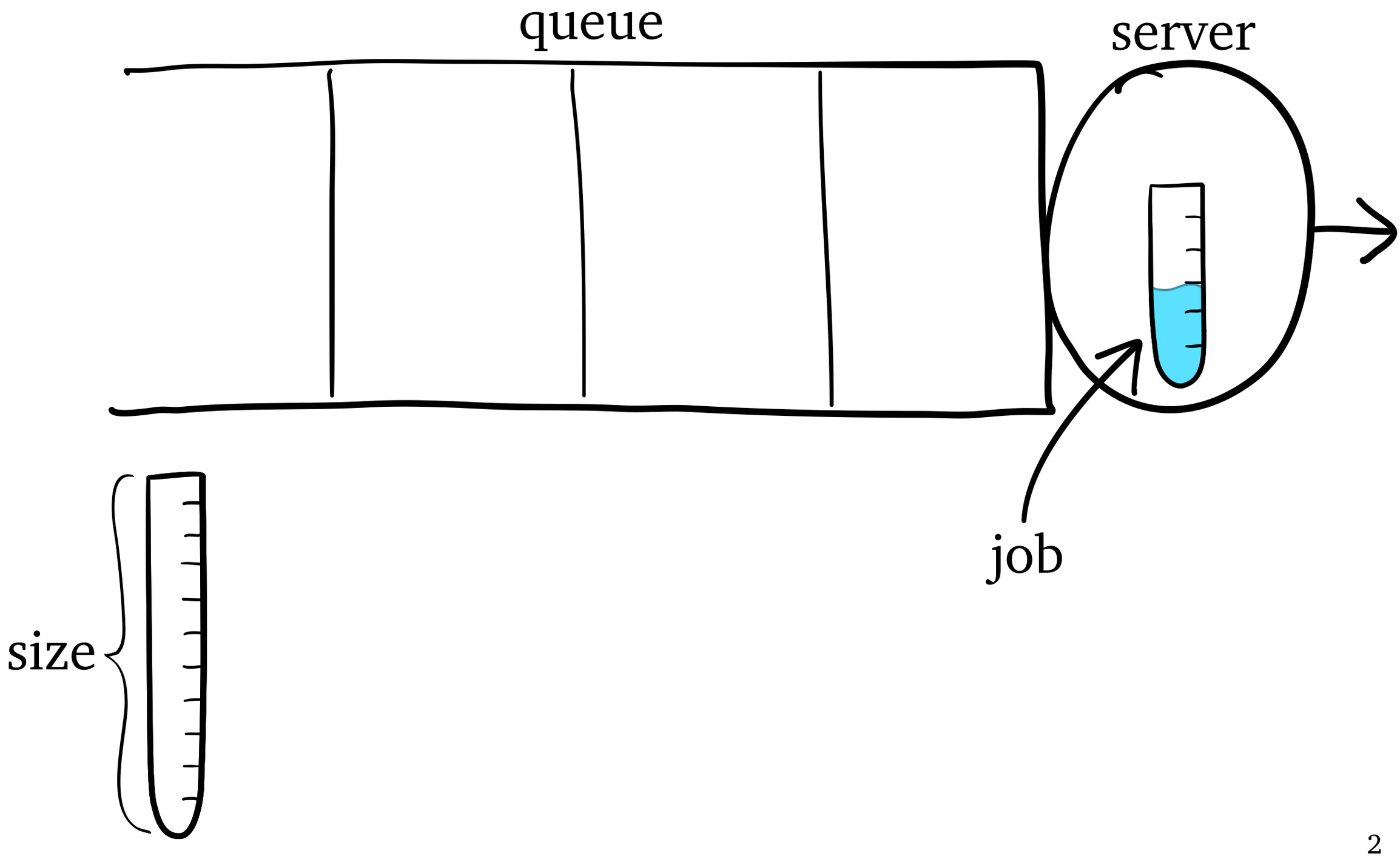

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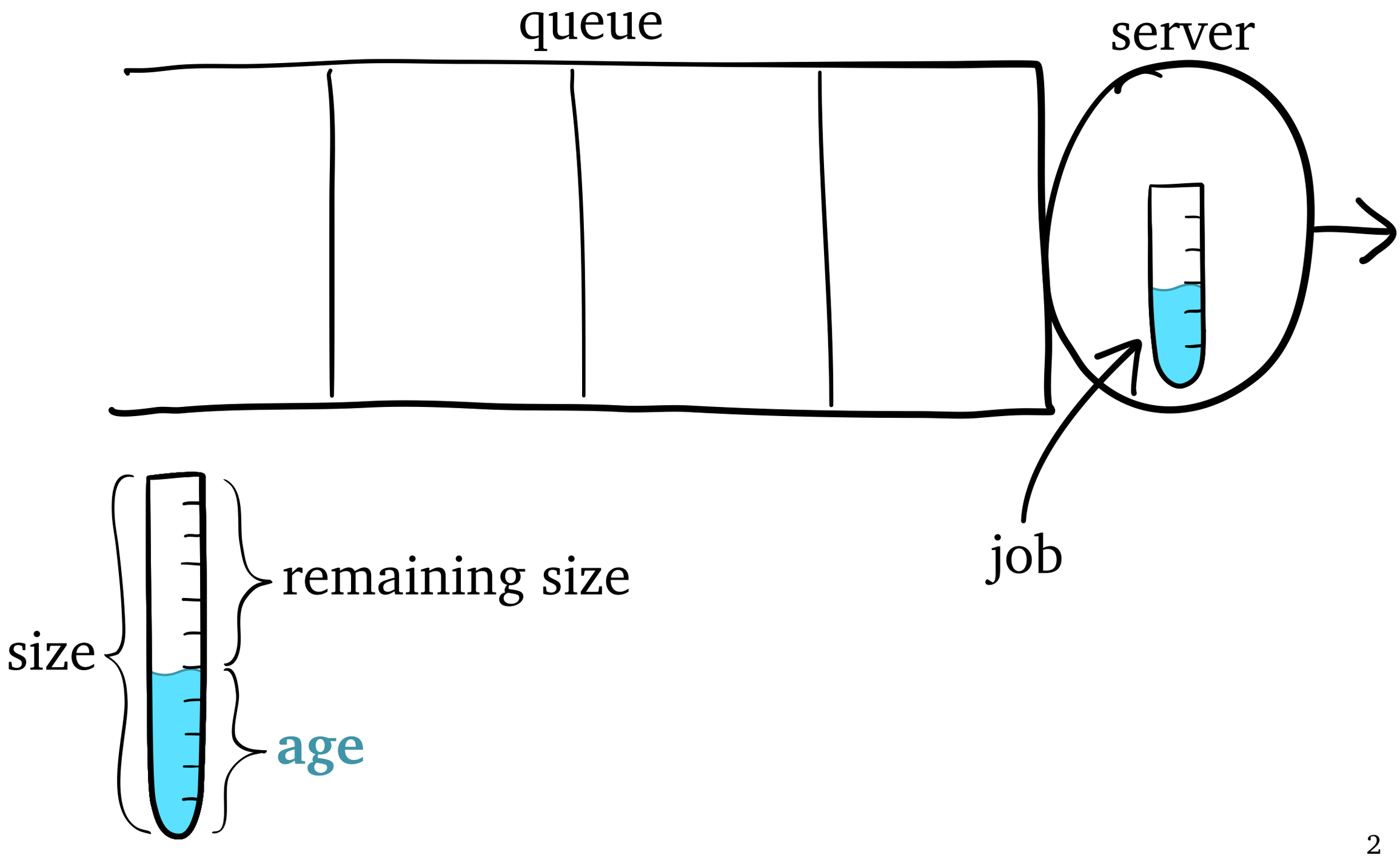

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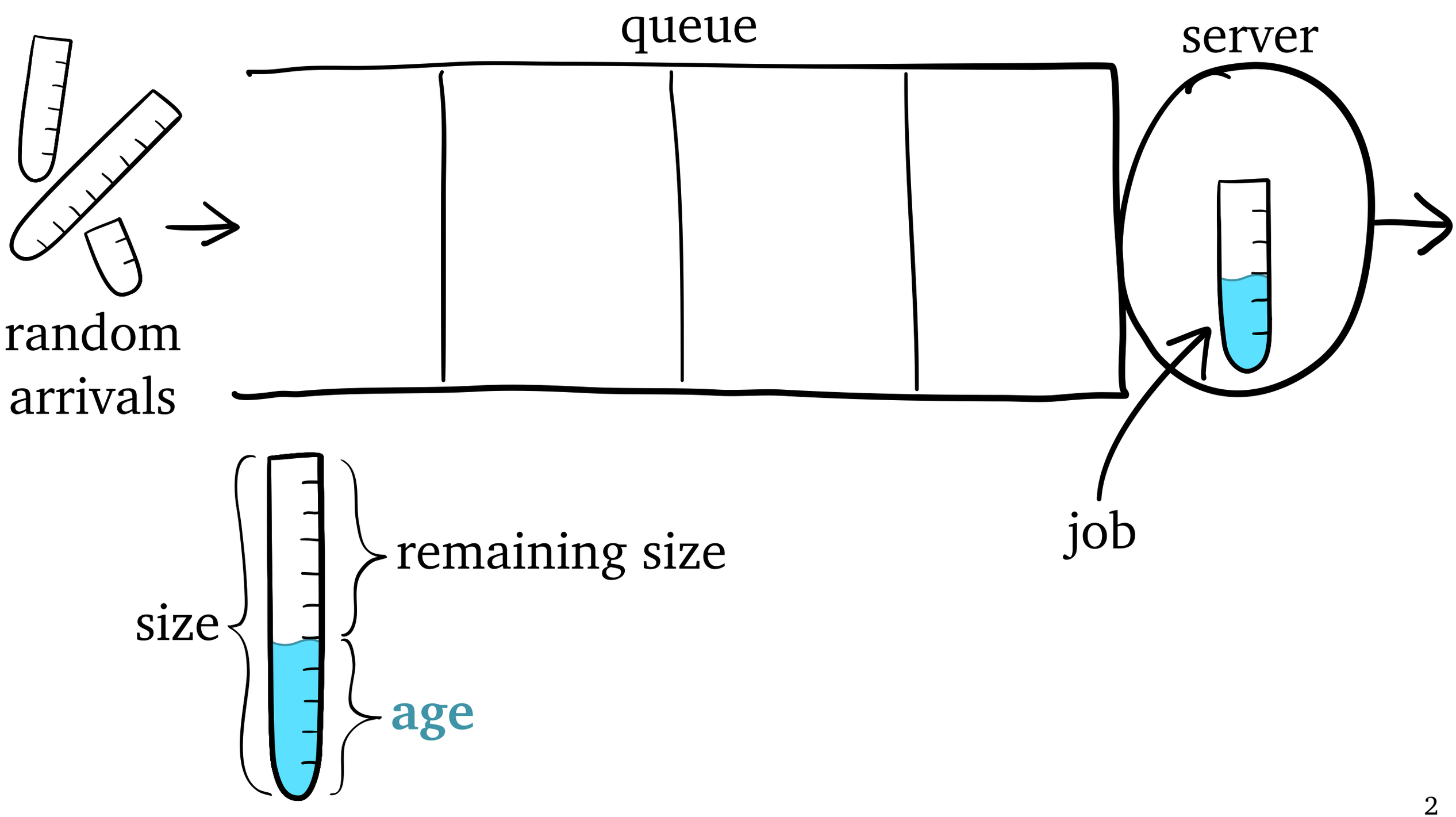

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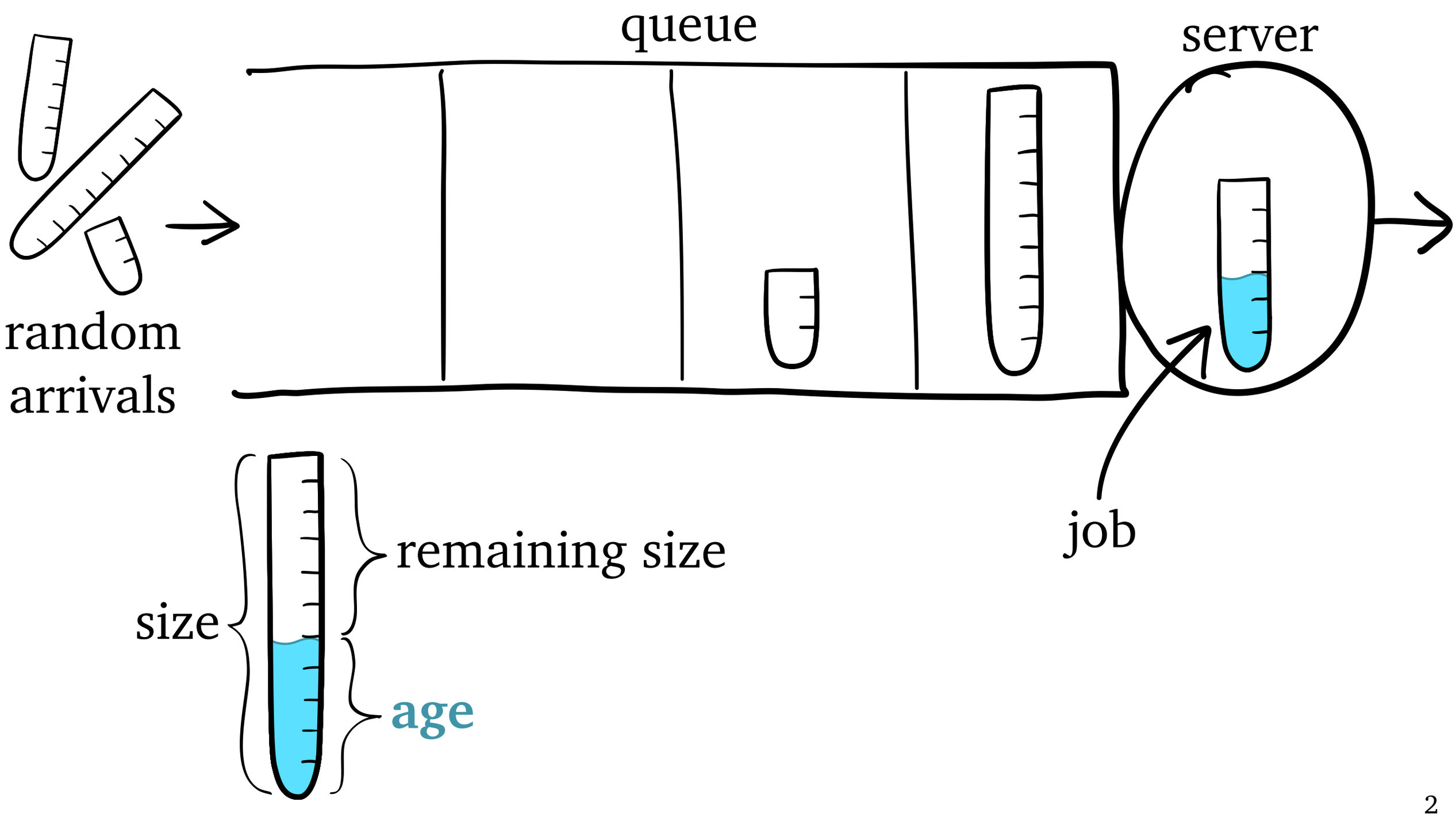

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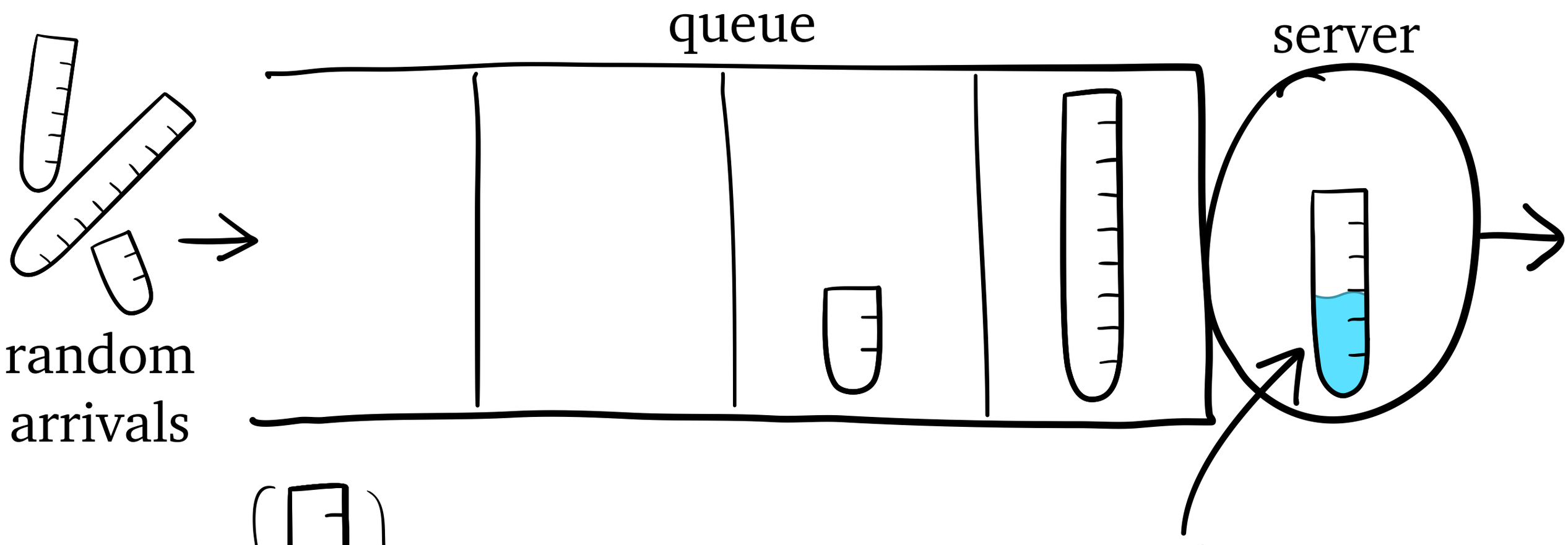

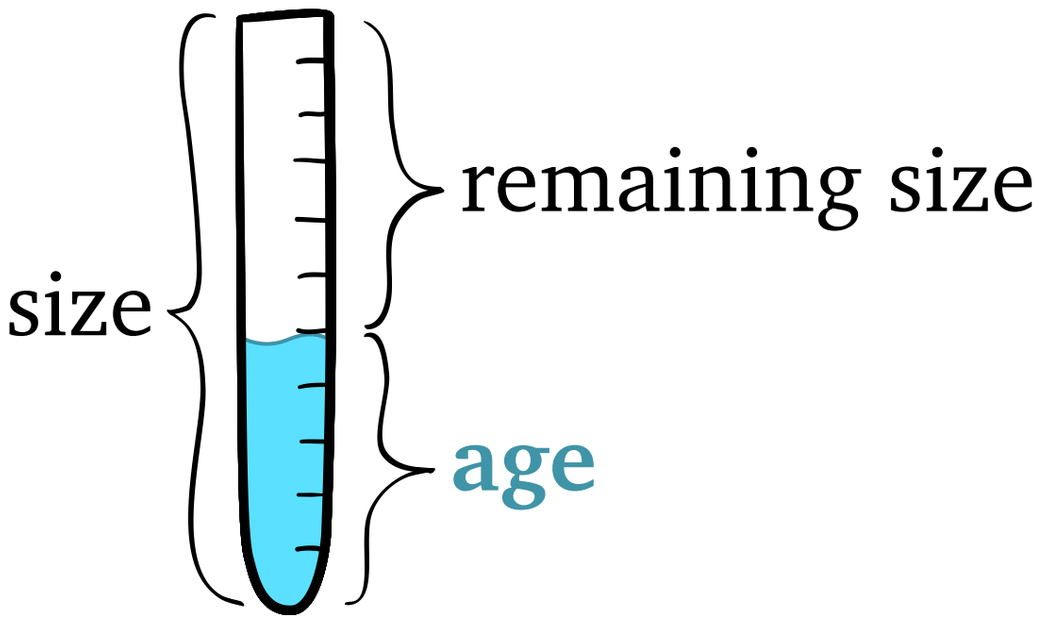

Scheduling policy:
picks which job to serve

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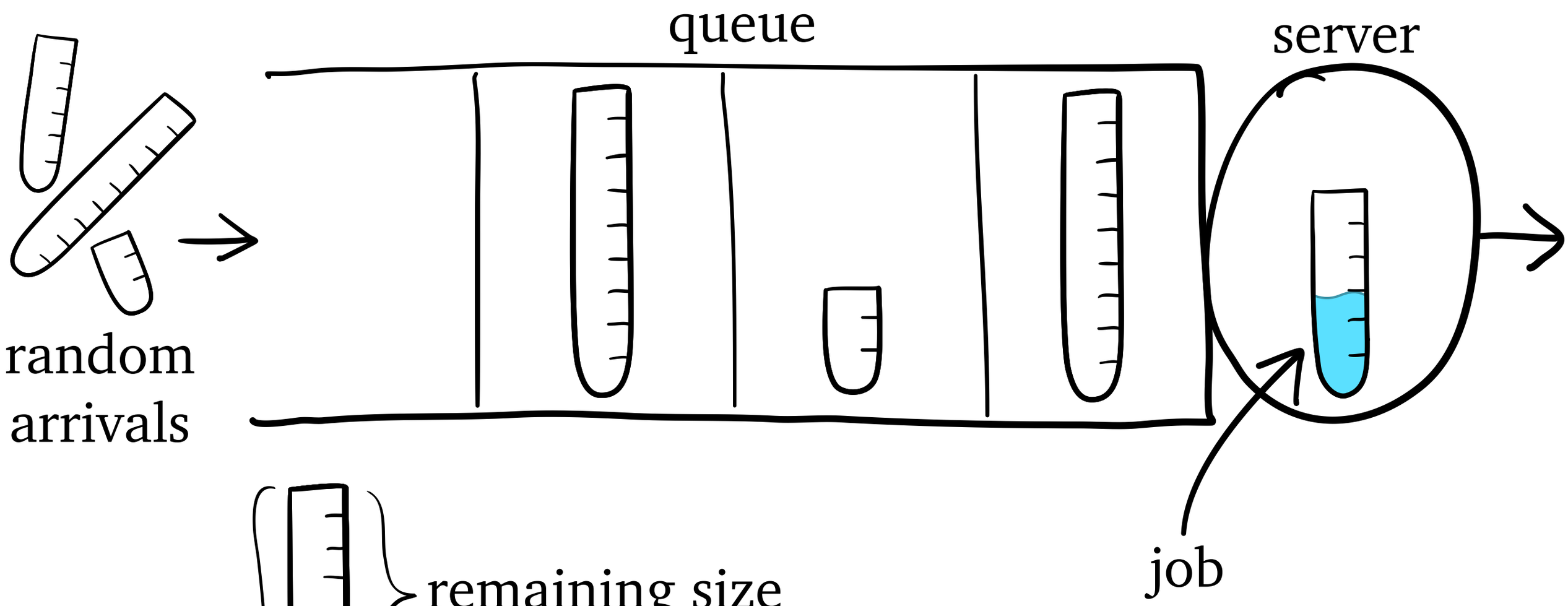

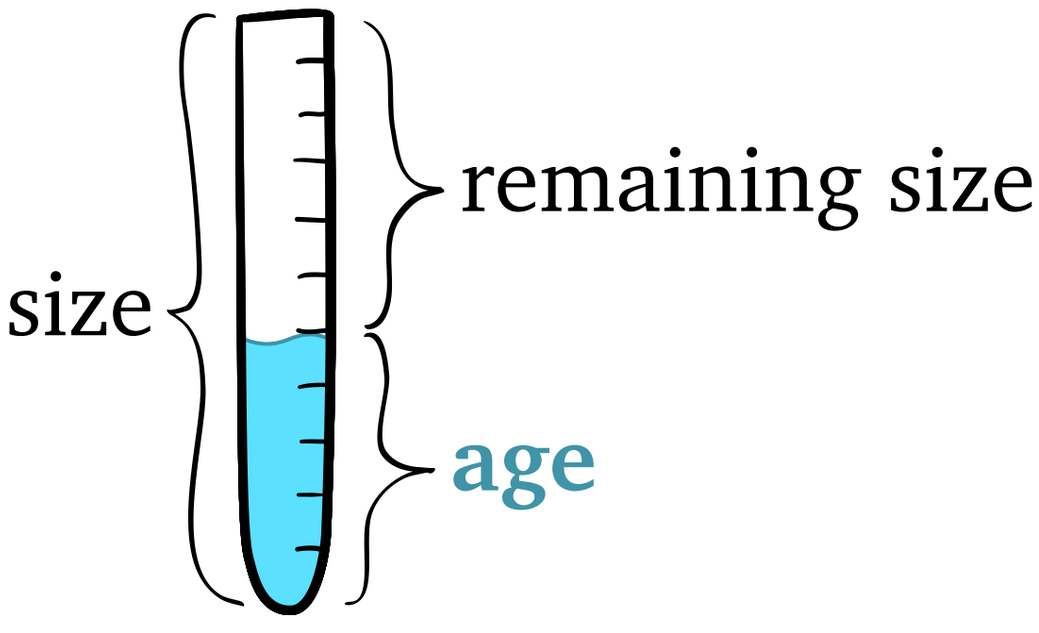

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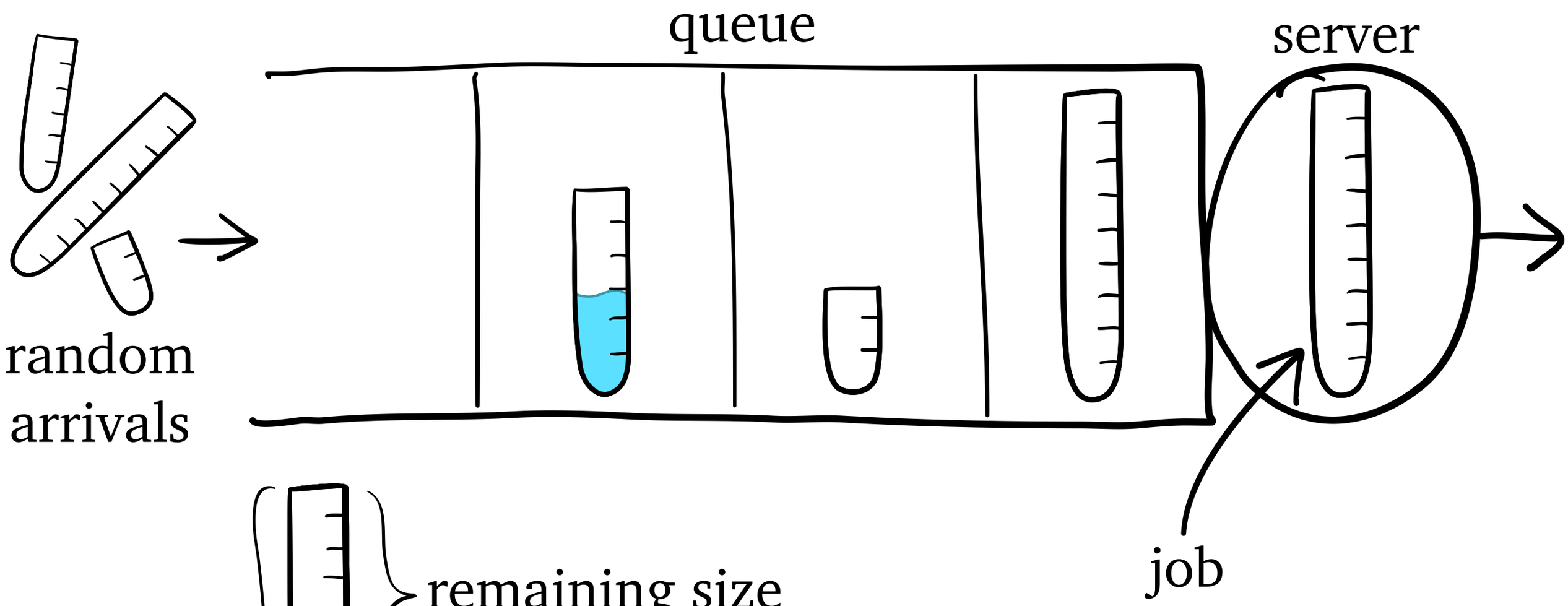

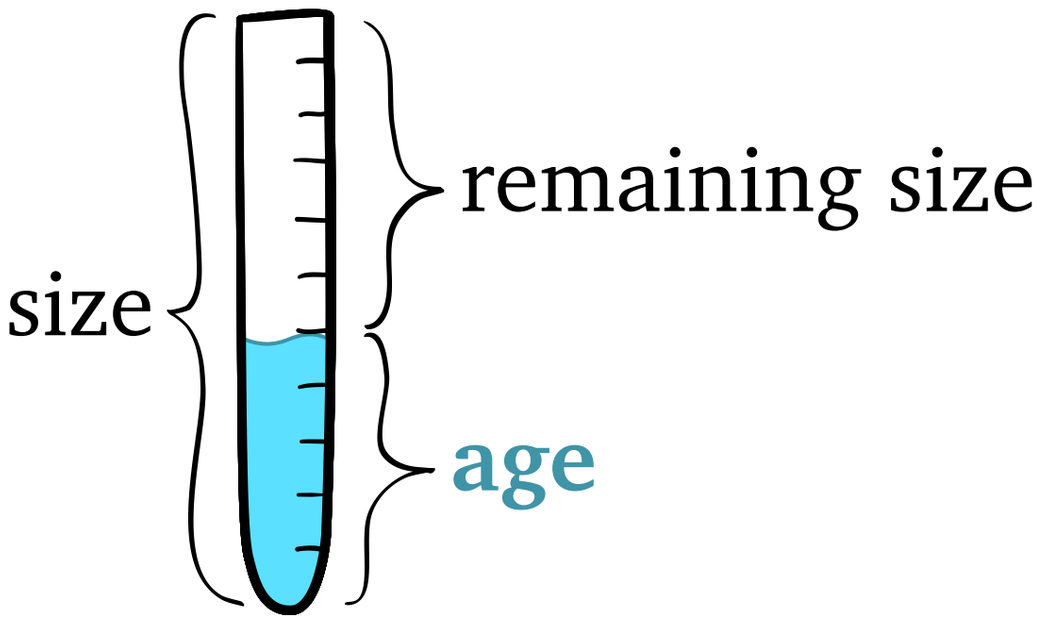

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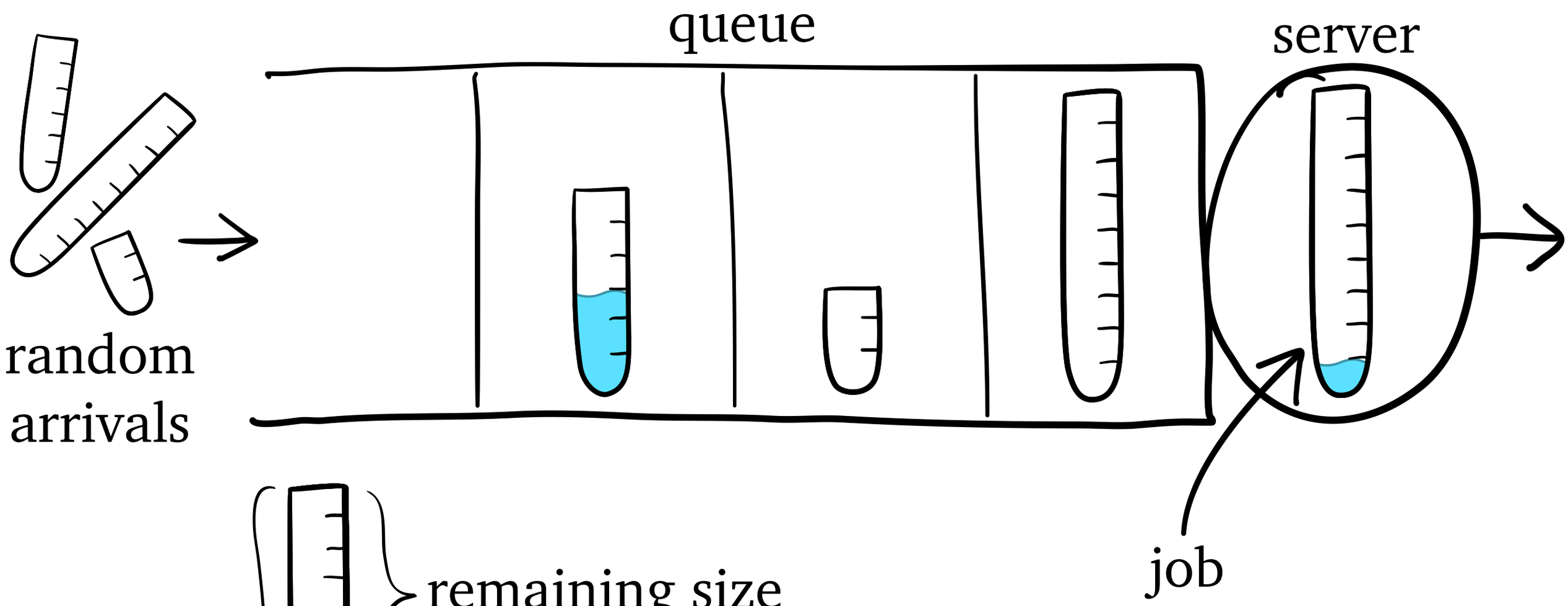

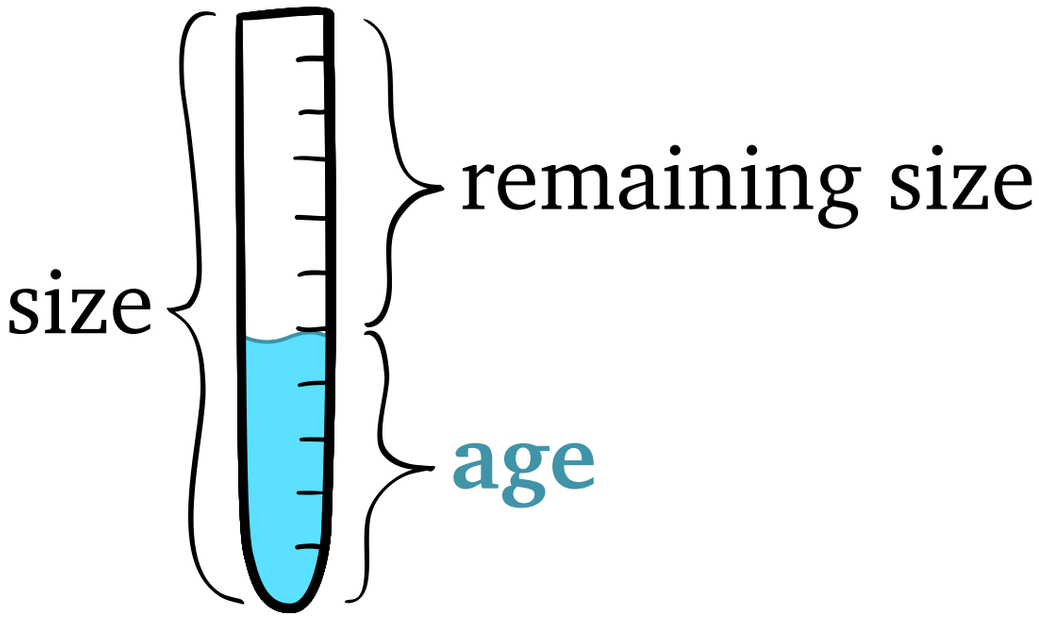

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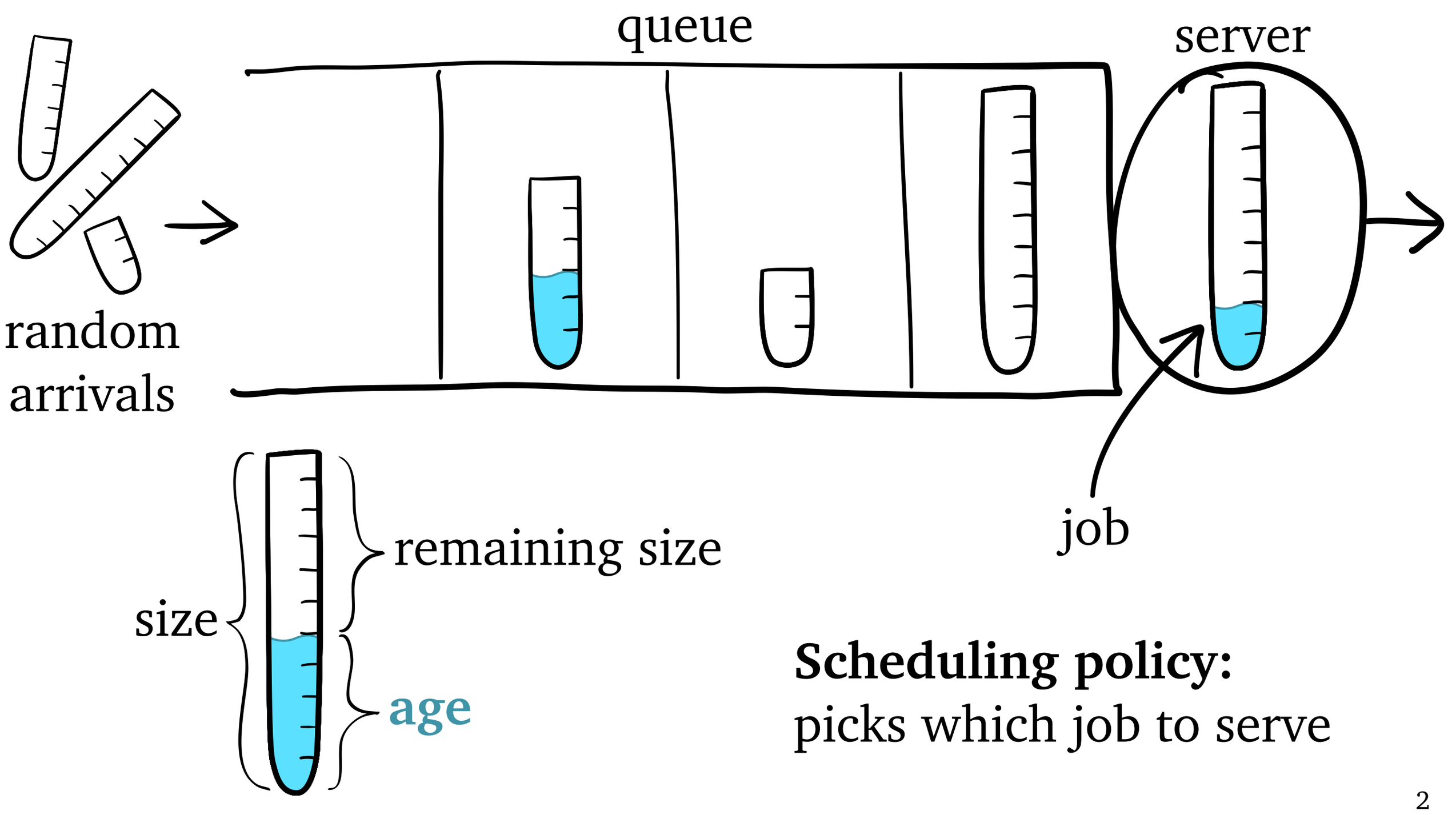

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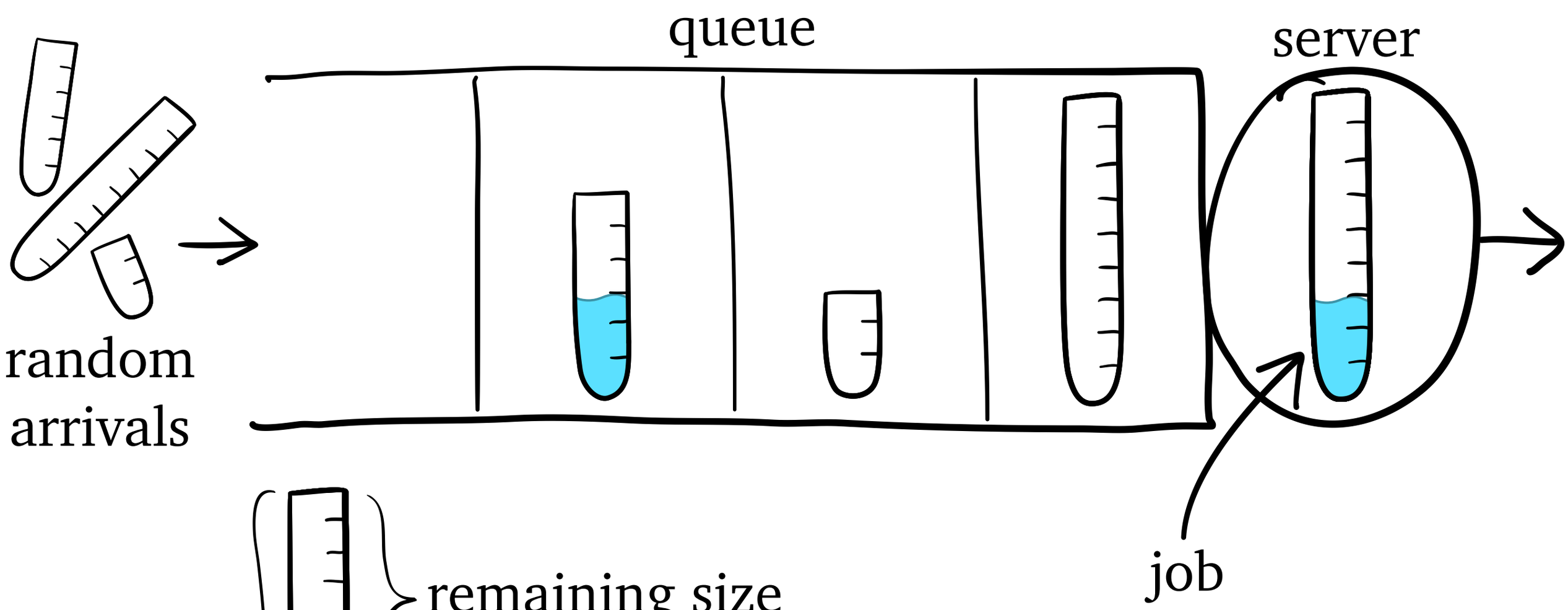

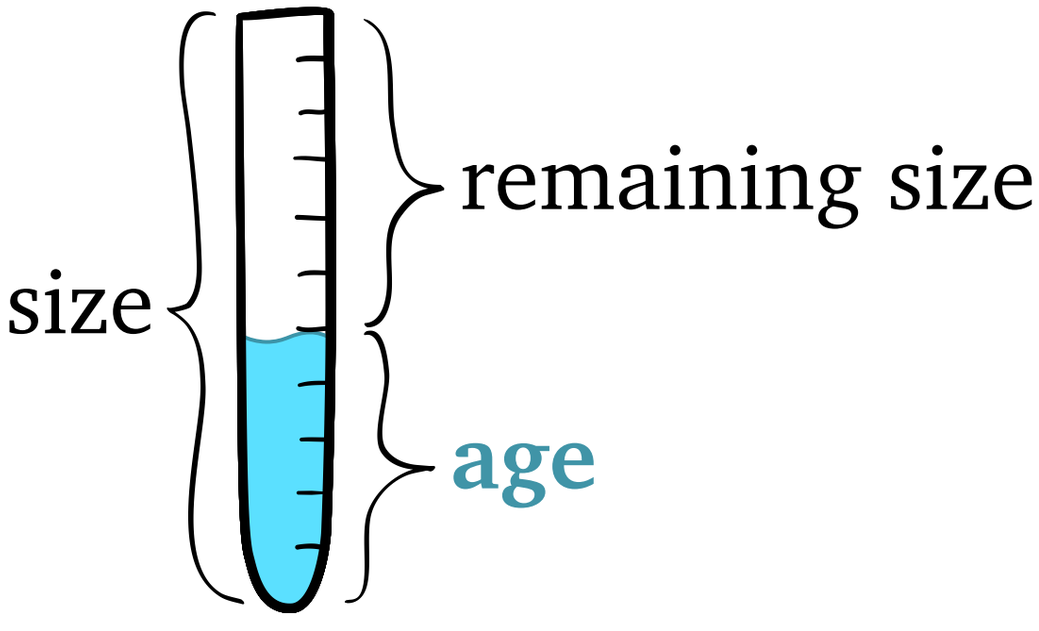

Scheduling policy:
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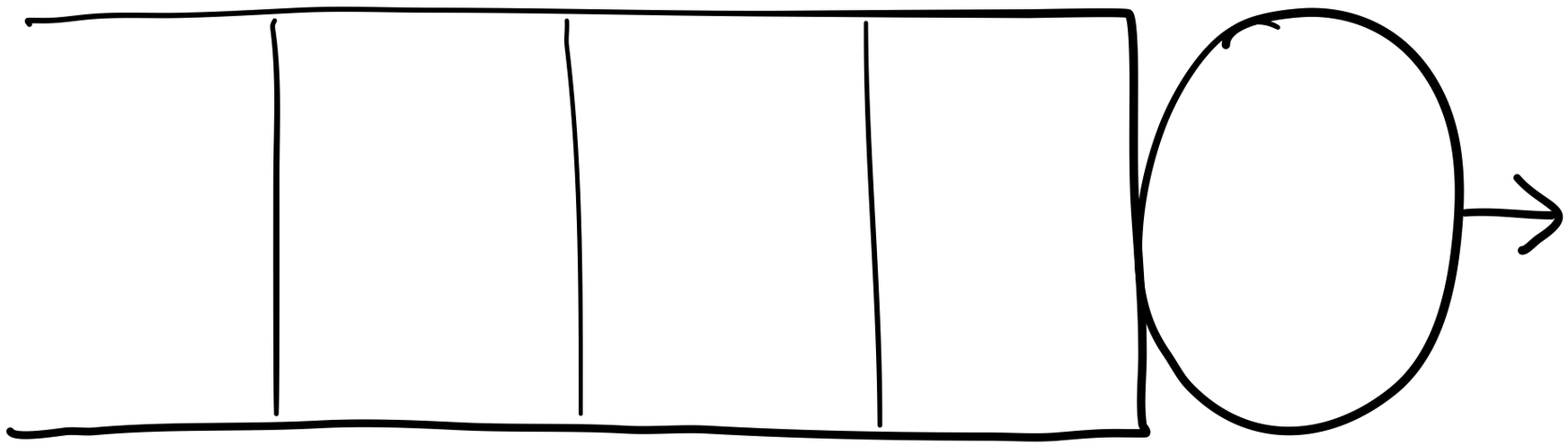
Response Time



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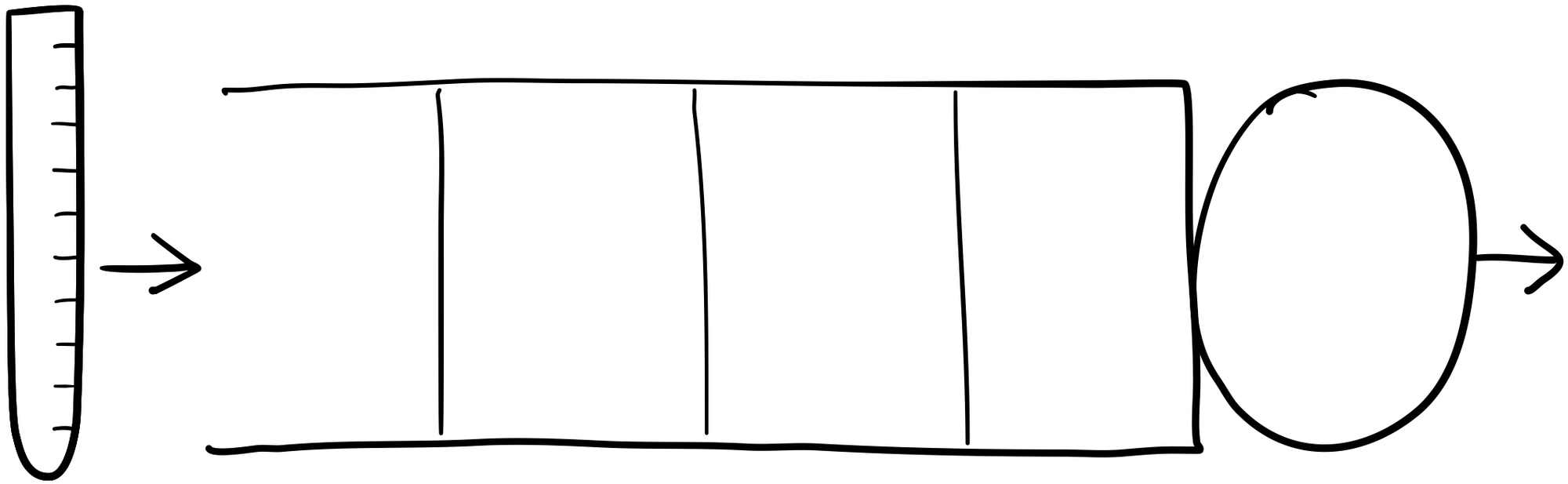

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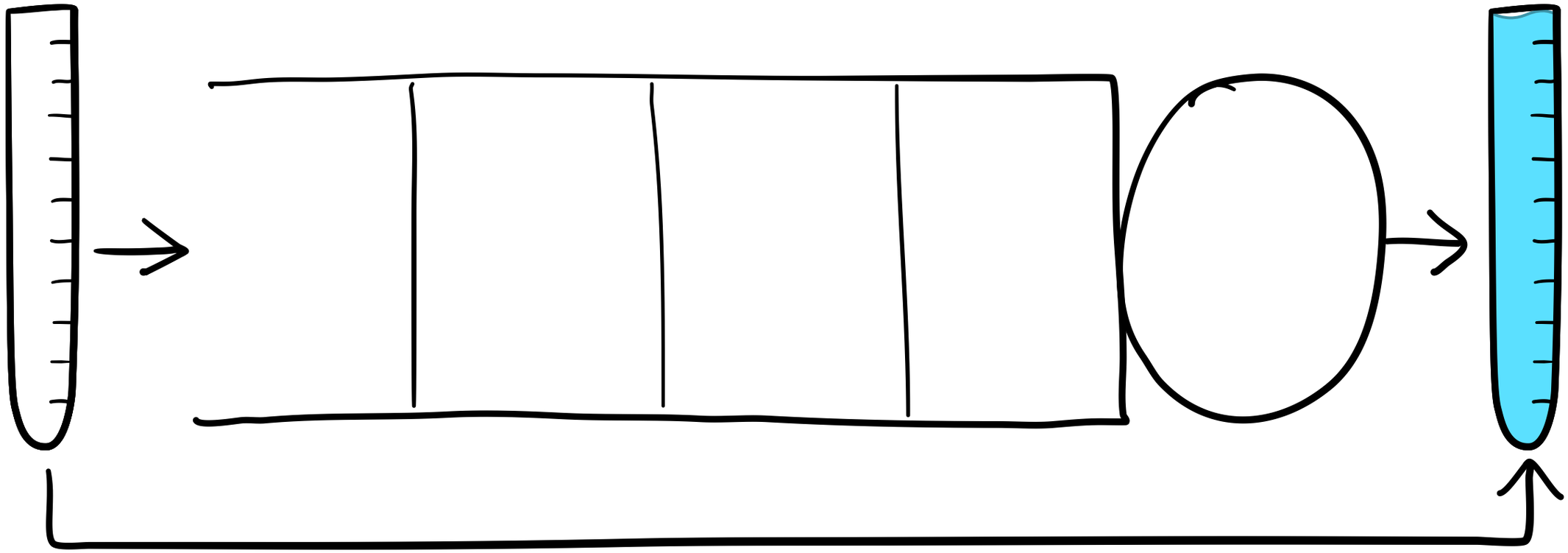
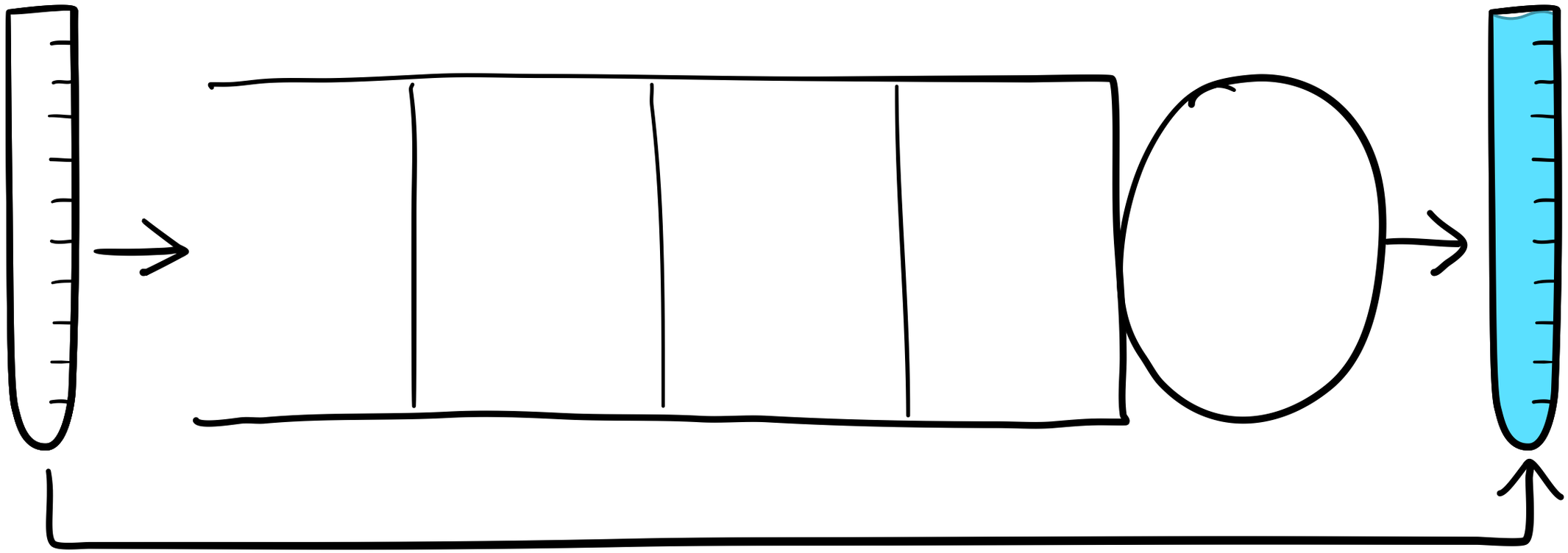


$= T =$ *response time*

Response Time



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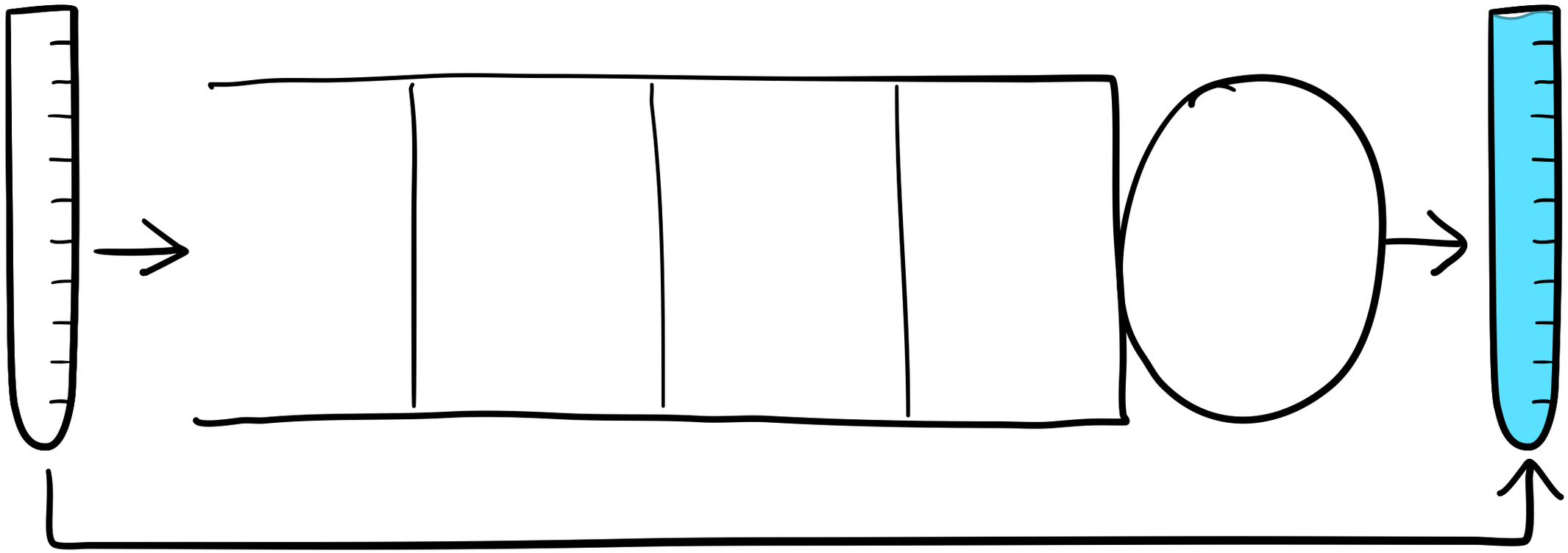
Depends on *scheduling policy*

Response Time



$= T =$ *response time*

Depends on *scheduling policy*

Goal: analyze *mean response time* $\mathbf{E}[T]$

Scheduling Policies

Scheduling Policies

FCFS

Scheduling Policies

$\mathbf{E}[T]$ known

FCFS

Scheduling Policies

$\mathbf{E}[T]$ known

FCFS

priority classes

Scheduling Policies

$\mathbf{E}[T]$ known

FCFS

priority classes

FB

Scheduling Policies

$\mathbf{E}[T]$ known

FCFS

priority classes

FB

SRPT

Scheduling Policies

E[T] known

FCFS (1930)

priority classes (1954)

FB (1967)

SRPT (1966)

Scheduling Policies

E[T] known

FCFS (1930)

discrete FB

priority classes (1954)

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Scheduling Policies

E[T] known

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Gittins policy (optimal!)

mixture of policies

Scheduling Policies

E[T] known

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Gittins policy (optimal!)

mixture of policies

many more…

Scheduling Policies

E[T] known

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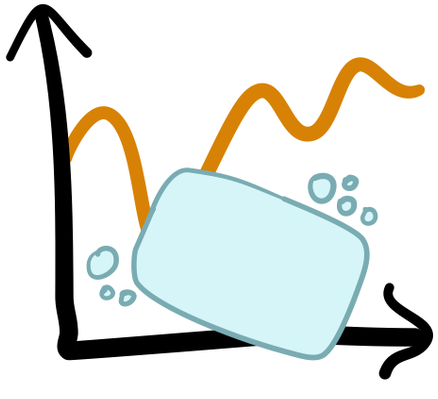
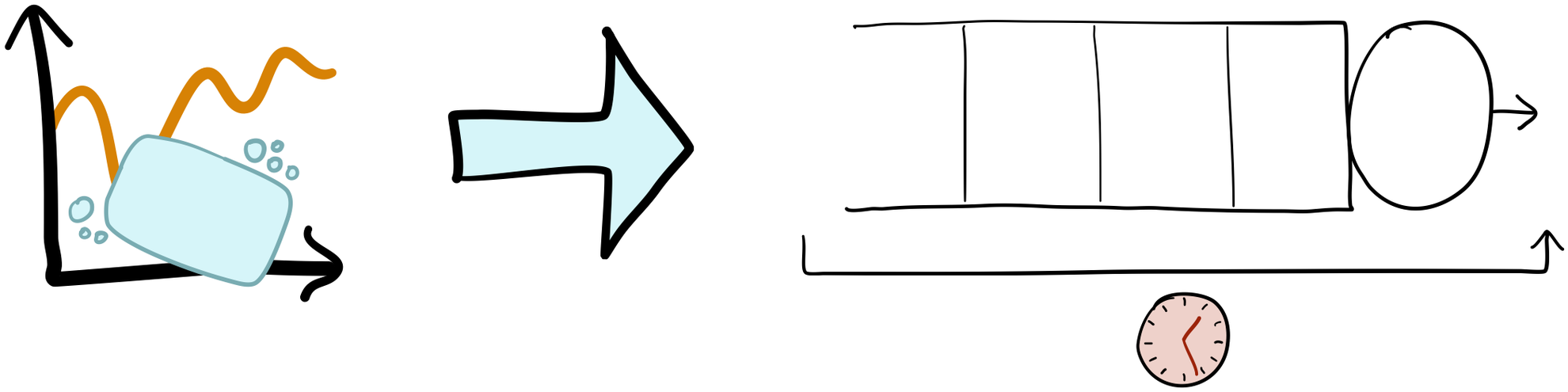
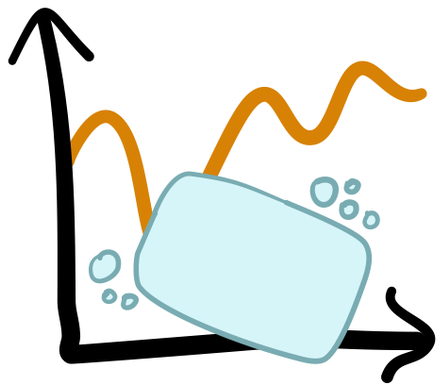


SOAP

Our contribution:

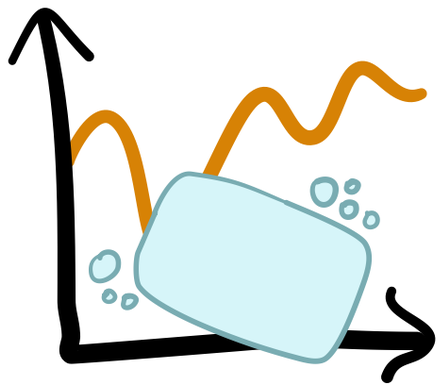
analyze $\mathbf{E}[T]$ of *any* **SOAP** policy





SOAP

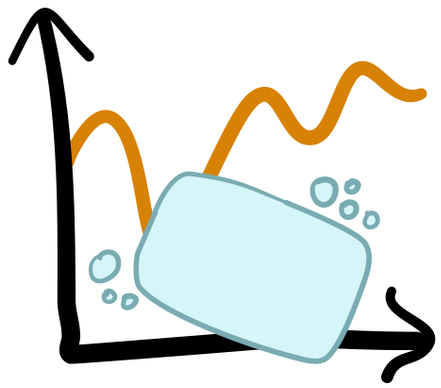
Schedule Ordered by Age-based Priority



SOAP

Schedule Ordered by Age-based Priority

Step 1: *unified framework* for defining scheduling policies

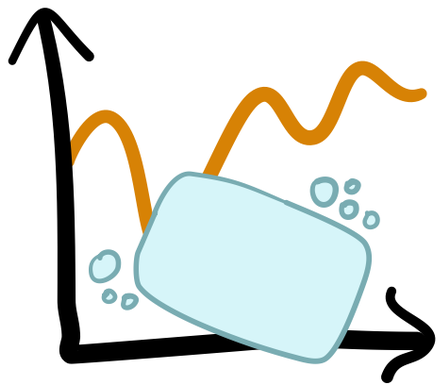


SOAP

Schedule Ordered by Age-based Priority

Step 1: *unified framework* for defining scheduling policies

Step 2: *universal analysis* of $\mathbf{E}[T]$



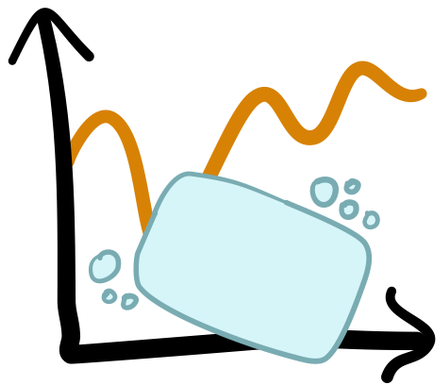
SOAP

Schedule Ordered by Age-based Priority

this talk

Step 1: *unified framework* for defining scheduling policies

Step 2: *universal analysis* of $\mathbf{E}[T]$



SOAP

Schedule Ordered by Age-based Priority

this talk

Step 1: *unified framework* for defining scheduling policies

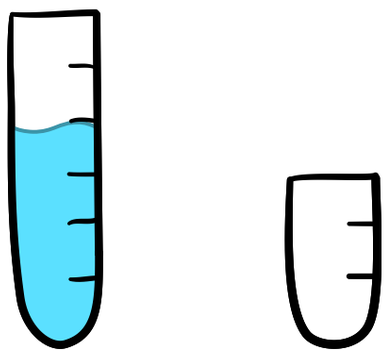
see paper

Step 2: *universal analysis* of $\mathbf{E}[T]$

How is FB Like SRPT?

How is FB Like SRPT?

FB



least age

How is FB Like SRPT?

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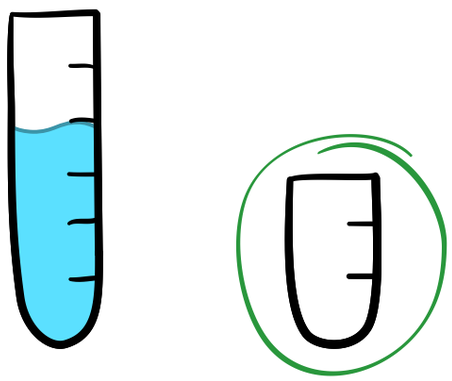

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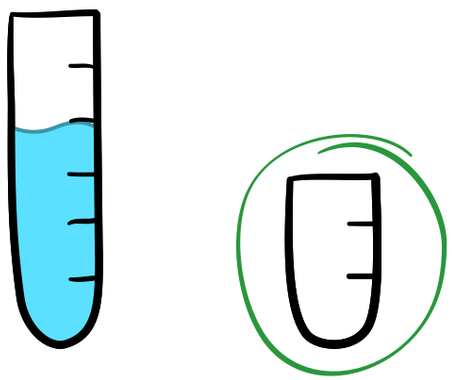

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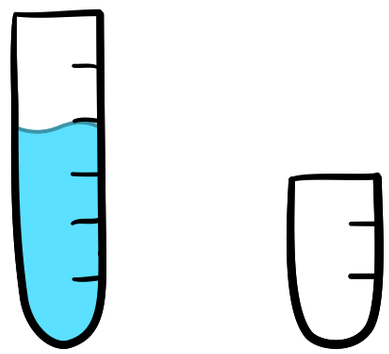
SRPT



least remaining size

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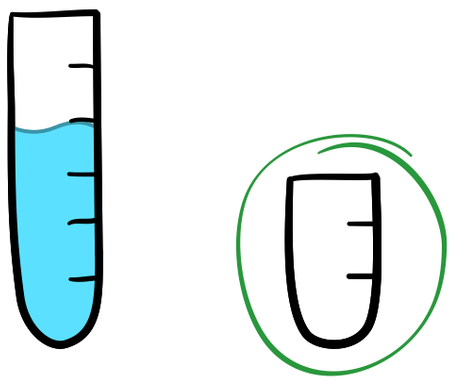

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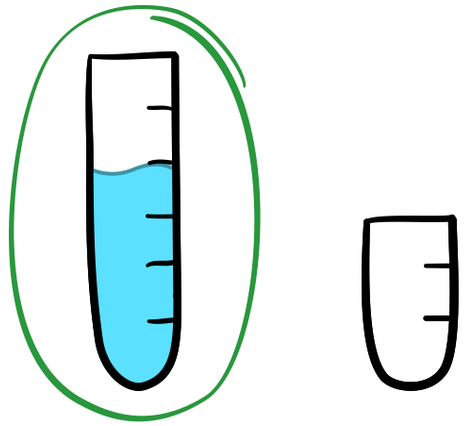

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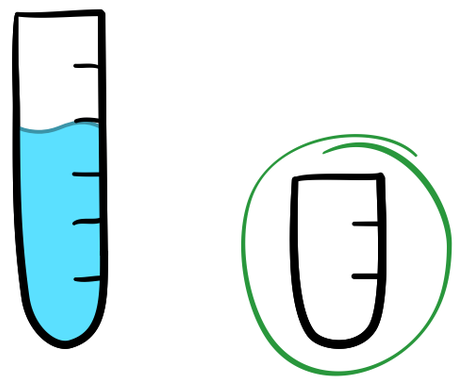

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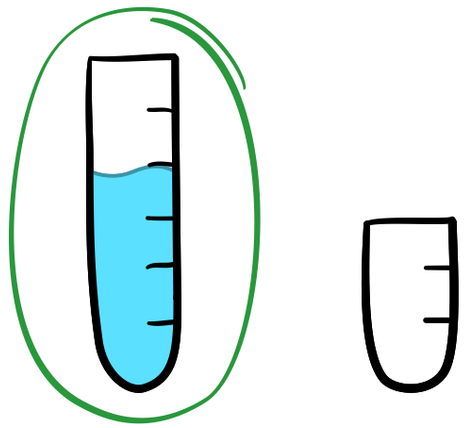

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Common theme: a job's **rank** depends on its **age**

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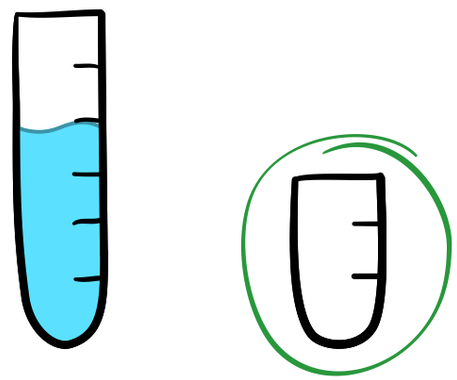

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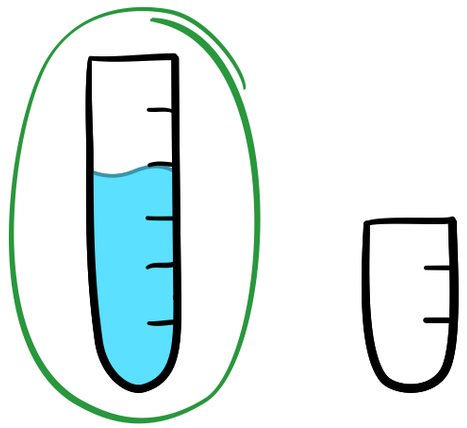

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rank = priority

Common theme: a job's **rank** depends on its **age**

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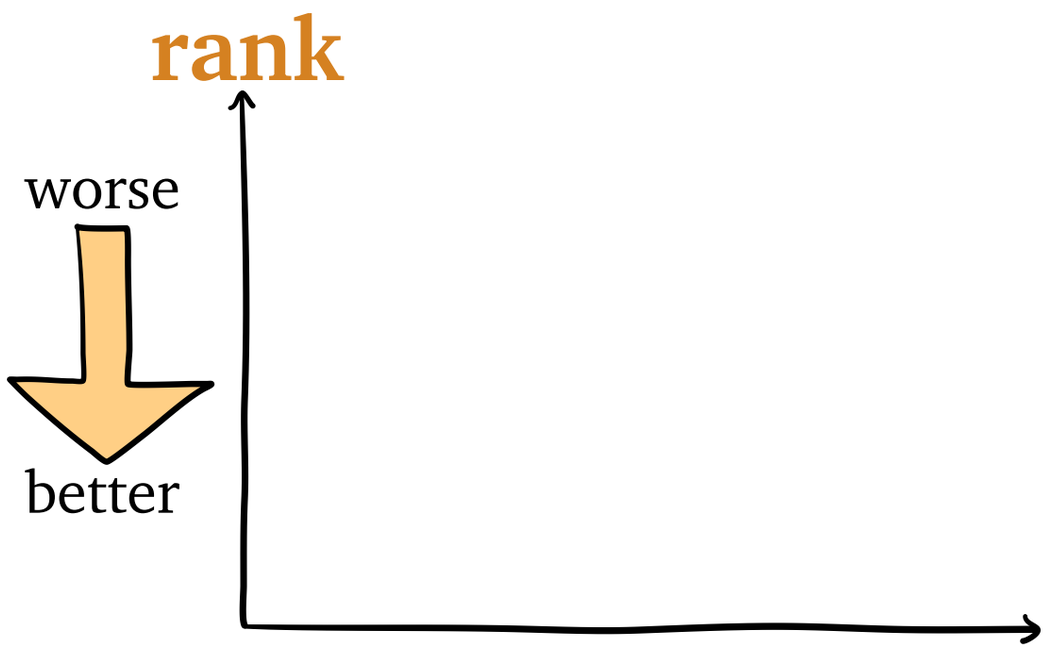

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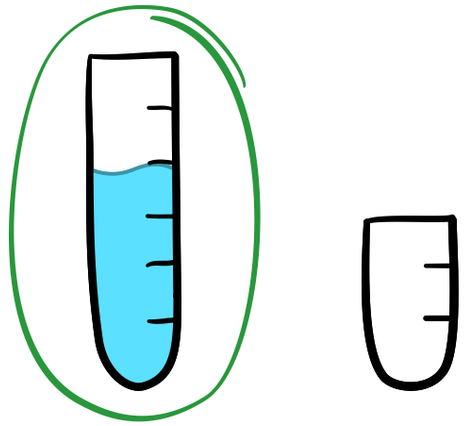

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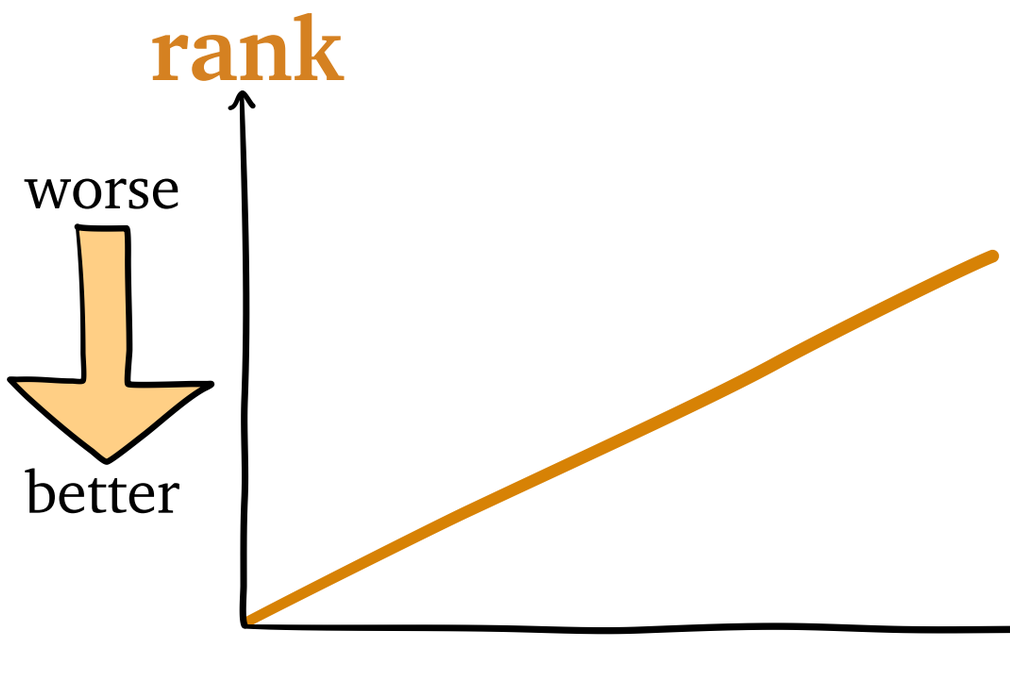

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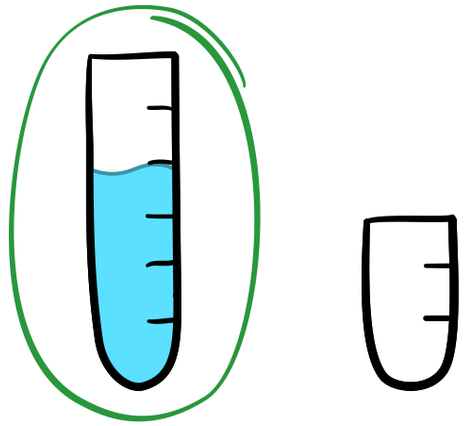
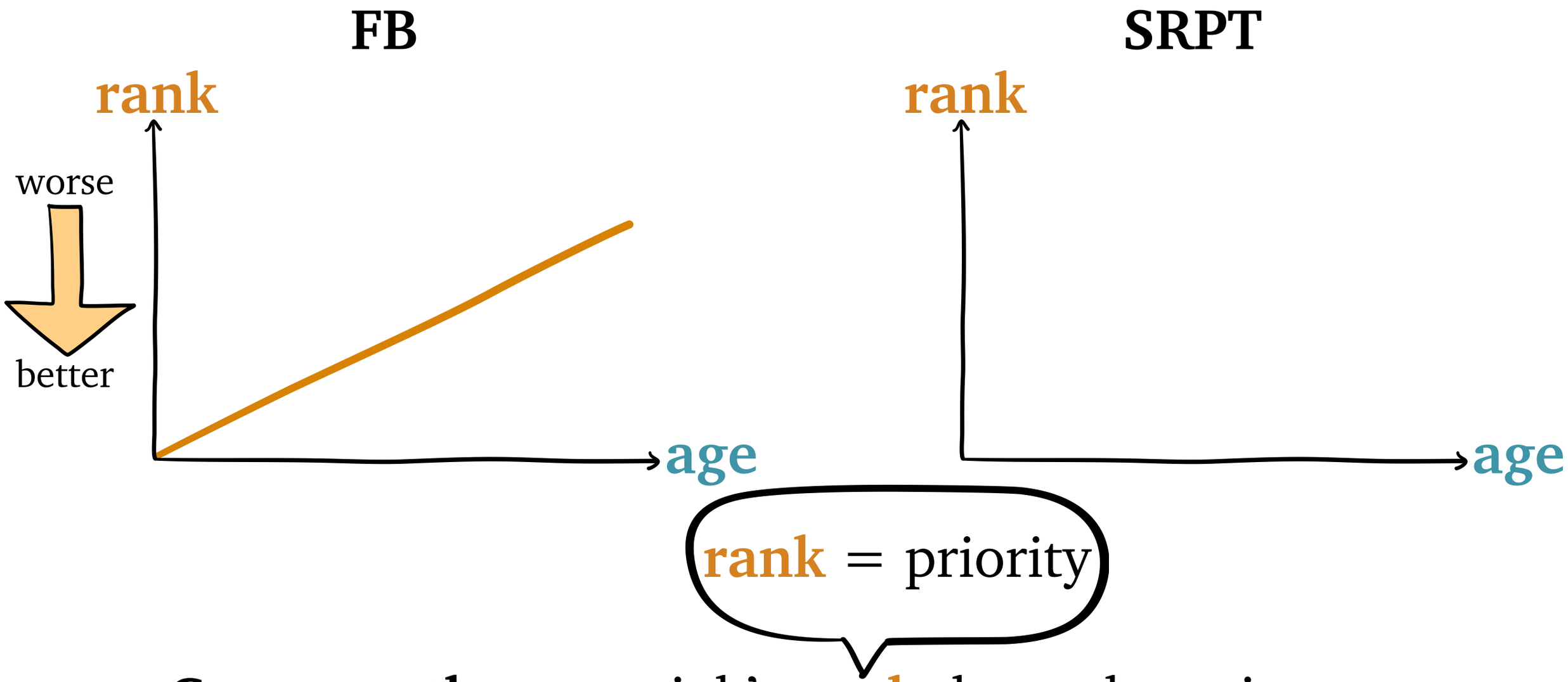


least remaining size

rank = priority

Common theme: a job's **rank** depends on its **age**

How is FB Like SRPT?



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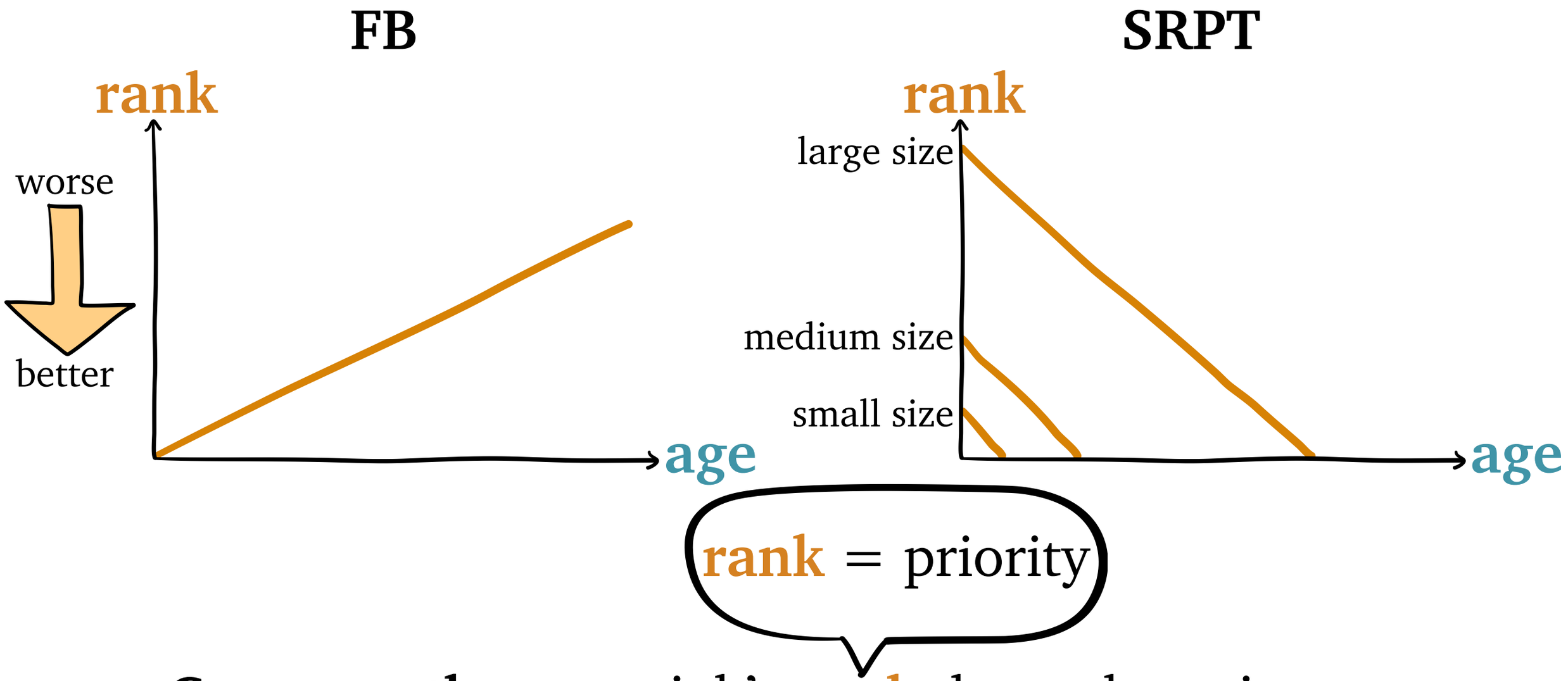

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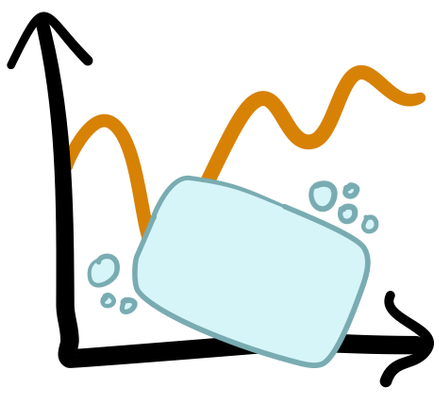
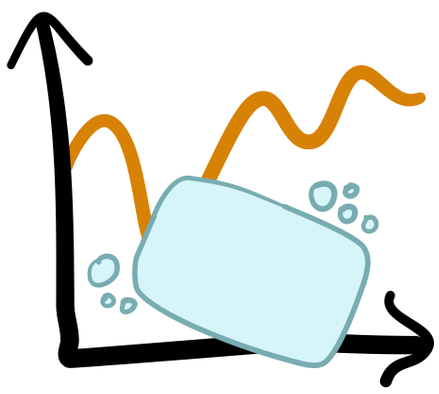


A ***SOAP*** *policy* is a ***rank*** *function* with one rule:



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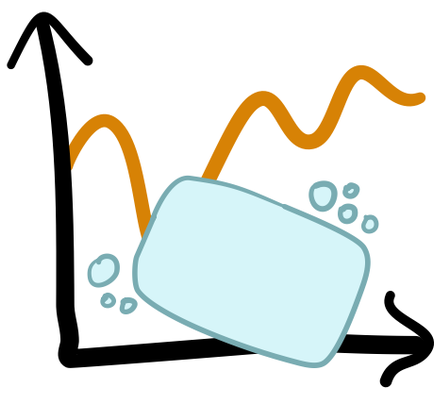
always serve the job of *minimum* ***rank***



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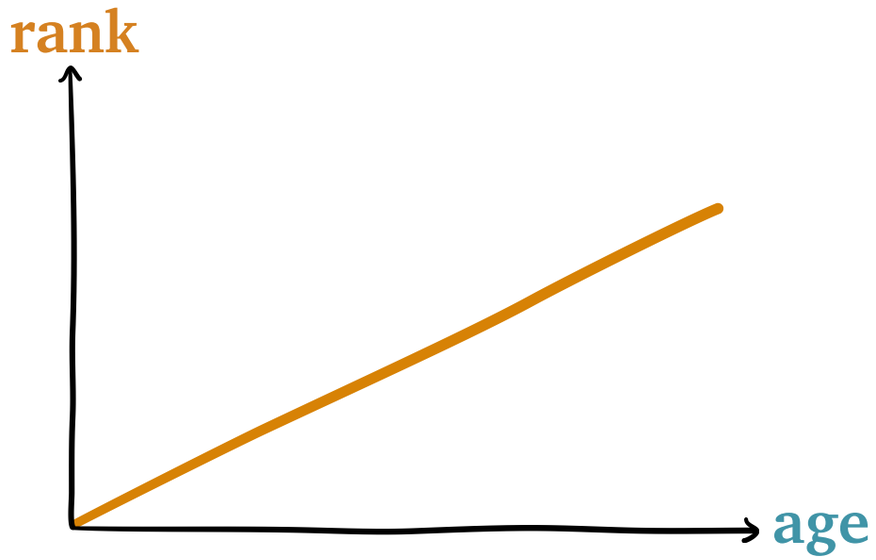
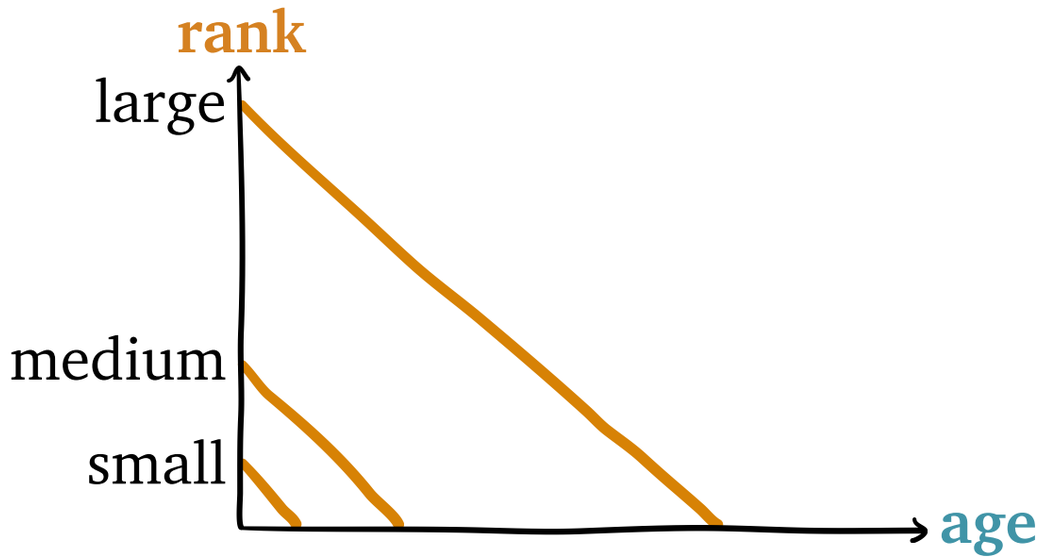
(break ties FCFS)

Classic SOAP Policies

FB



SRPT



Classic SOAP Policies

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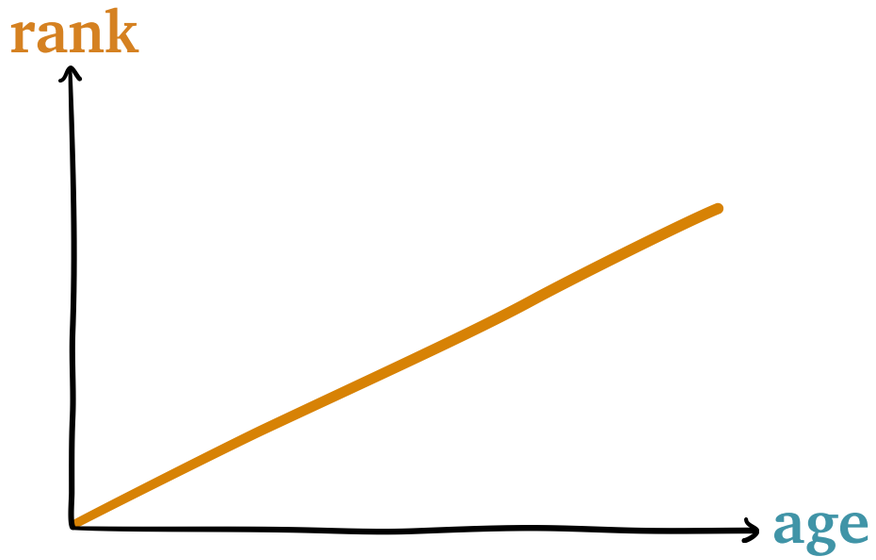

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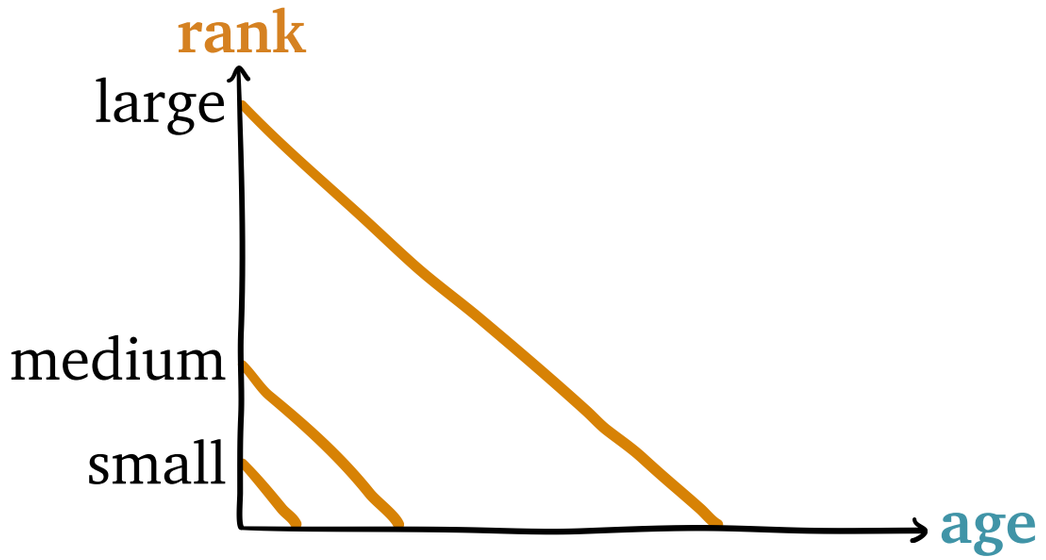

FCFS

Classic SOAP Policies

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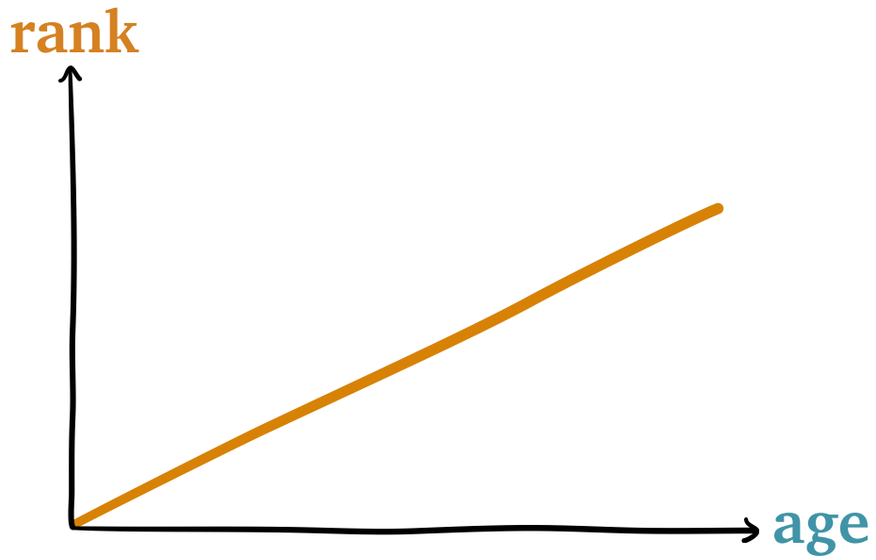

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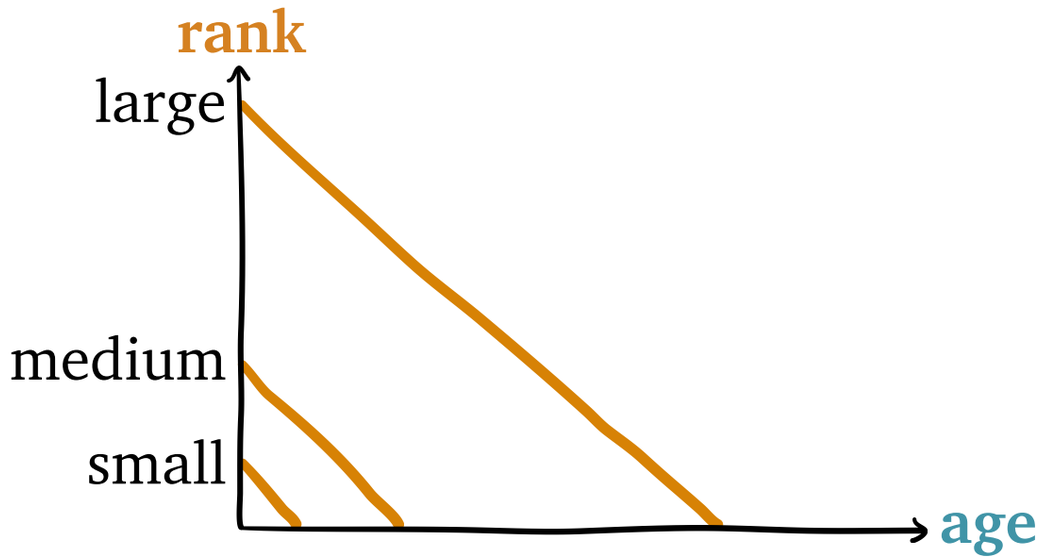

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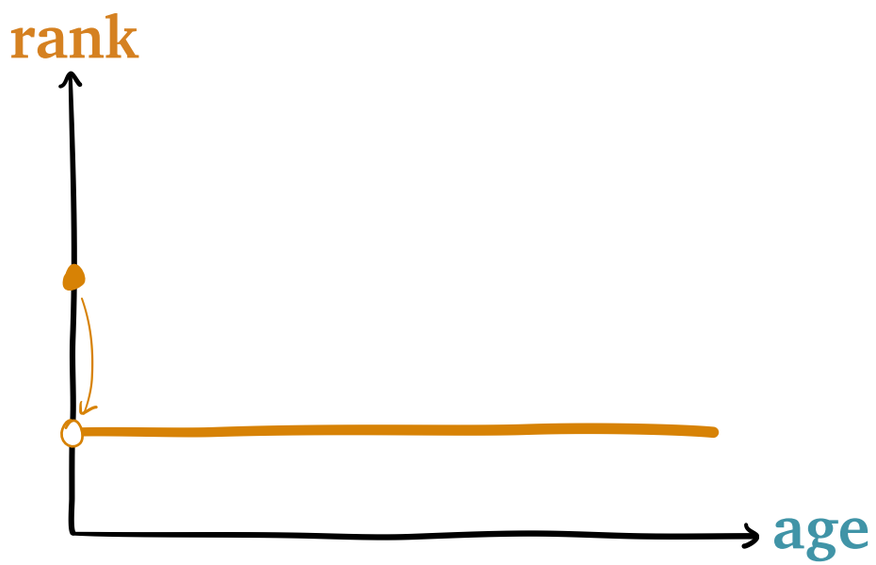
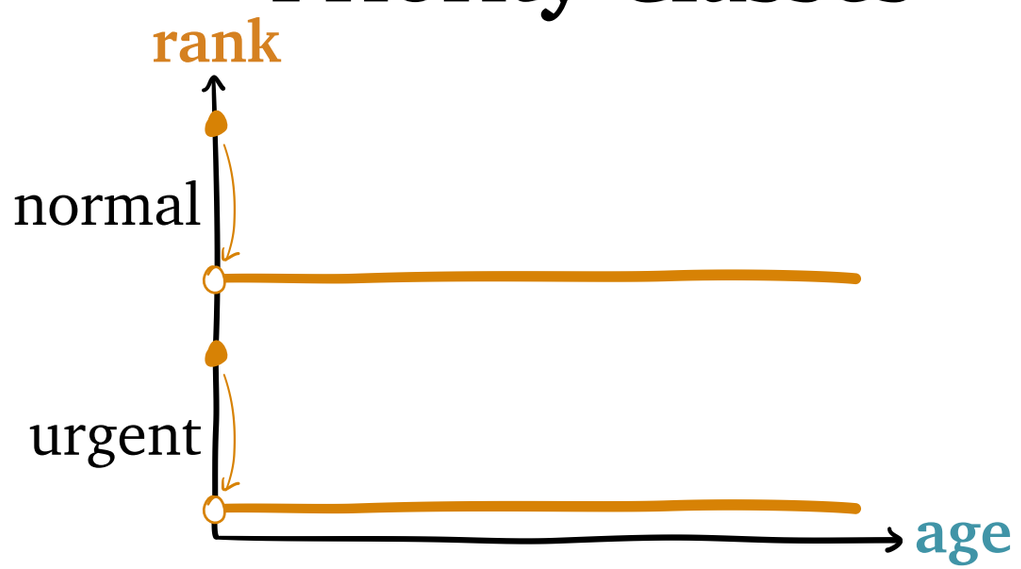
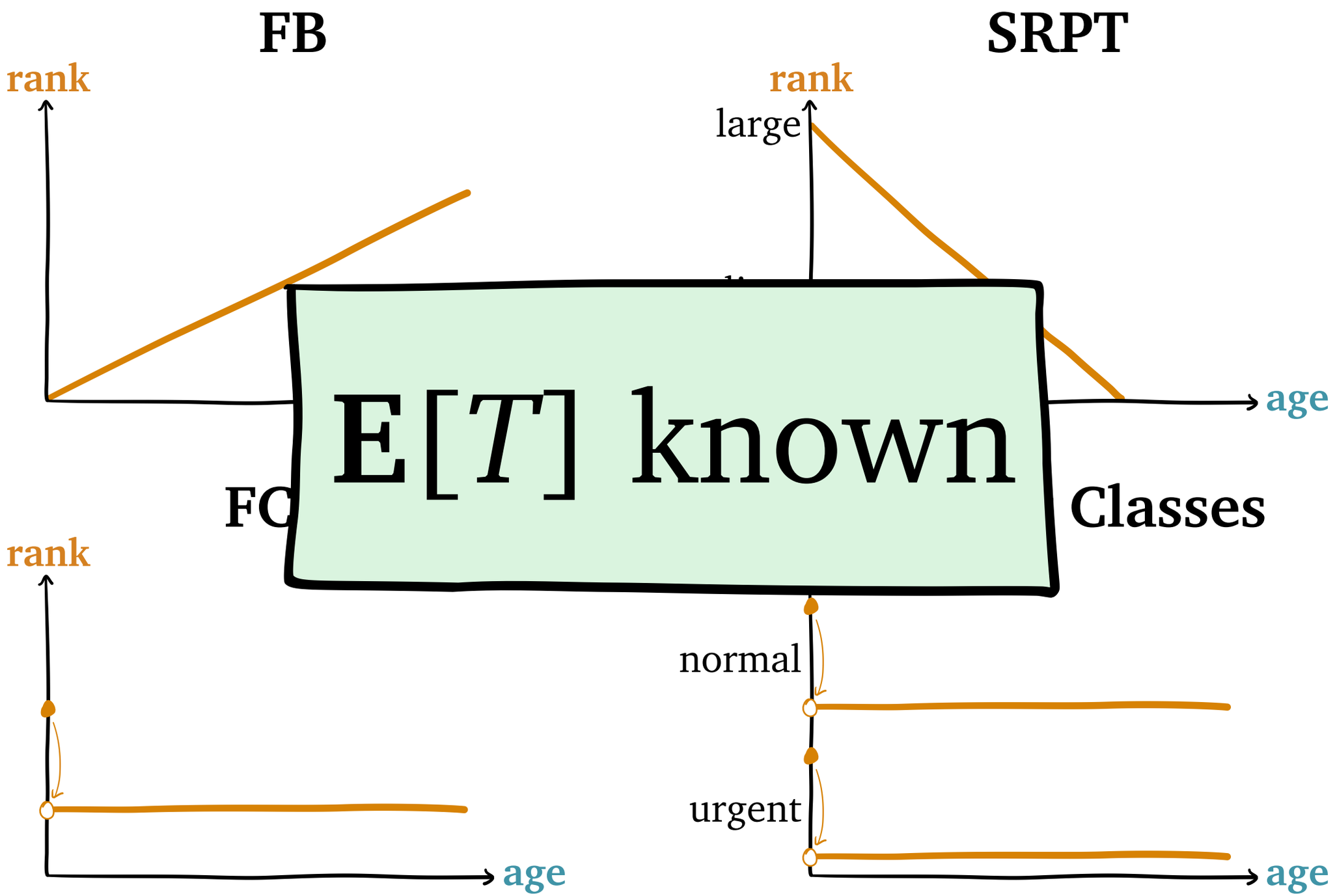


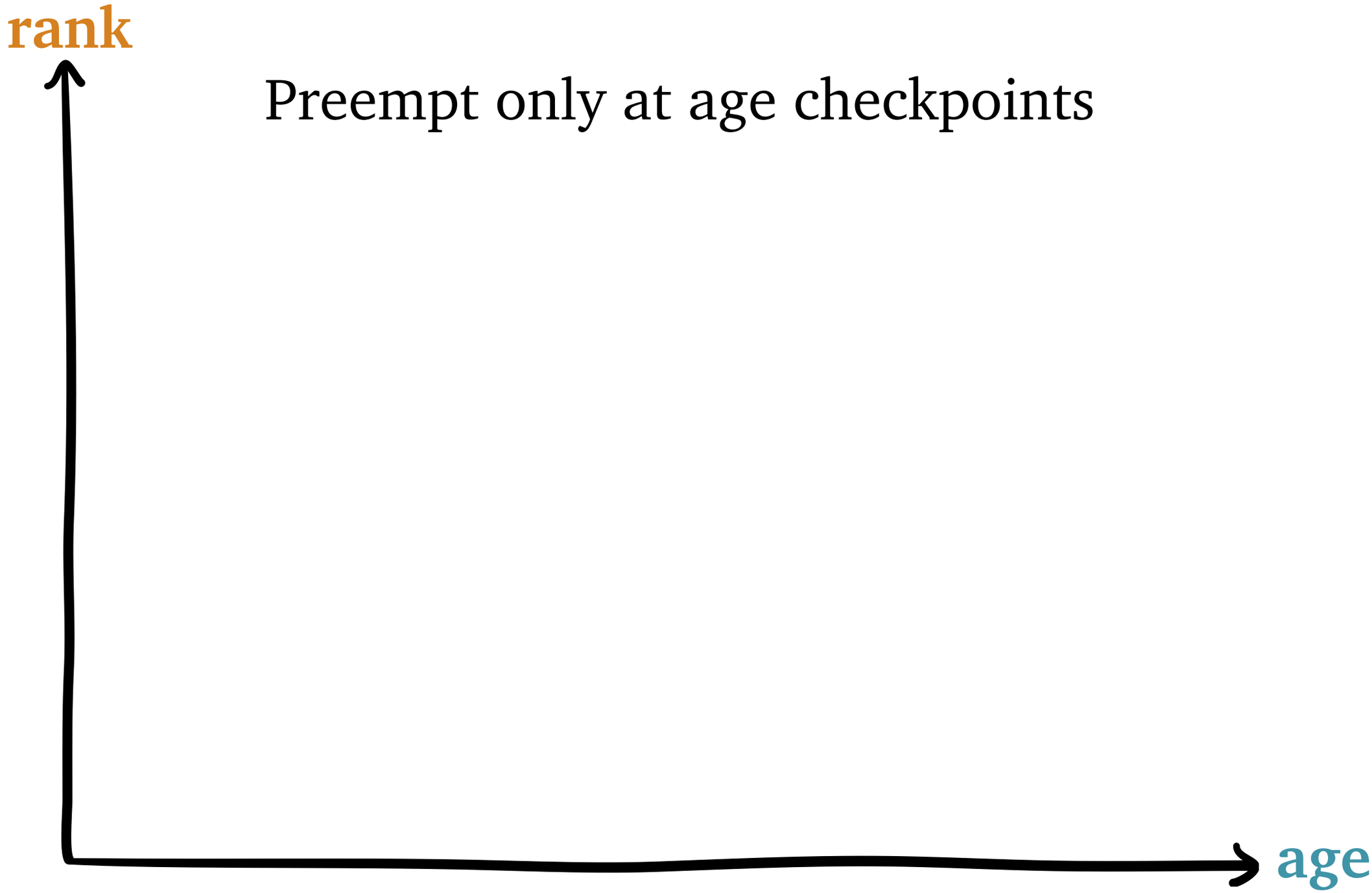
Priority Classes



Classic SOAP Policies



SOAP Policy: Discrete FB



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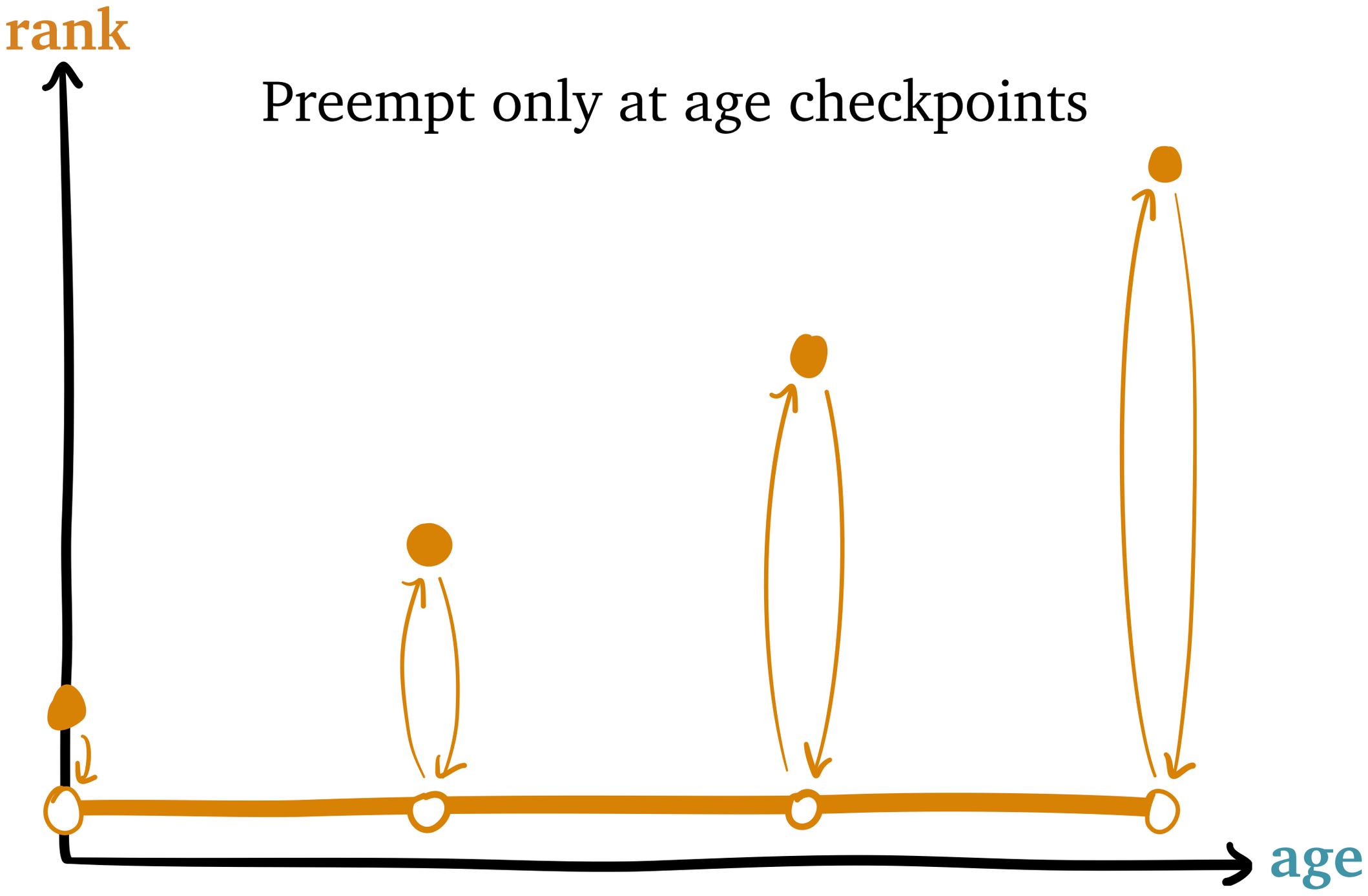
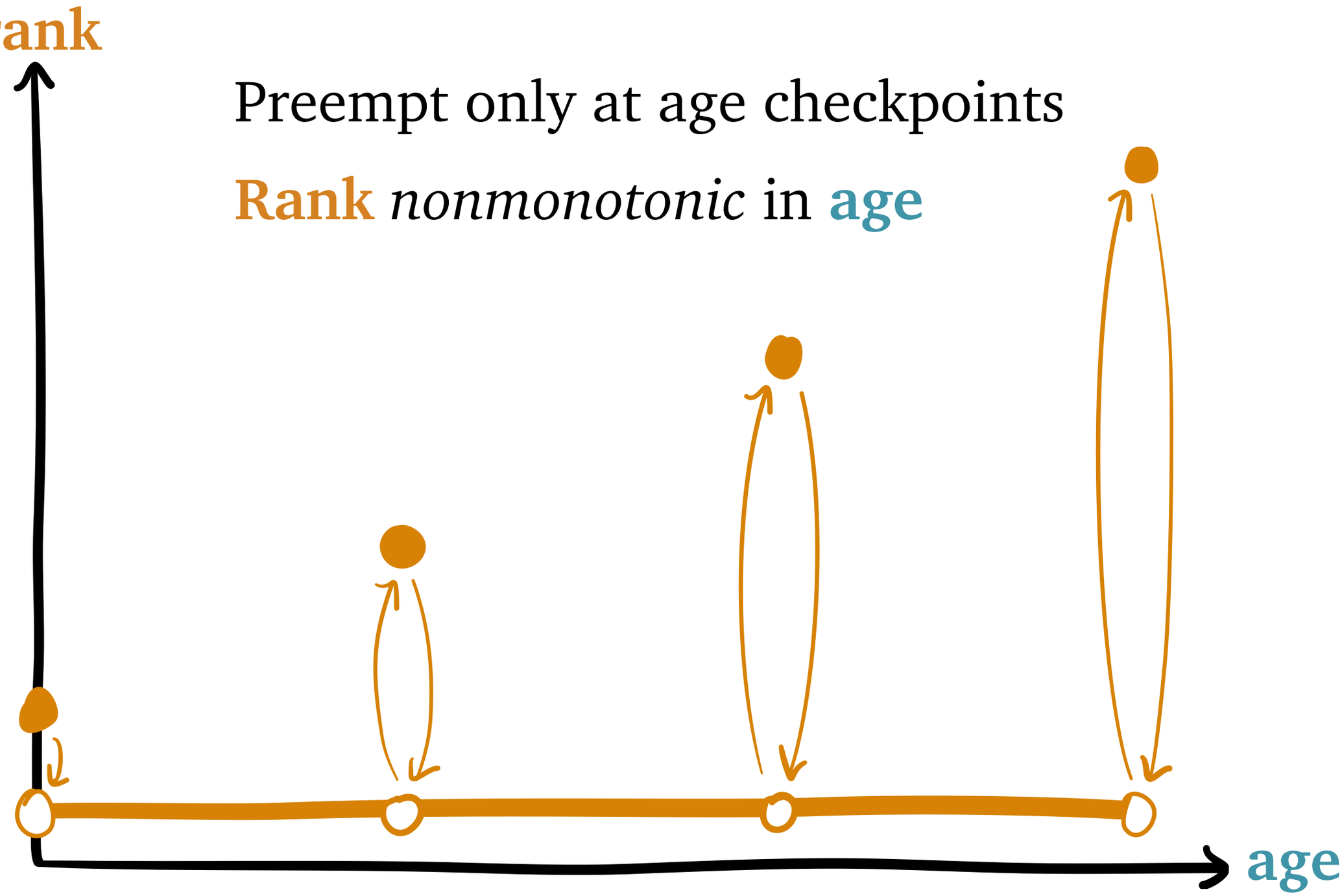


SOAP Policy: Discrete FB

Preempt only at age checkpoints

Rank *nonmonotonic* in **age**



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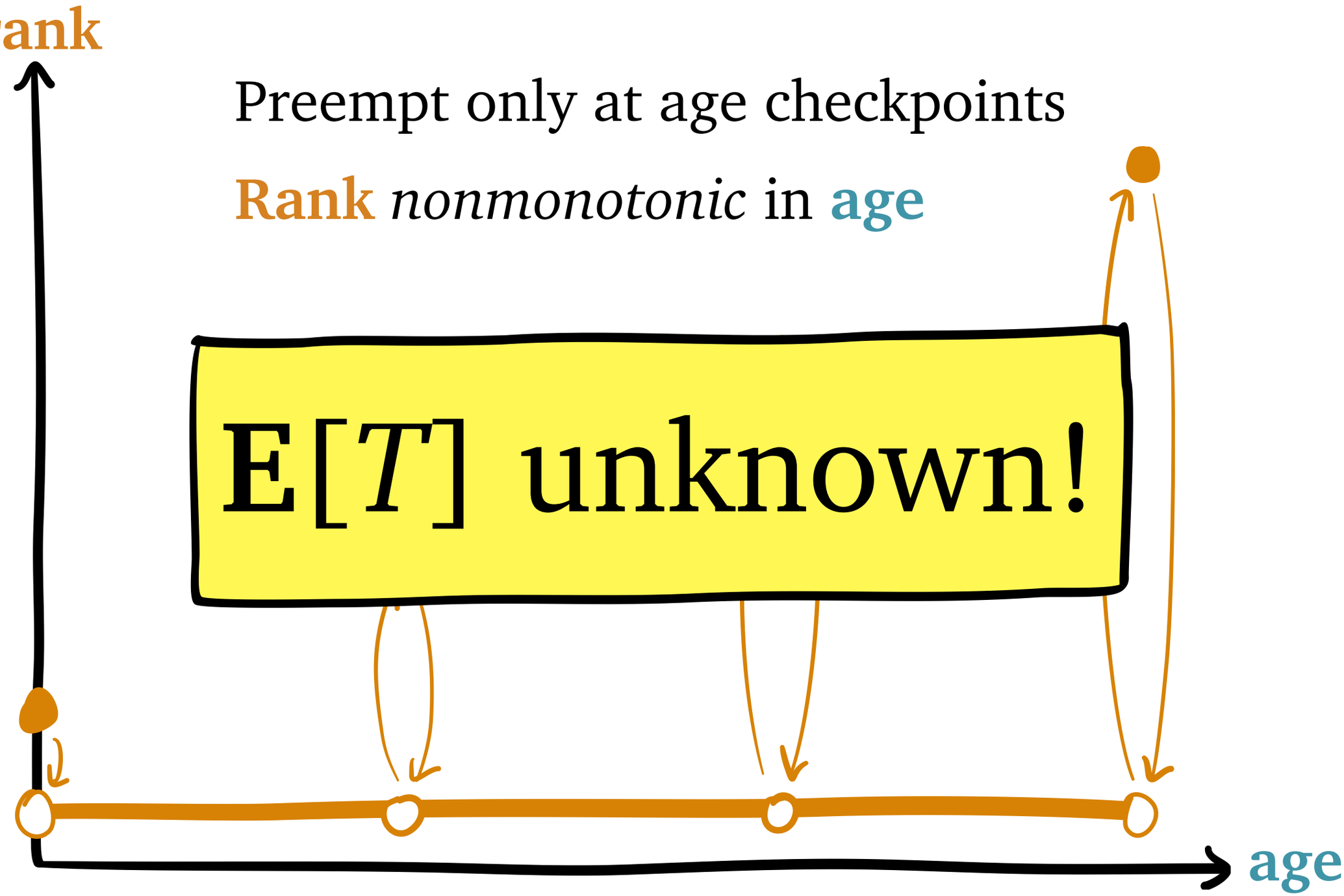

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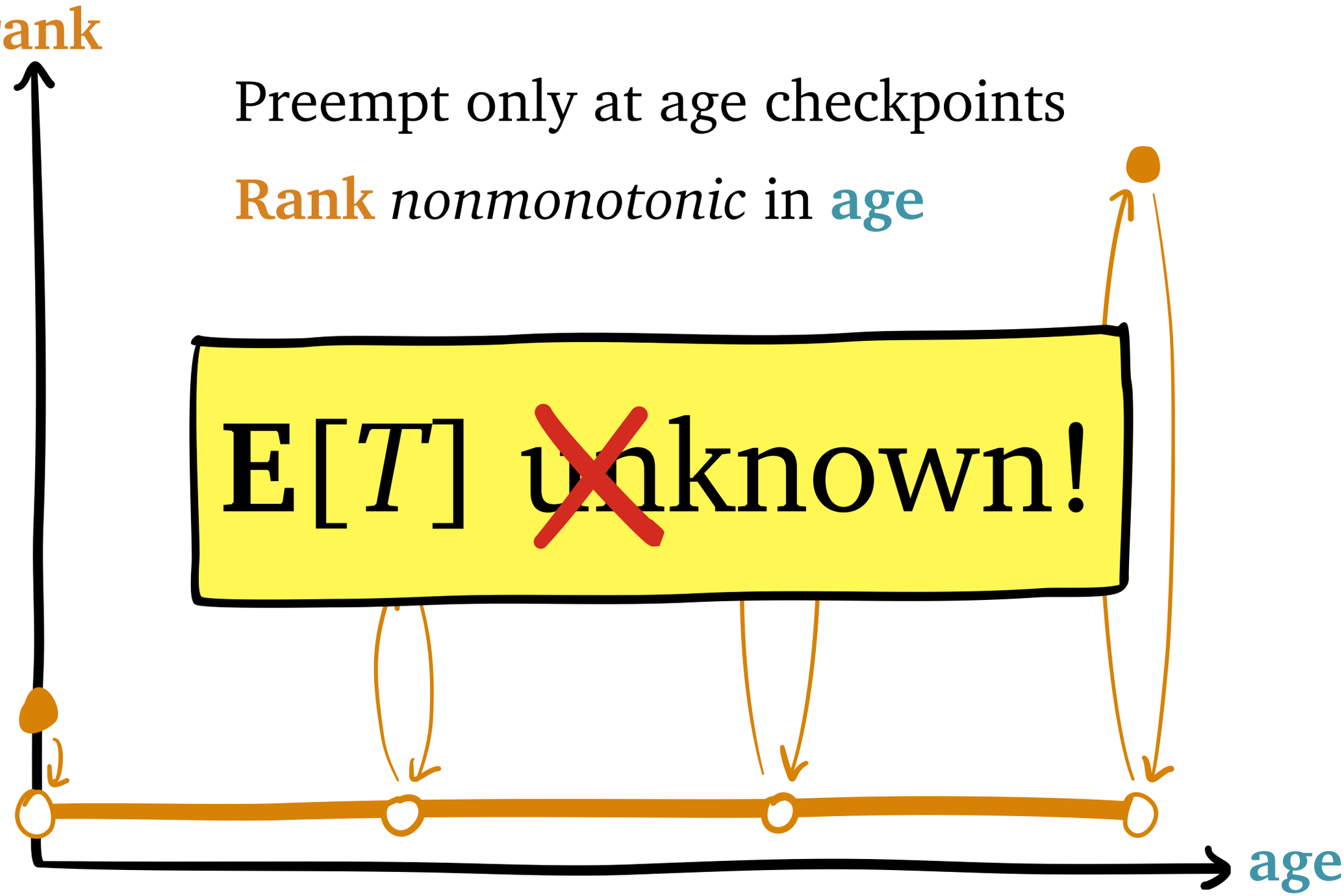
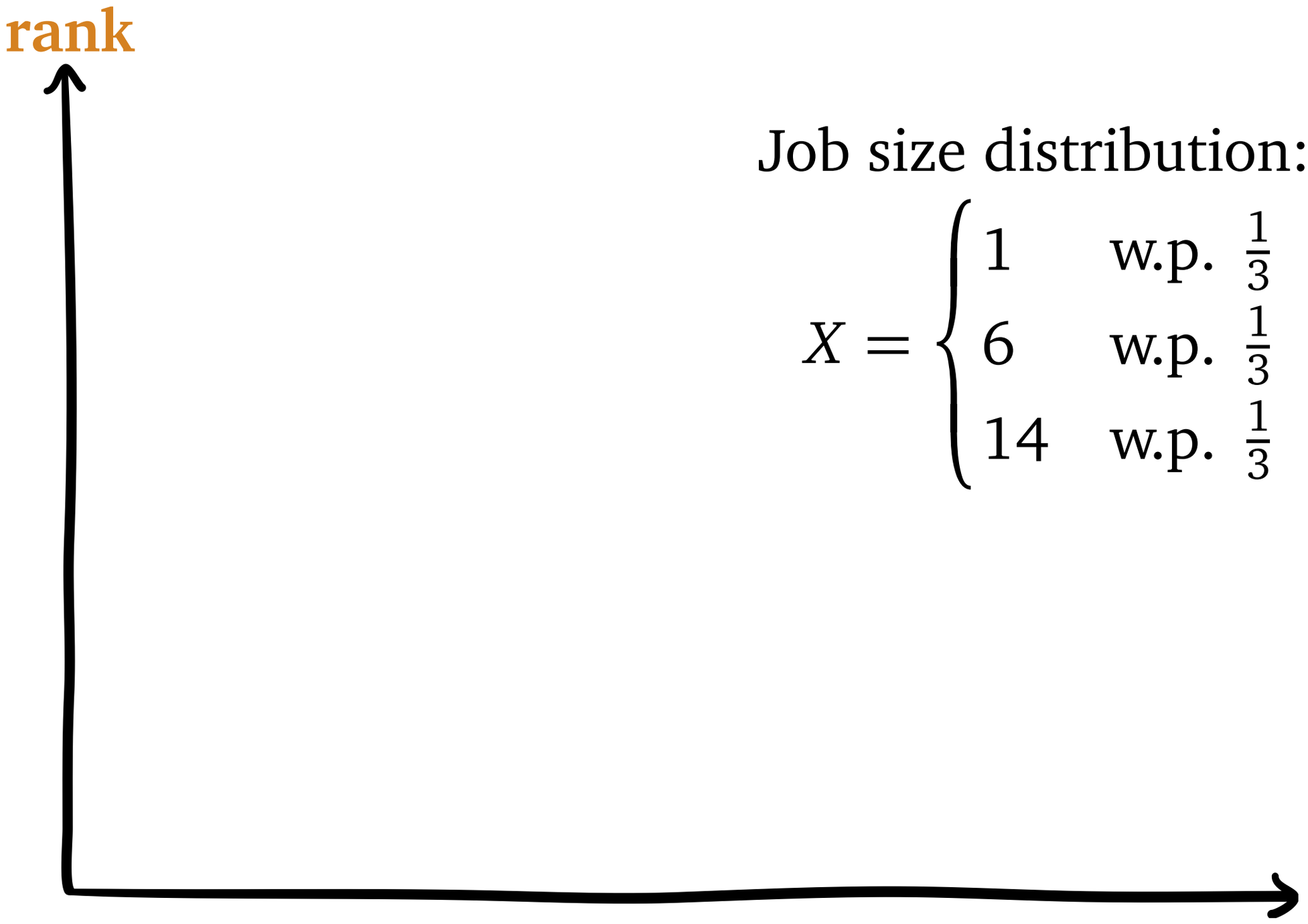


SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

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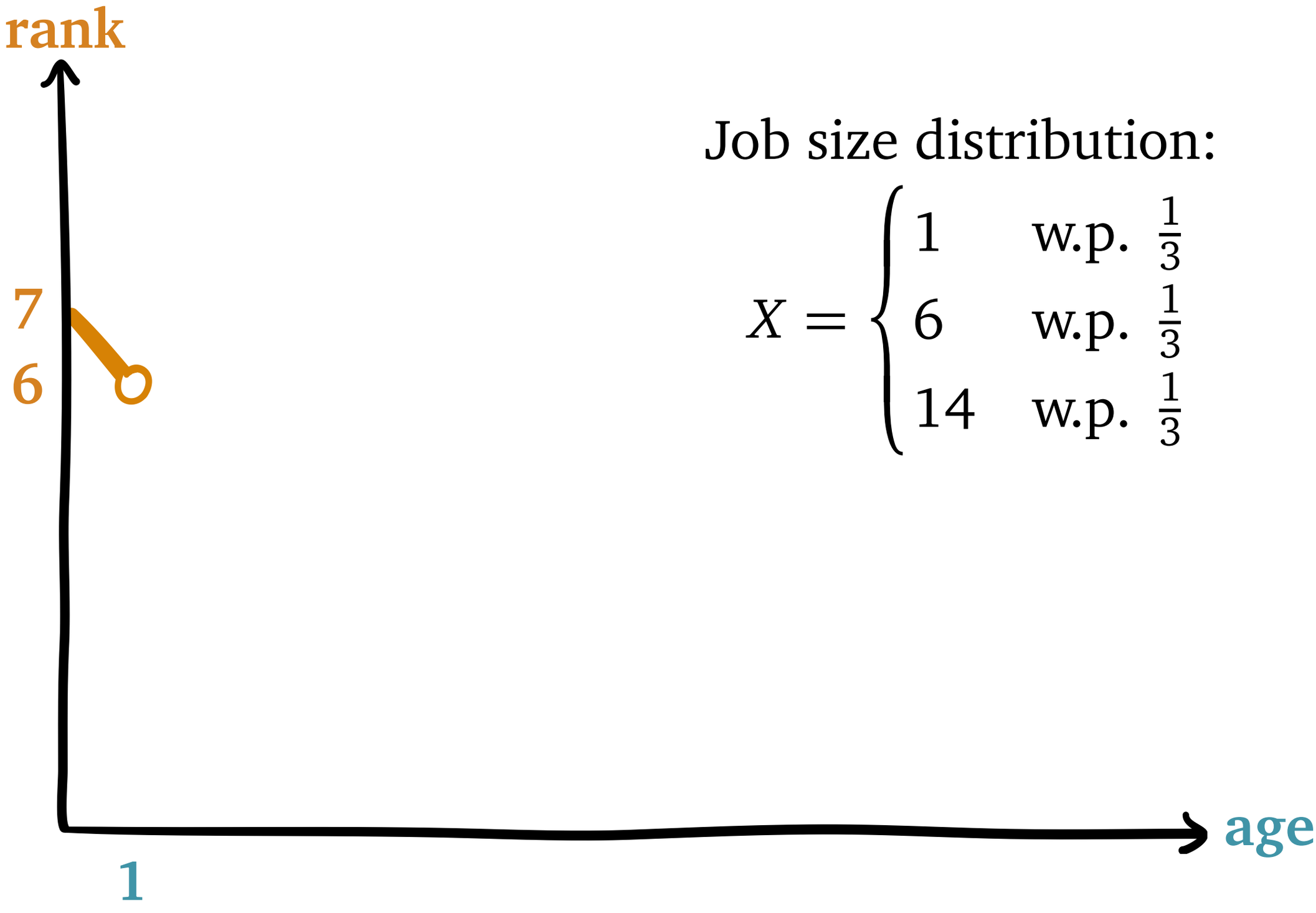

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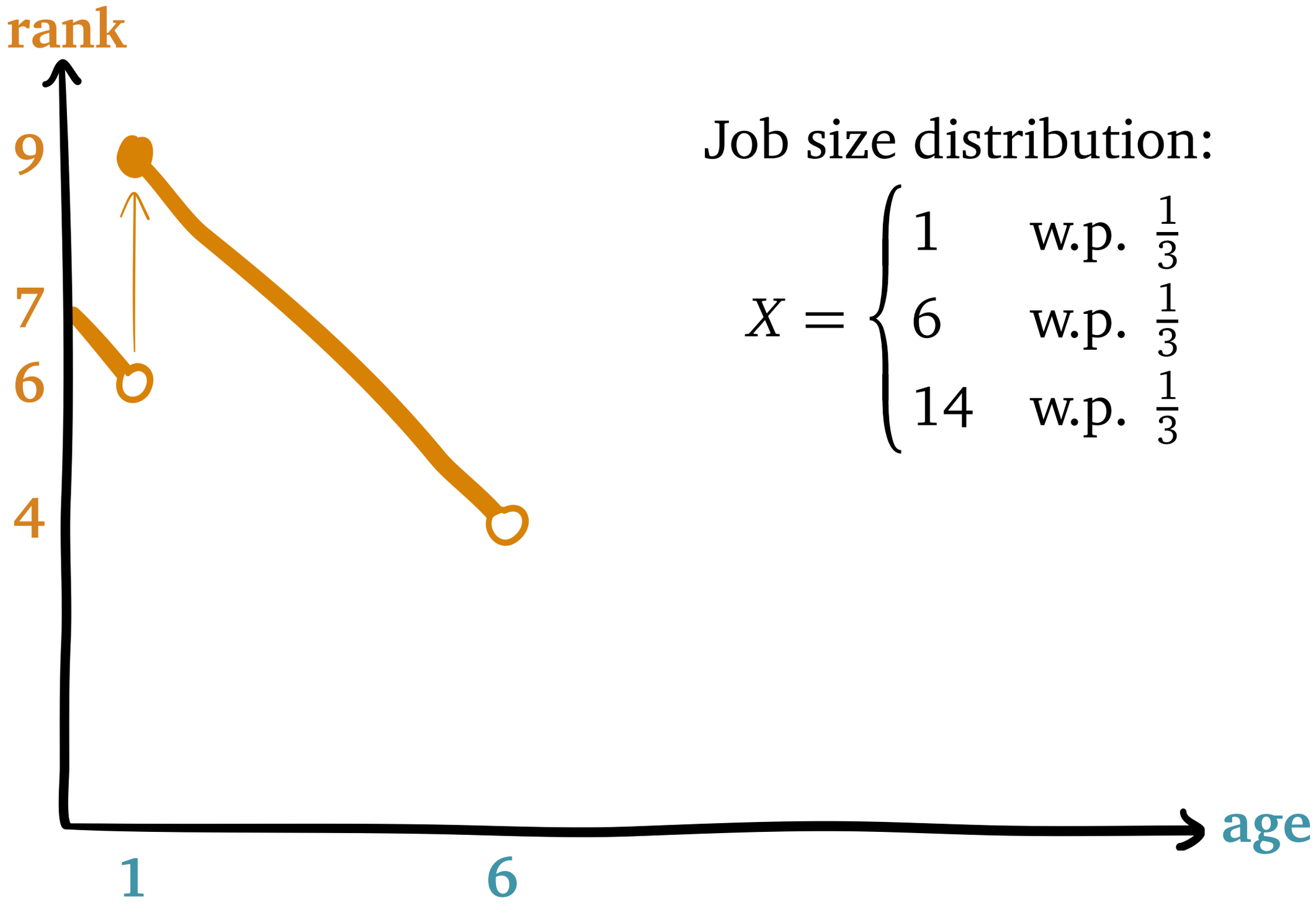

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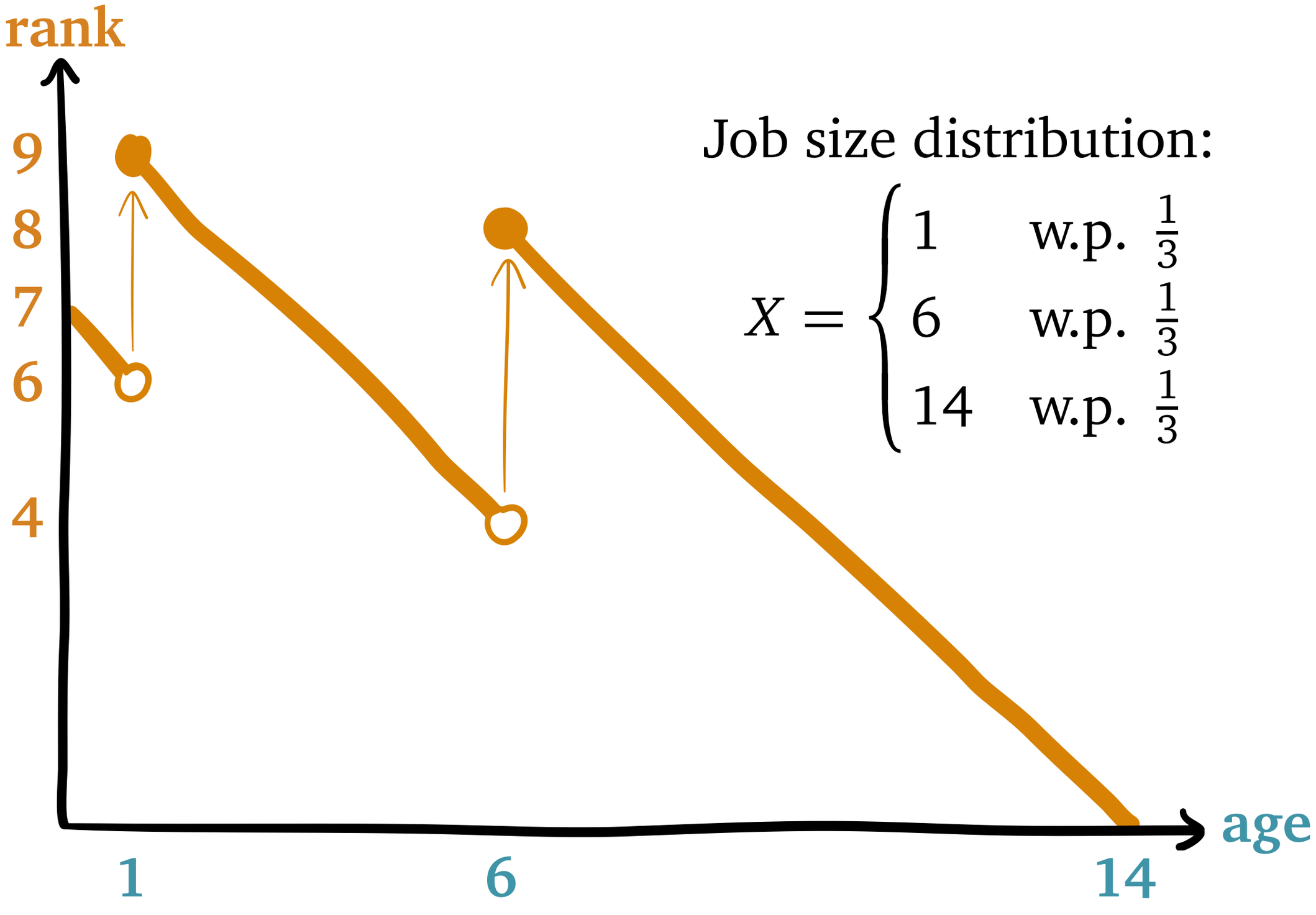

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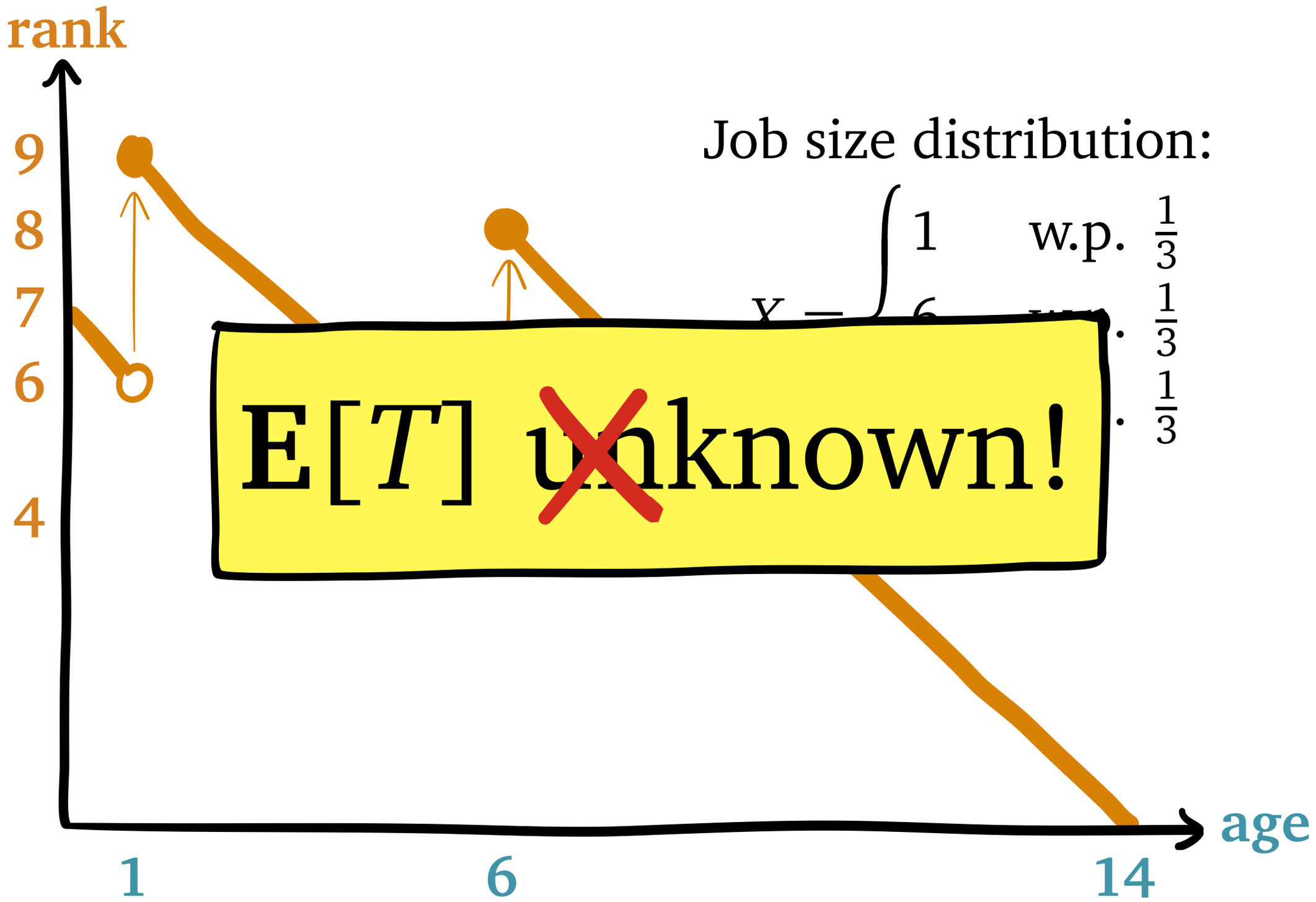
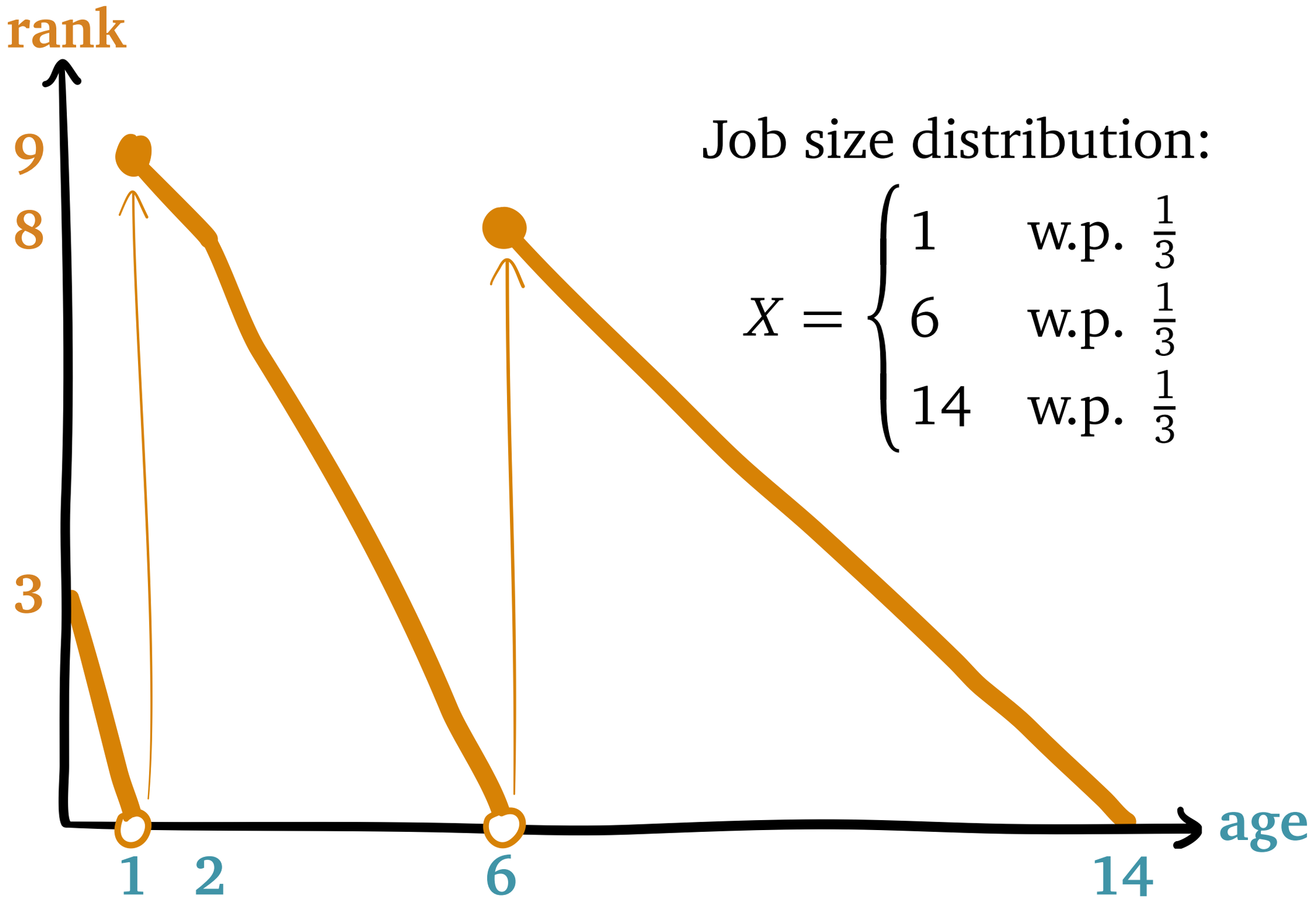


SOAP Policy: Gittins



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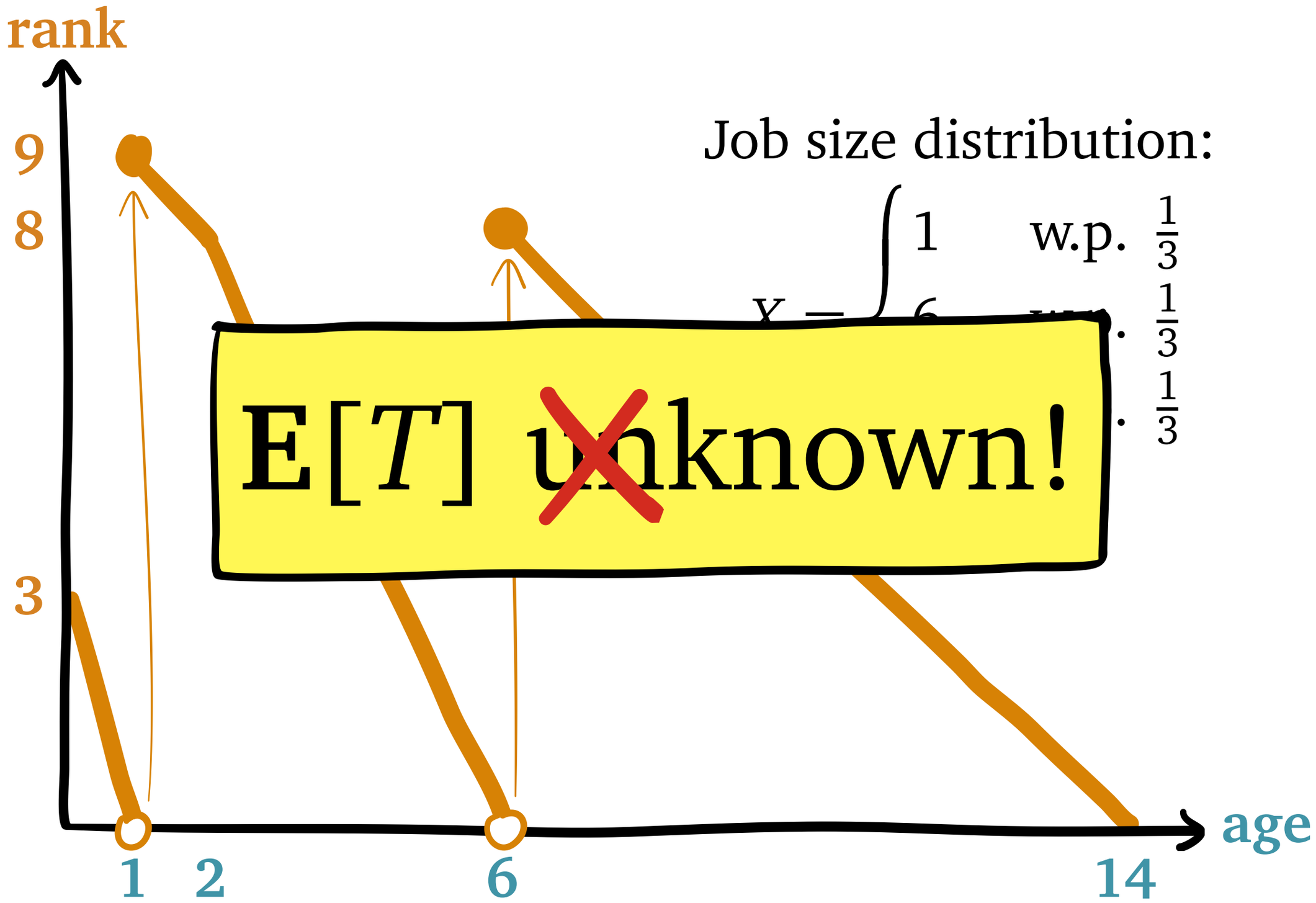

SOAP Policy: Mixture

Two customer classes: *humans* and *robots*

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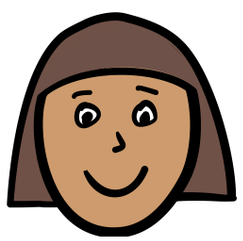

Humans

- nonpreemptible
- unknown size
- FCFS

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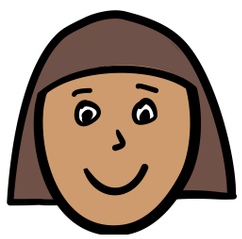

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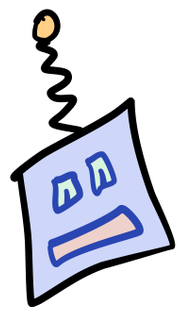

Robots

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SOAP Policy: Mixture

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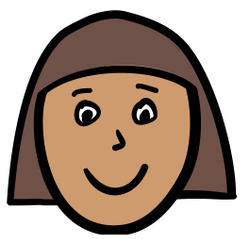

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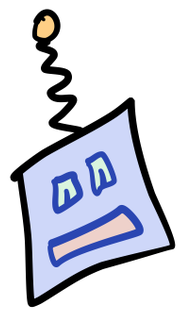

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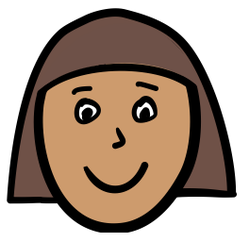

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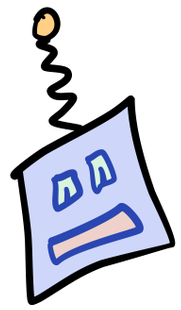

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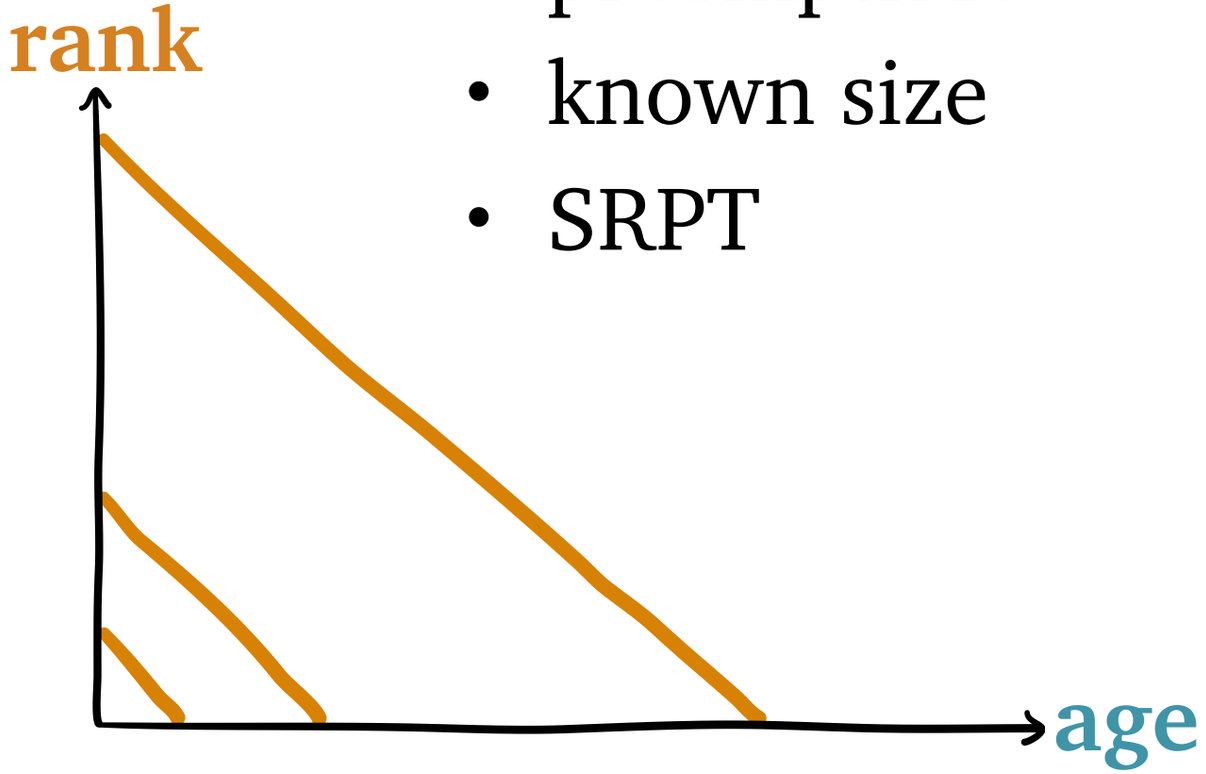

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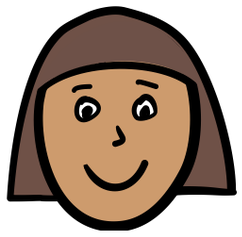

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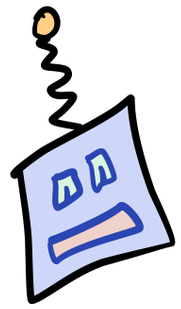

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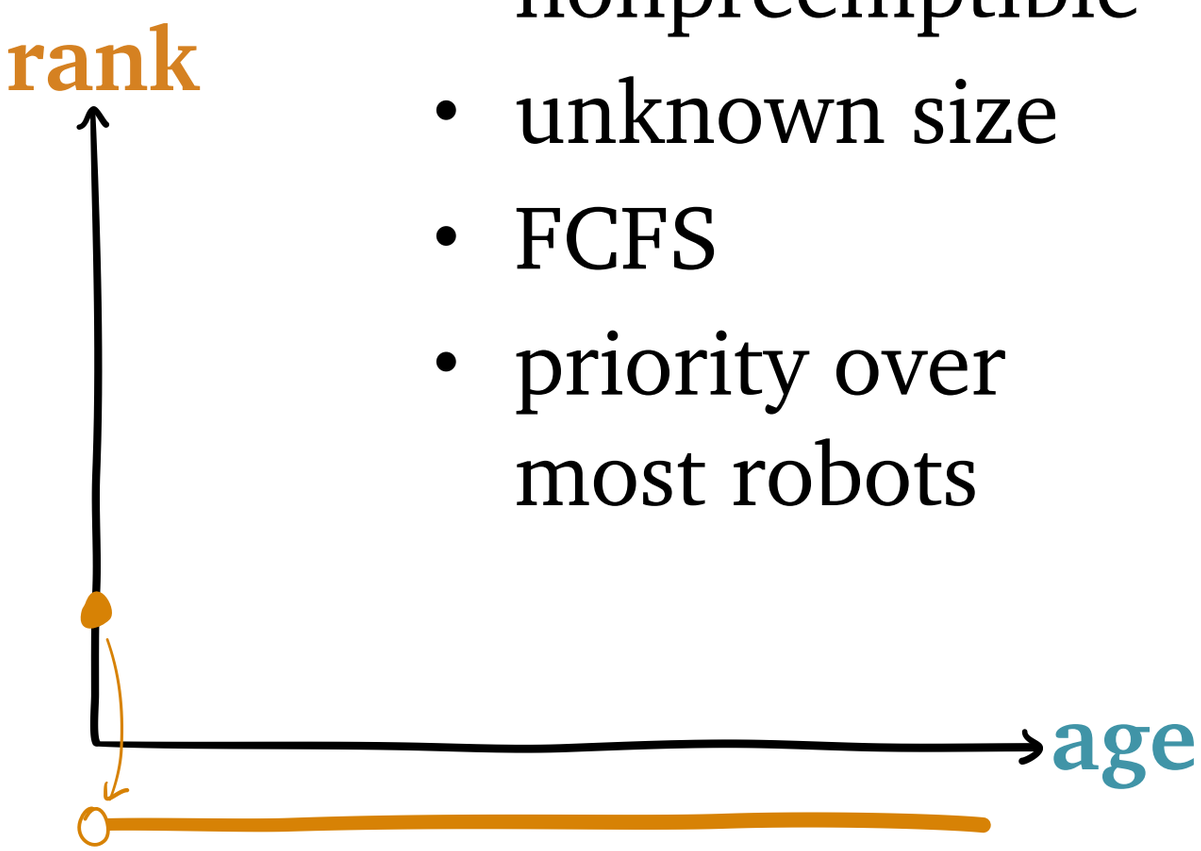

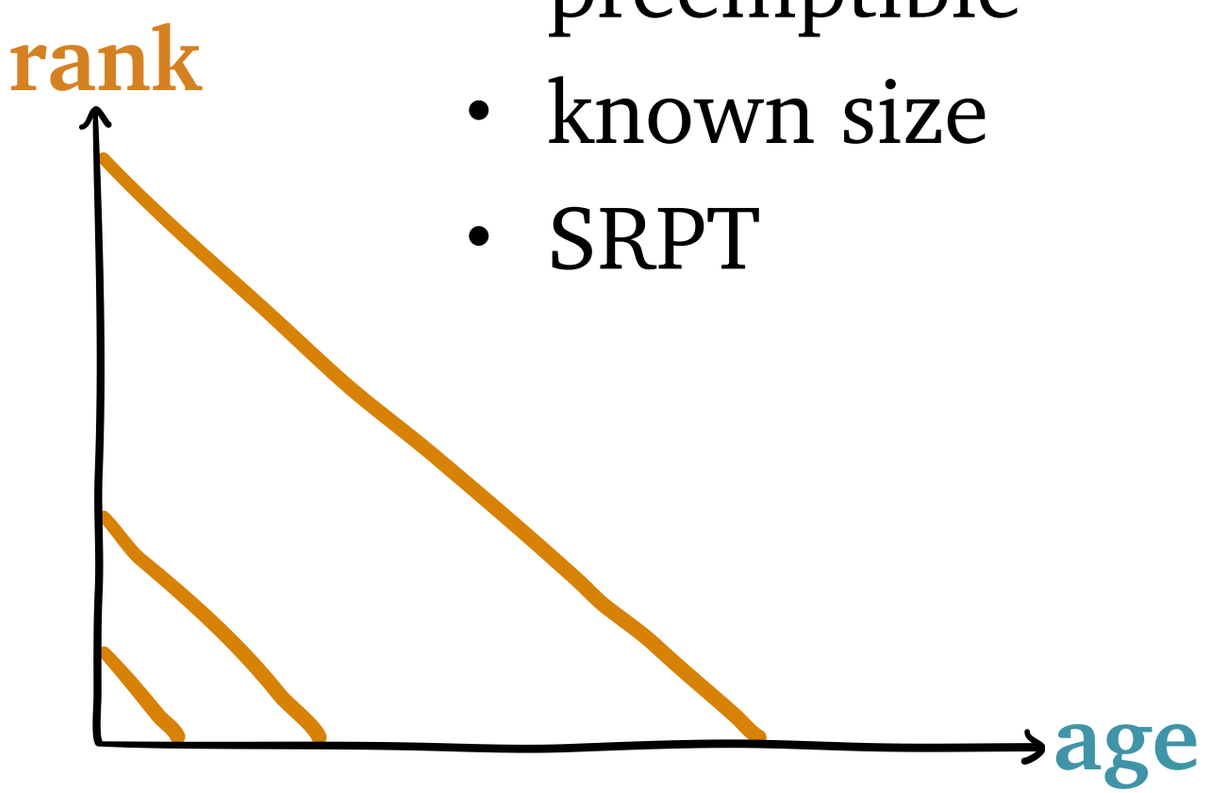

SOAP Policy: Mixture

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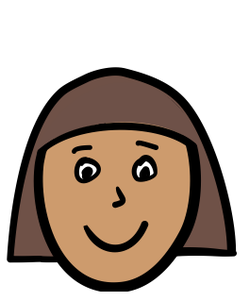

Humans

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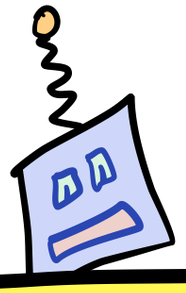
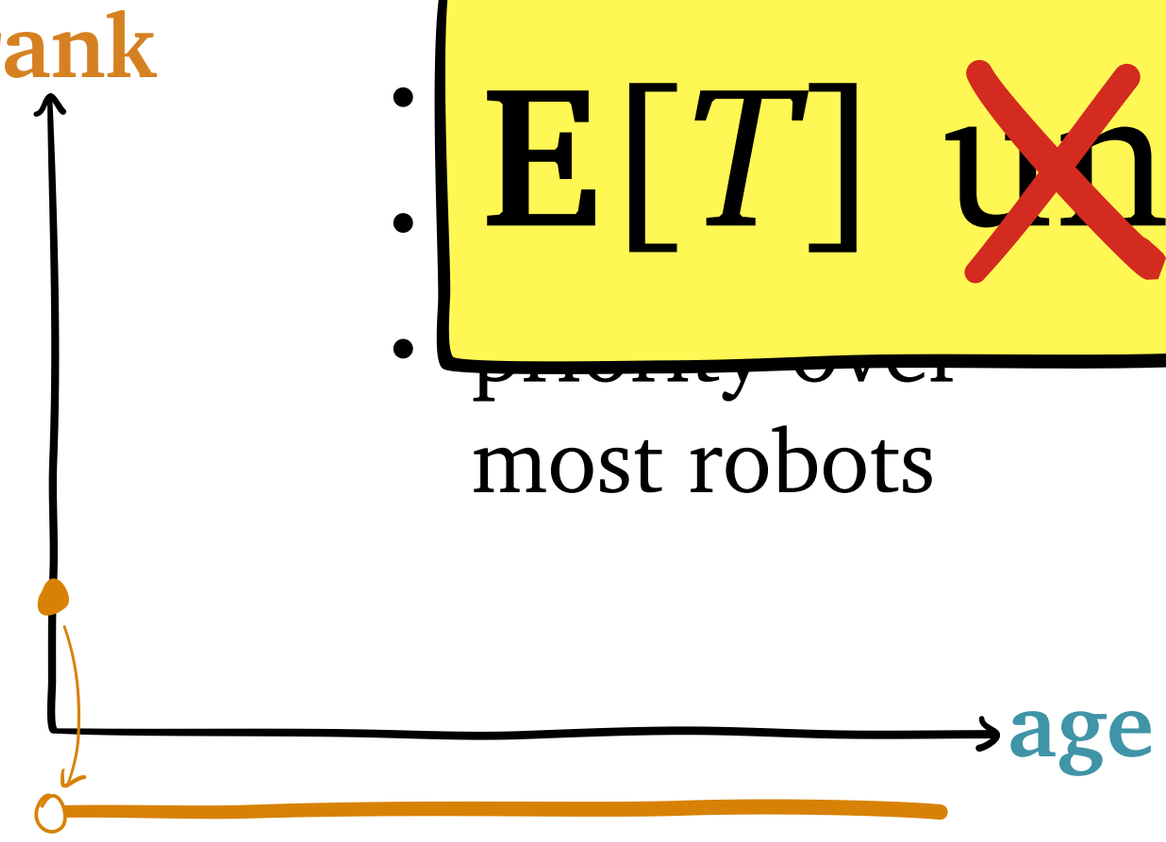
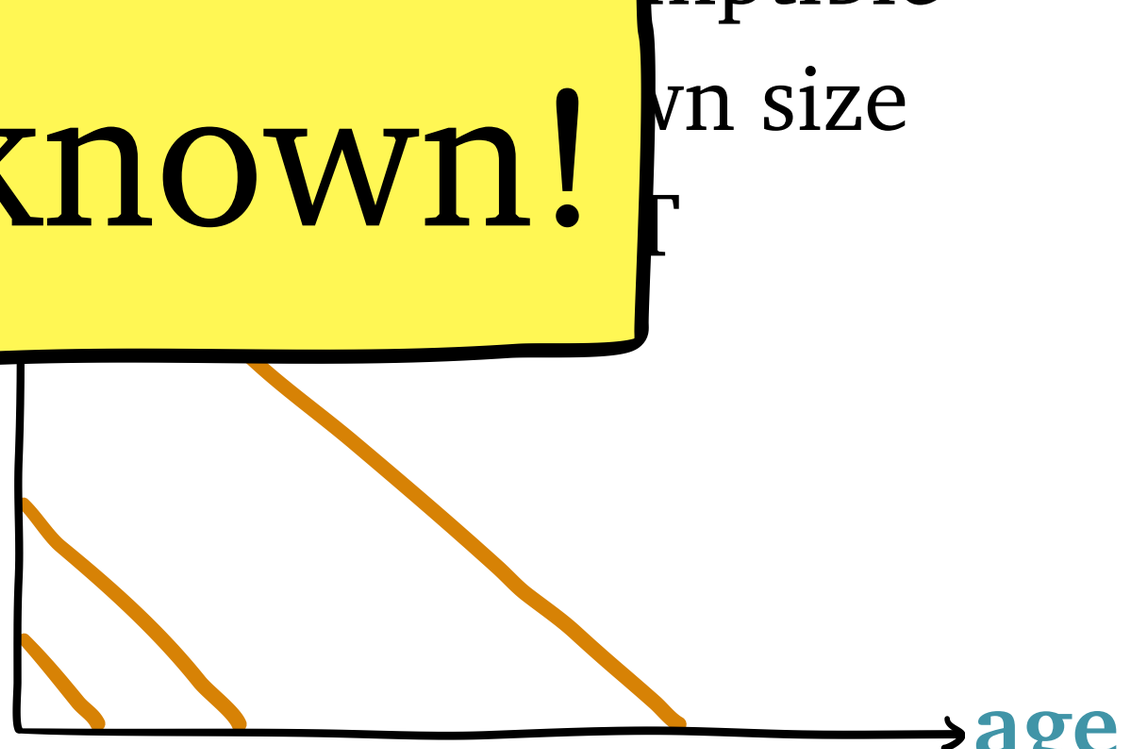
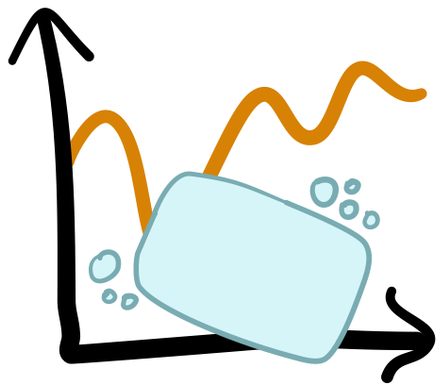


Robots

E[*T*] ~~un~~known!







SOAP

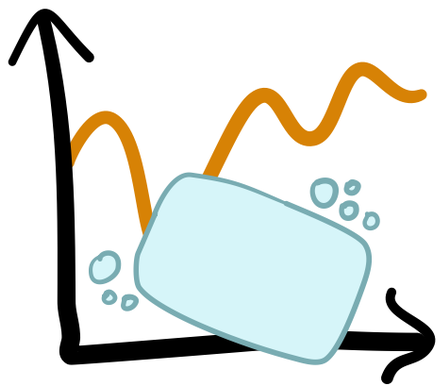
Schedule Ordered by Age-based Priority

this talk

Step 1: *unified framework* for defining scheduling policies

see paper

Step 2: *universal analysis* of $\mathbf{E}[T]$



SOAP

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this talk

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Two Key Ideas

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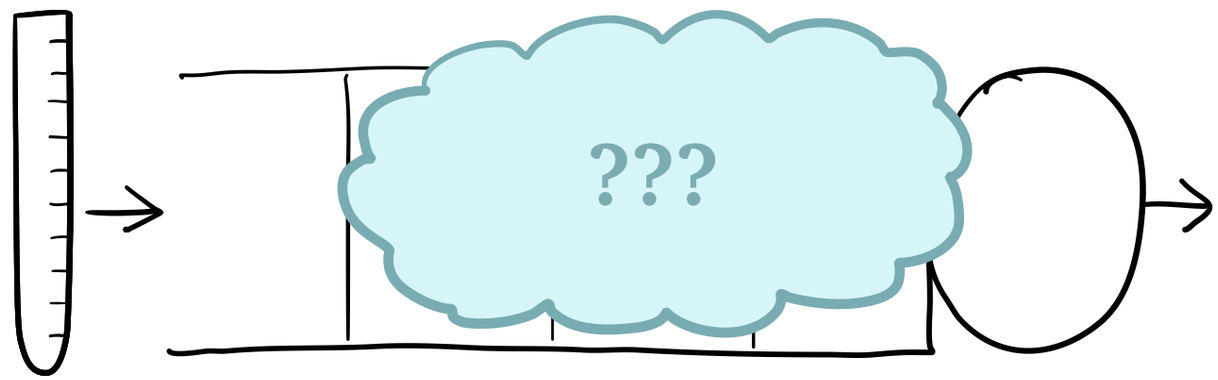
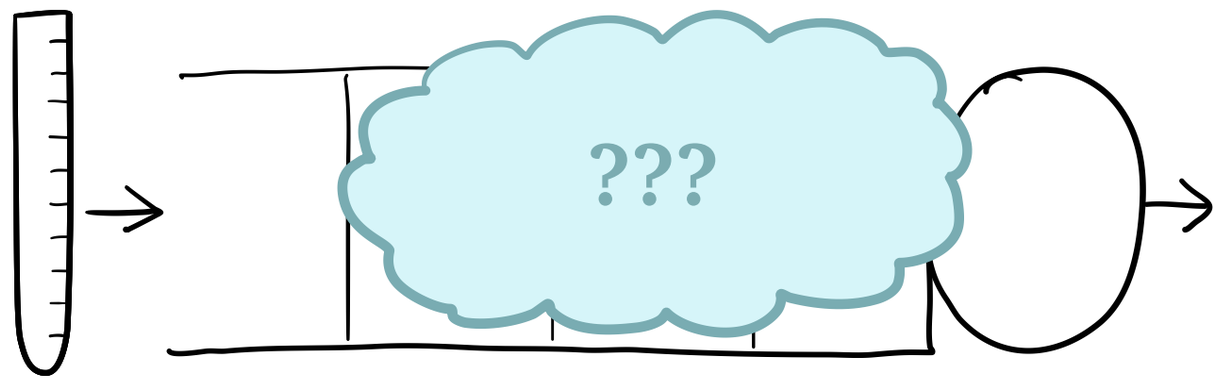


“me” random system state

Two Key Ideas



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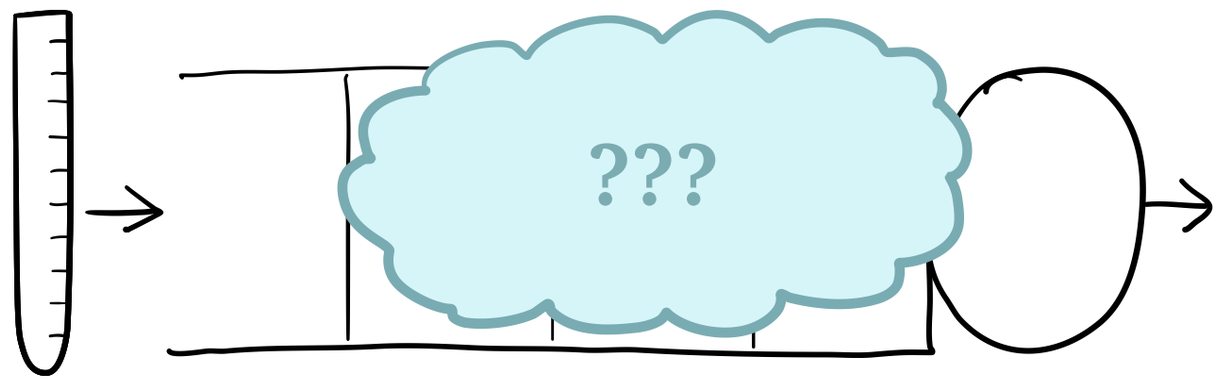
Problem: my nonmonotonicity

Two Key Ideas

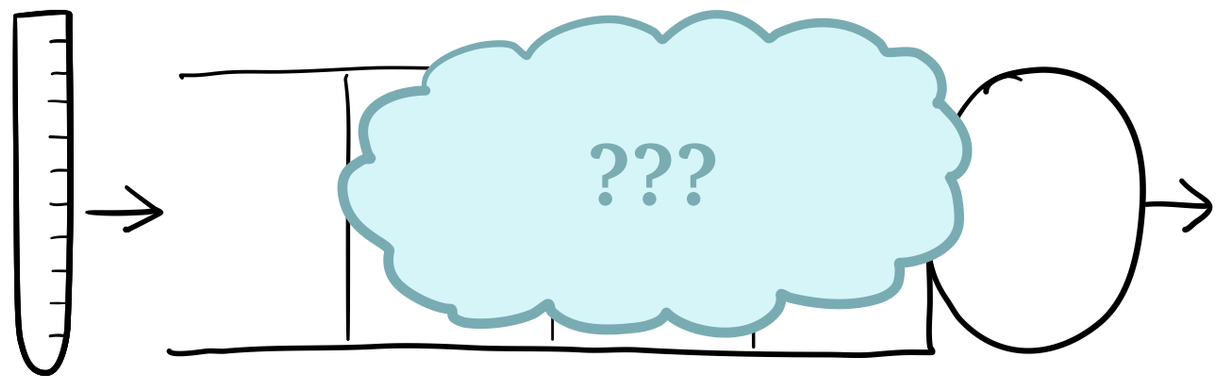


“me” random system state

Problem: my nonmonotonicity

Solution: *Pessimism Principle*

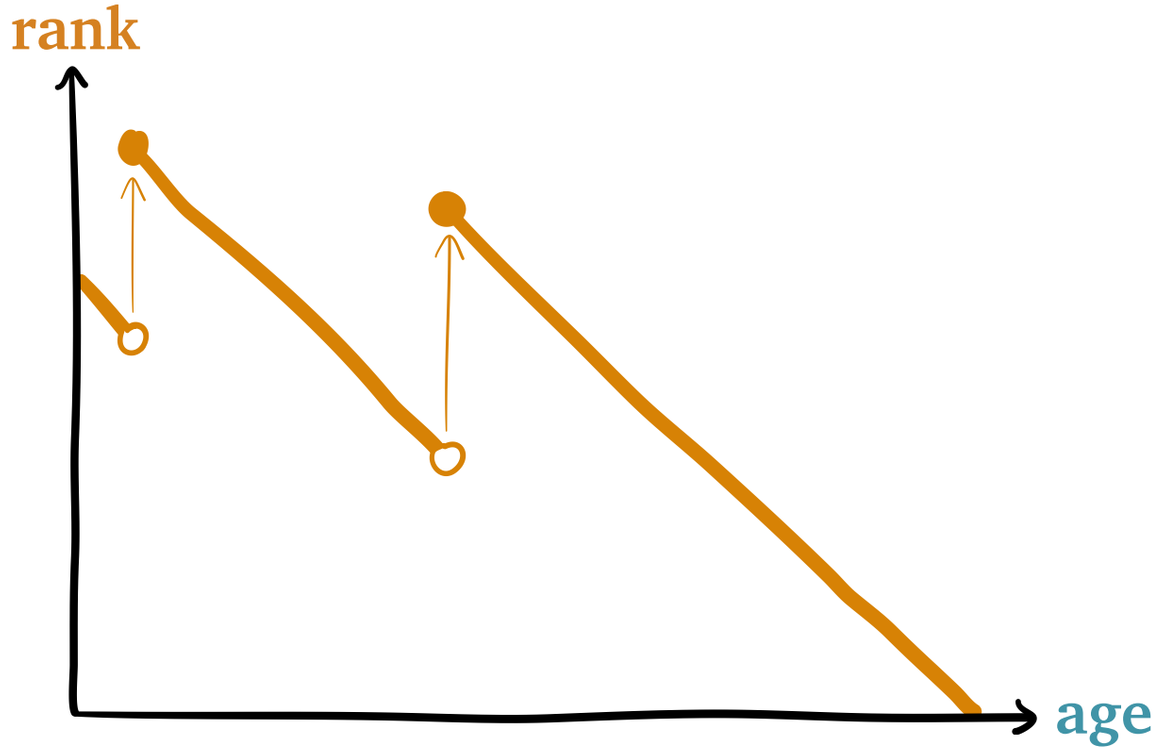
Two Key Ideas



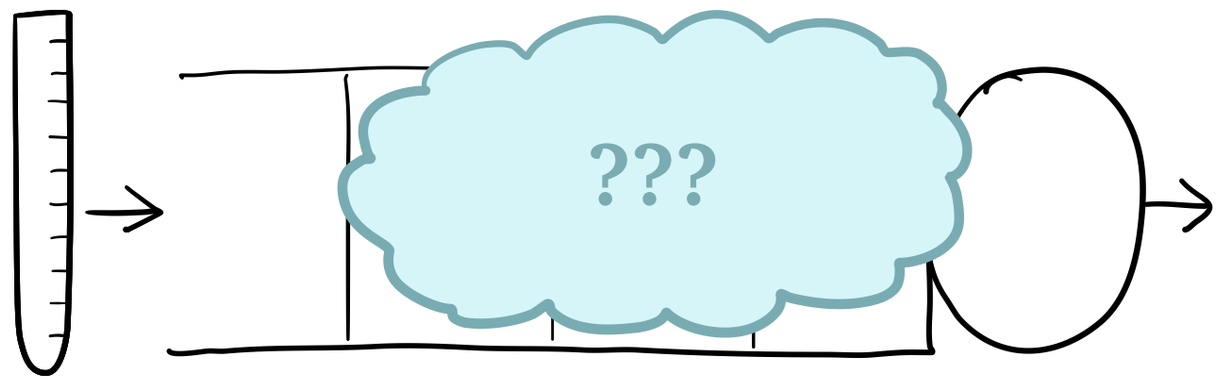
“me” random system state

Problem: my nonmonotonicity

Solution: *Pessimism Principle*



Two Key Ideas



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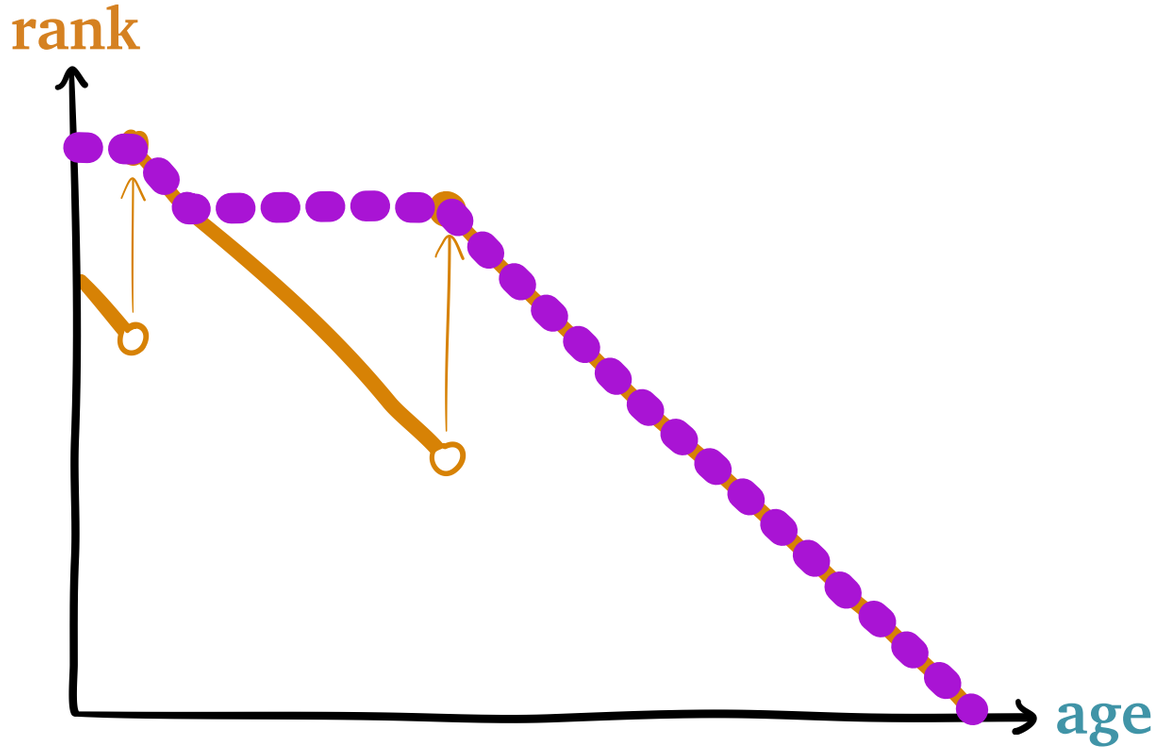
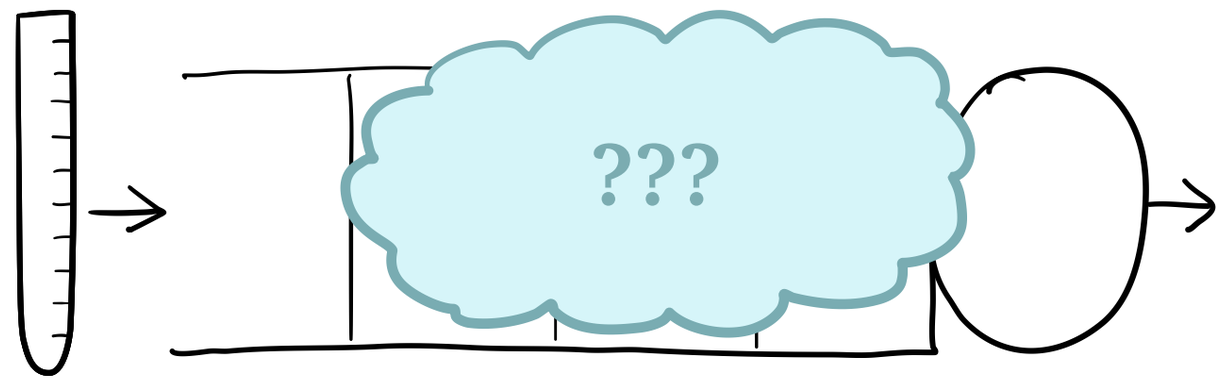


Two Key Ideas

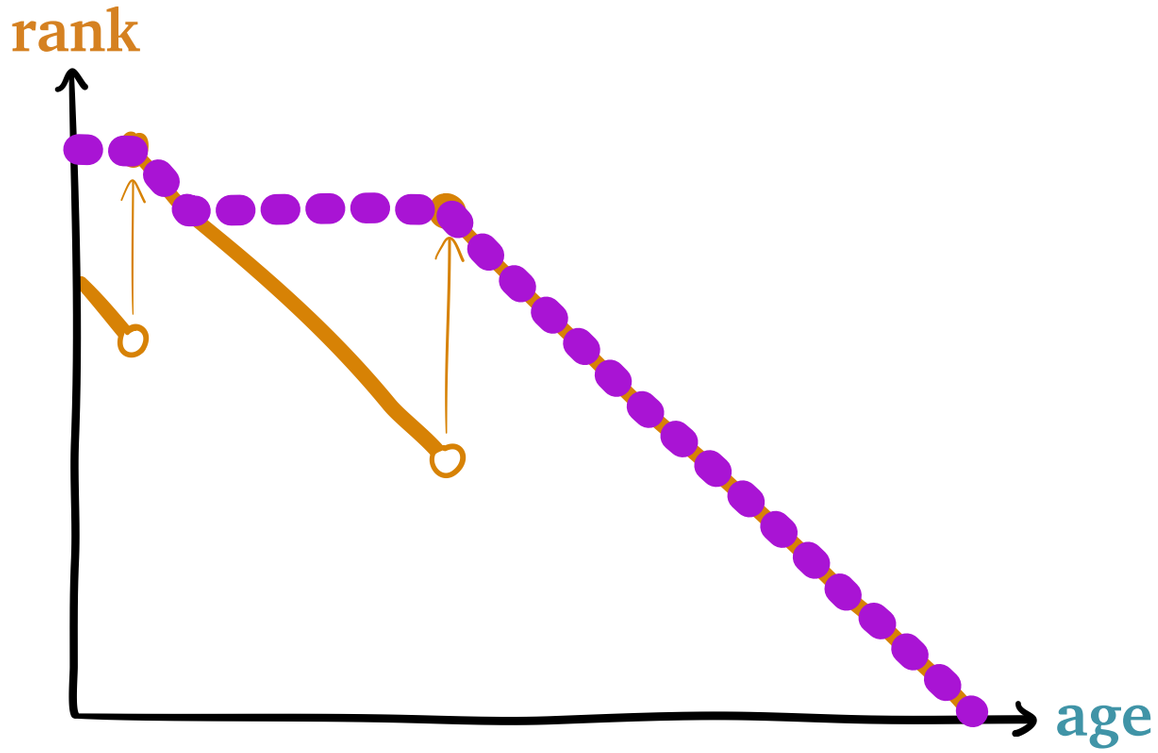


"me" random system state

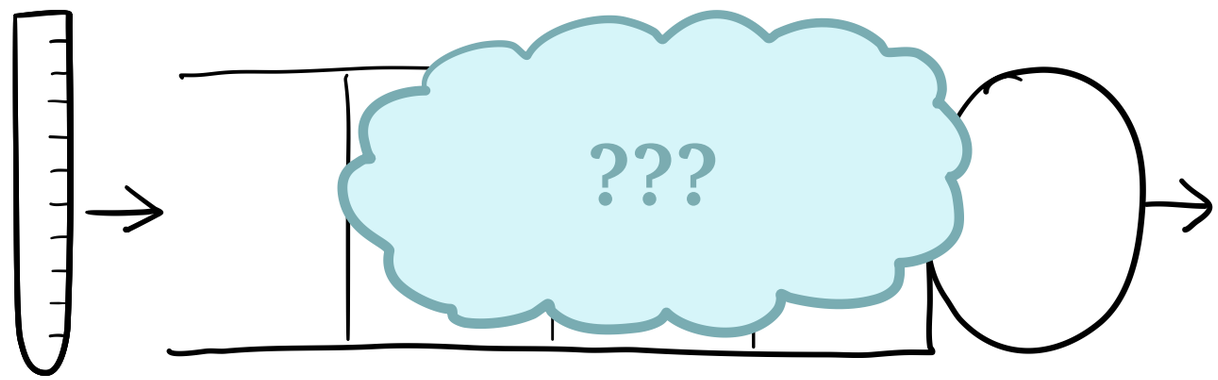
Problem: my nonmonotonicity

Solution: *Pessimism Principle*

Problem: their nonmonotonicity



Two Key Ideas



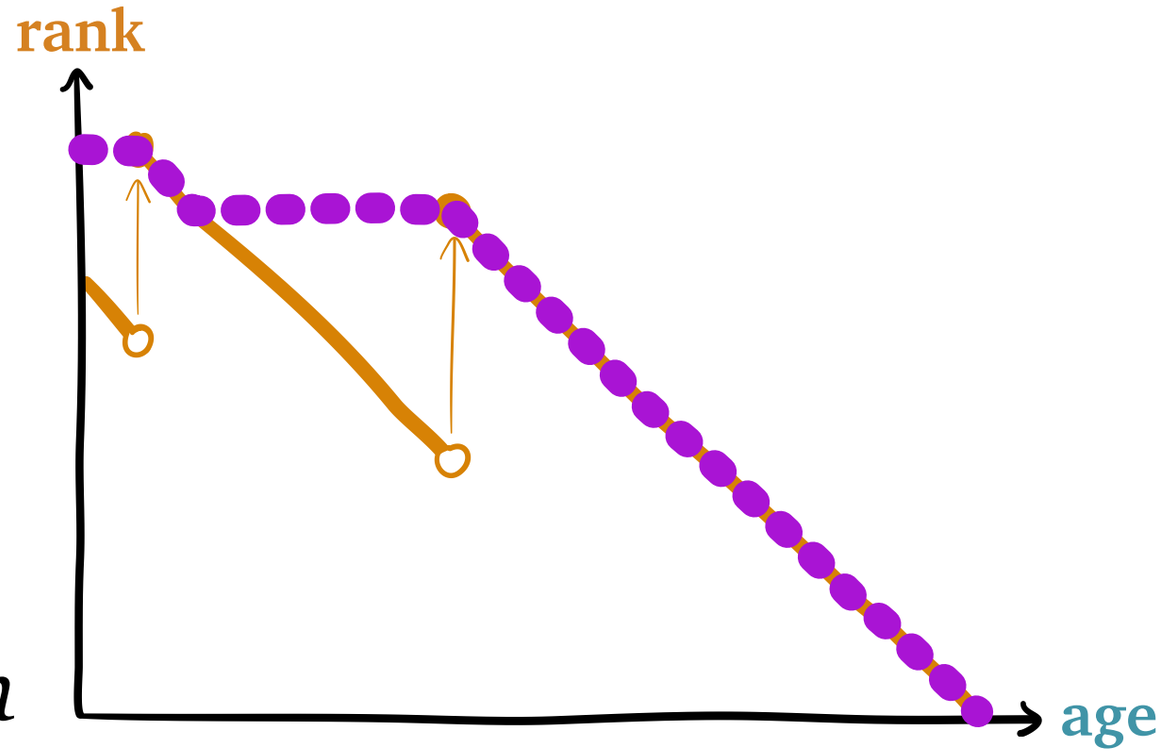
“me” random system state

Problem: my nonmonotonicity

Solution: *Pessimism Principle*

Problem: their nonmonotonicity

Solution: *Vacation Transformation*

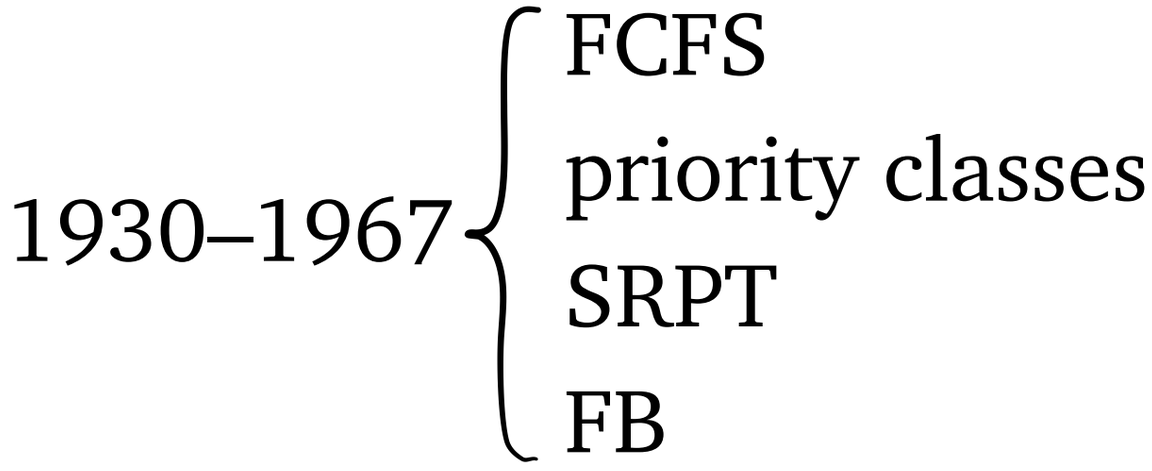


FCFS

priority classes

SRPT

FB


1930–1967
FCFS
priority classes
SRPT
FB

1930–1967
- FCFS
- priority classes
- SRPT
- FB

discrete FB

SERPT

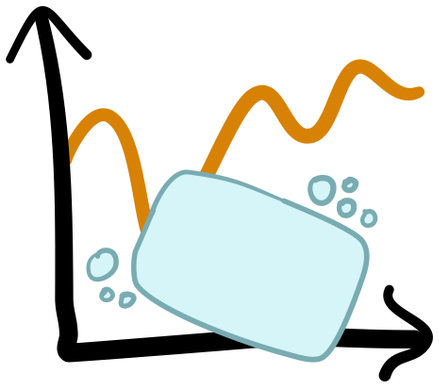
Gittins policy (optimal!)

mixture of policies

many more…

SOAP

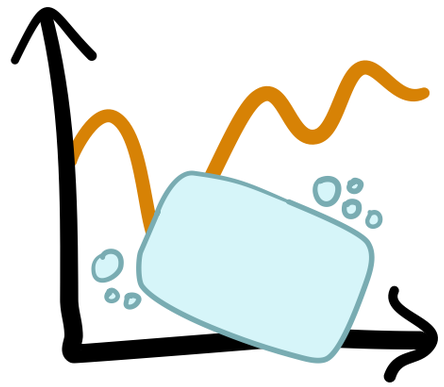
2018



- 1930–1967
  - FCFS
  - priority classes
  - SRPT
  - FB
- discrete FB
- SERPT
- Gittins policy (optimal!)
- mixture of policies
- many more…

SOAP

2018



- 1930–1967
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- many more…

$$\mathbf{E}[T(x)] = \frac{\lambda \sum_{i=0}^{\infty} \mathbf{E}[X_i^{\text{old}}[r]^2]}{2(1-\rho_0^{\text{old}}[r])(1-\rho^{\text{new}}[r])} + \int_0^x \frac{1}{1-\rho^{\text{new}}[r(a)]}\, da$$