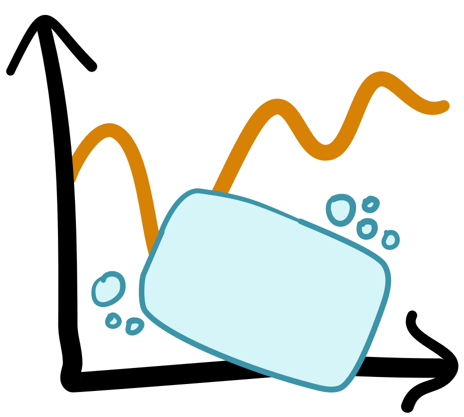
SOAP:

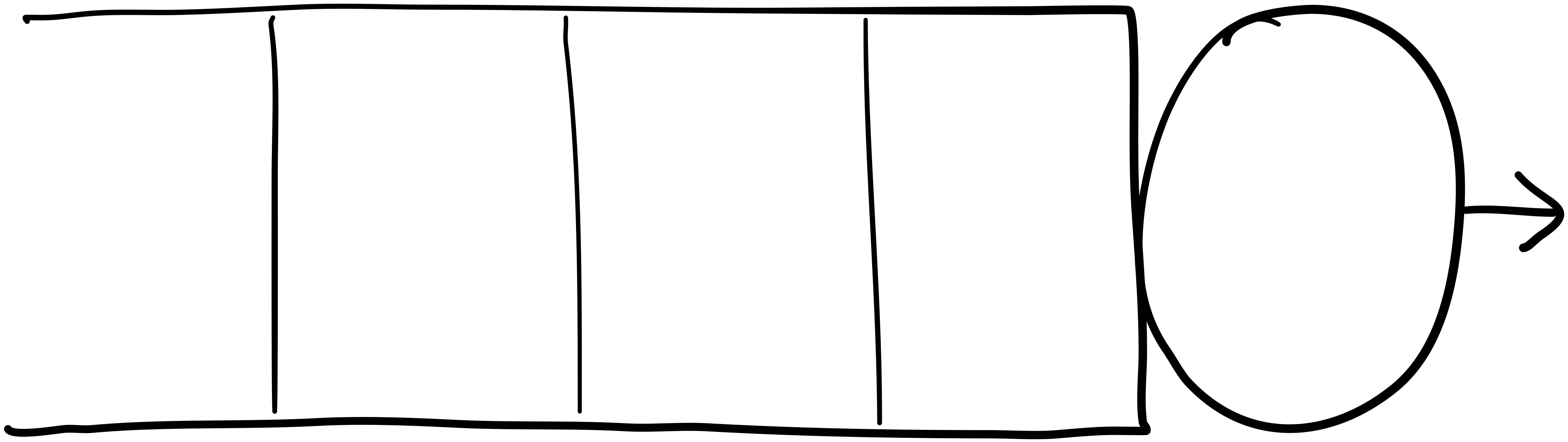
One Clean Analysis of All Age-Based Scheduling Policies

Ziv Scully
Mor Harchol-Balter
Alan Scheller-Wolf

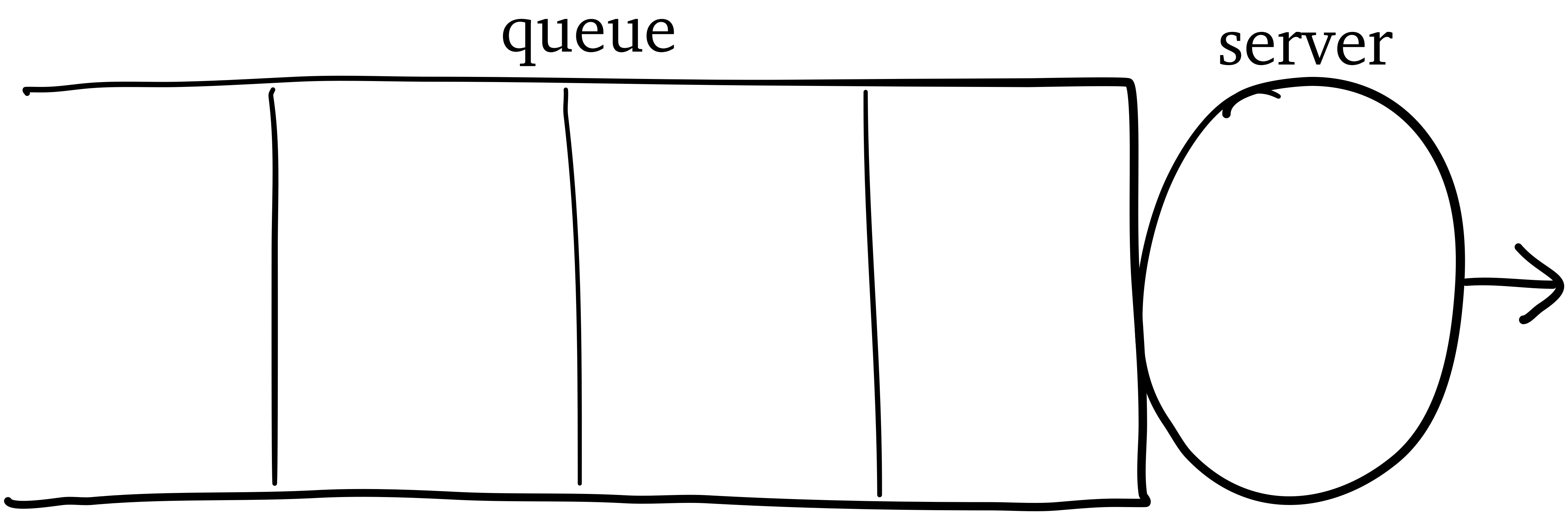
Carnegie Mellon University



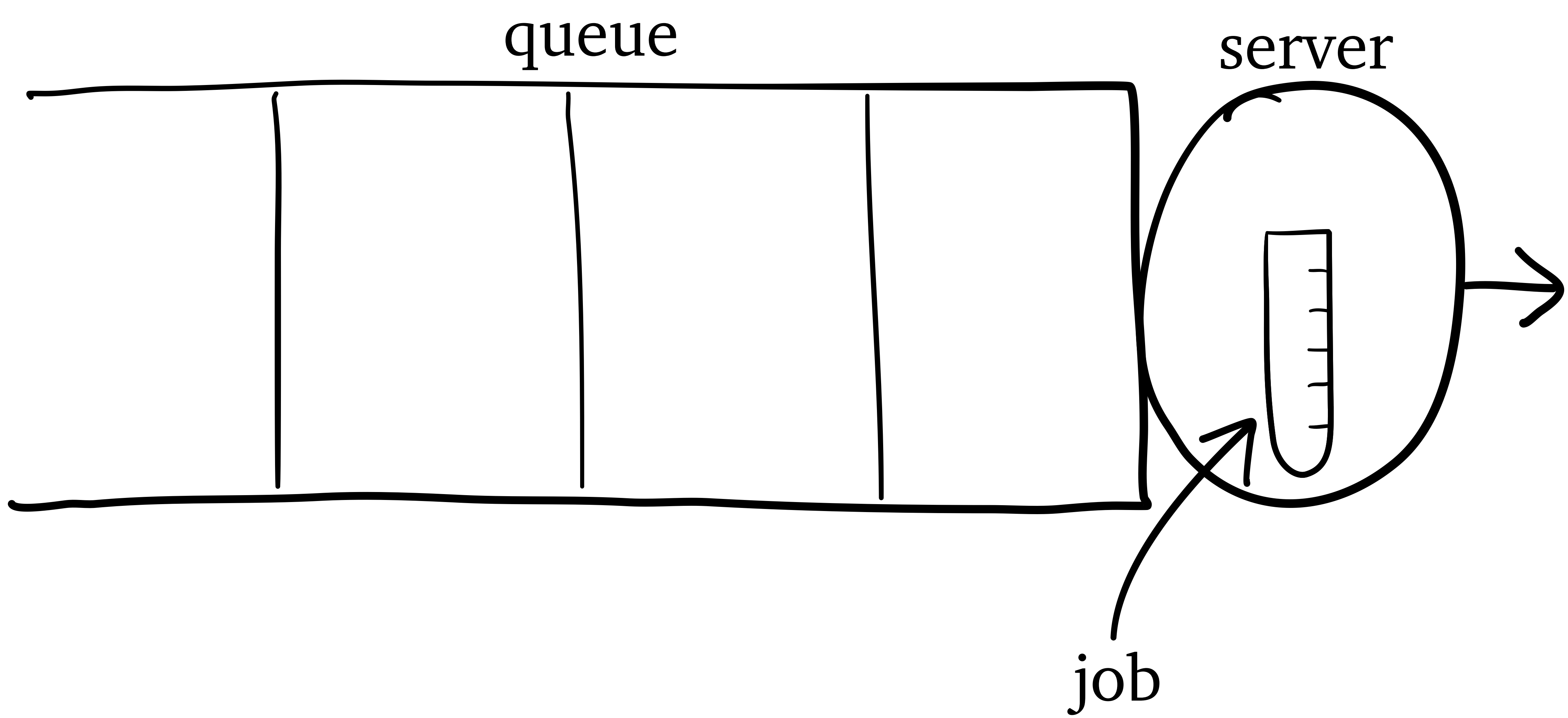
M/G/1 Queue



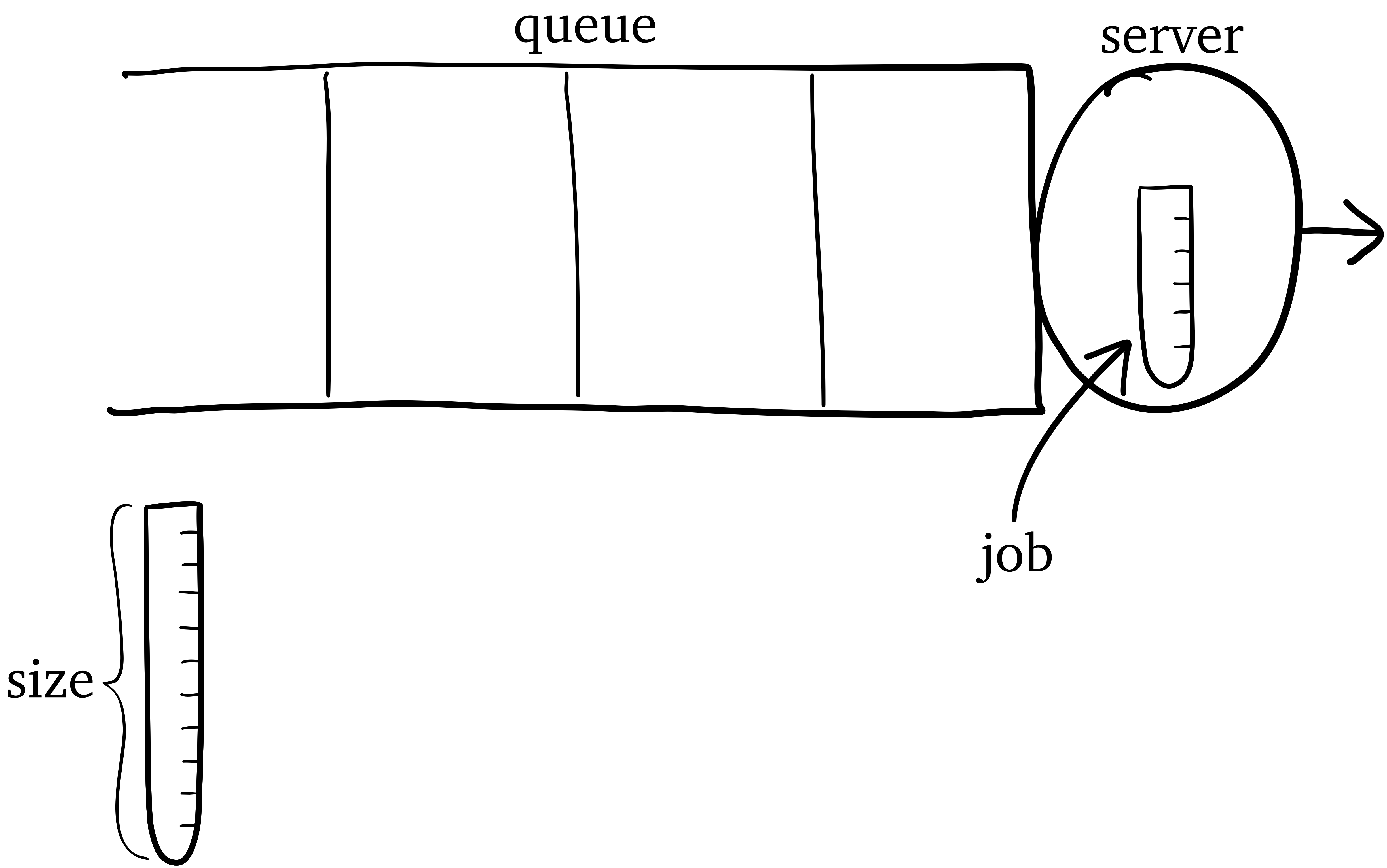
M/G/1 Queue



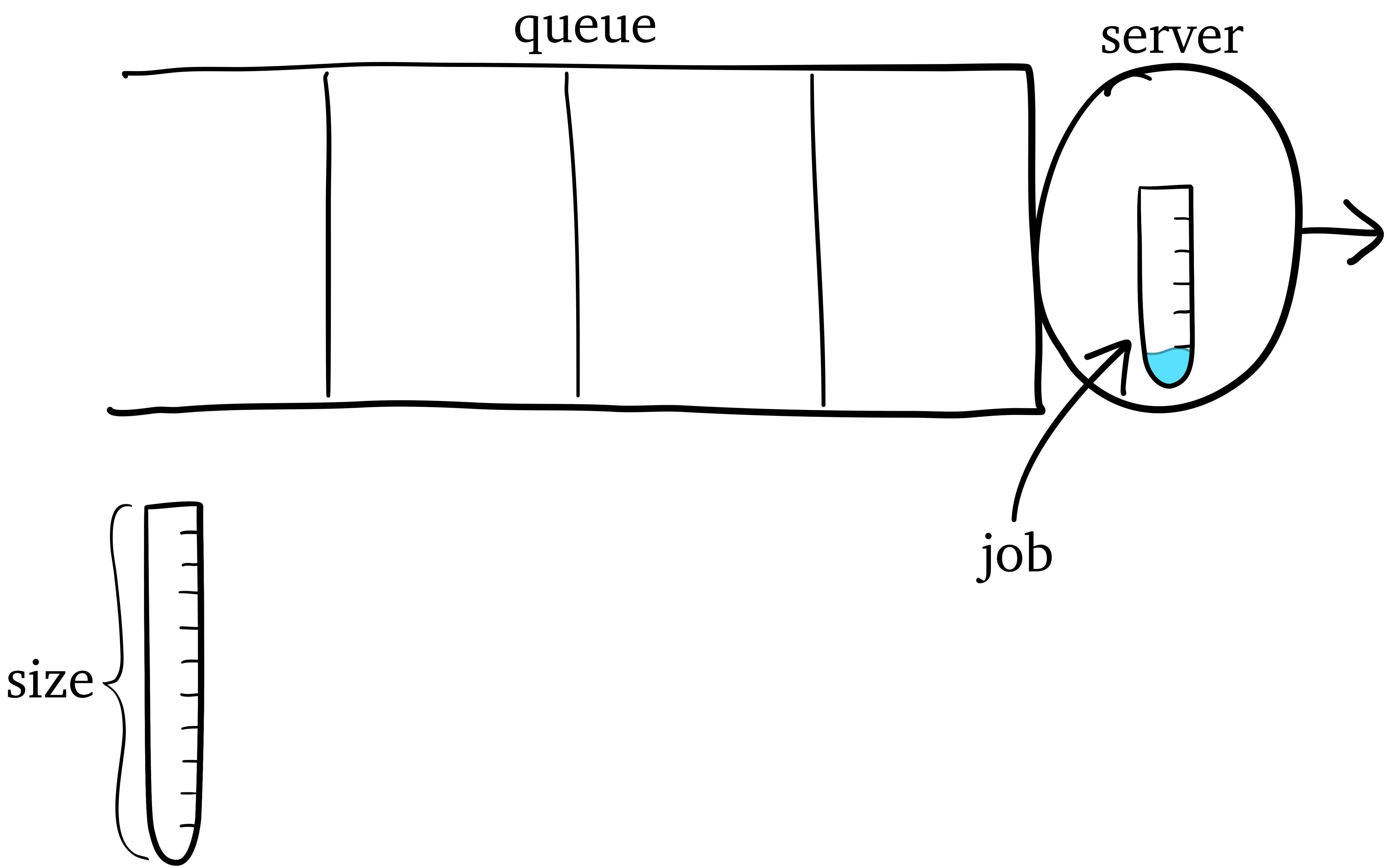
M/G/1 Queue



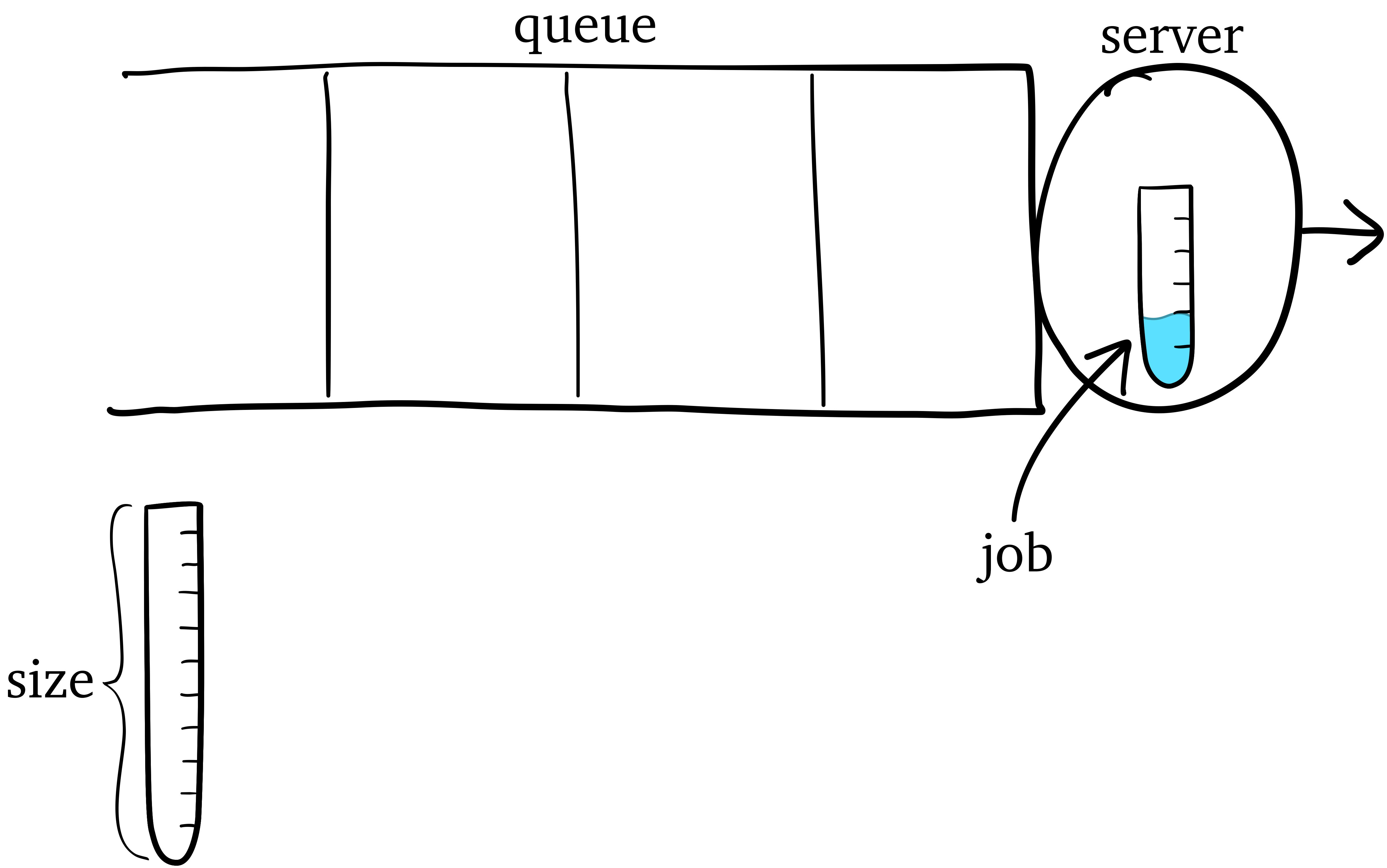
M/G/1 Queue



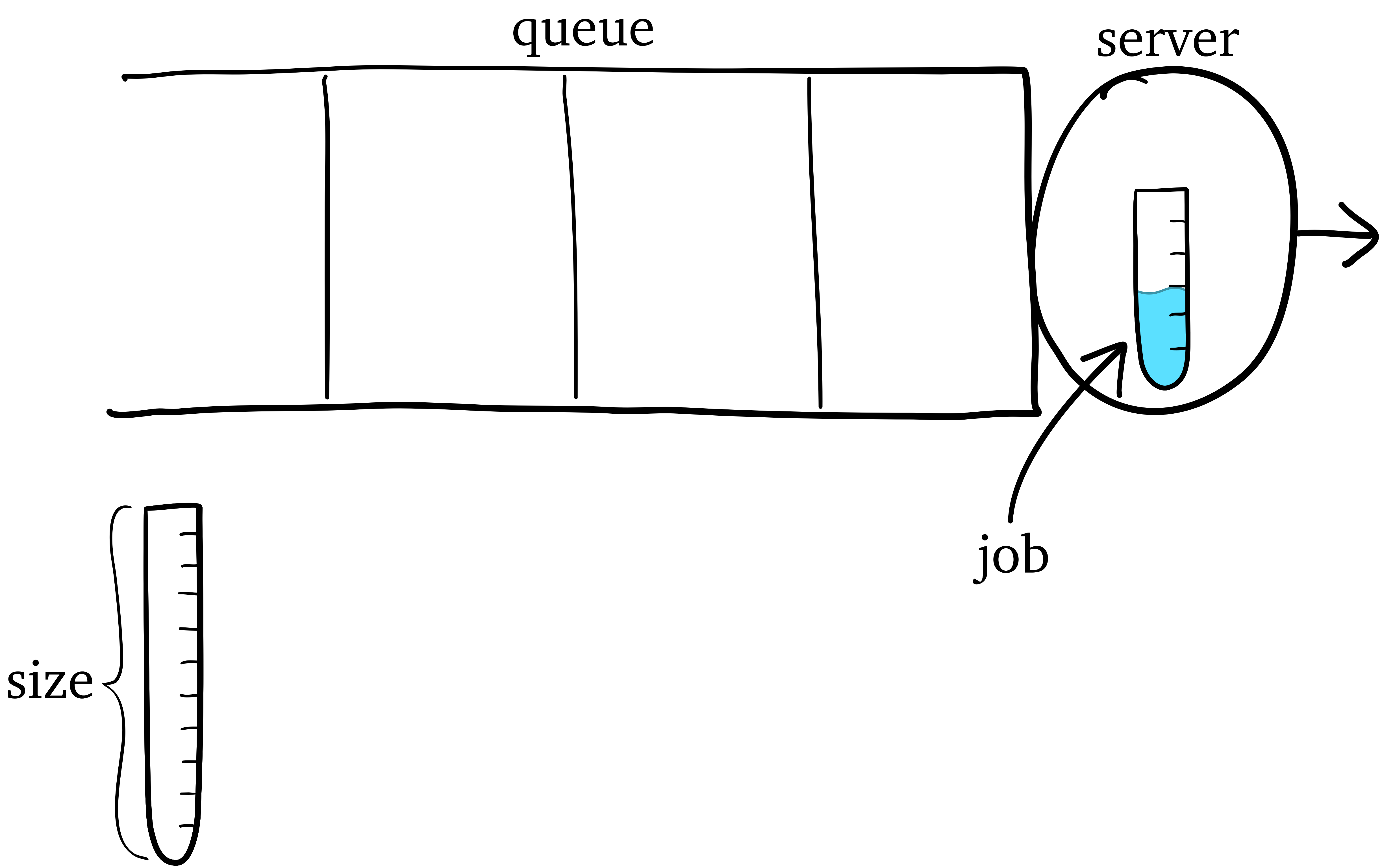
M/G/1 Queue



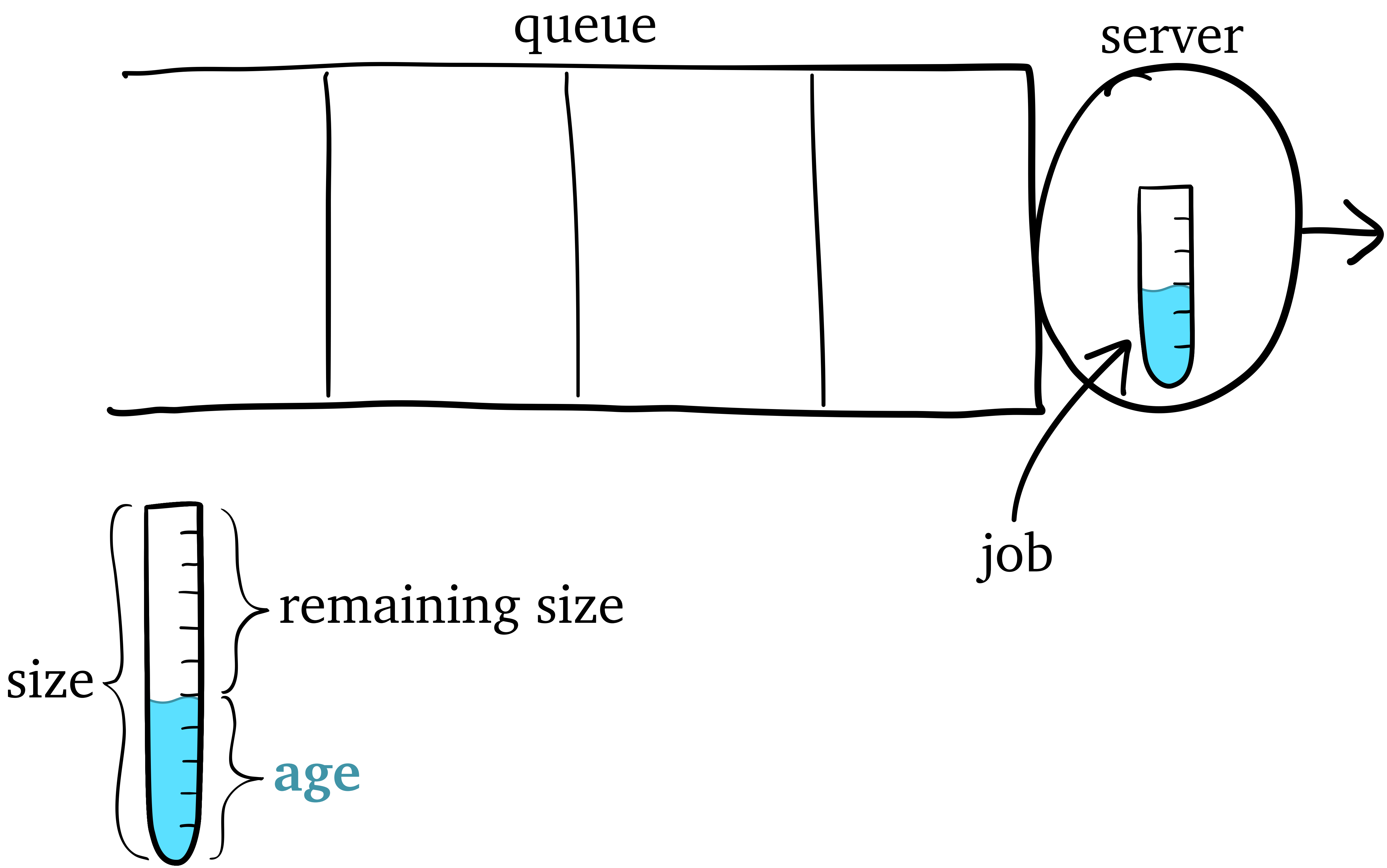
M/G/1 Queue



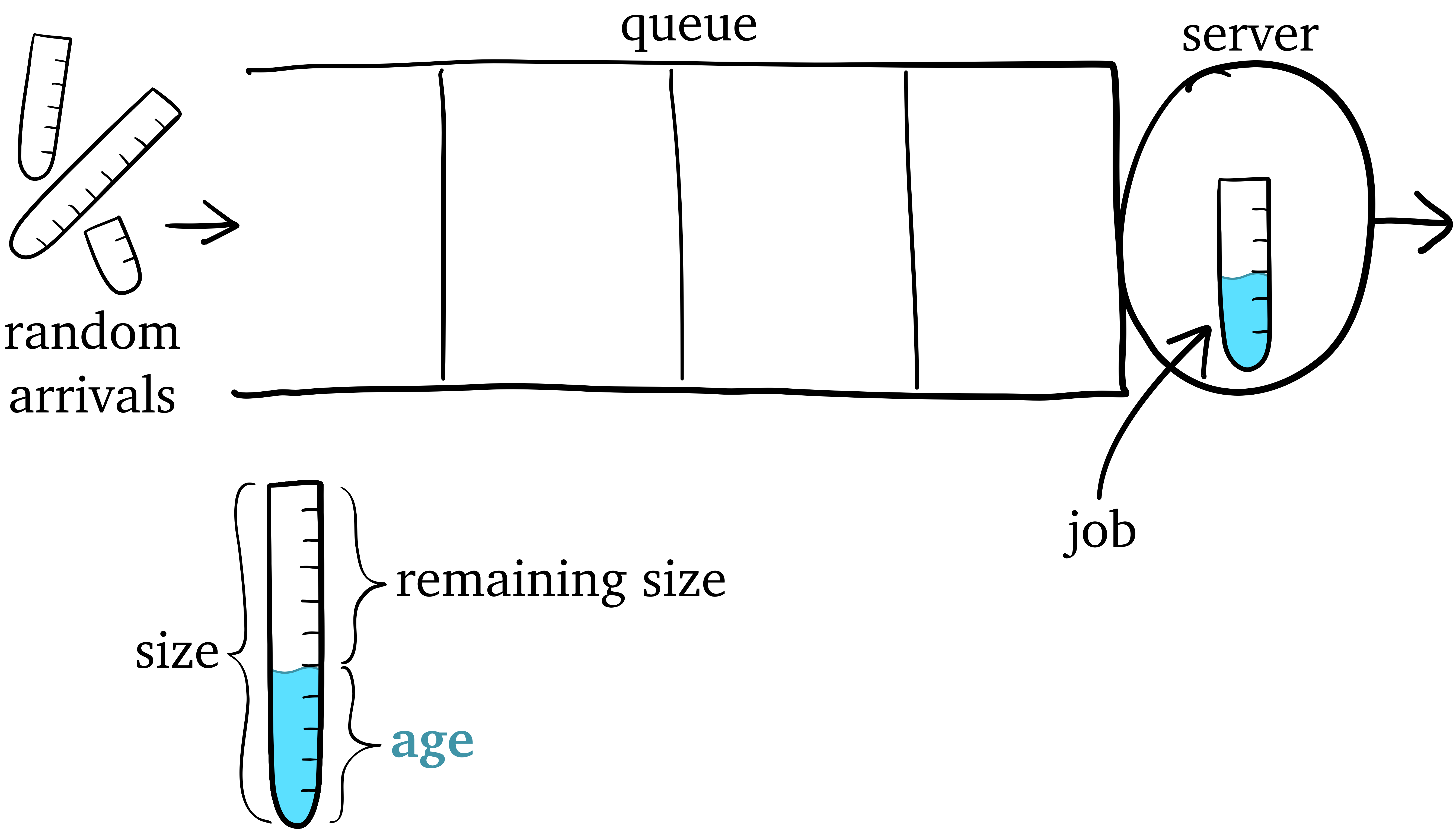
M/G/1 Queue



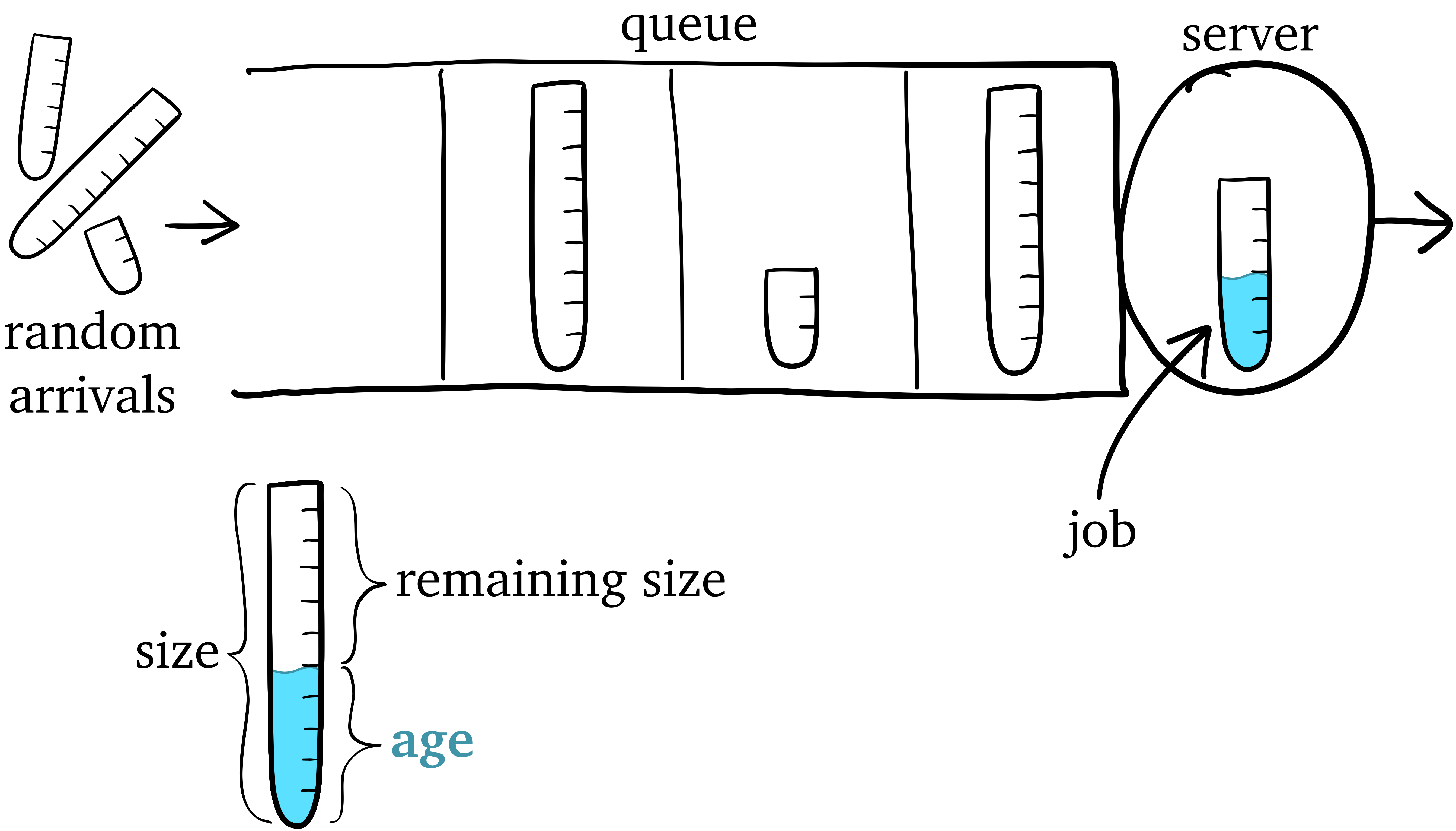
M/G/1 Queue



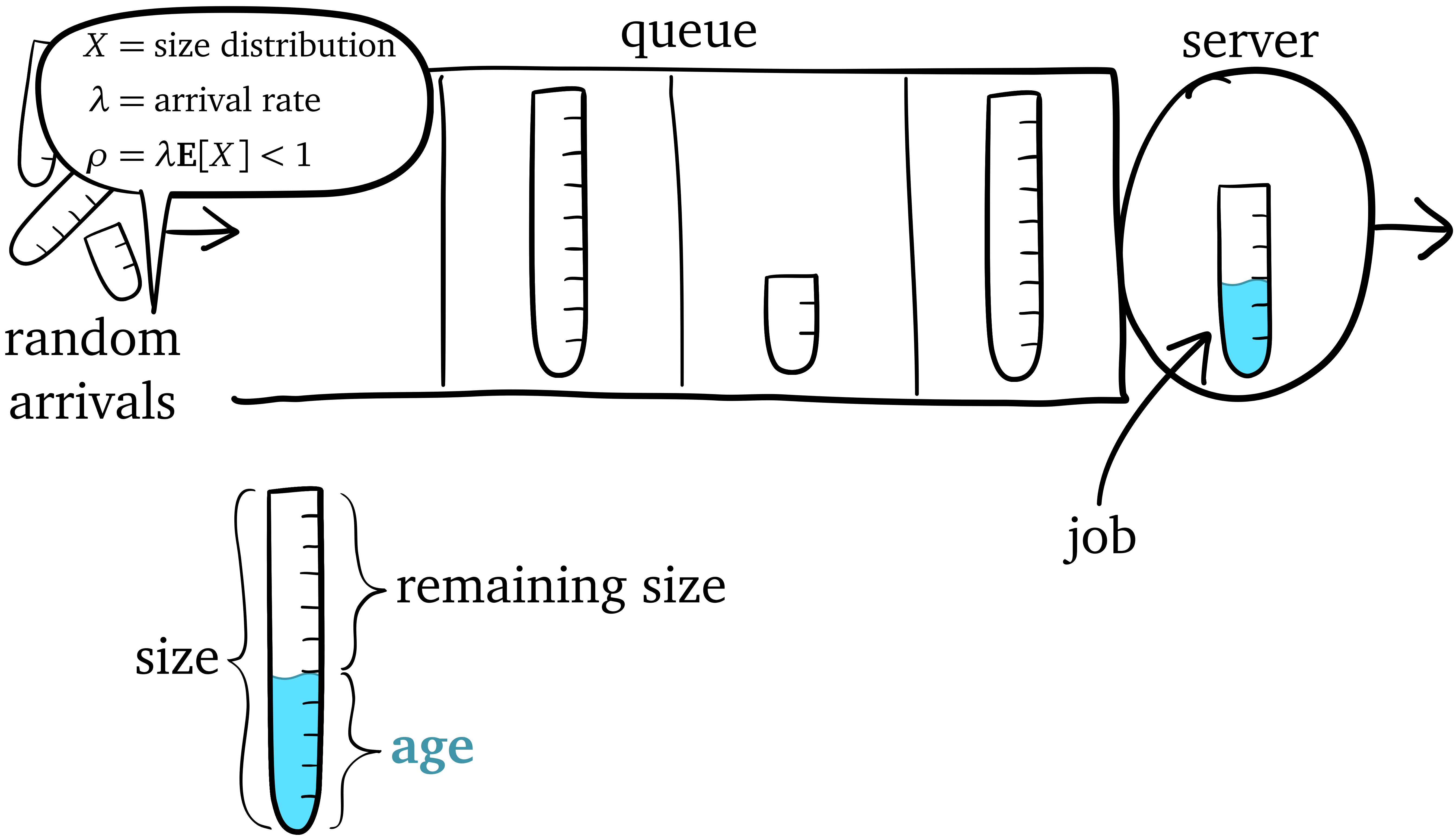
M/G/1 Queue



M/G/1 Queue

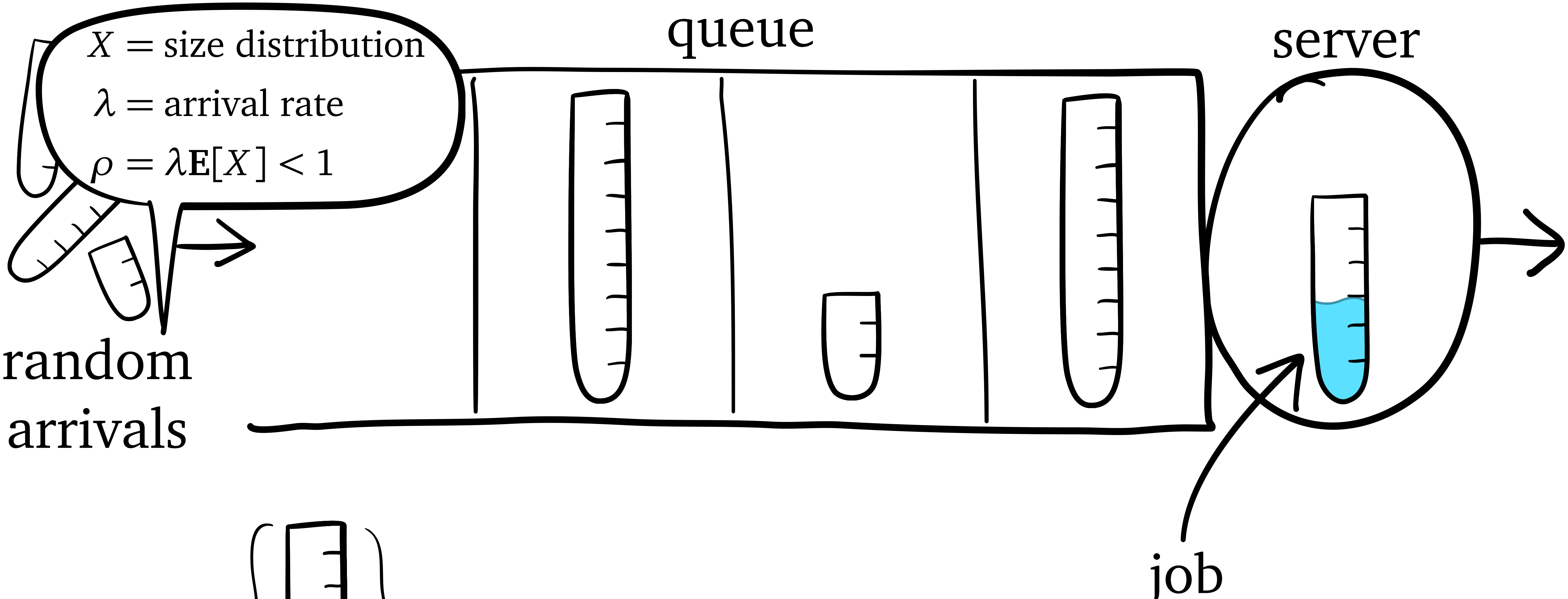


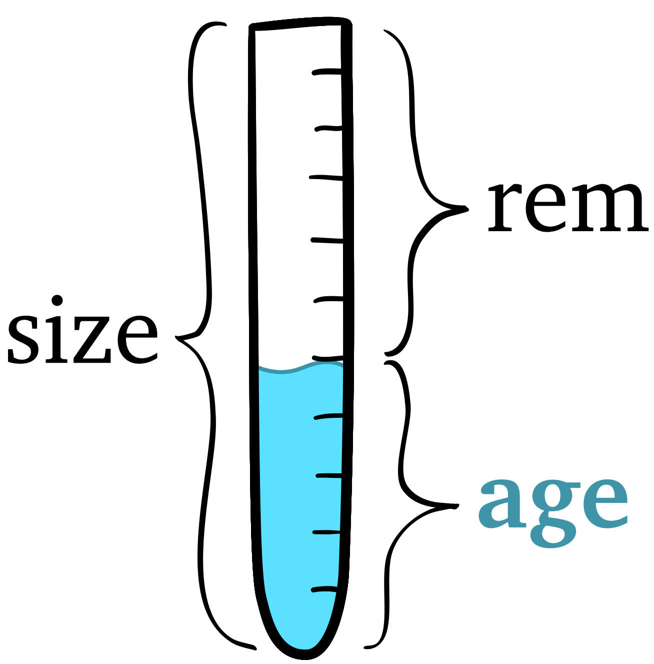
M/G/1 Queue



X = size distribution
λ = arrival rate
$\rho = \lambda \mathbf{E}[X] < 1$
queue
server
random arrivals
job
size
remaining size
age

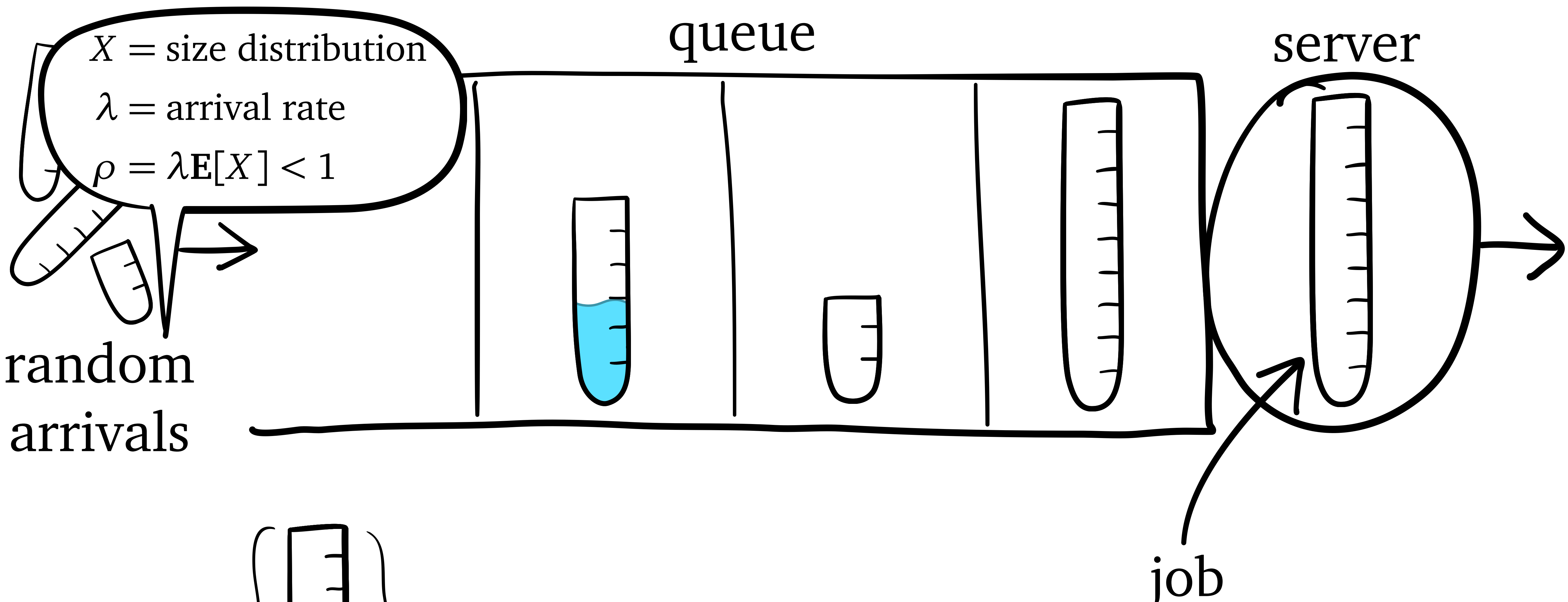
M/G/1 Queue

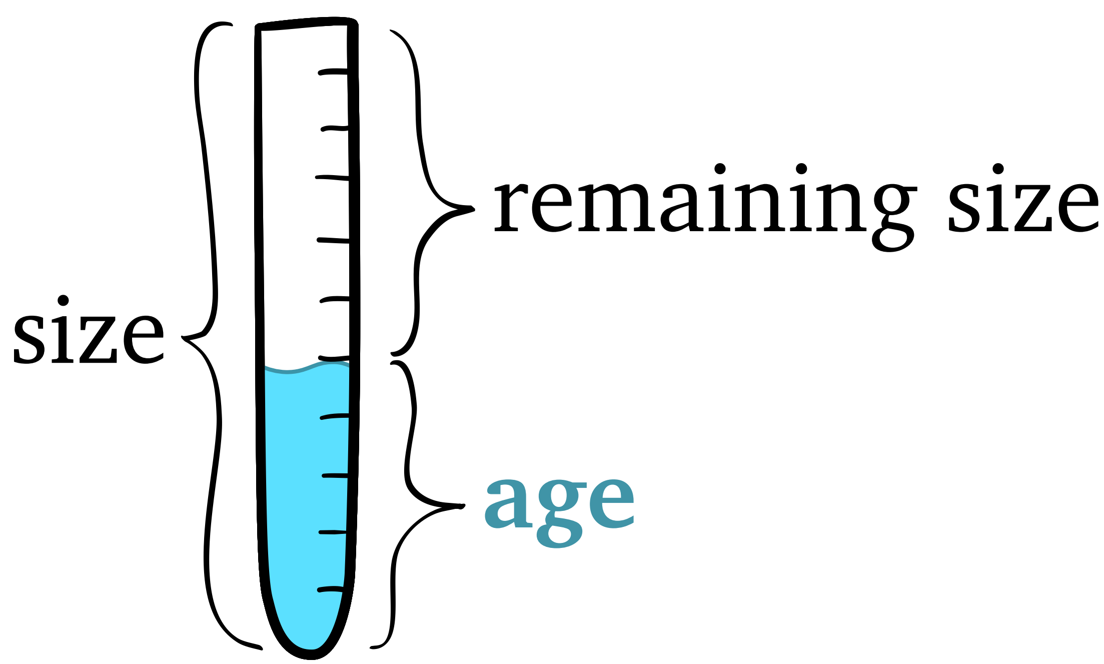




Scheduling policy:
picks which job to serve

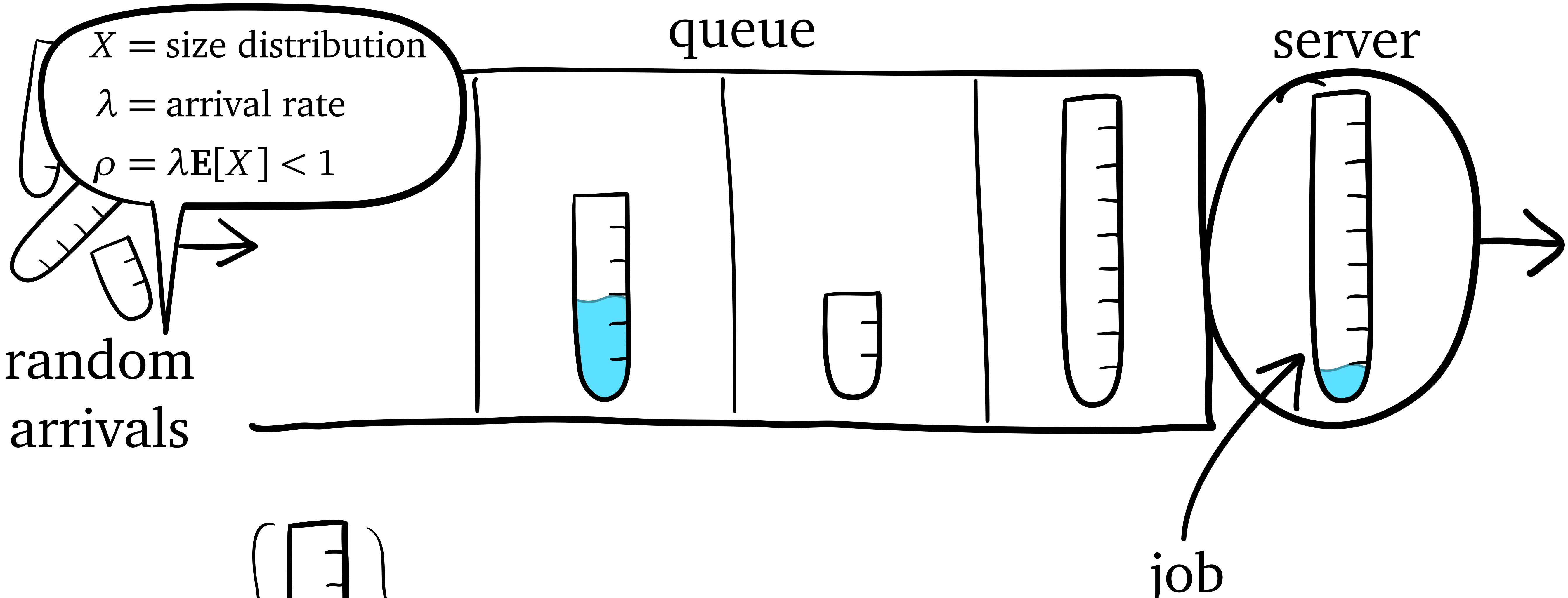
M/G/1 Queue

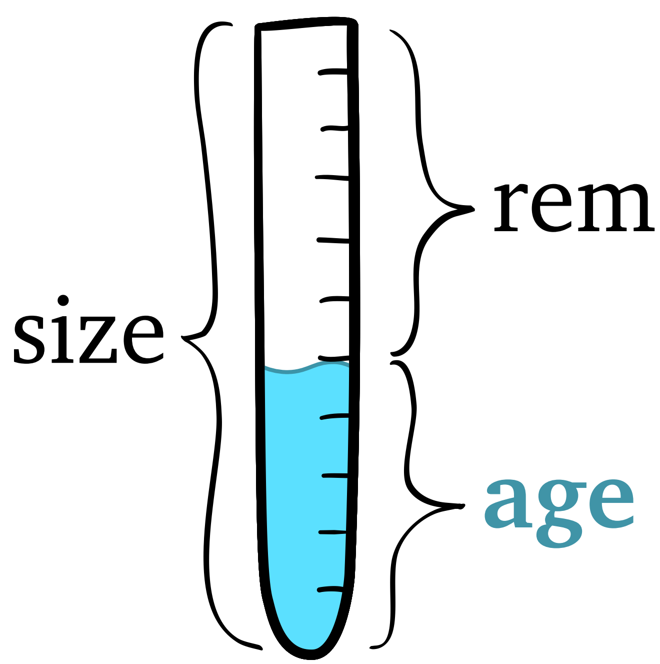




Scheduling policy:
picks which job to serve

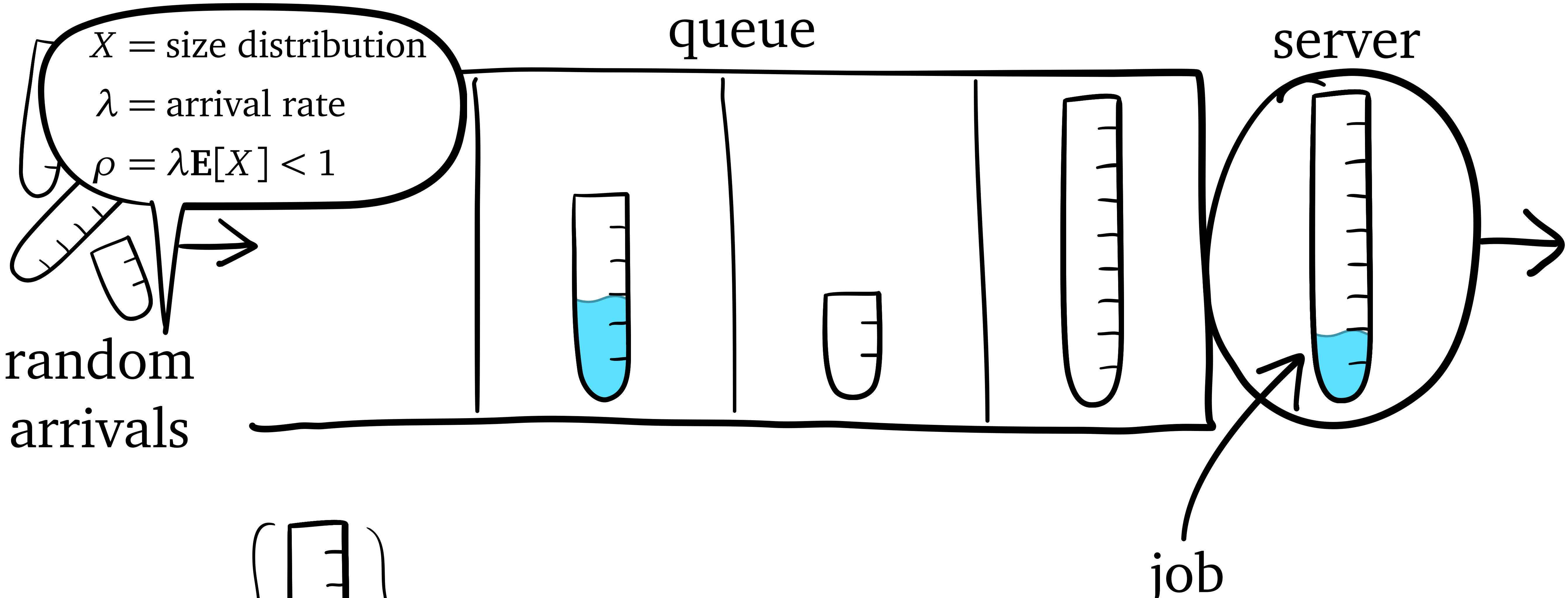
M/G/1 Queue

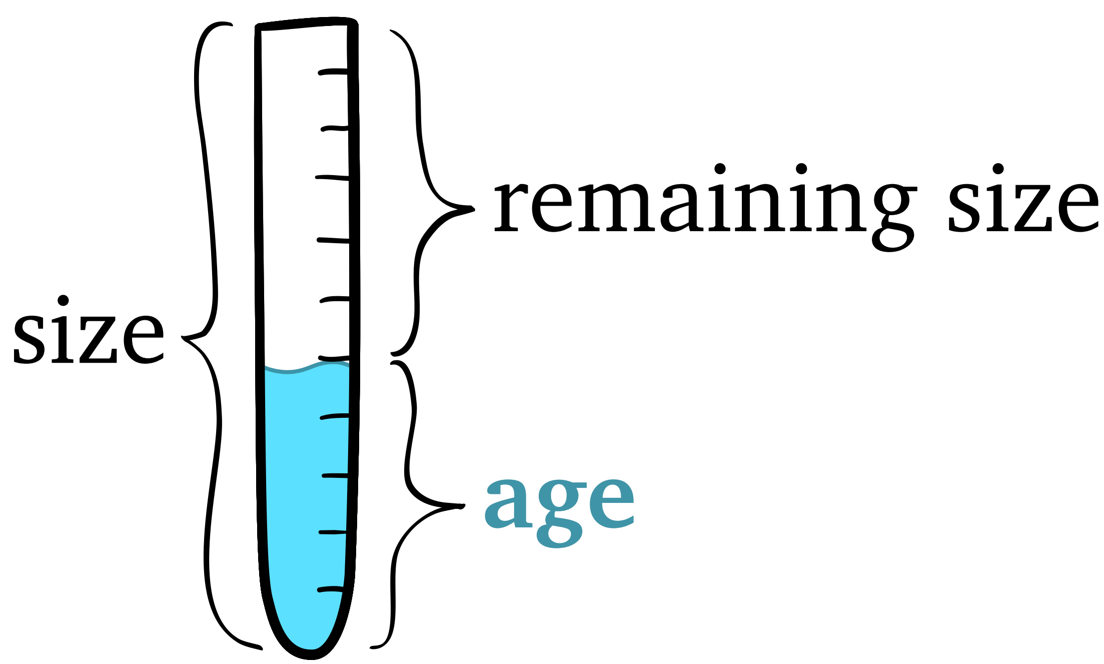




Scheduling policy:
picks which job to serve

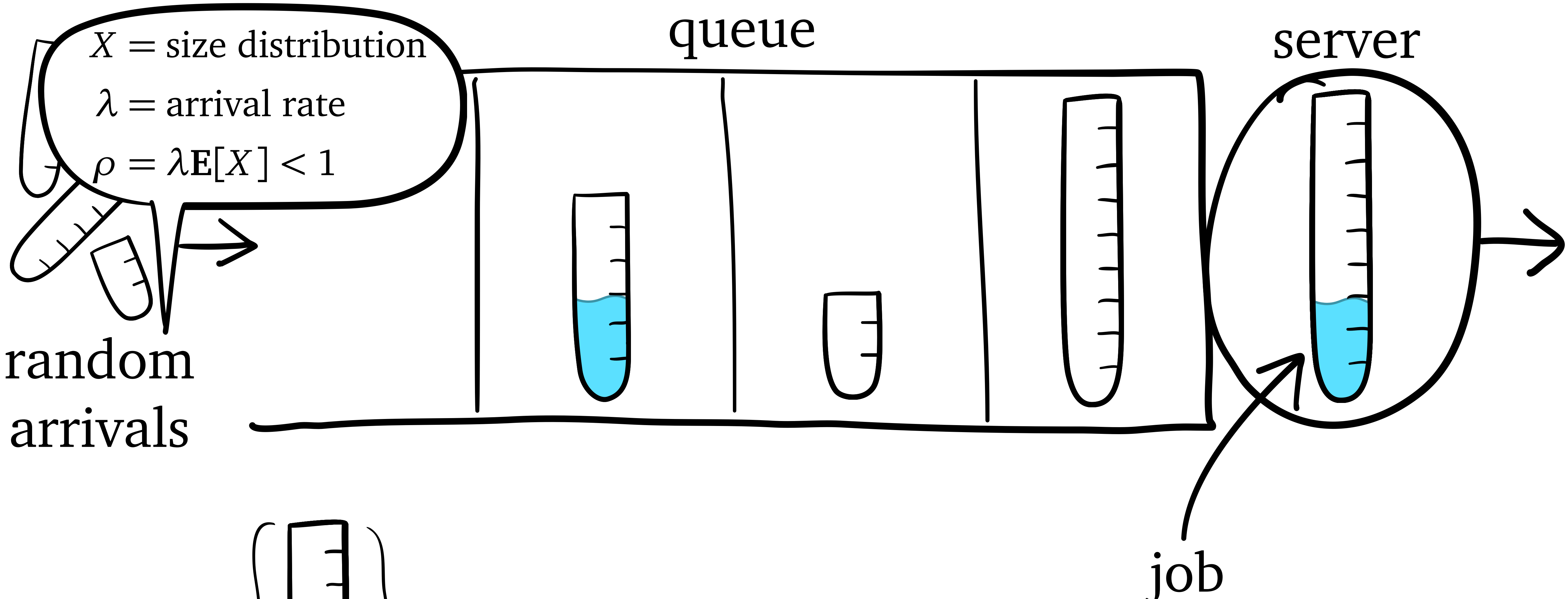
M/G/1 Queue

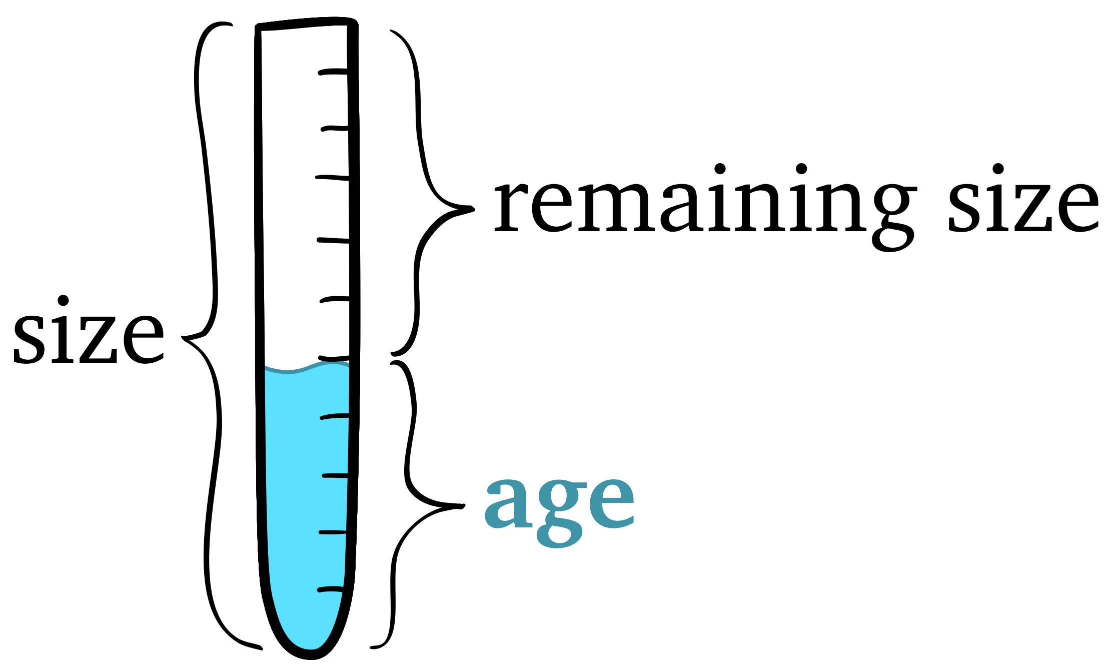




Scheduling policy:
picks which job to serve

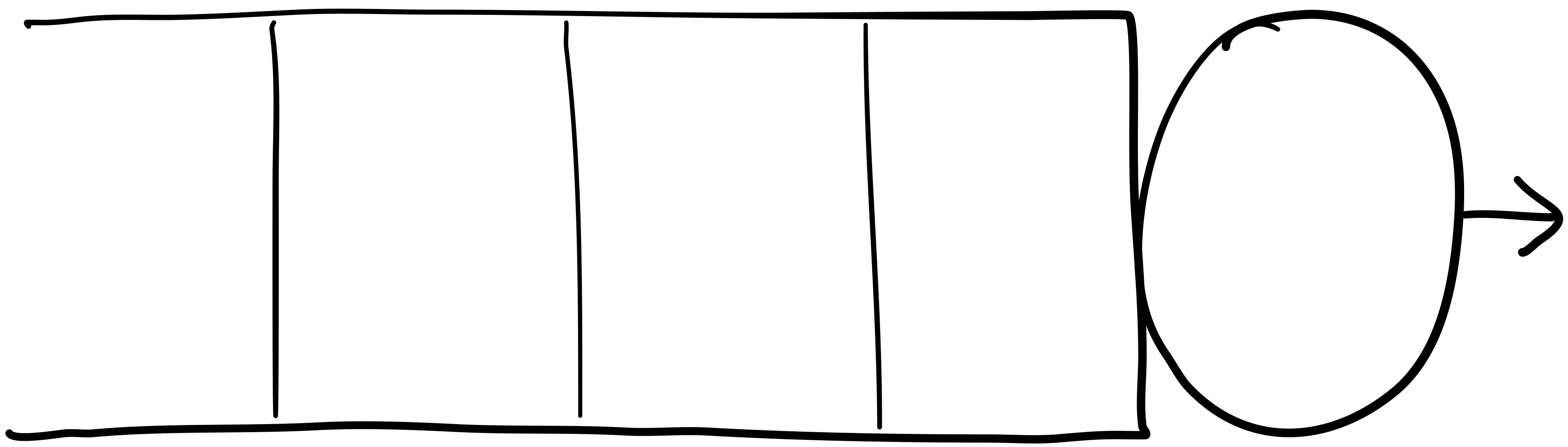
M/G/1 Queue



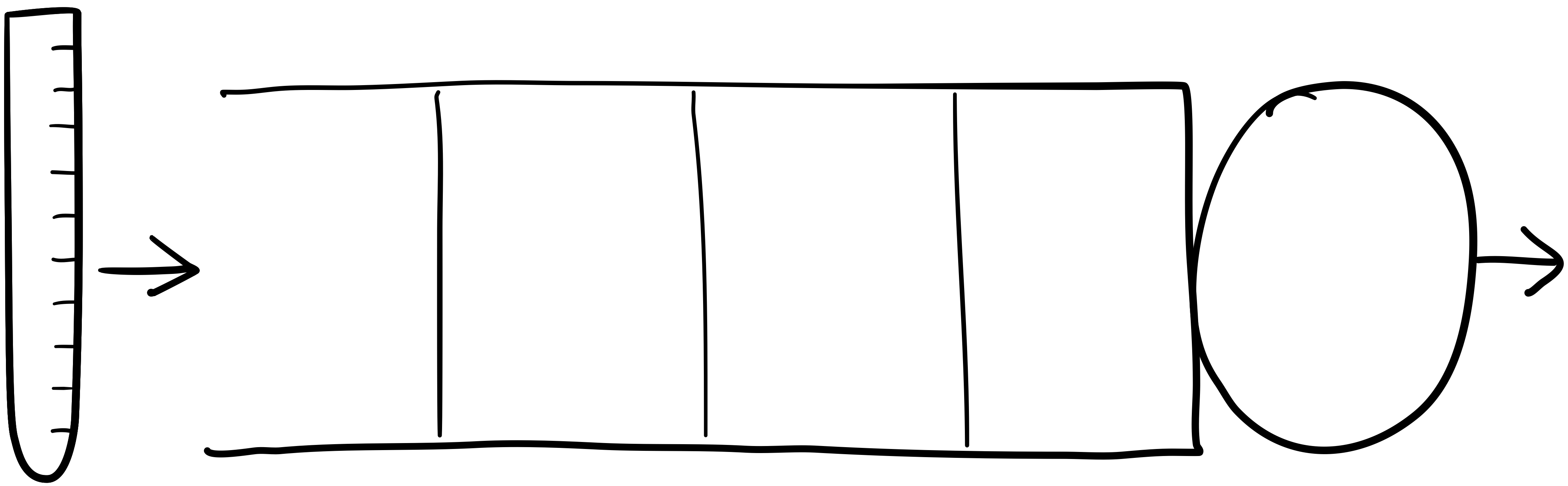


Scheduling policy:
picks which job to serve

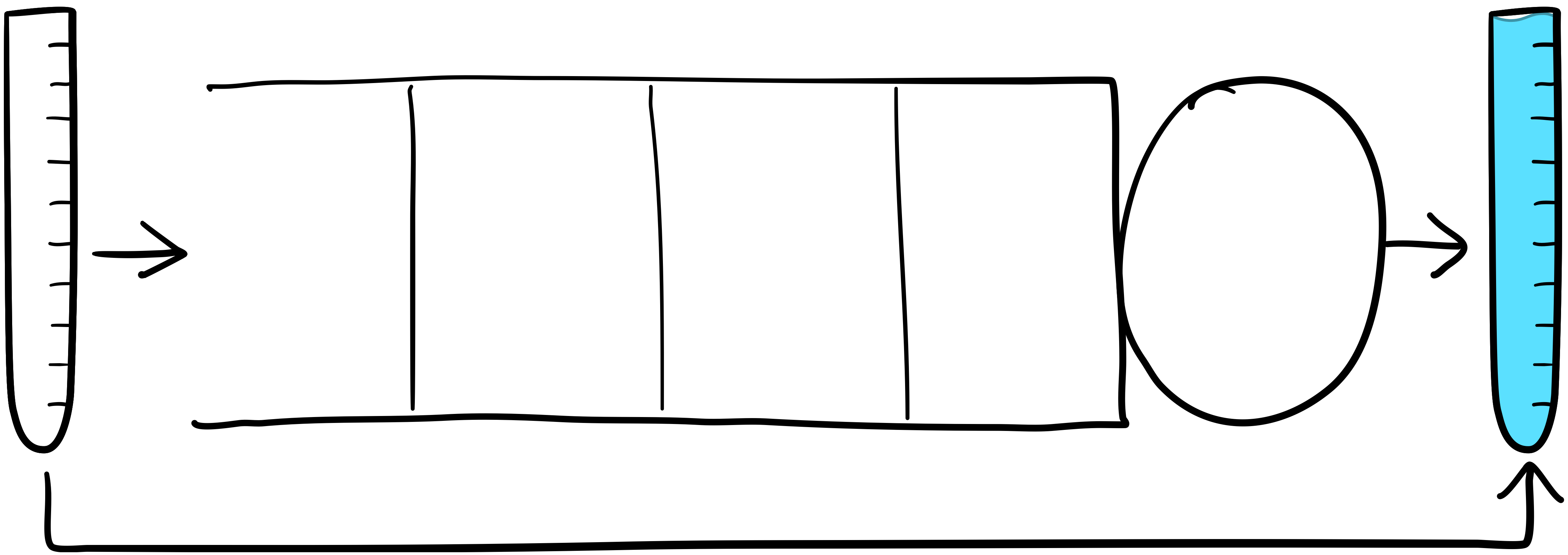
Response Time

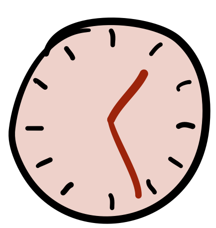


Response Time

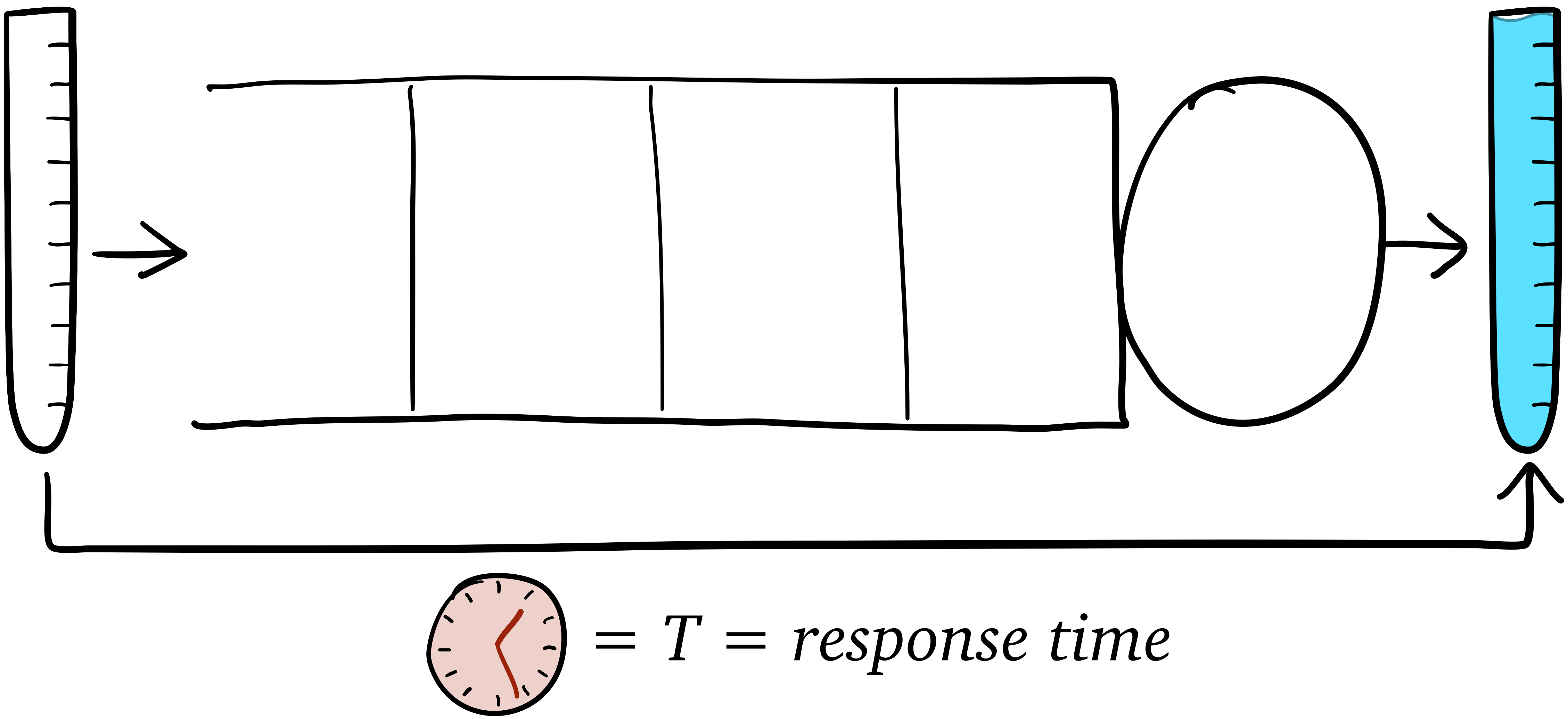


Response Time



 $= T =$ *response time*

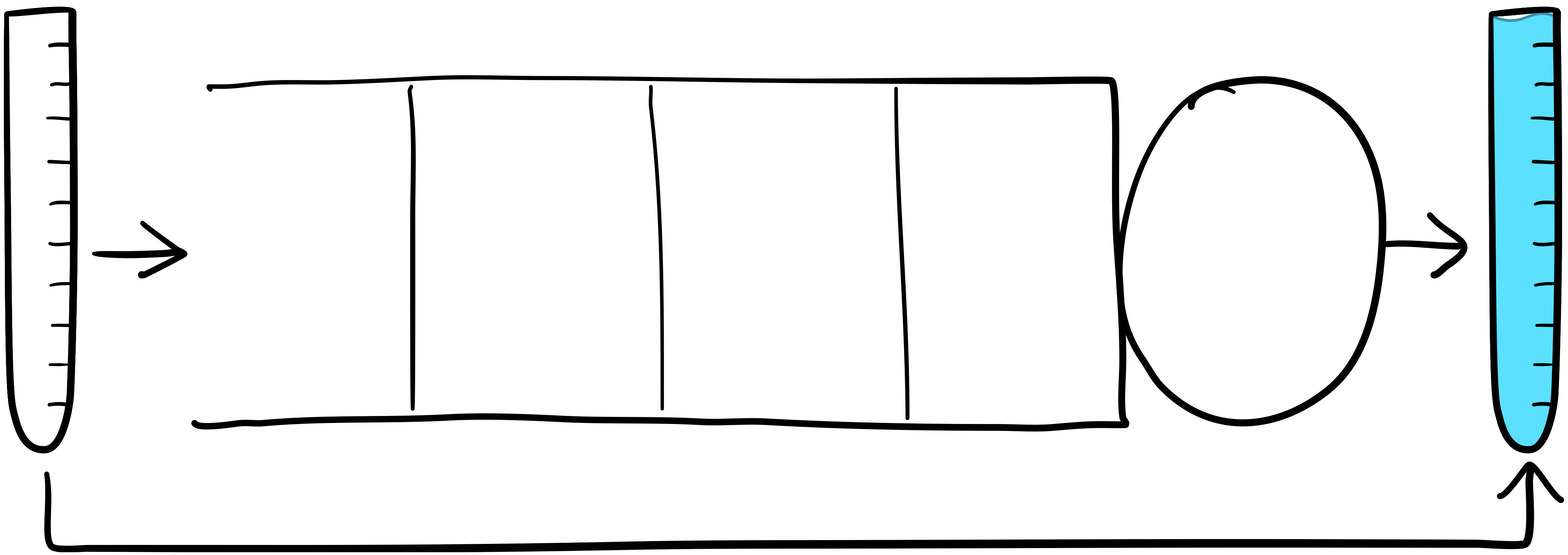
Response Time

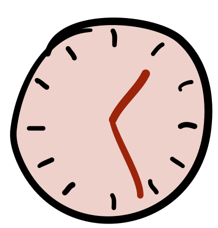


$= T =$ *response time*

Goal: analyze *mean response time* $\mathbf{E}[T]$

Response Time



 $= T =$ *response time*

Goal: analyze *mean response time* $\mathbf{E}[T]$

Depends on *scheduling policy*

Impact of Scheduling

What scheduling policy minimizes $\mathbf{E}[T]$?

Impact of Scheduling

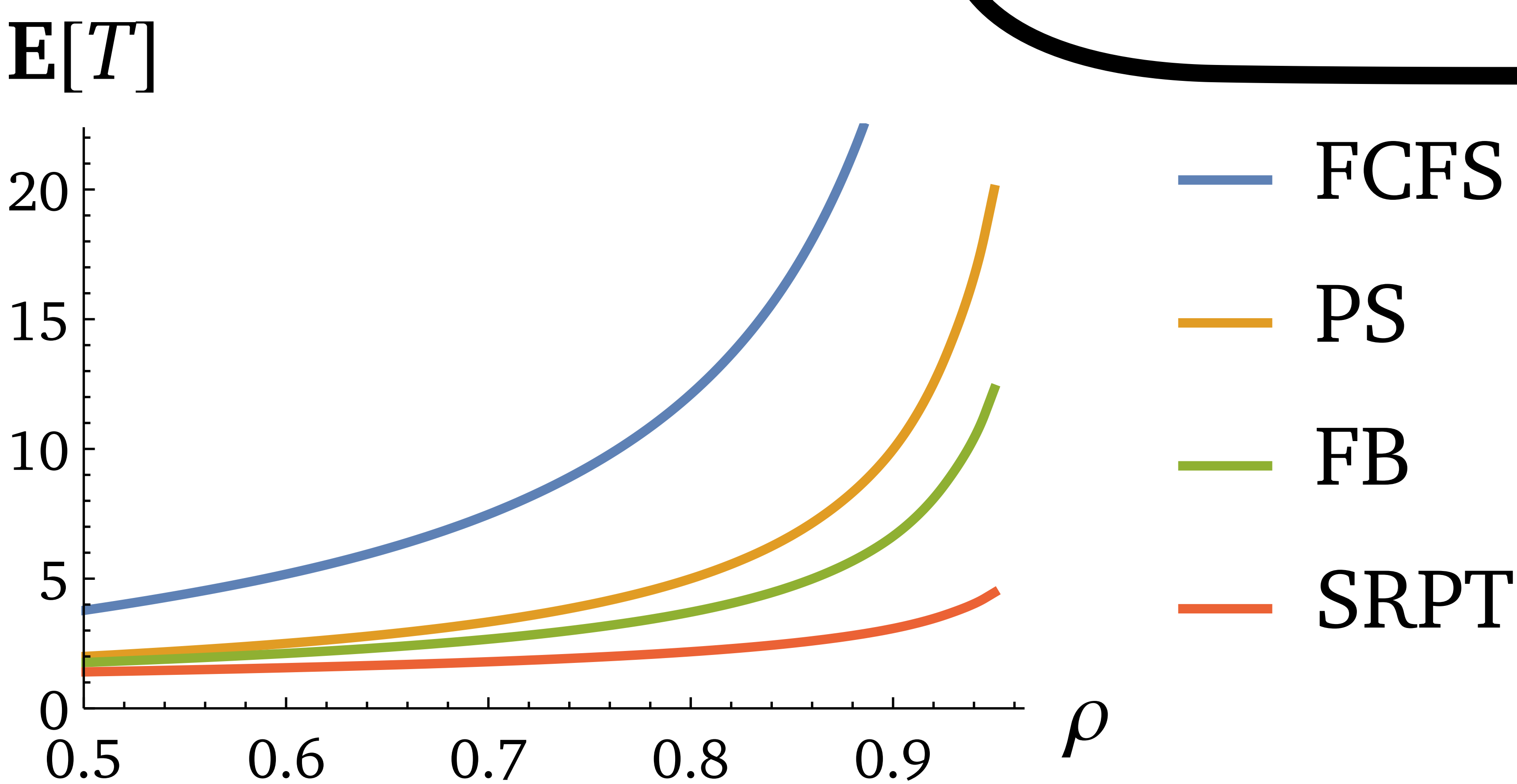
What scheduling policy minimizes $\mathbf{E}[T]$?

Shortest remaining processing time (SRPT)

Impact of Scheduling

What scheduling policy minimizes $\mathbf{E}[T]$?

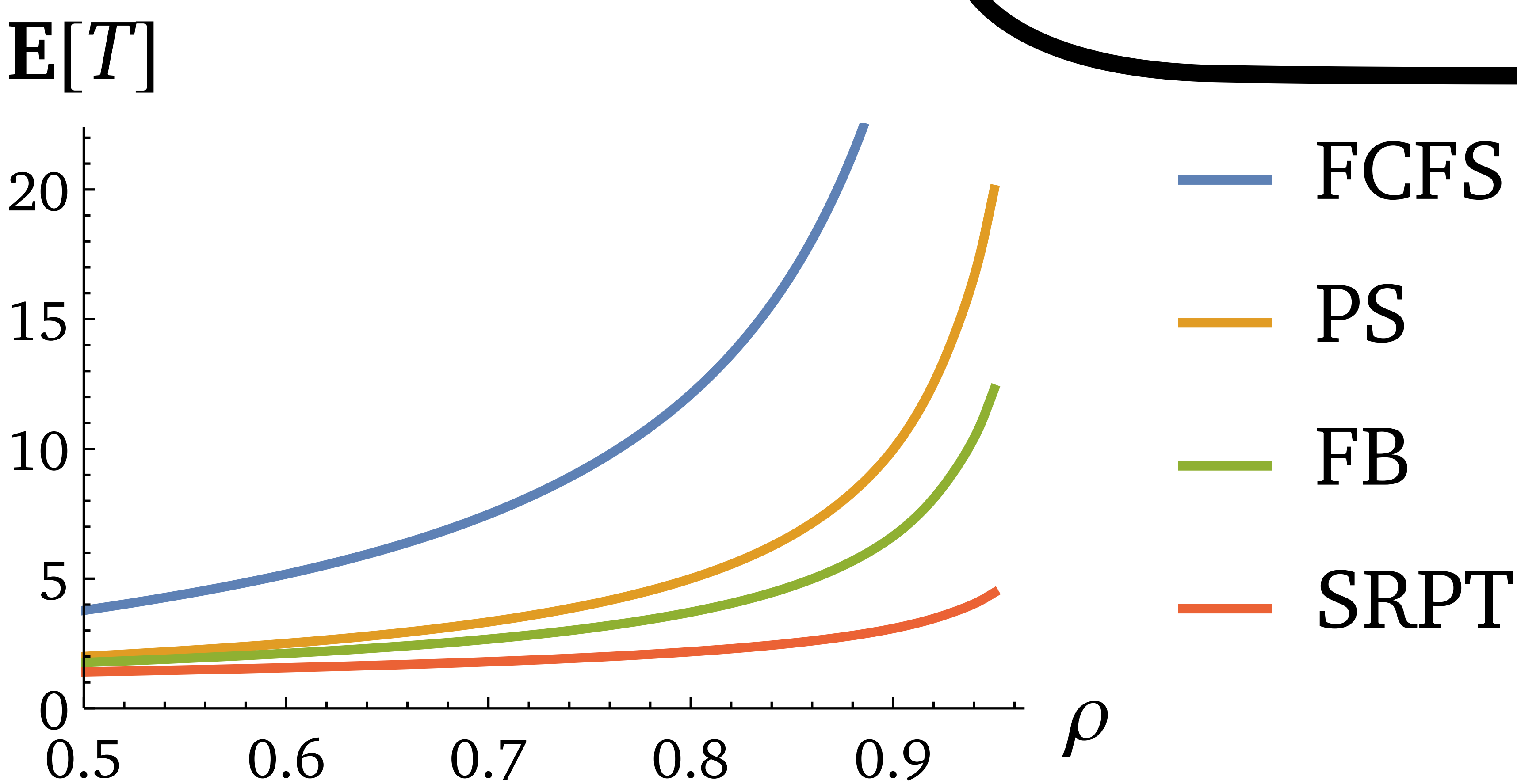
Shortest remaining processing time (SRPT)



Impact of Scheduling

What scheduling policy minimizes $\mathbf{E}[T]$?

Shortest remaining processing time (SRPT)



… but nobody uses SRPT!

Why Not SRPT?

Why Not SRPT?

Unknown job sizes

Why Not SRPT?

Unknown job sizes

FCFS (first come, first served)

Why Not SRPT?

Unknown job sizes $\left\{ \begin{array}{l} \text{FCFS (first come, first served)} \\ \text{FB (foreground-background: least age)} \\ \\ \end{array} \right.$

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)

Why Not SRPT?

Unknown job sizes $\left\{\begin{array}{l}\text{FCFS (first come, first served)}\\ \text{FB (foreground-background: least age)}\\ \text{SERPT (least } \textit{expected} \text{ remaining size)}\\ \text{Gittins (optimal!)}\end{array}\right.$

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

- "Discrete" SRPT
  (preempt only at checkpoints)

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

- "Discrete" SRPT (preempt only at checkpoints)
- "Bucketed" SRPT (limited number of priority levels)

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

- "Discrete" SRPT, FB, etc.
  (preempt only at checkpoints)
- "Bucketed" SRPT, FB, etc.
  (limited number of priority levels)

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

- “Discrete” SRPT, FB, etc. (preempt only at checkpoints)
- “Bucketed” SRPT, FB, etc. (limited number of priority levels)

Metric other than $\mathbf{E}[T]$

Why Not SRPT?

Unknown job sizes
- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints
- "Discrete" SRPT, FB, etc.
  (preempt only at checkpoints)
- "Bucketed" SRPT, FB, etc.
  (limited number of priority levels)

Metric other than $\mathbf{E}[T]$
- Priority classes

Why Not SRPT?

Unknown job sizes

- FCFS (first come, first served)
- FB (foreground-background: least age)
- SERPT (least *expected* remaining size)
- Gittins (optimal!)

Hardware constraints

- "Discrete" SRPT, FB, etc. (preempt only at checkpoints)
- "Bucketed" SRPT, FB, etc. (limited number of priority levels)

Metric other than $\mathbf{E}[T]$

- Priority classes
- RS (optimal for mean slowdown)

Many Scheduling Policies

Many Scheduling Policies

$\mathbf{E}[T]$ known

Many Scheduling Policies

$\mathbf{E}[T]$ known

SRPT

Many Scheduling Policies

$\boxed{\mathbf{E}[T] \text{ known}}$

SRPT

FCFS

Many Scheduling Policies

$\boxed{\mathbf{E}[T] \text{ known}}$

SRPT

FCFS

FB

Many Scheduling Policies

$\boxed{\mathbf{E}[T] \text{ known}}$

SRPT

FCFS

FB

Simple priority classes

Many Scheduling Policies

$\mathbf{E}[T]$ known

SRPT

FCFS

FB

Simple priority classes

$\mathbf{E}[T]$ unknown!

Many Scheduling Policies

$\mathbf{E}[T]$ known

SRPT

FCFS

FB

Simple priority classes

$\mathbf{E}[T]$ unknown!

SERPT

Gittins

Discrete SRPT

Discrete FB

Bucketed SRPT

Bucketed FB

RS*

Complex priority classes

… and more!

Many Scheduling Policies

E[T] known

SRPT

FCFS

FB

Simple priority classes

E[T] unknown!

SERPT

Gittins

Discrete SRPT

Discrete FB

Bucketed SRPT

Bucketed FB

RS*

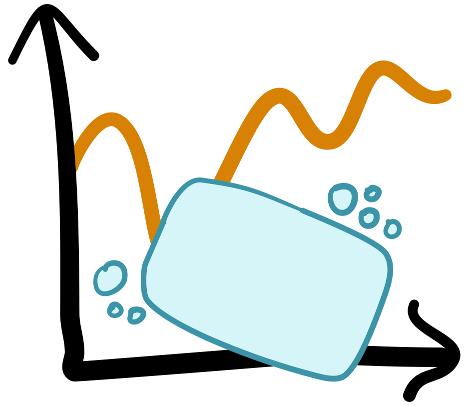
Complex priority classes

… and more!

SOAP

SOAP

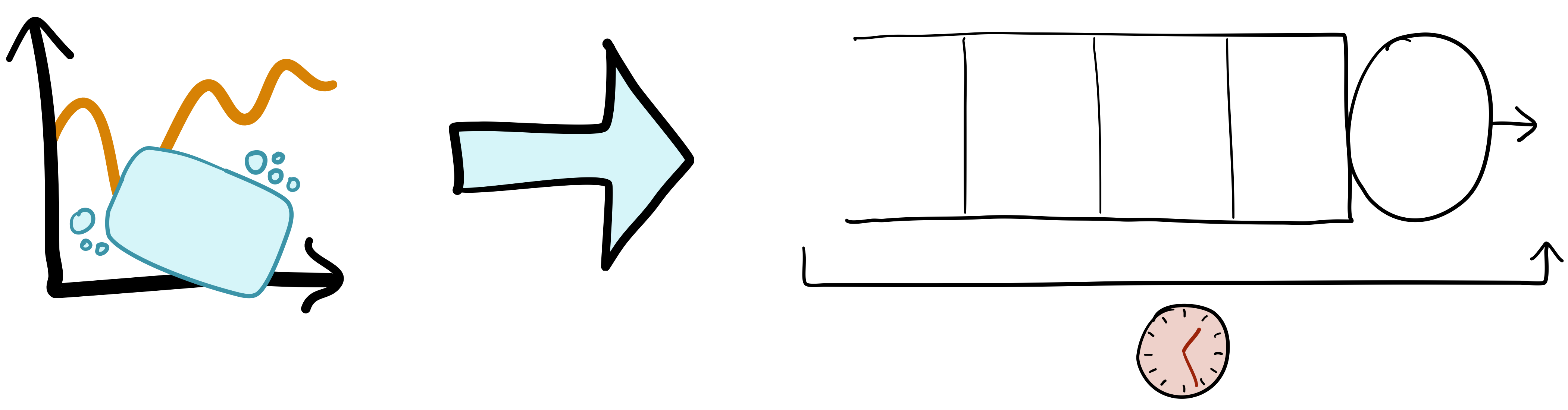
Broad *class* of scheduling policies…



SOAP

Broad *class* of scheduling policies…

… with *universal* response time analysis

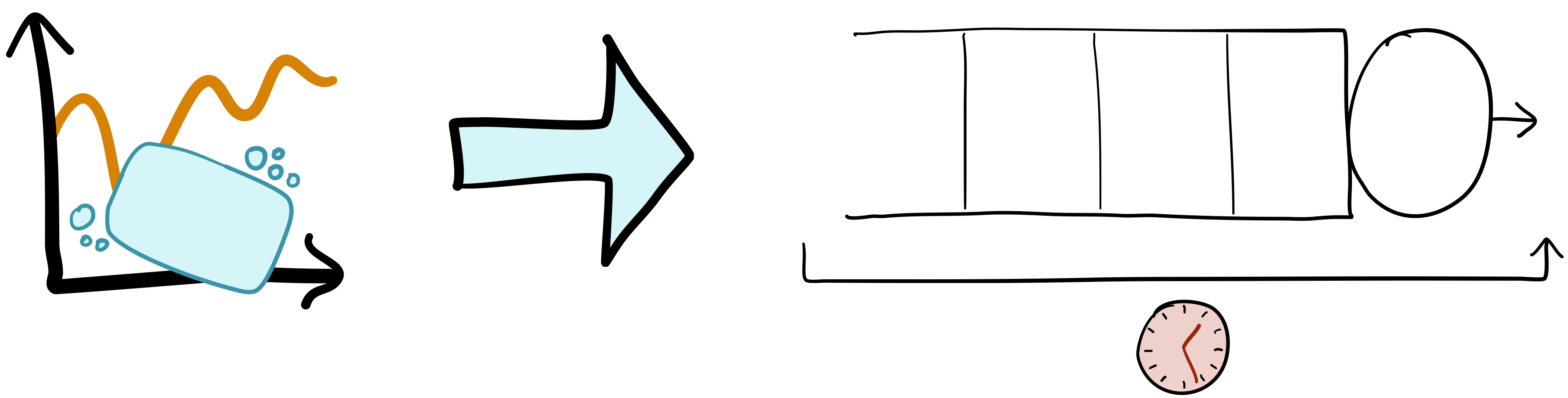


SOAP

Schedule Ordered by Age-based Priority

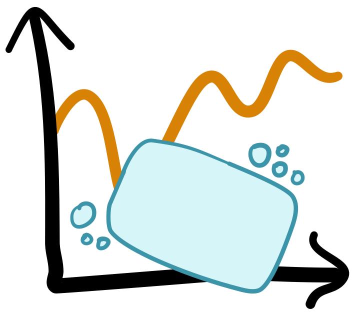
Broad *class* of scheduling policies…

… with *universal* response time analysis



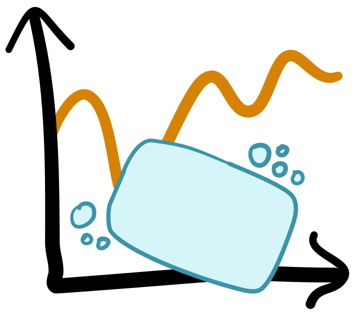
Outline

Outline

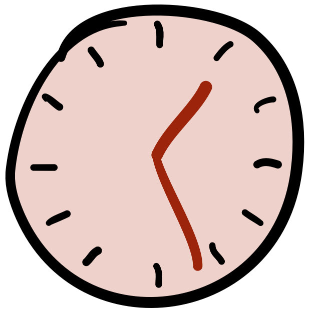


Part 1: *defining* **SOAP** policies

Outline

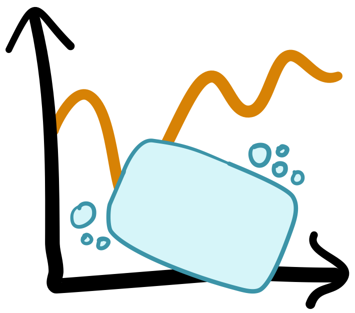


Part 1: *defining* **SOAP** policies

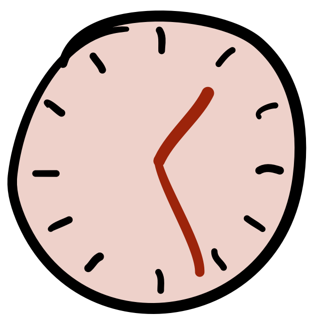


Part 2: *analyzing* **SOAP** policies

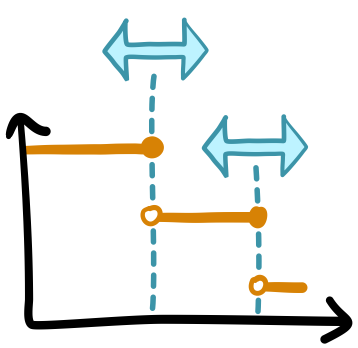
Outline



Part 1: *defining* **SOAP** policies

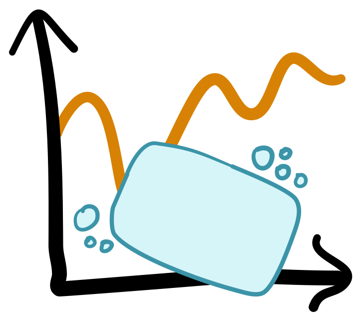


Part 2: *analyzing* **SOAP** policies

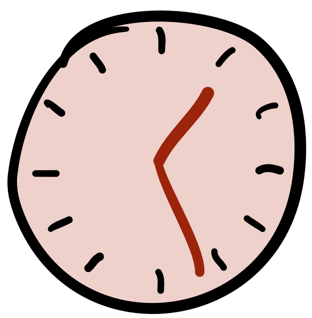


Part 3: *policy design* with **SOAP**

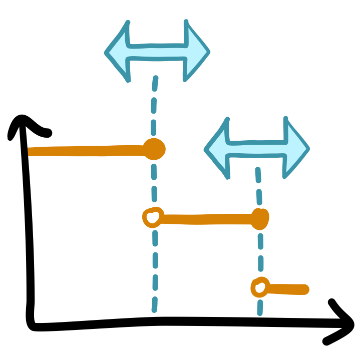
Outline



Part 1: *defining* **SOAP** policies

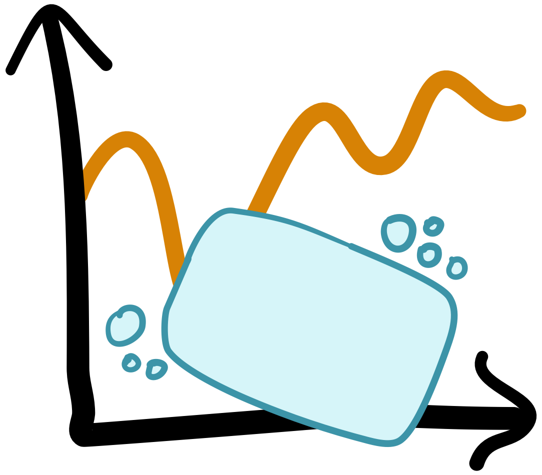


Part 2: *analyzing* **SOAP** policies



Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**



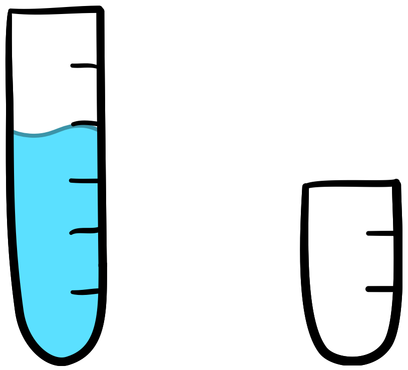
Part 1:

defining **SOAP** policies

Scheduling with **Ranks**

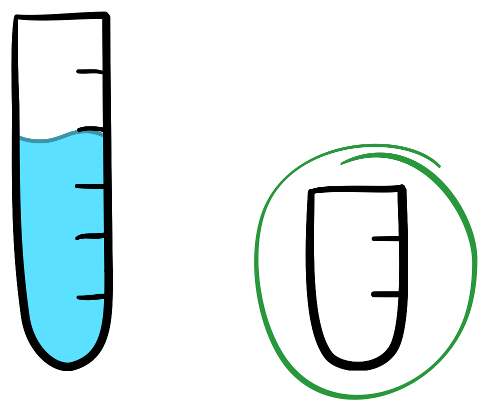
Scheduling with **Ranks**

FB
serve by least age



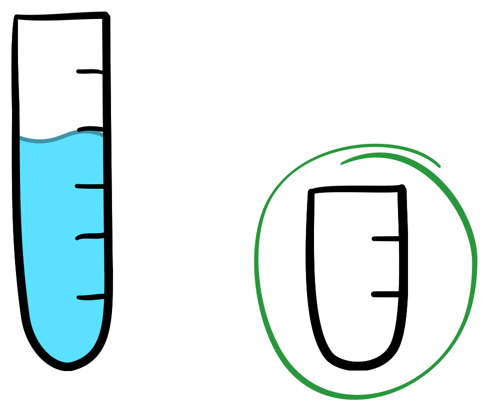
Scheduling with **Ranks**

FB
serve by least age

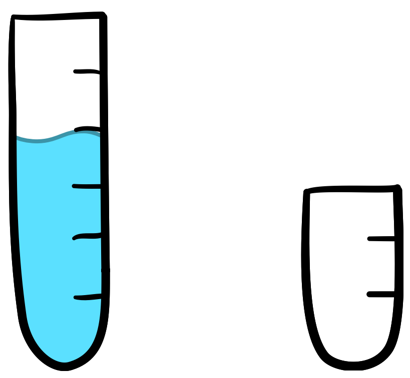


Scheduling with **Ranks**

FB
serve by least age

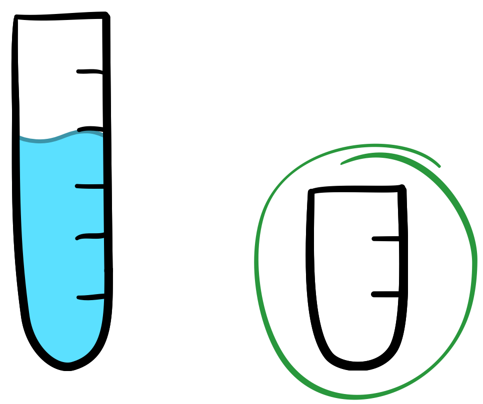


SRPT
serve by least remaining size

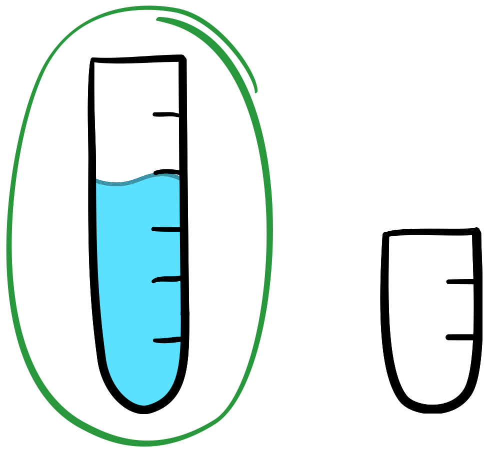


Scheduling with **Ranks**

FB
serve by least age

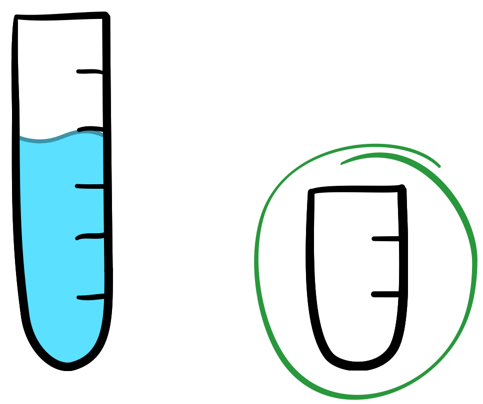


SRPT
serve by least remaining size

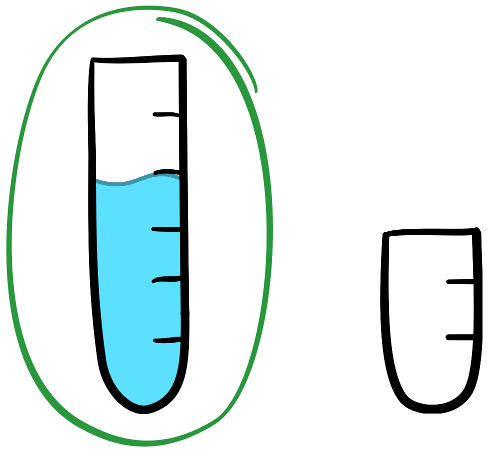


Scheduling with Ranks

FB
serve by least age



SRPT
serve by least remaining size

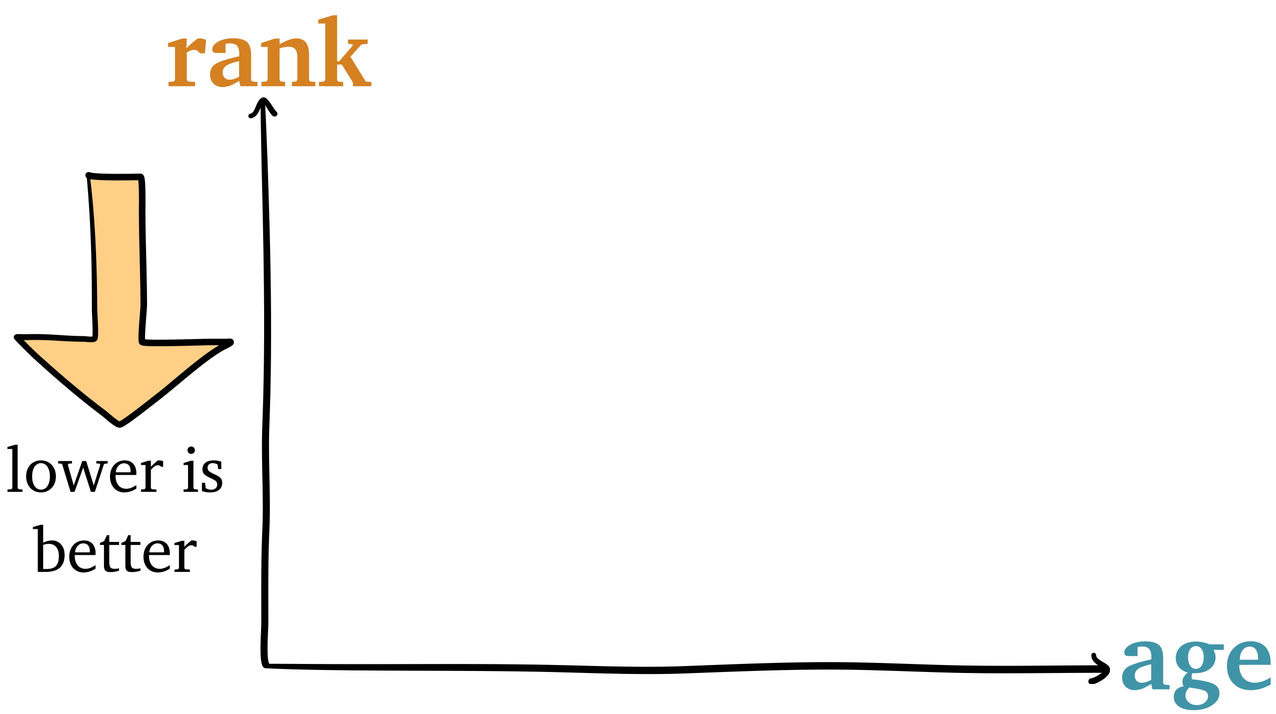


Common theme: a job's **rank** (priority) depends on its **age**

Scheduling with **Ranks**

FB

serve by least age



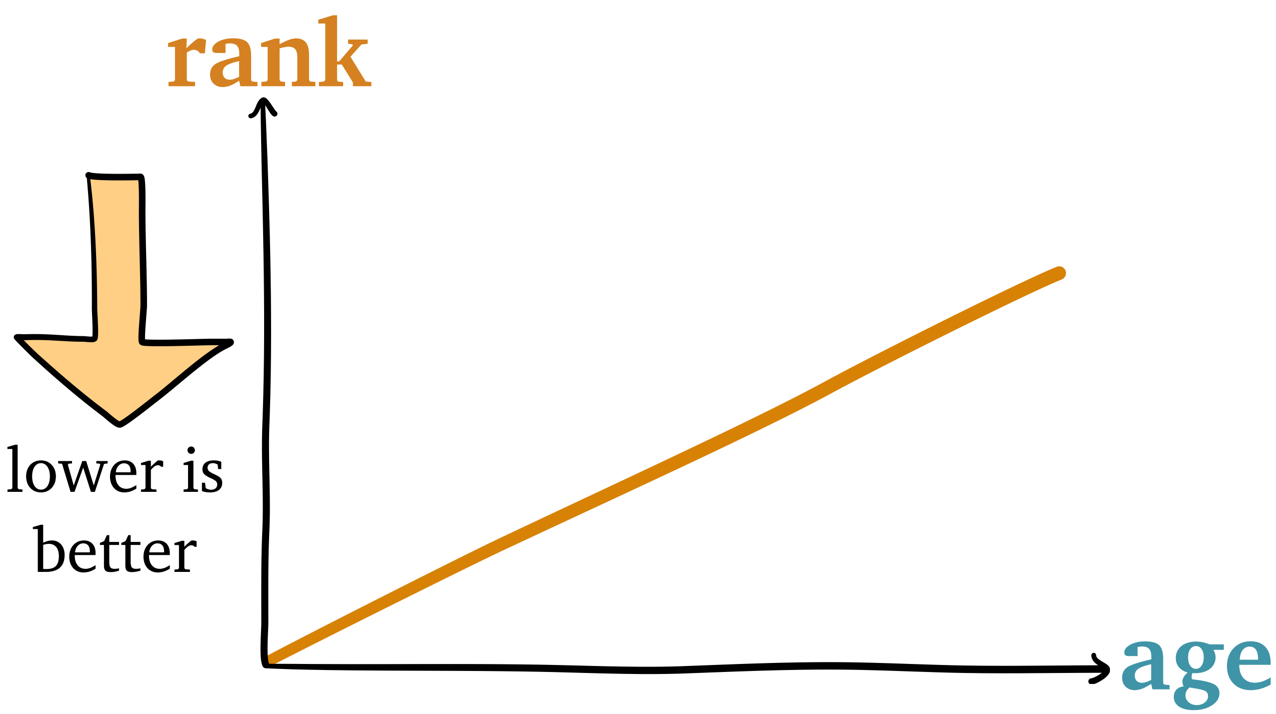
SRPT

serve by least remaining size

Common theme: a job's **rank** (priority) depends on its **age**

Scheduling with Ranks

FB
serve by least age

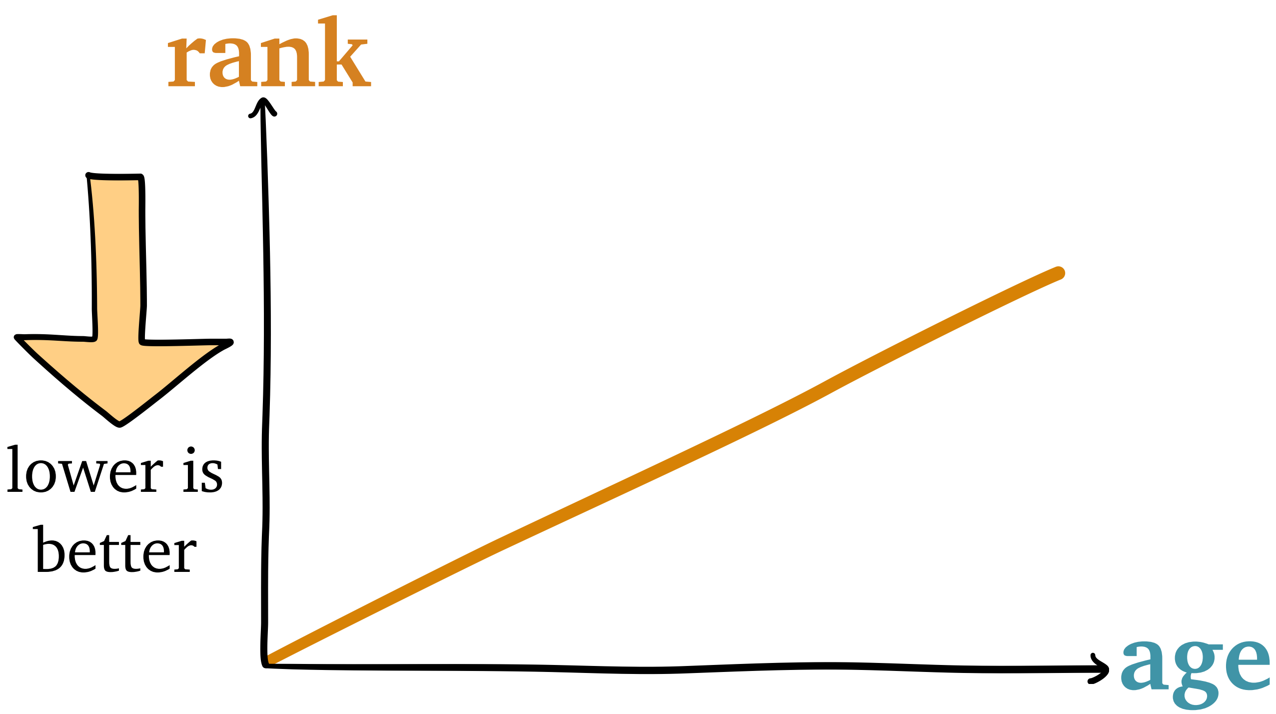


SRPT
serve by least remaining size

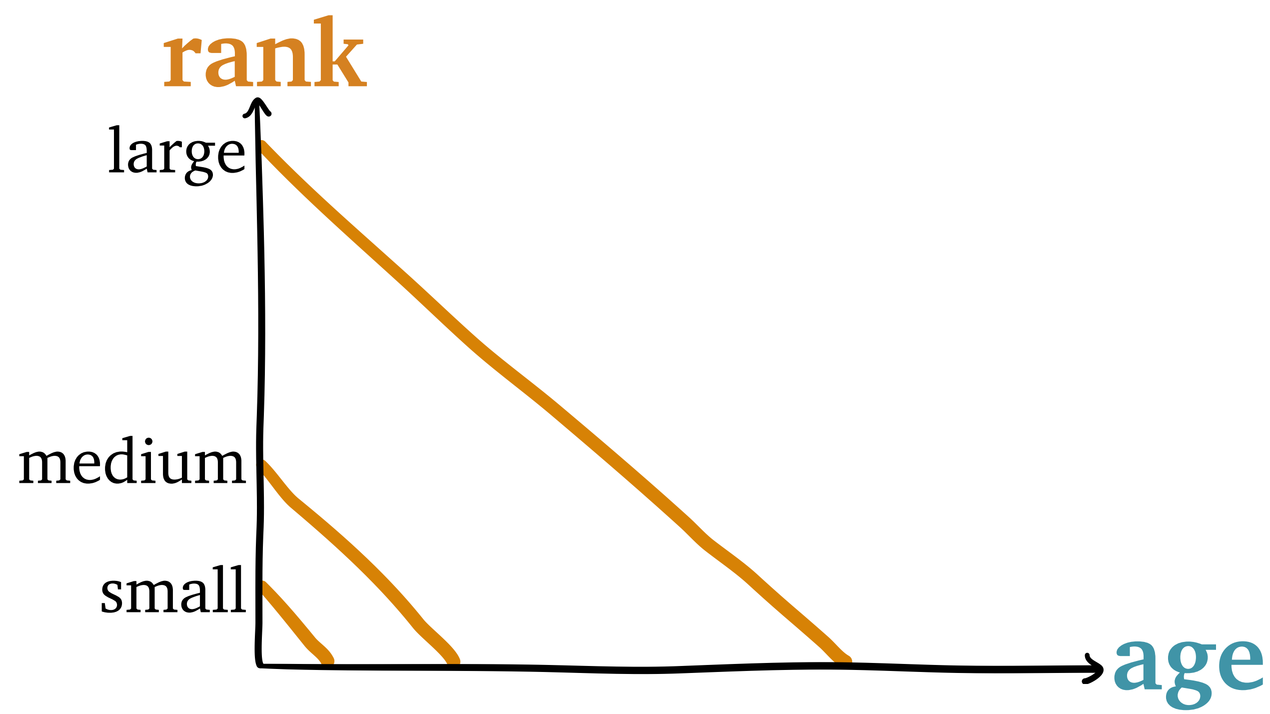
Common theme: a job's **rank** (priority) depends on its **age**

Scheduling with Ranks

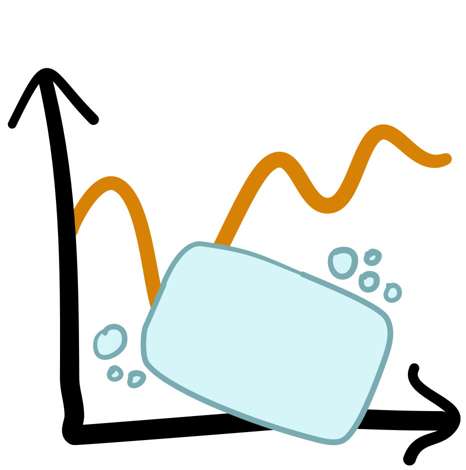
FB
serve by least age



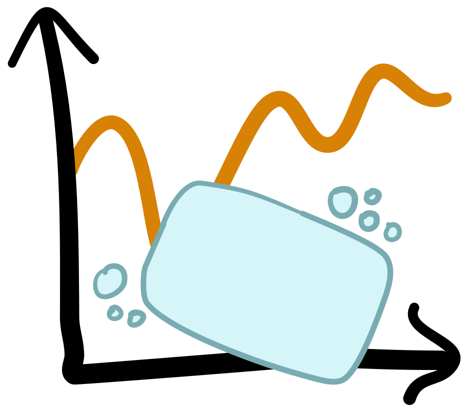
SRPT
serve by least remaining size



Common theme: a job's rank (priority) depends on its age

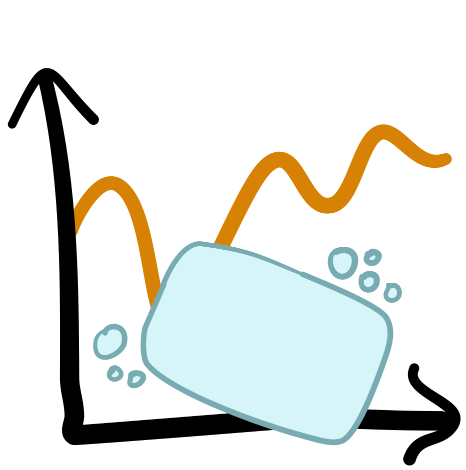


A ***SOAP*** *policy* is a ***rank*** *function* with one rule:



A ***SOAP*** *policy* is a ***rank*** *function* with one rule:

always serve the job of *minimum* ***rank***

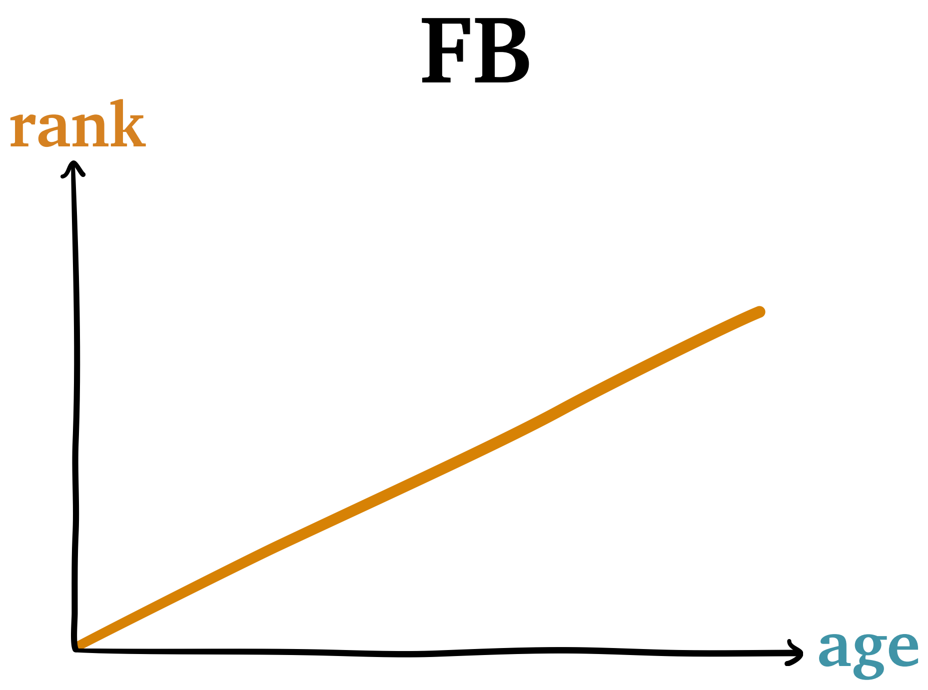


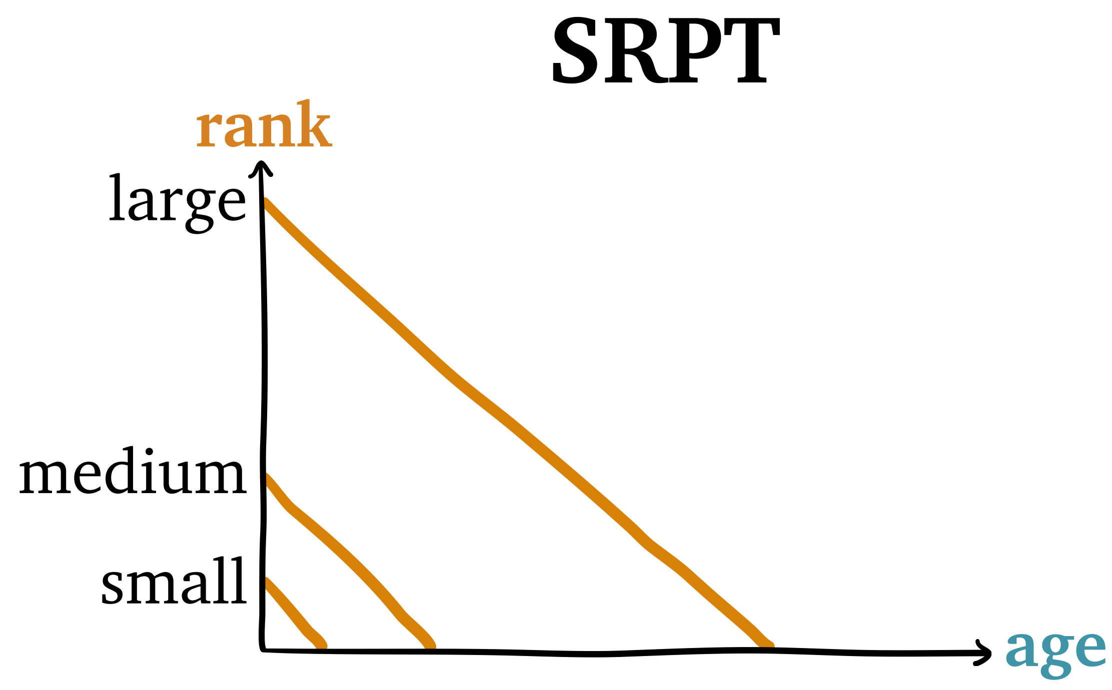
A ***SOAP*** *policy* is a **rank** *function* with one rule:

always serve the job of *minimum* **rank**

(break ties FCFS)

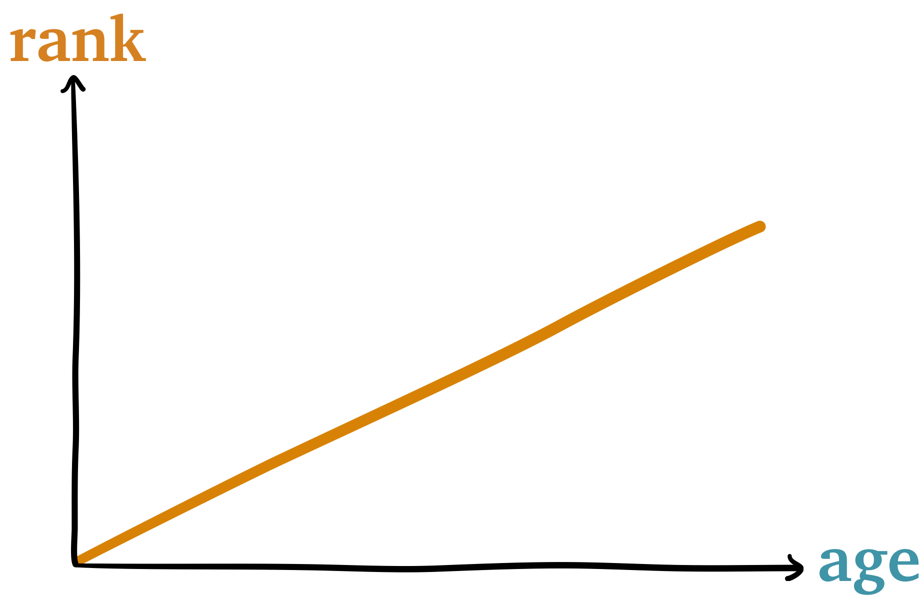
Classic SOAP Policies



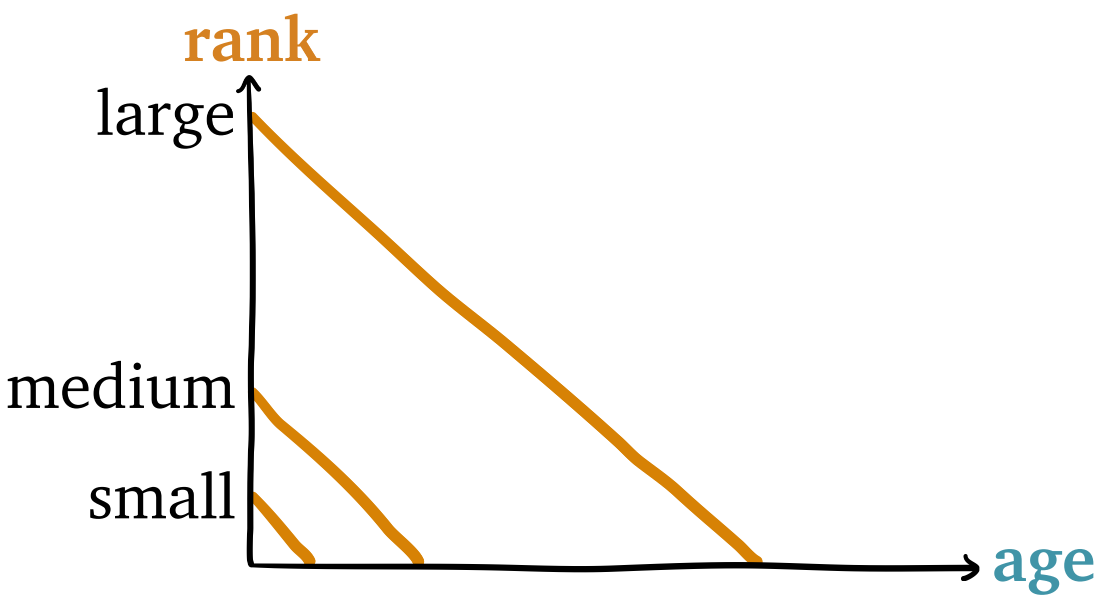


Classic **SOAP** Policies

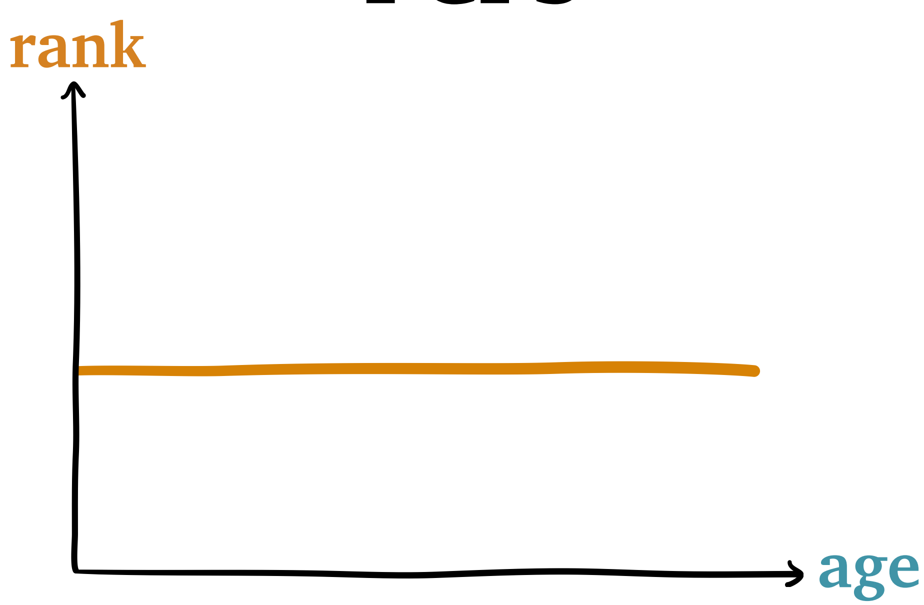
FB



SRPT

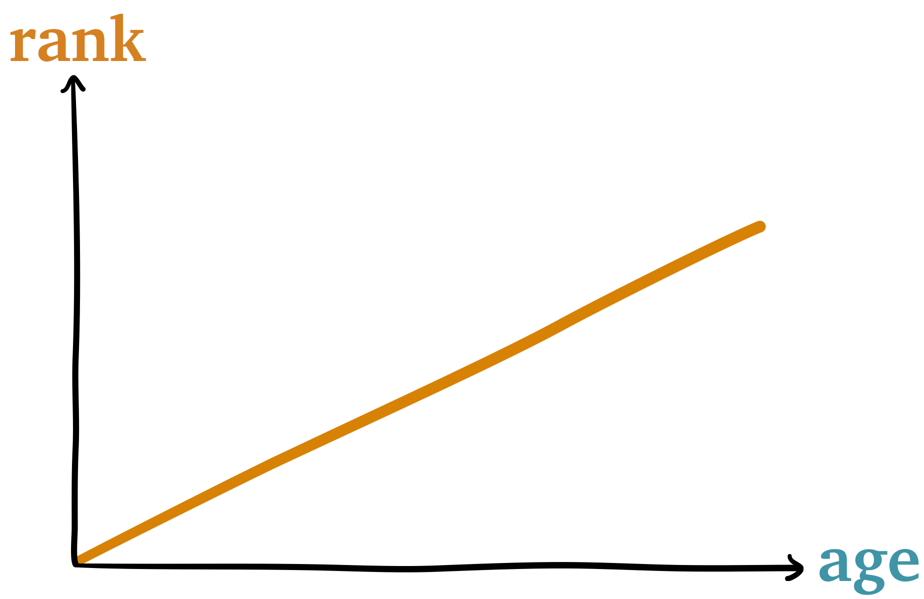


FCFS

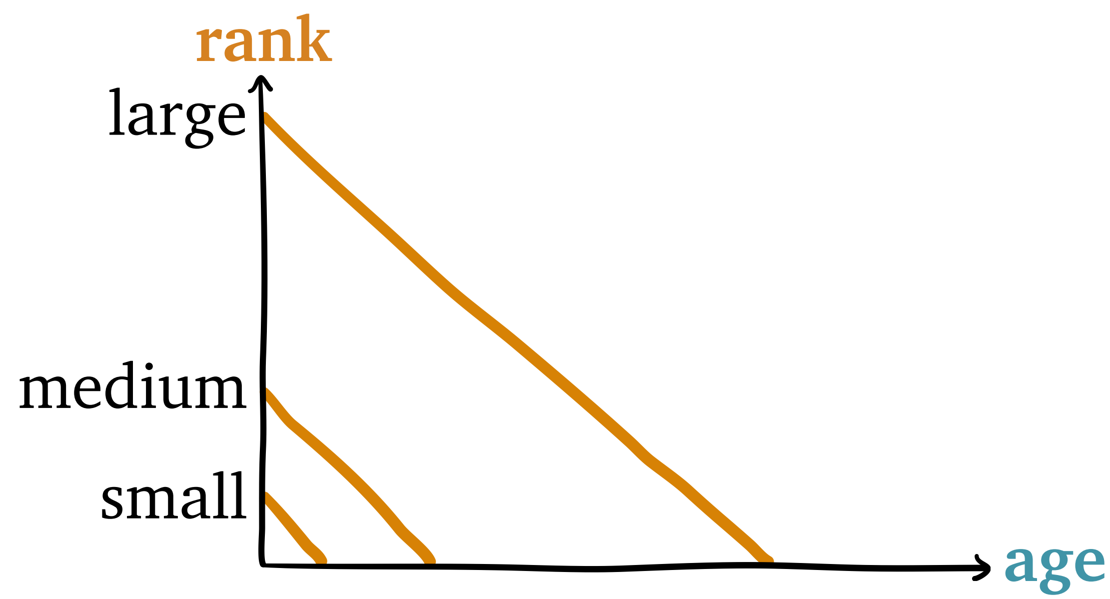


Classic SOAP Policies

FB

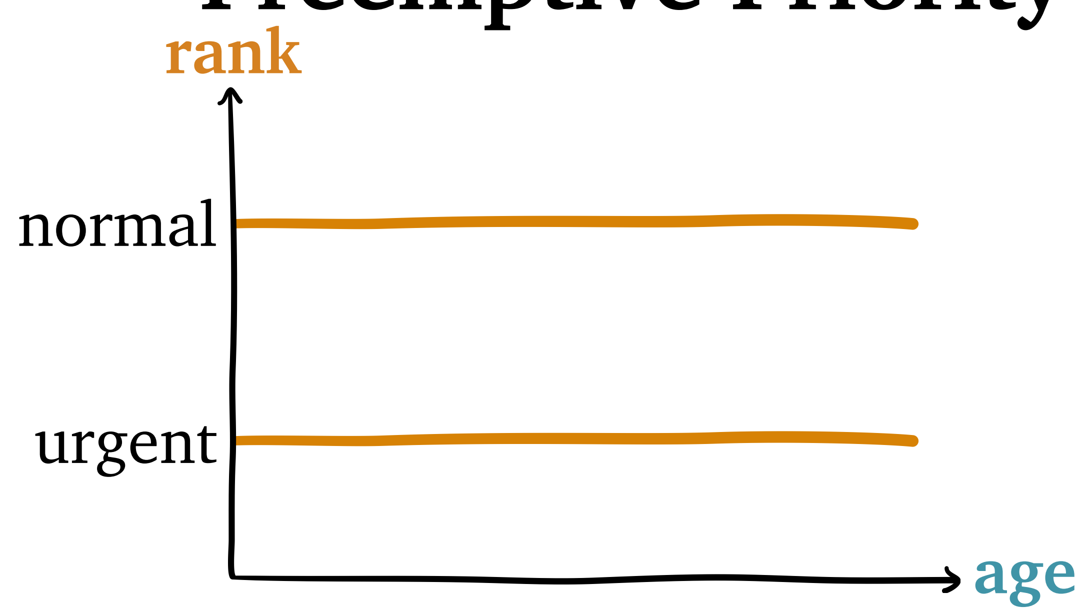


SRPT

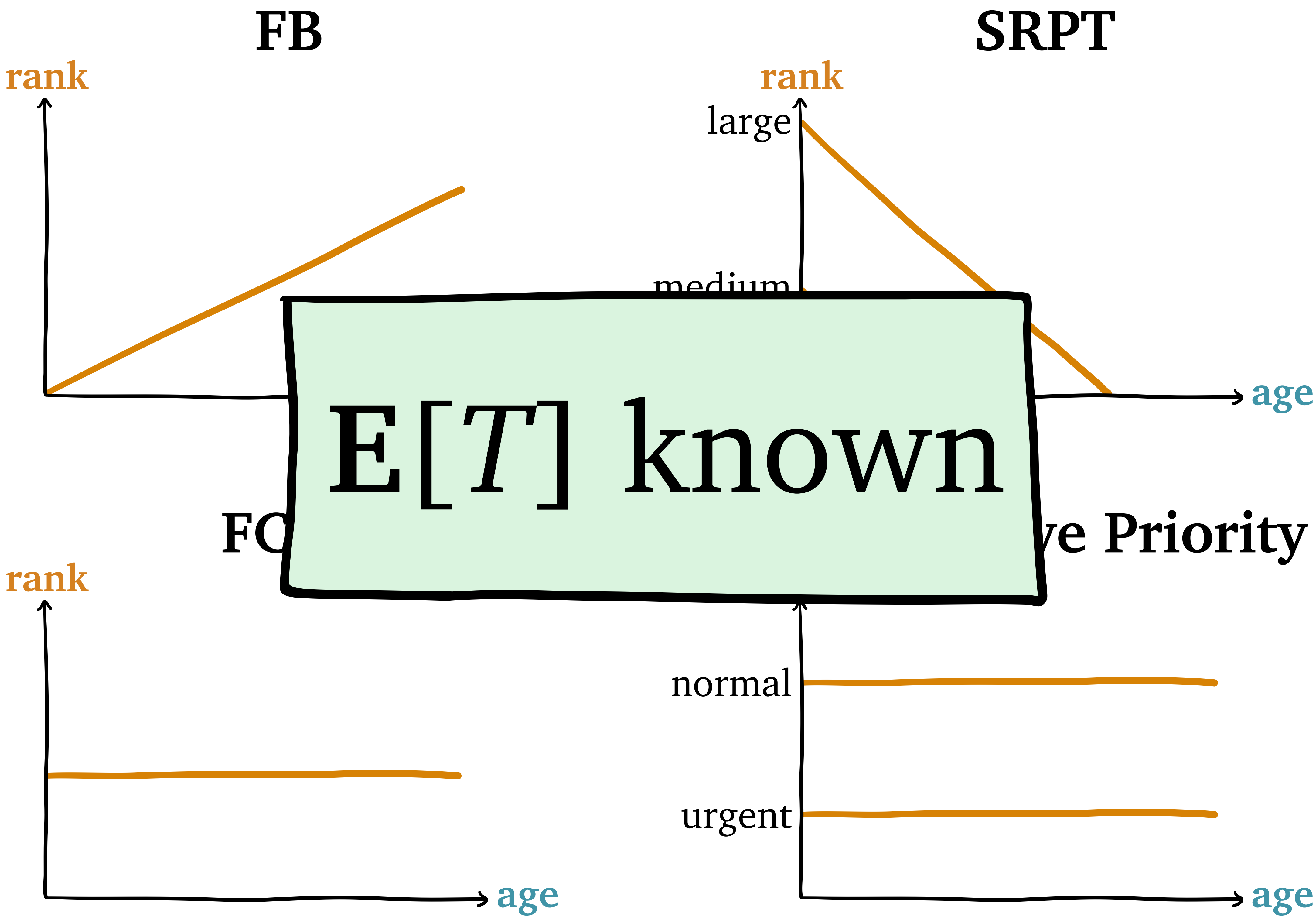


FCFS

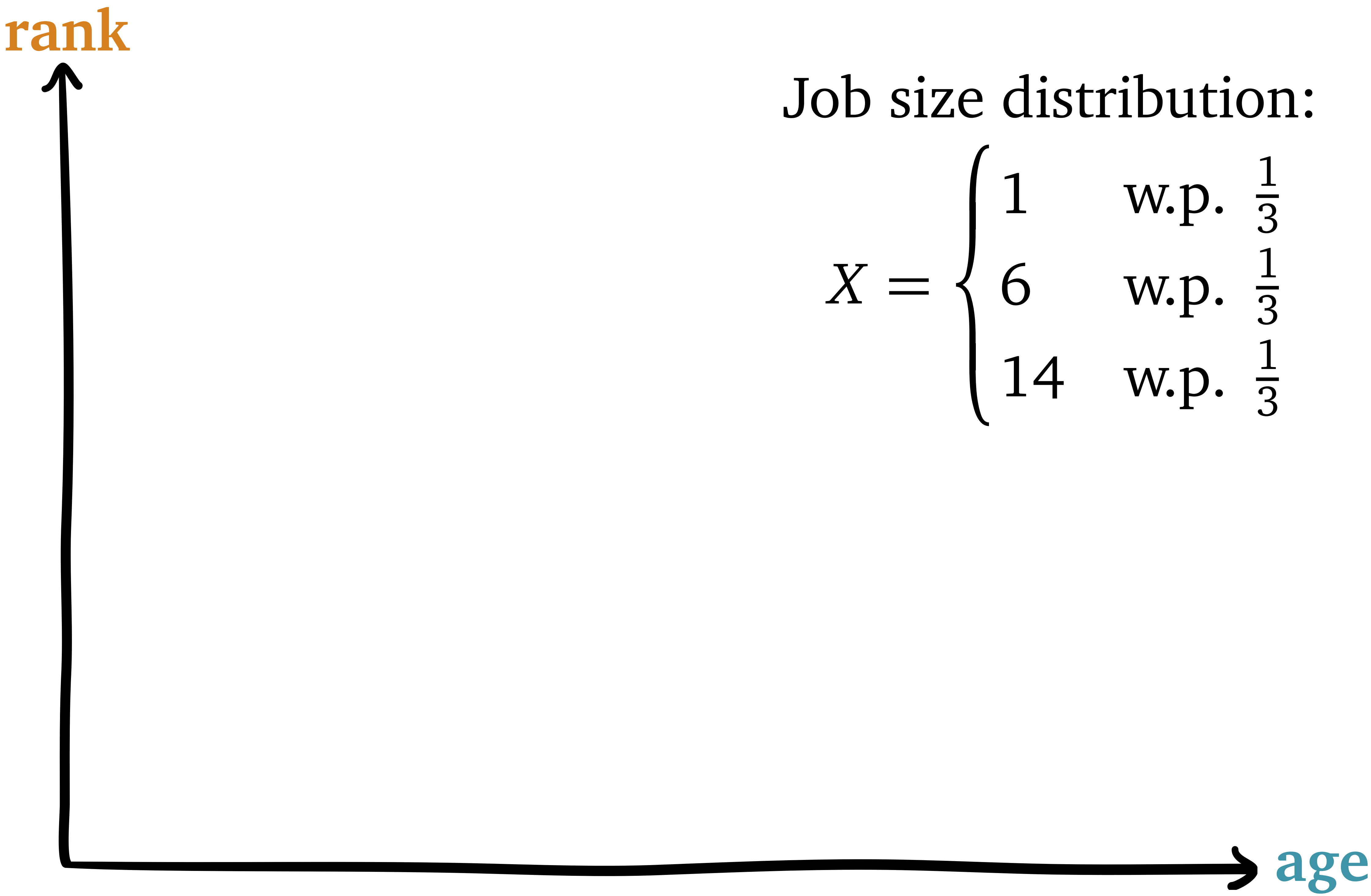
Preemptive Priority



Classic SOAP Policies



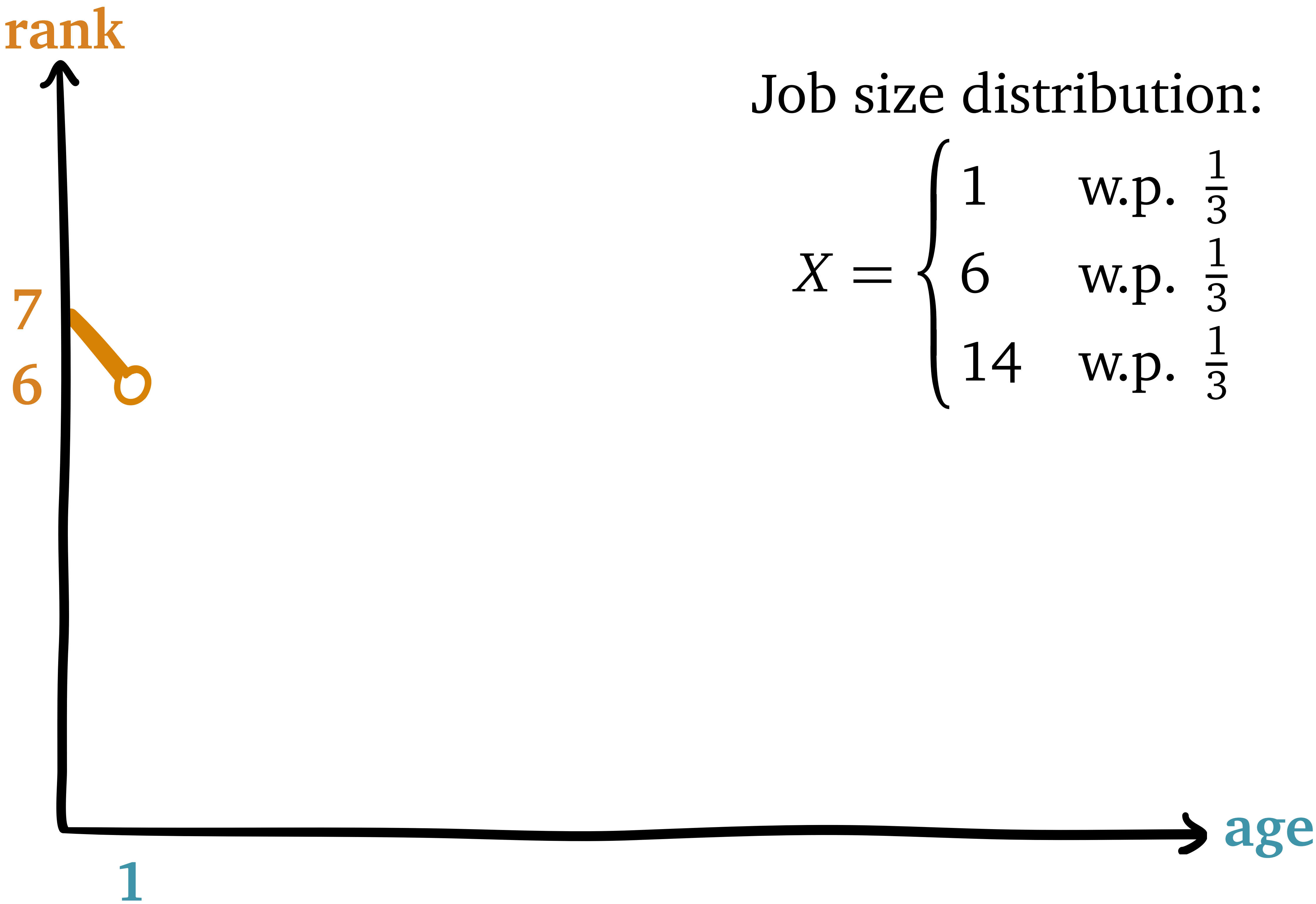
SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

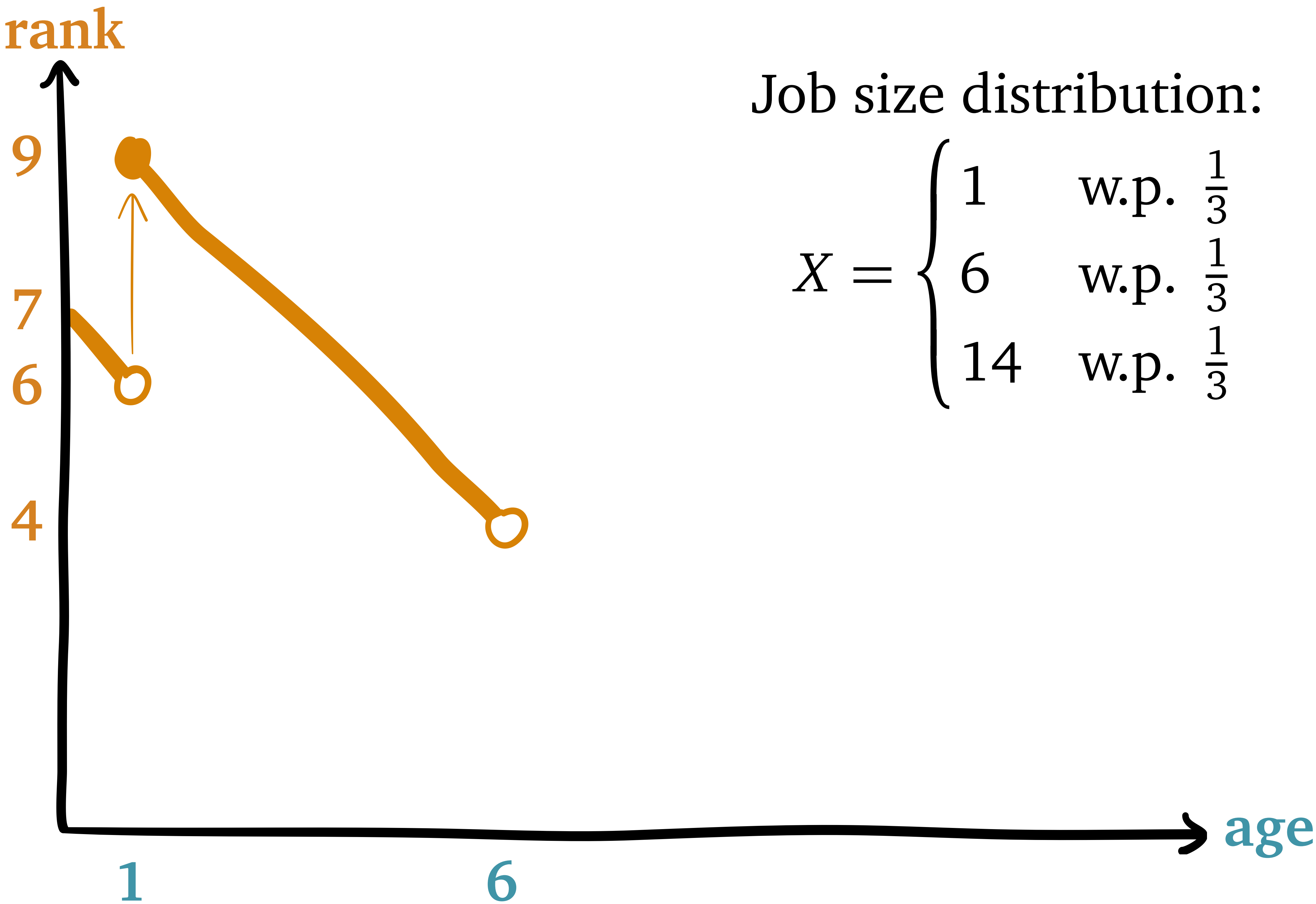
SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

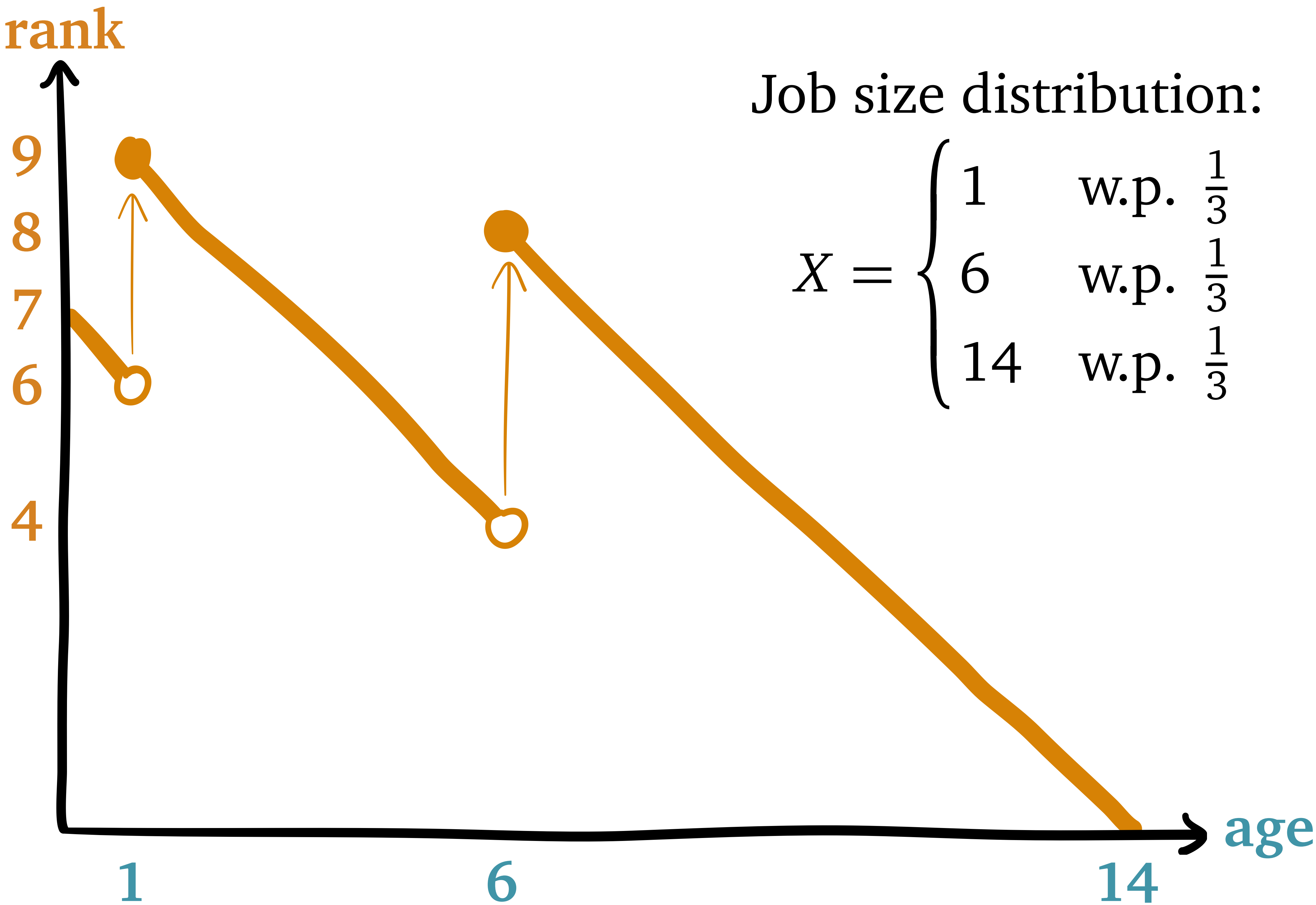
SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

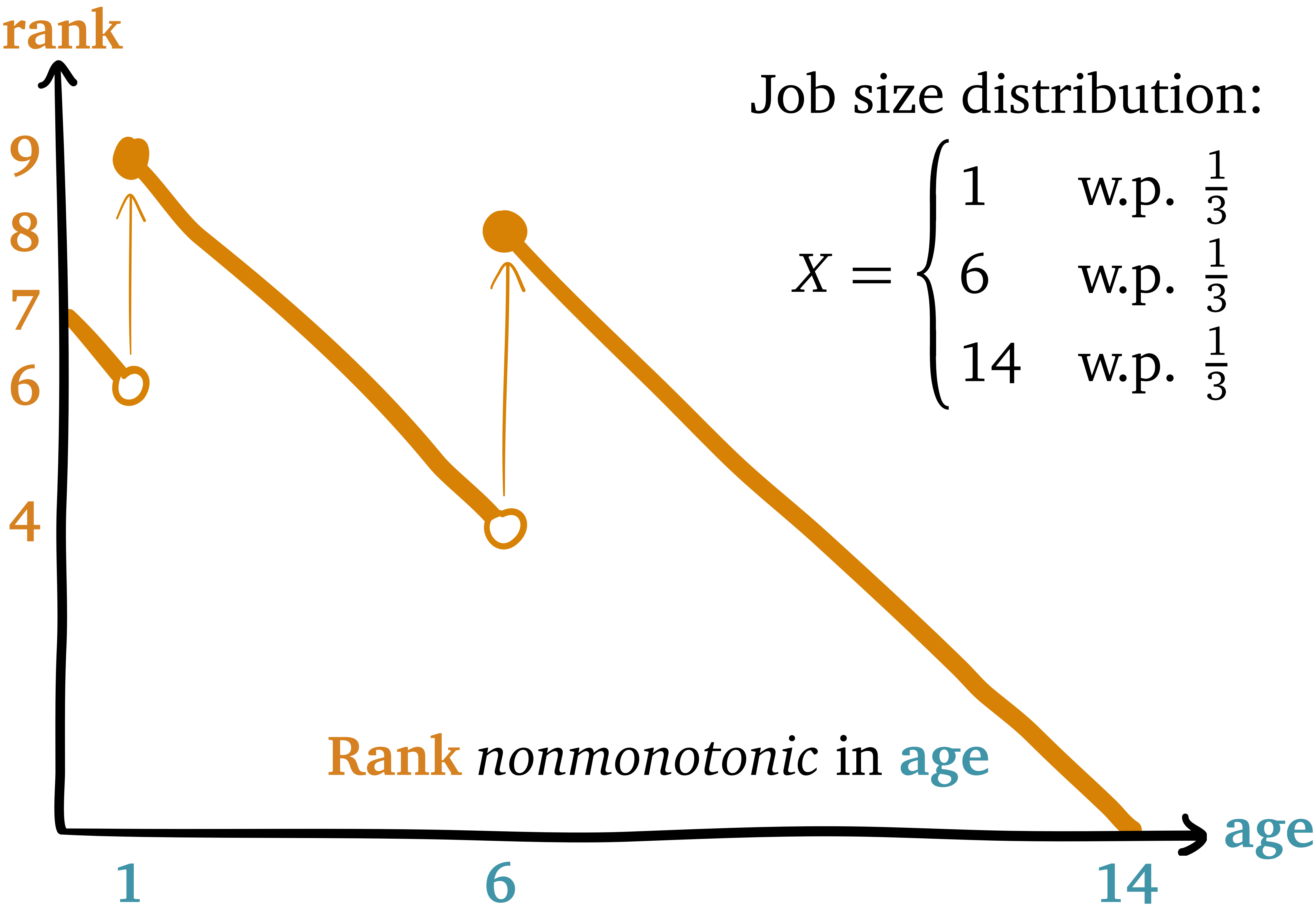
SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

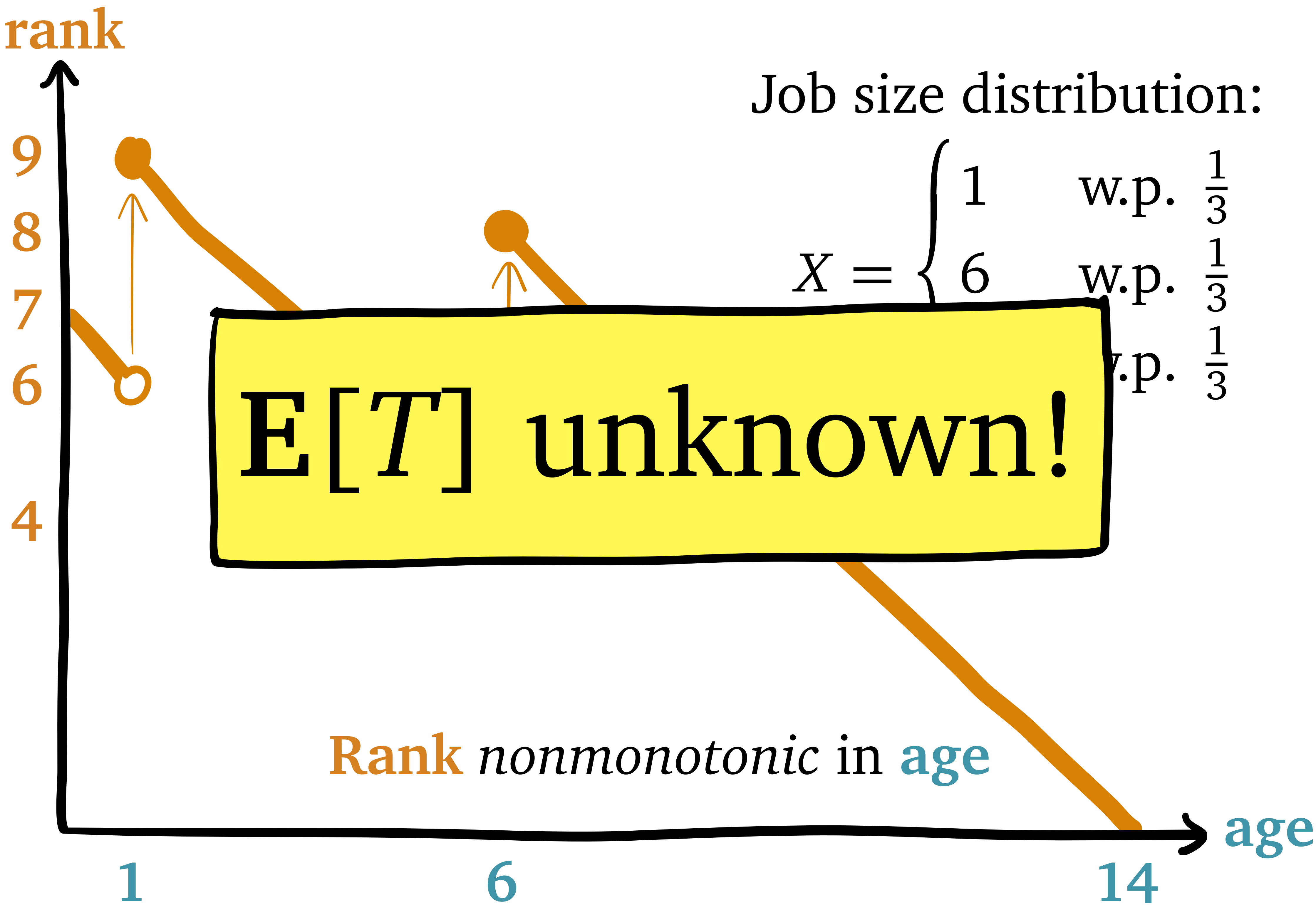
SOAP Policy: SERPT



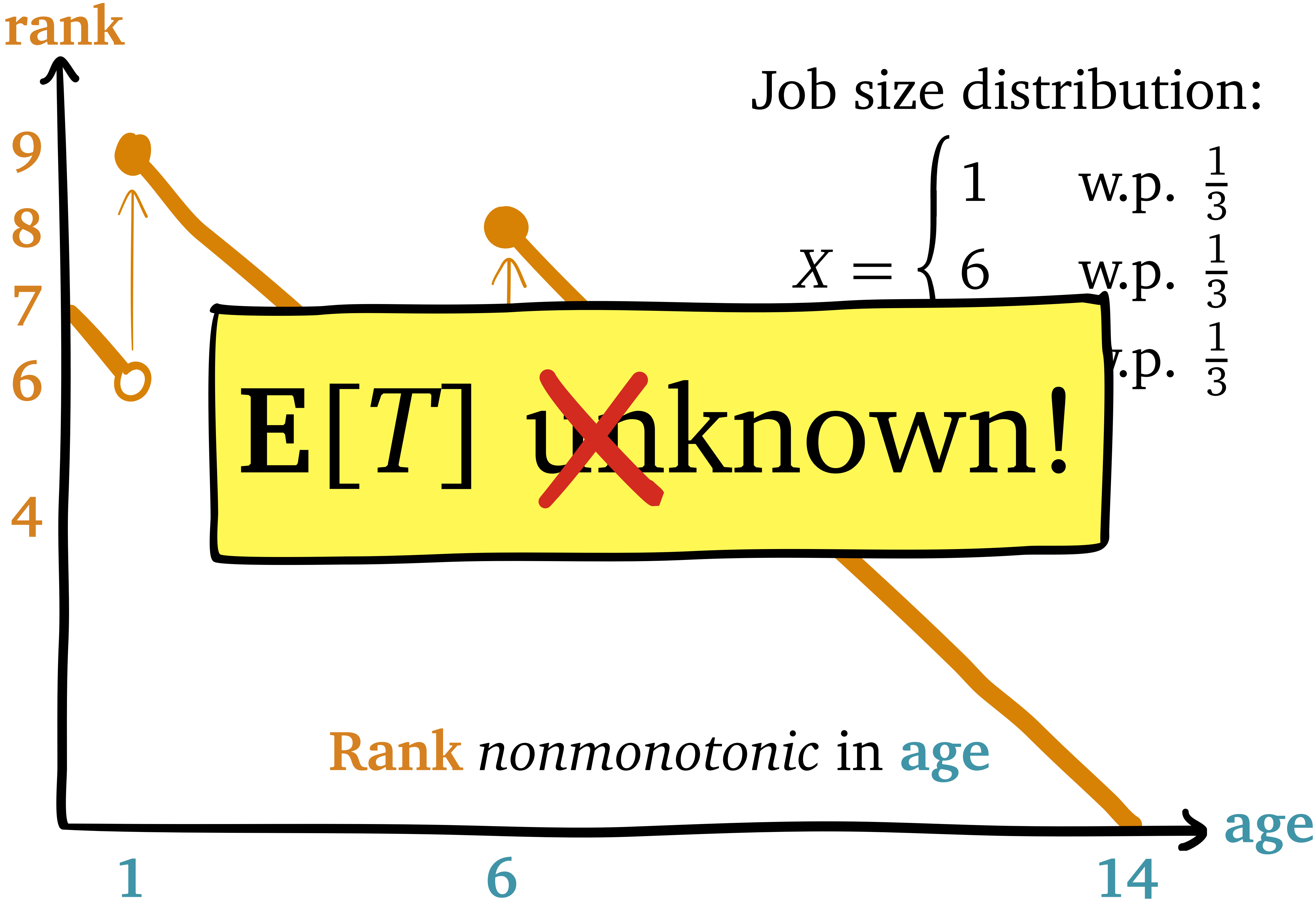
Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

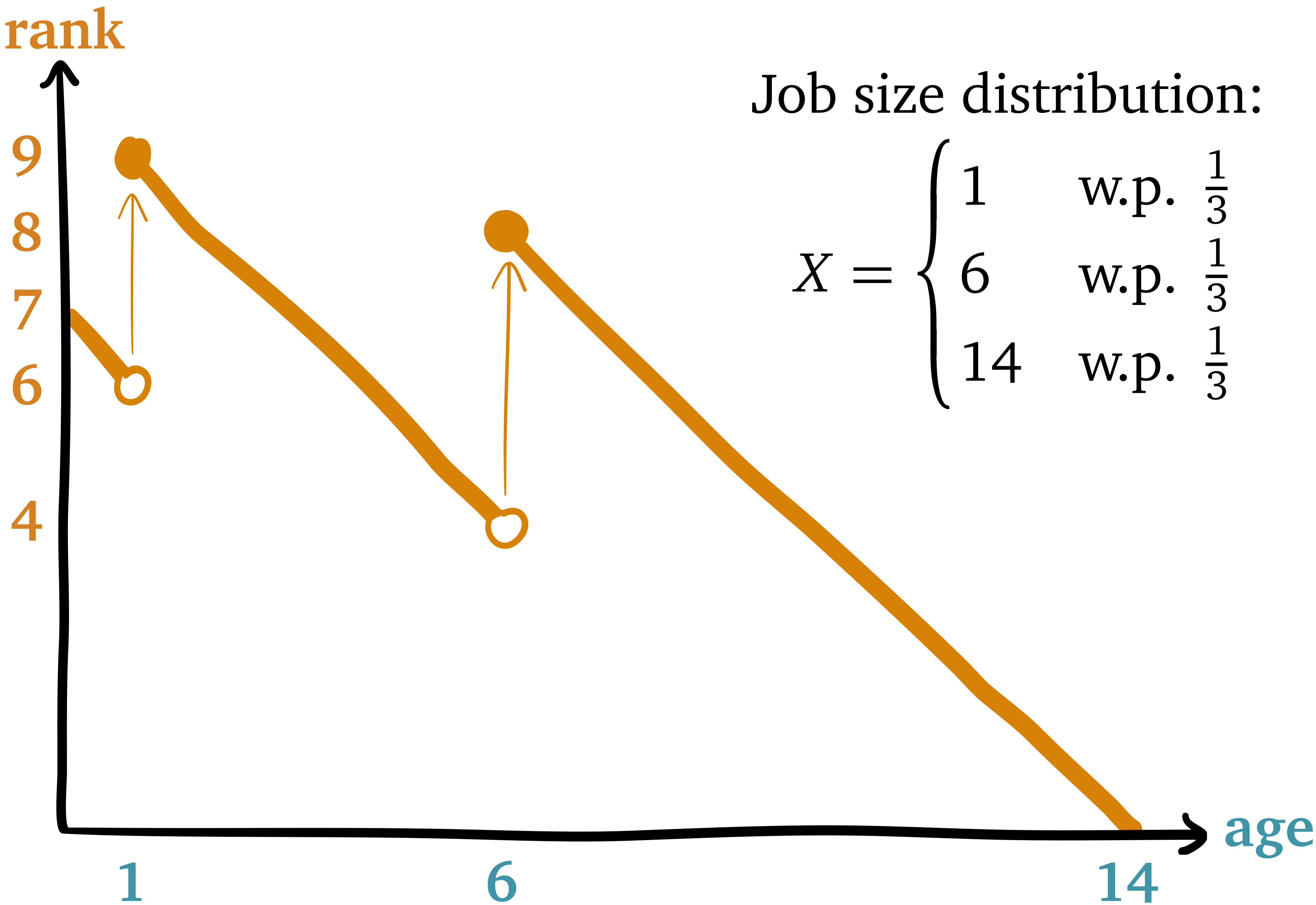
SOAP Policy: SERPT



SOAP Policy: SERPT



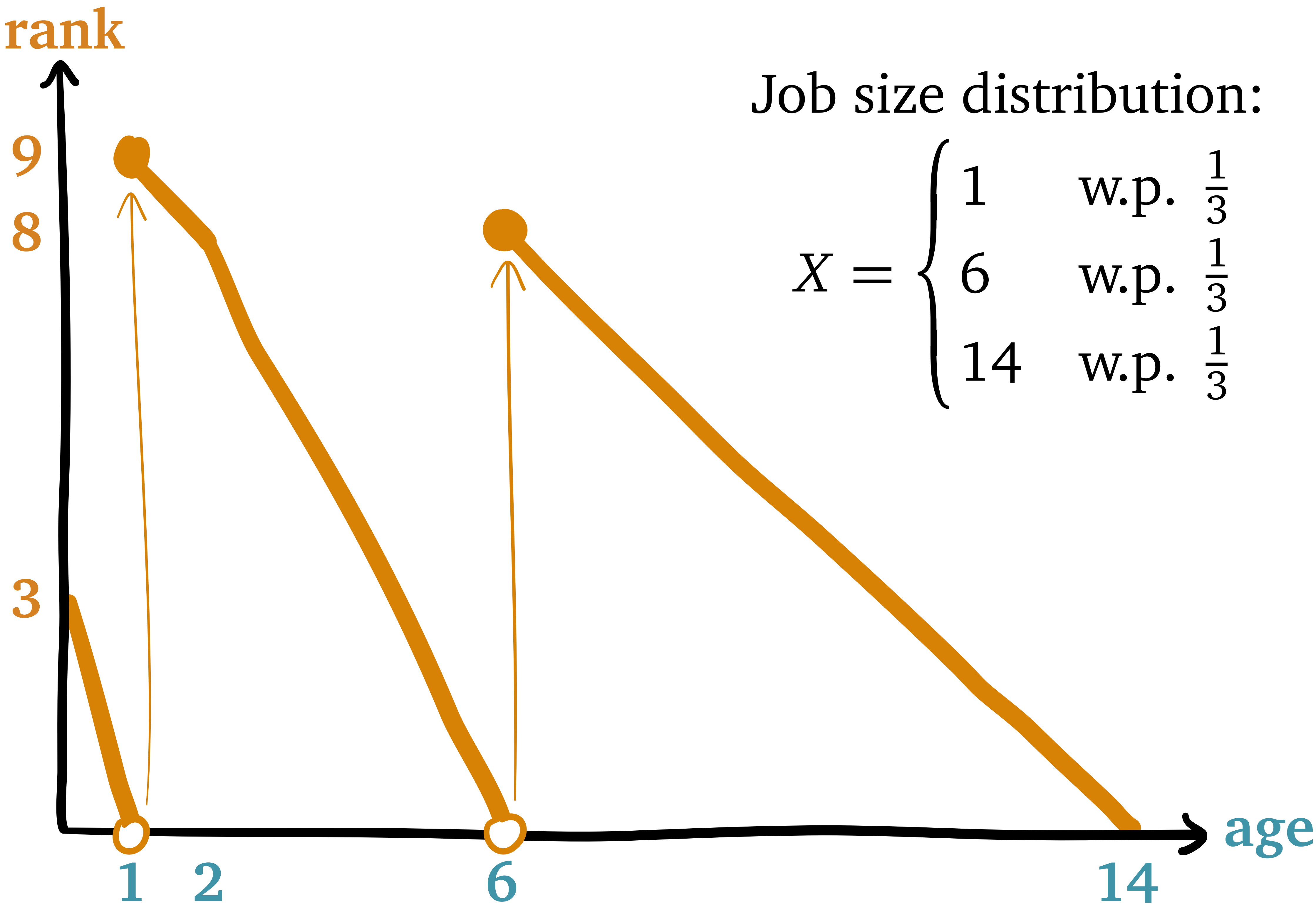
SOAP Policy: SERPT



Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

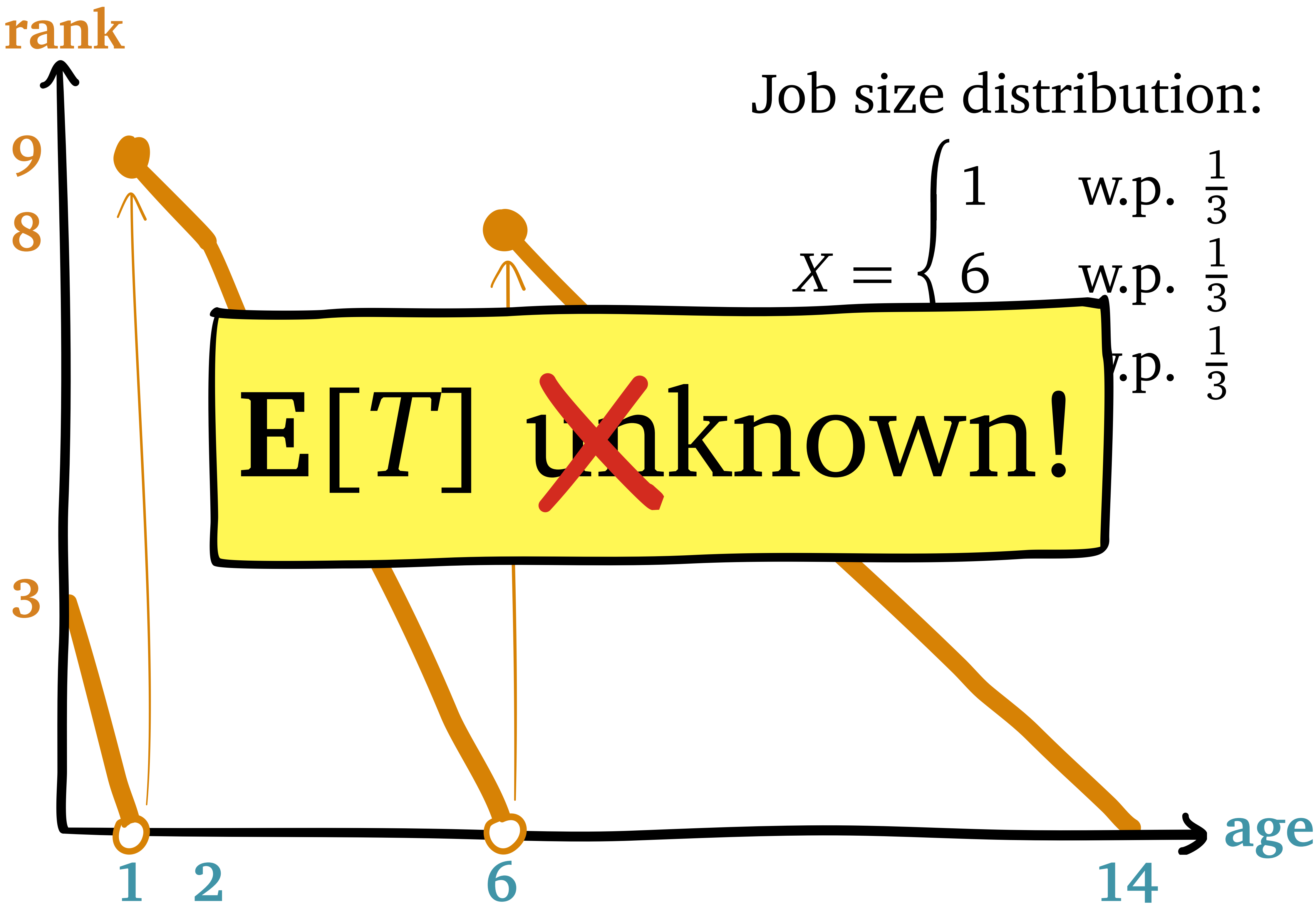
SOAP Policy: Gittins



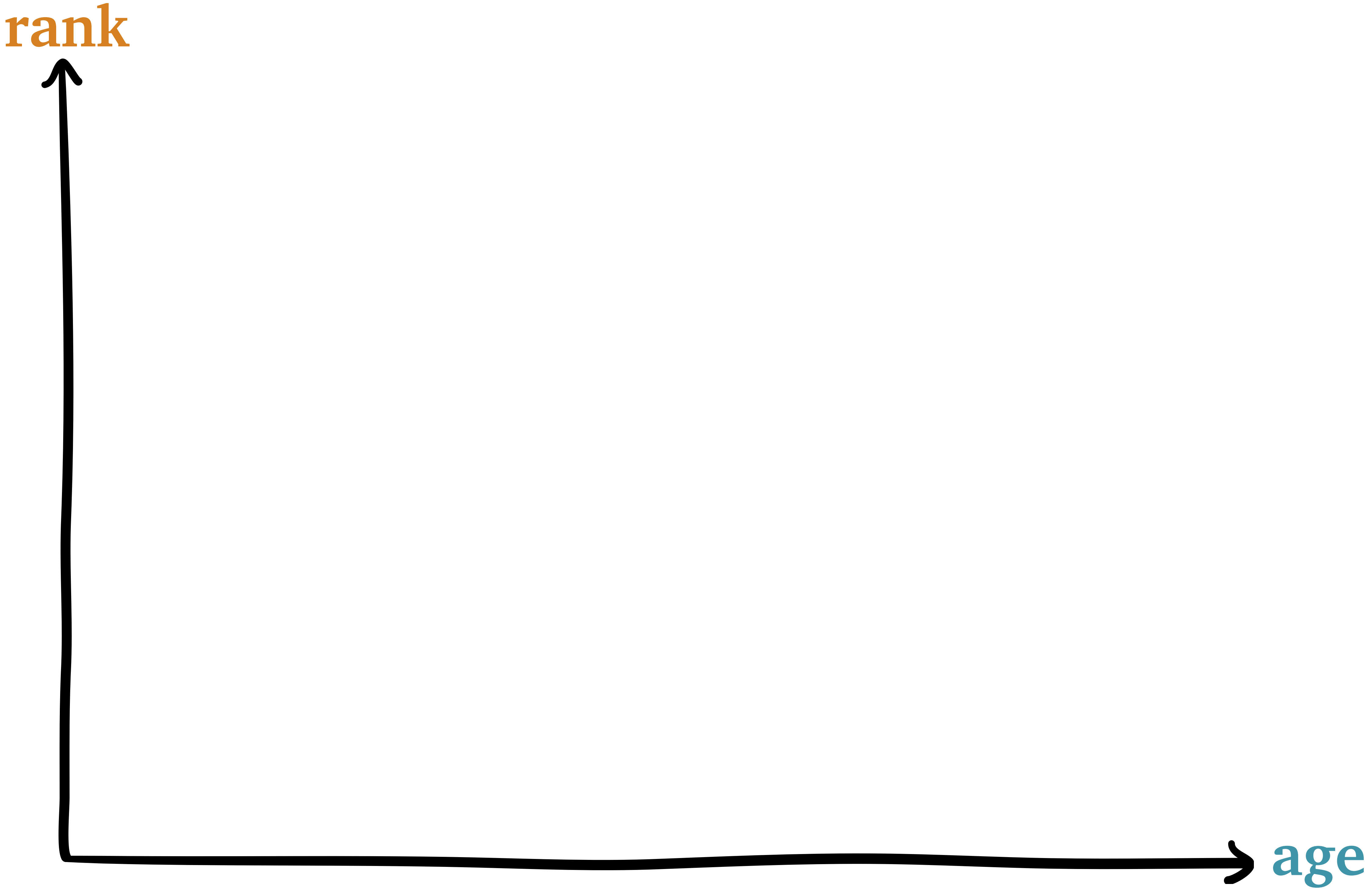
Job size distribution:

$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

SOAP Policy: Gittins

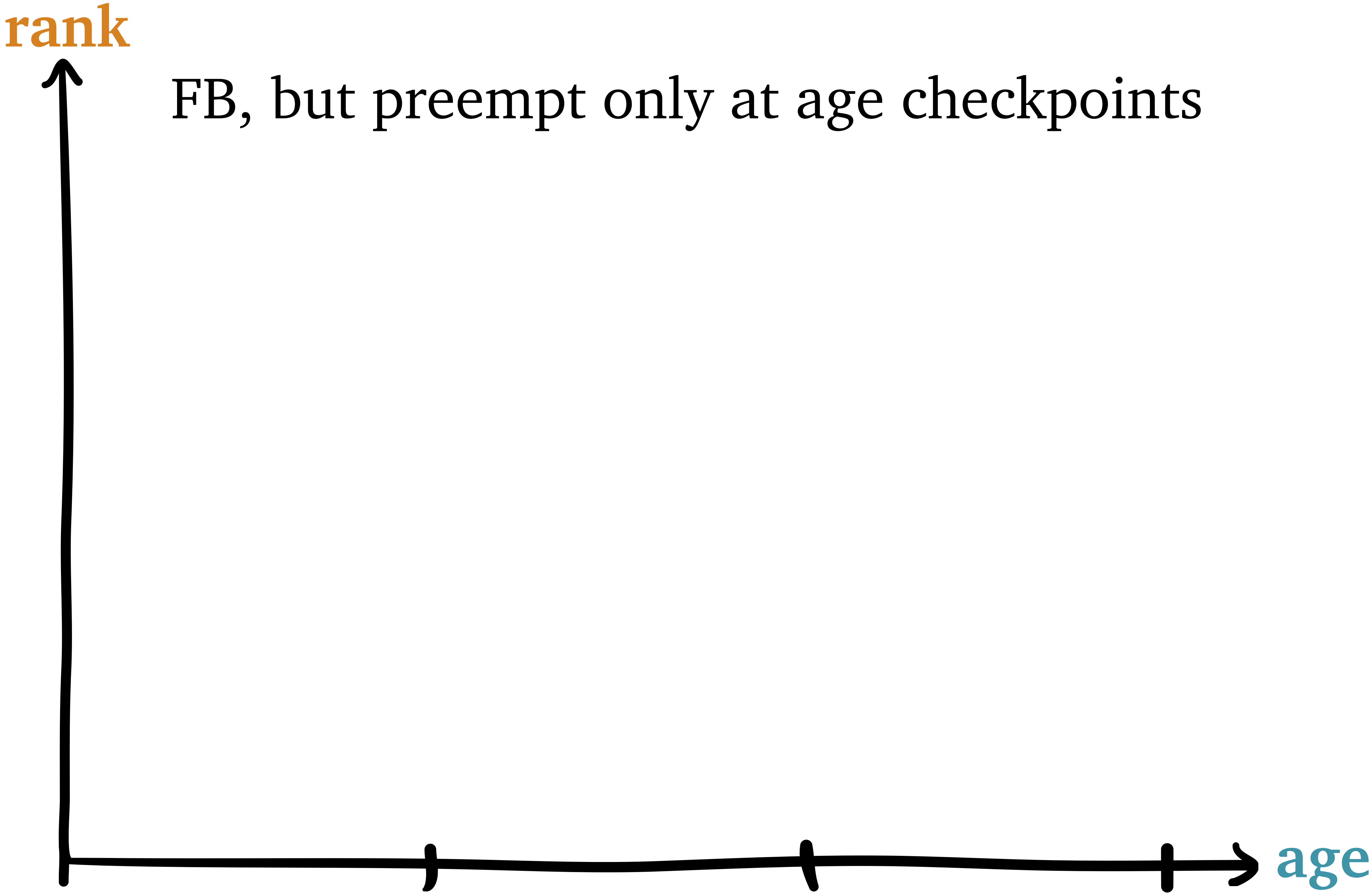


SOAP Policy: Discrete FB



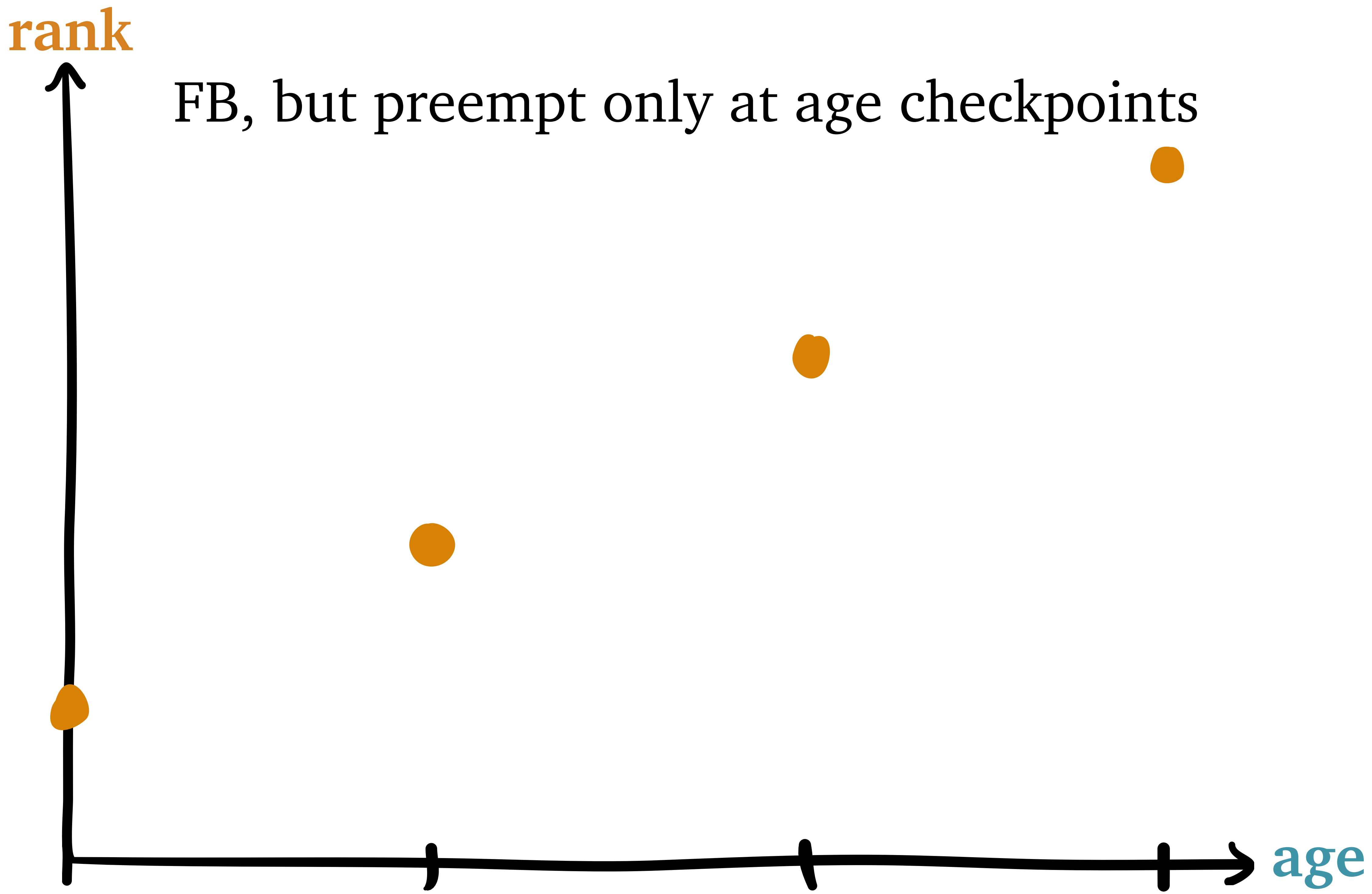
SOAP Policy: Discrete FB

FB, but preempt only at age checkpoints

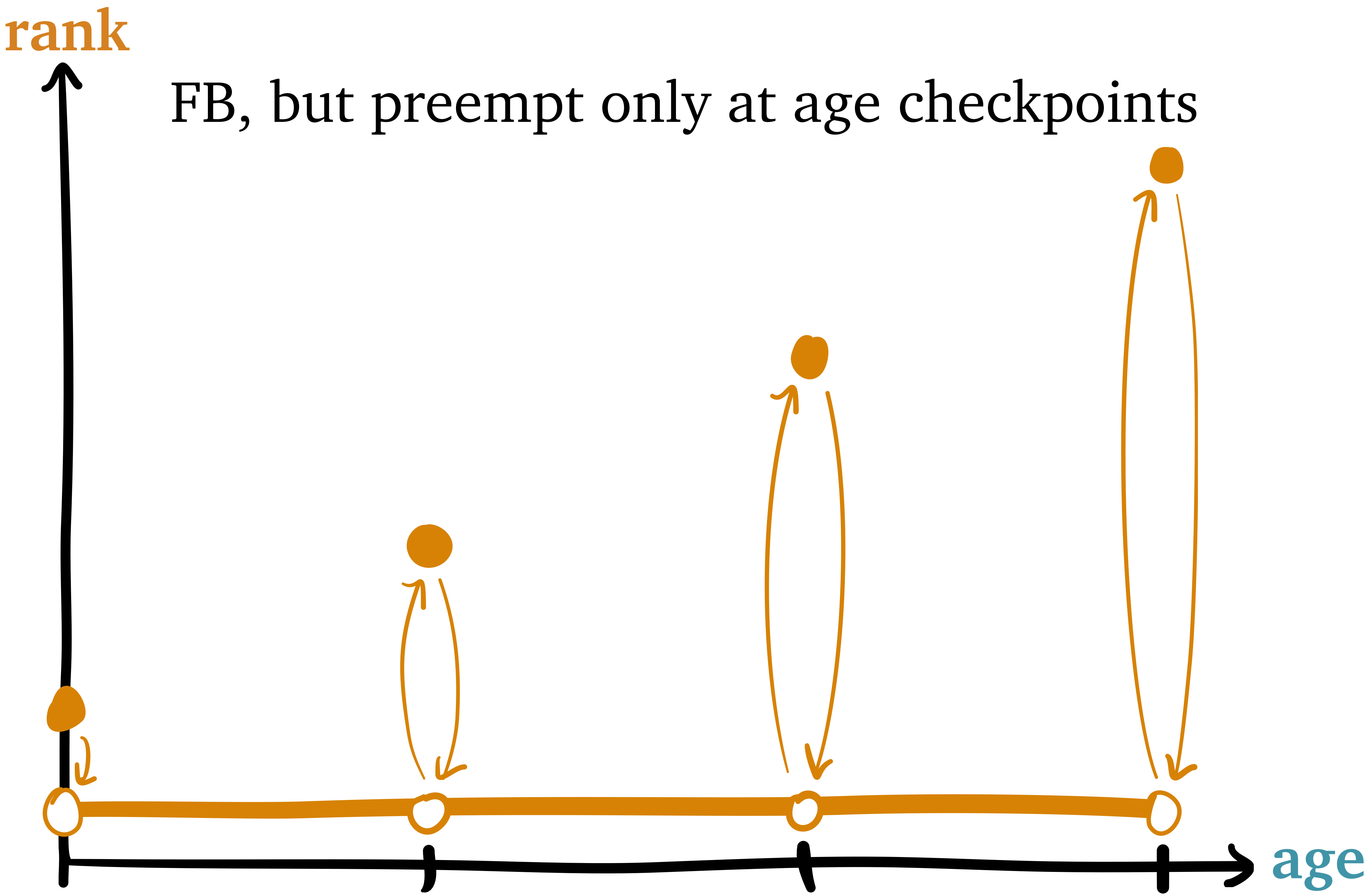


SOAP Policy: Discrete FB

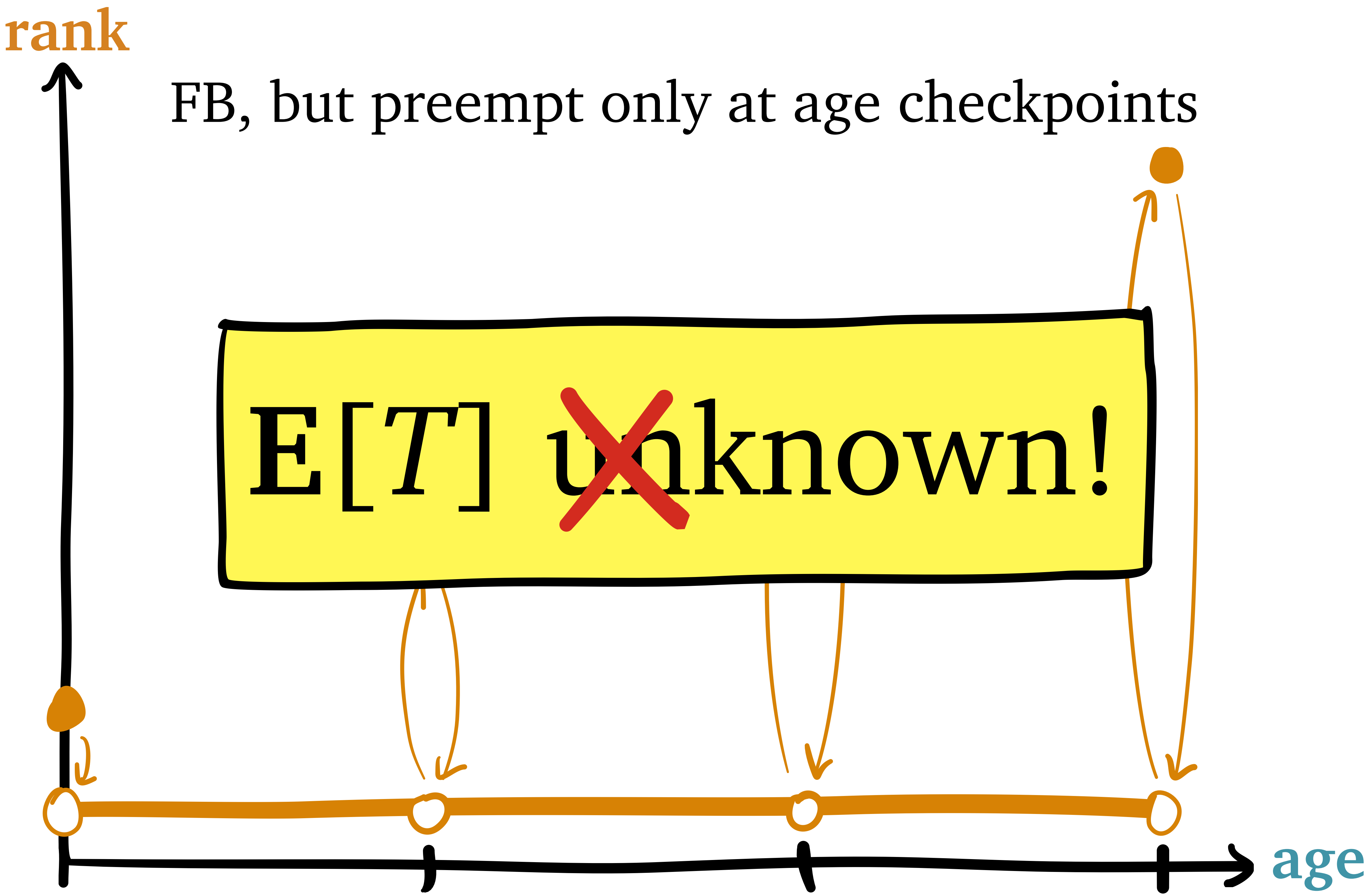
FB, but preempt only at age checkpoints



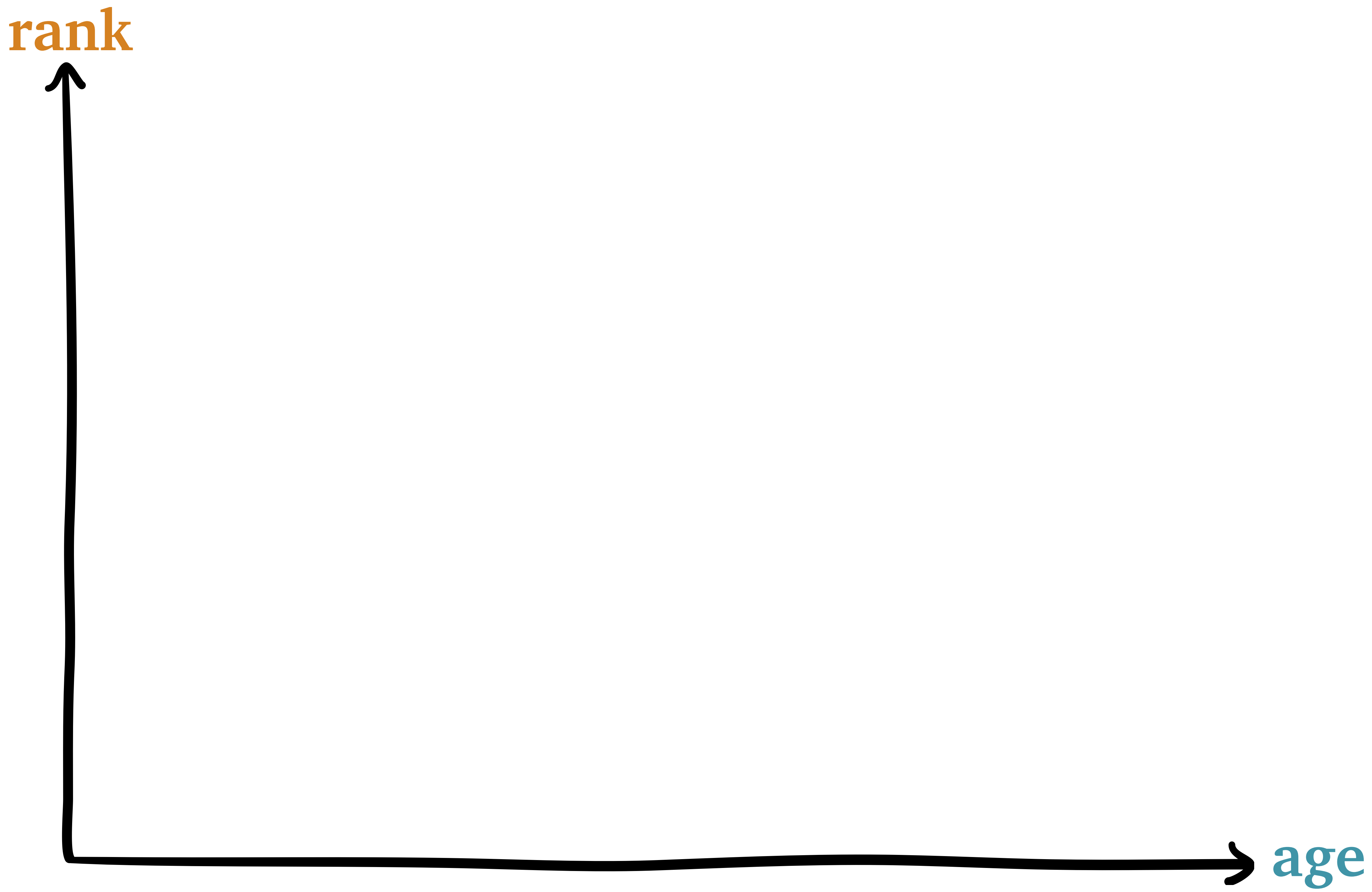
SOAP Policy: Discrete FB



SOAP Policy: Discrete FB

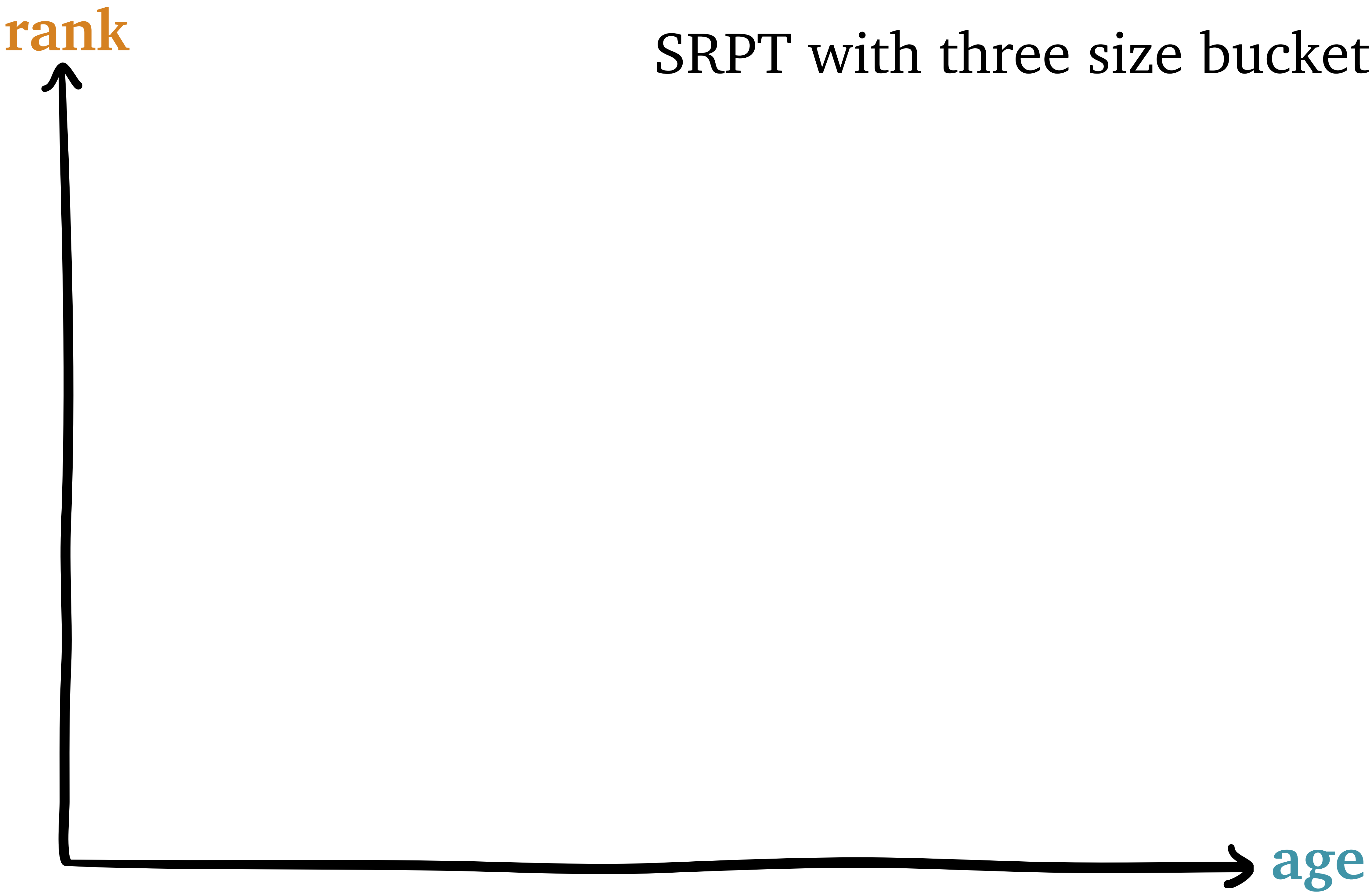


SOAP Policy: Bucketed SRPT

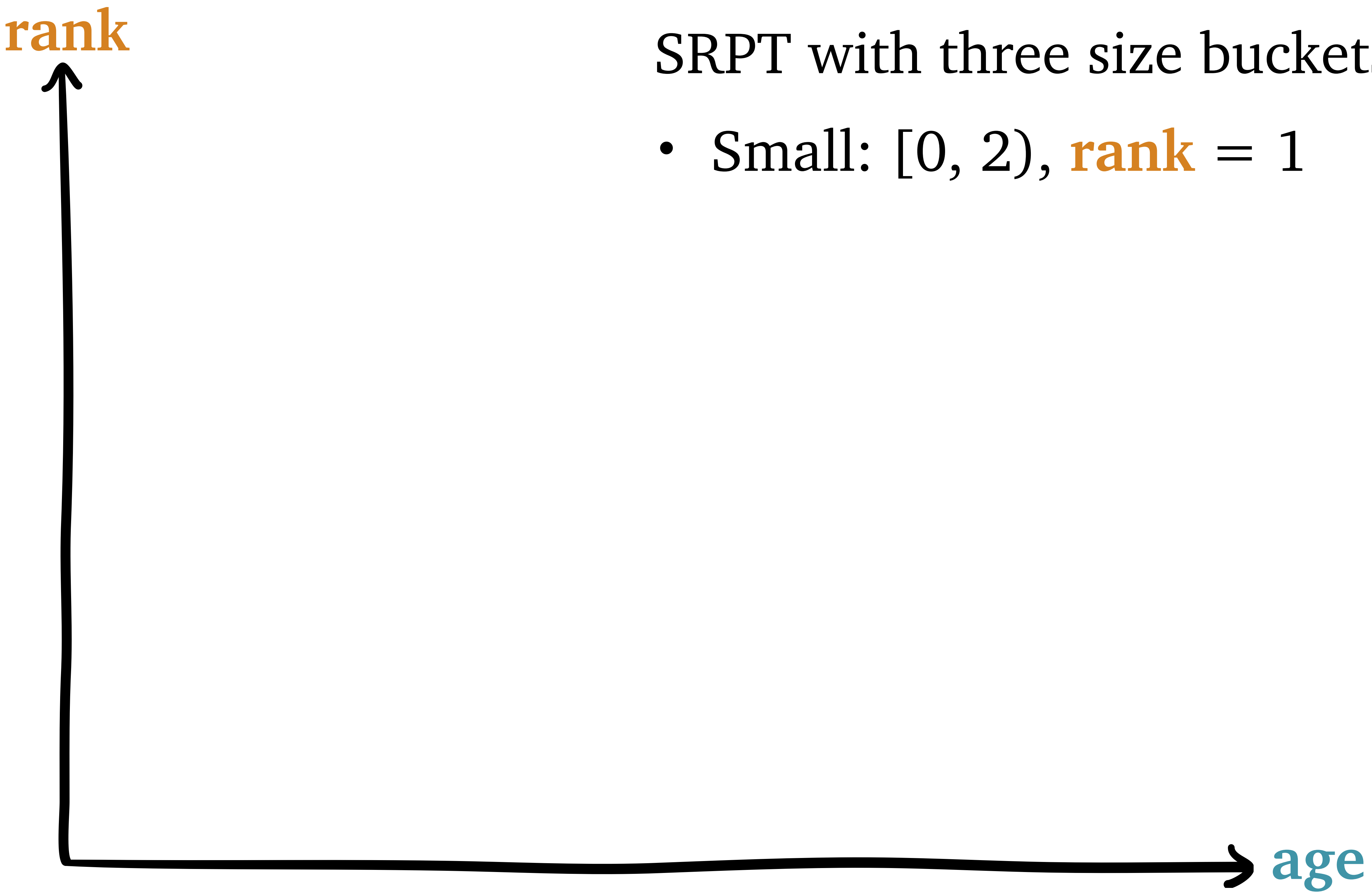


SOAP Policy: Bucketed SRPT

SRPT with three size buckets:



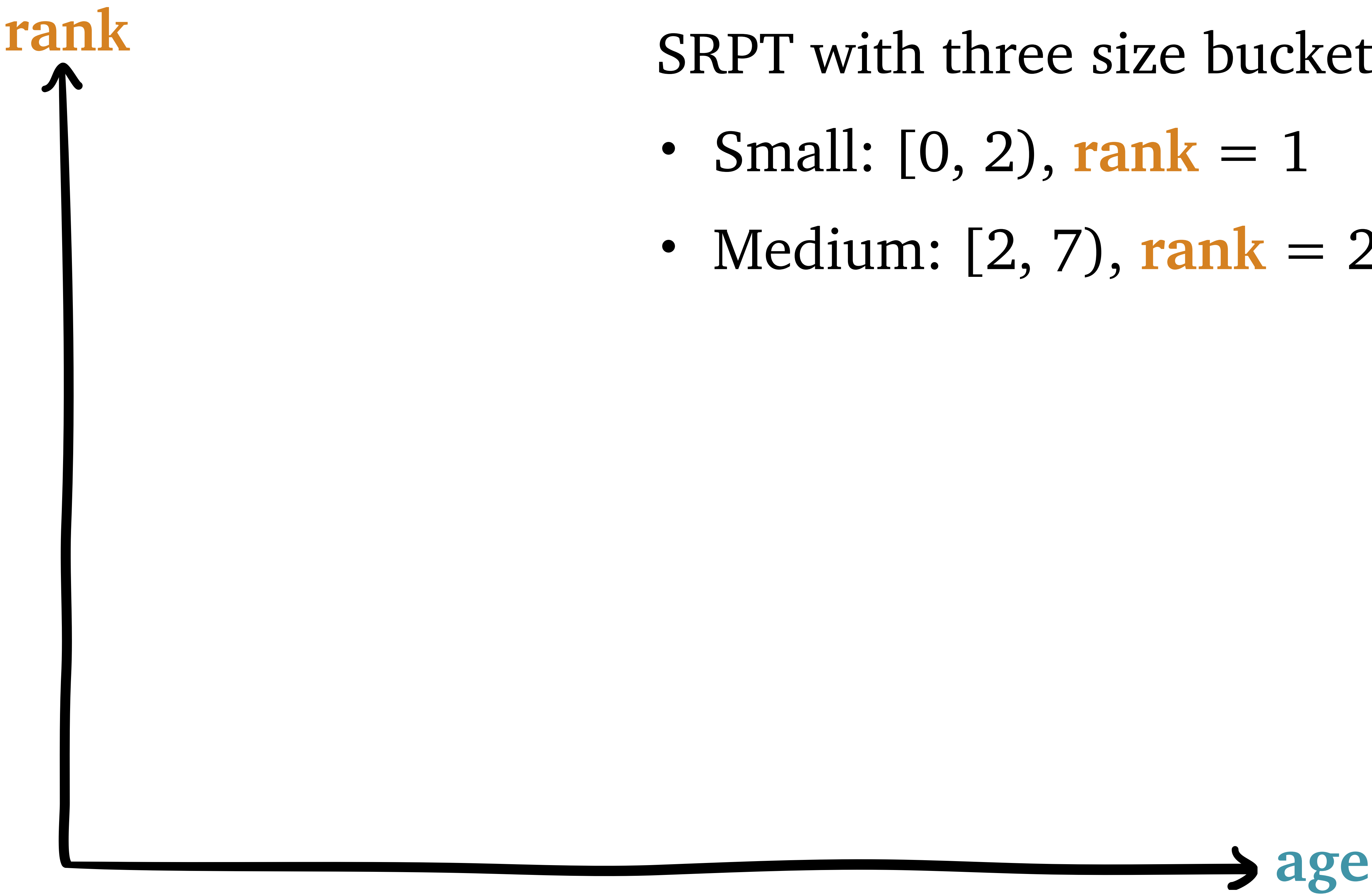
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: [0, 2), **rank** = 1

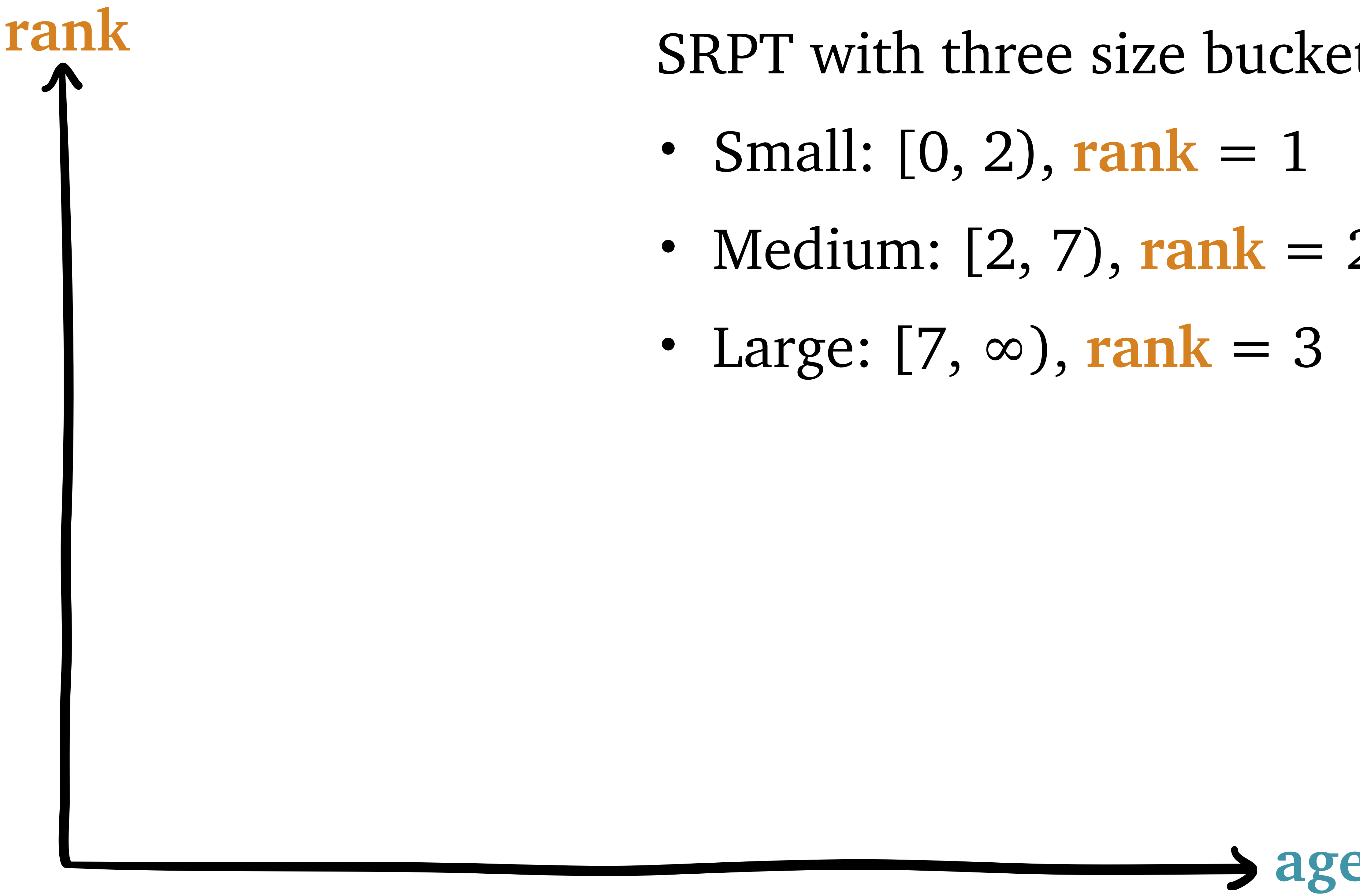
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: $[0, 2)$, **rank** $= 1$
- Medium: $[2, 7)$, **rank** $= 2$

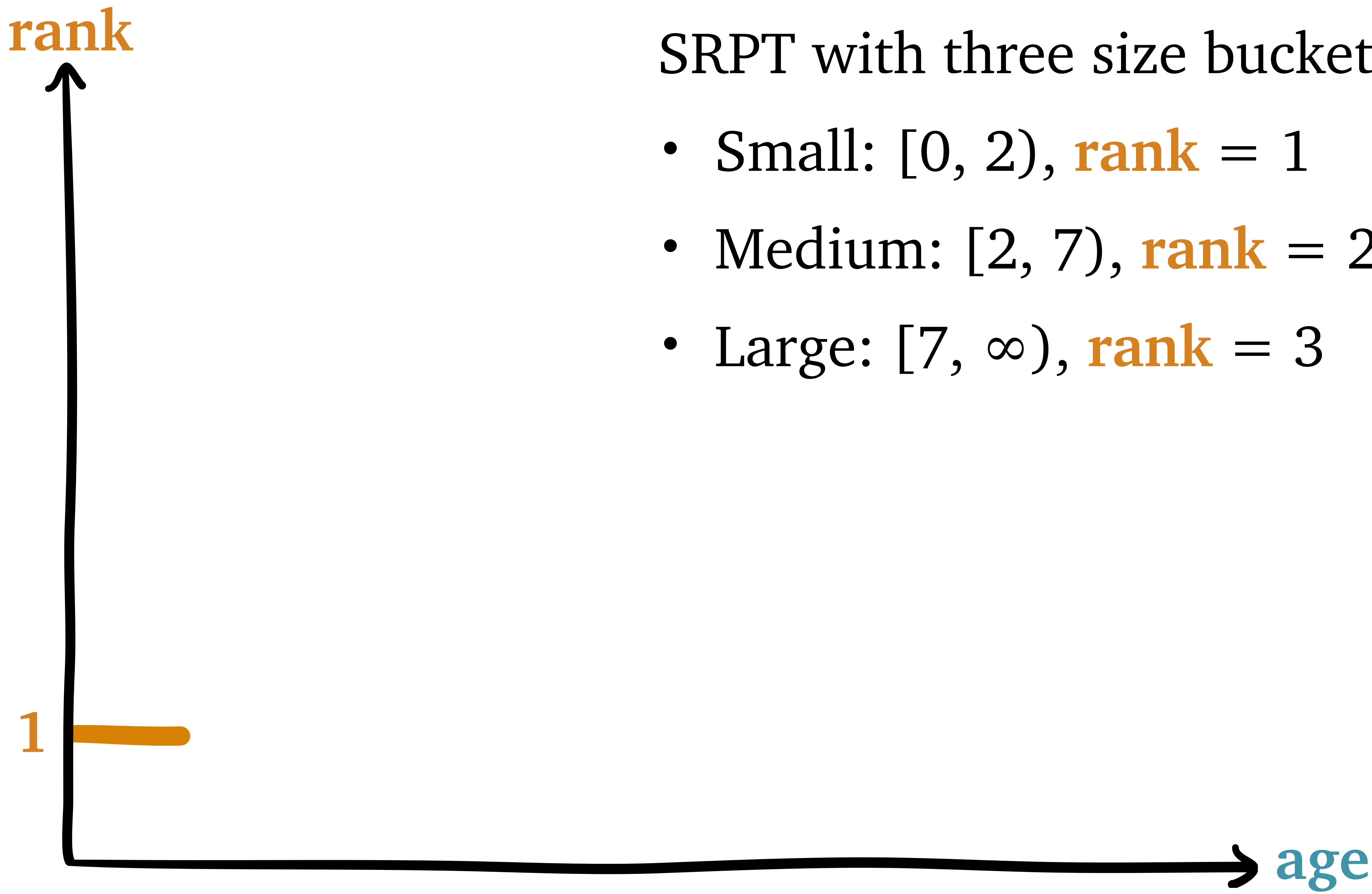
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: $[0, 2)$, rank $= 1$
- Medium: $[2, 7)$, rank $= 2$
- Large: $[7, \infty)$, rank $= 3$

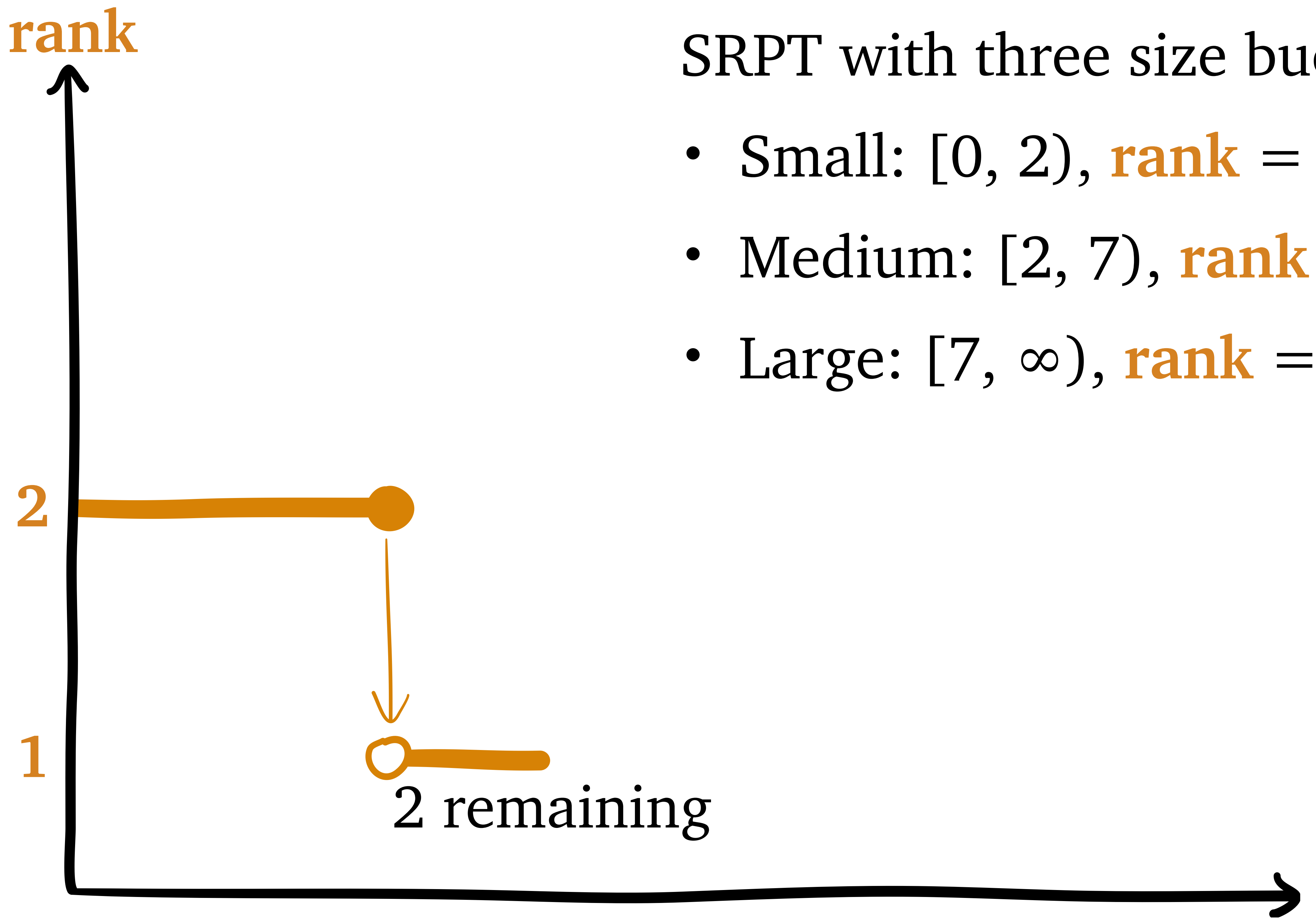
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: $[0, 2)$, **rank** $= 1$
- Medium: $[2, 7)$, **rank** $= 2$
- Large: $[7, \infty)$, **rank** $= 3$

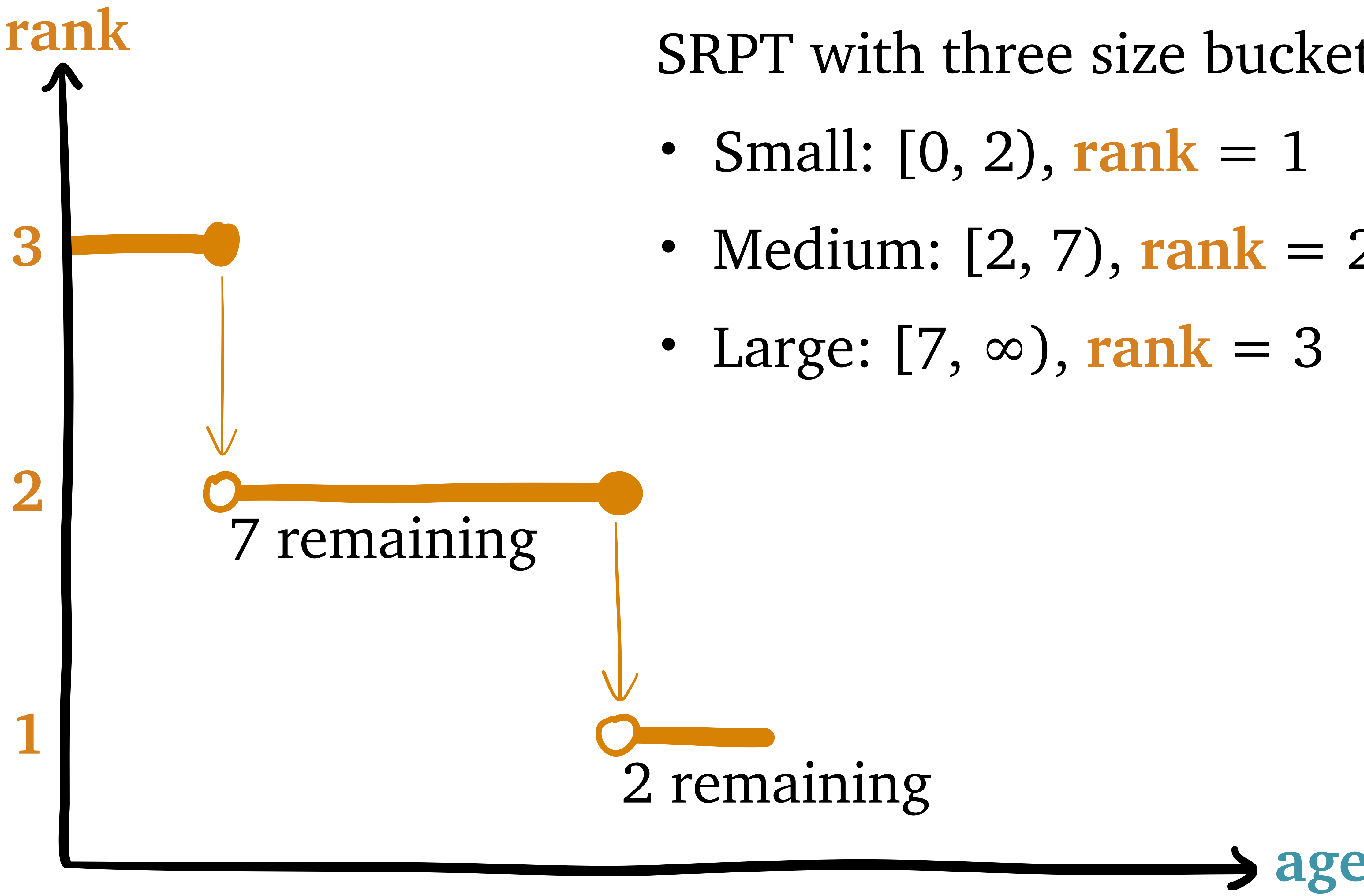
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: [0, 2), **rank** = 1
- Medium: [2, 7), **rank** = 2
- Large: [7, ∞), **rank** = 3

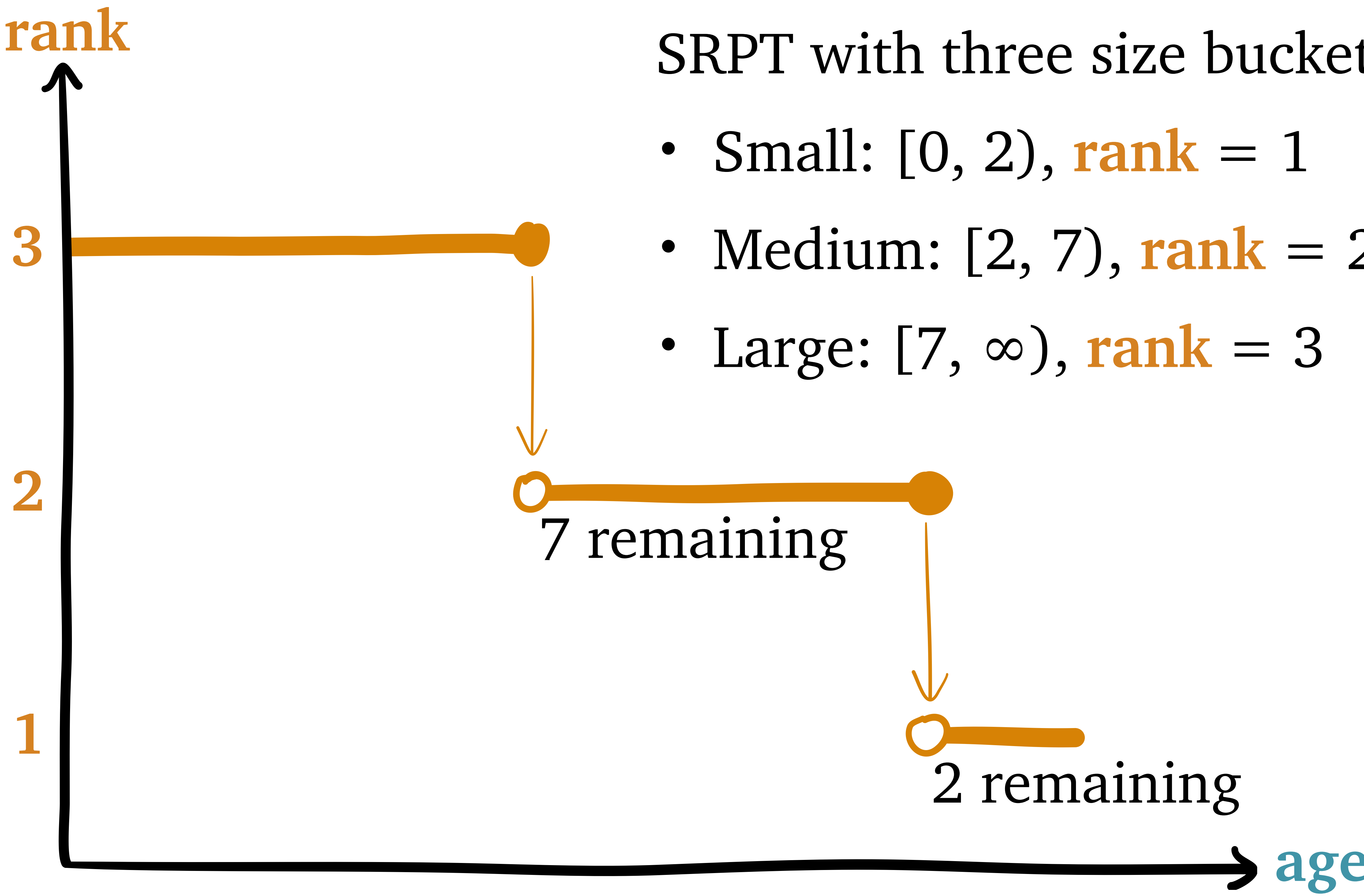
SOAP Policy: Bucketed SRPT



SRPT with three size buckets:

- Small: $[0, 2)$, **rank** $= 1$
- Medium: $[2, 7)$, **rank** $= 2$
- Large: $[7, \infty)$, **rank** $= 3$

SOAP Policy: Bucketed SRPT



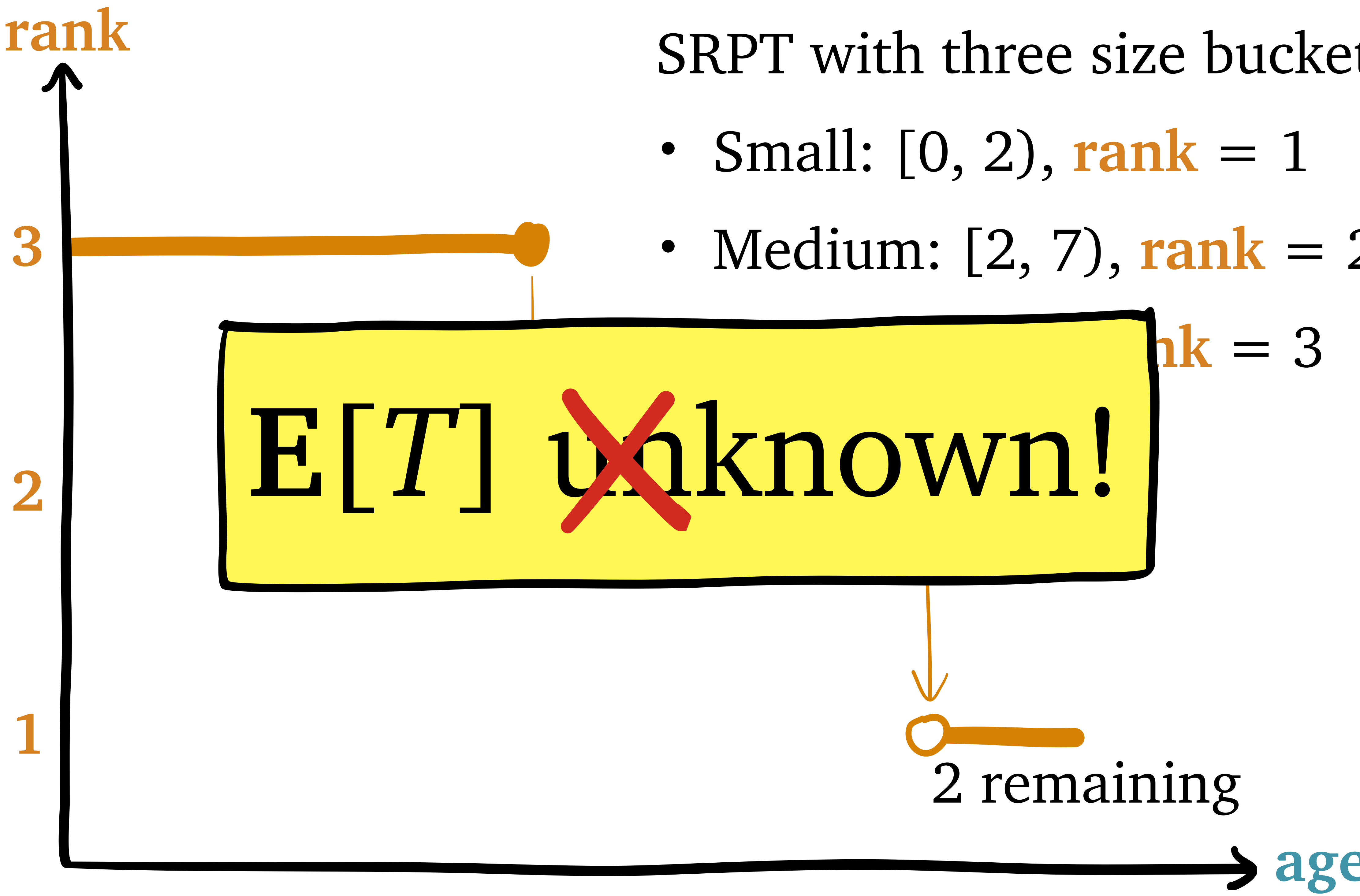
SRPT with three size buckets:

- Small: $[0, 2)$, **rank** $= 1$
- Medium: $[2, 7)$, **rank** $= 2$
- Large: $[7, \infty)$, **rank** $= 3$

SOAP Policy: Bucketed SRPT

SRPT with three size buckets:

- Small: [0, 2), **rank** = 1
- Medium: [2, 7), **rank** = 2
- nk = 3



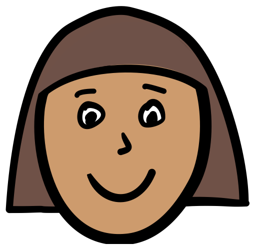
$\mathbf{E}[T]$ ~~un~~known!

SOAP Policy: Mixture

Two customer classes: *humans* and *robots*

SOAP Policy: Mixture

Two customer classes: *humans* and *robots*

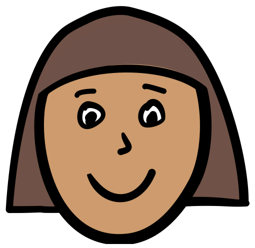


Humans

- unknown size
- nonpreemptible
- FCFS

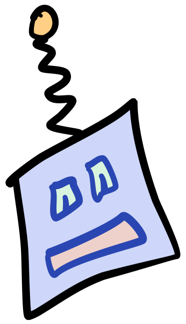
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

- unknown size
- nonpreemptible
- FCFS

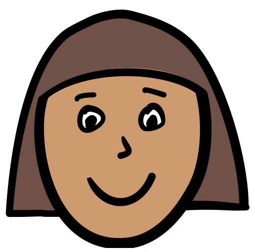


Robots

- known size
- preemptible
- SRPT

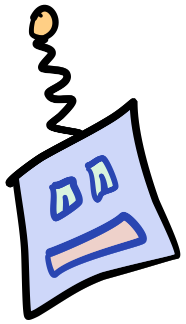
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

- unknown size
- nonpreemptible
- FCFS



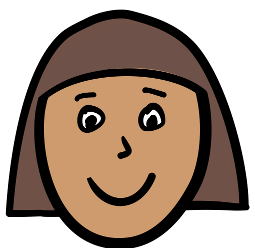
Robots

- known size
- preemptible
- SRPT

Priority: small robots, humans, large robots

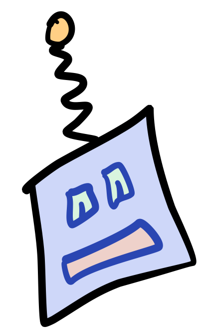
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

- unknown size
- nonpreemptible
- FCFS



Robots

- known size
- preemptible
- SRPT

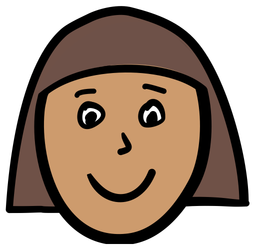
size $< x_{\mathrm{H}}$

size $\geq x_{\mathrm{H}}$

Priority: small robots, humans, large robots

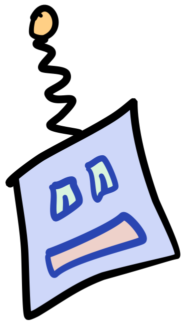
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

- unknown size
- nonpreemptible
- FCFS



Robots

- known size
- preemptible
- SRPT

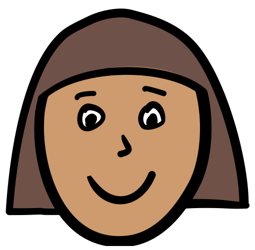
size $< x_{\mathrm{H}}$

size $\geq x_{\mathrm{H}}$

Priority: small robots, humans, large robots

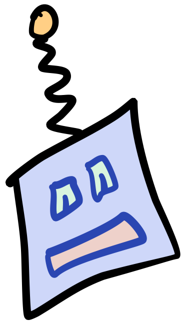
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

- unknown size
- nonpreemptible
- FCFS

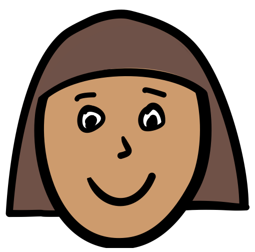


Robots

- known size
- preemptible
- SRPT

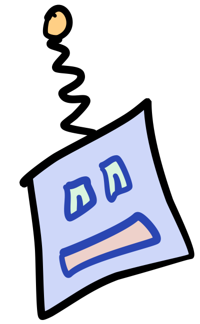
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



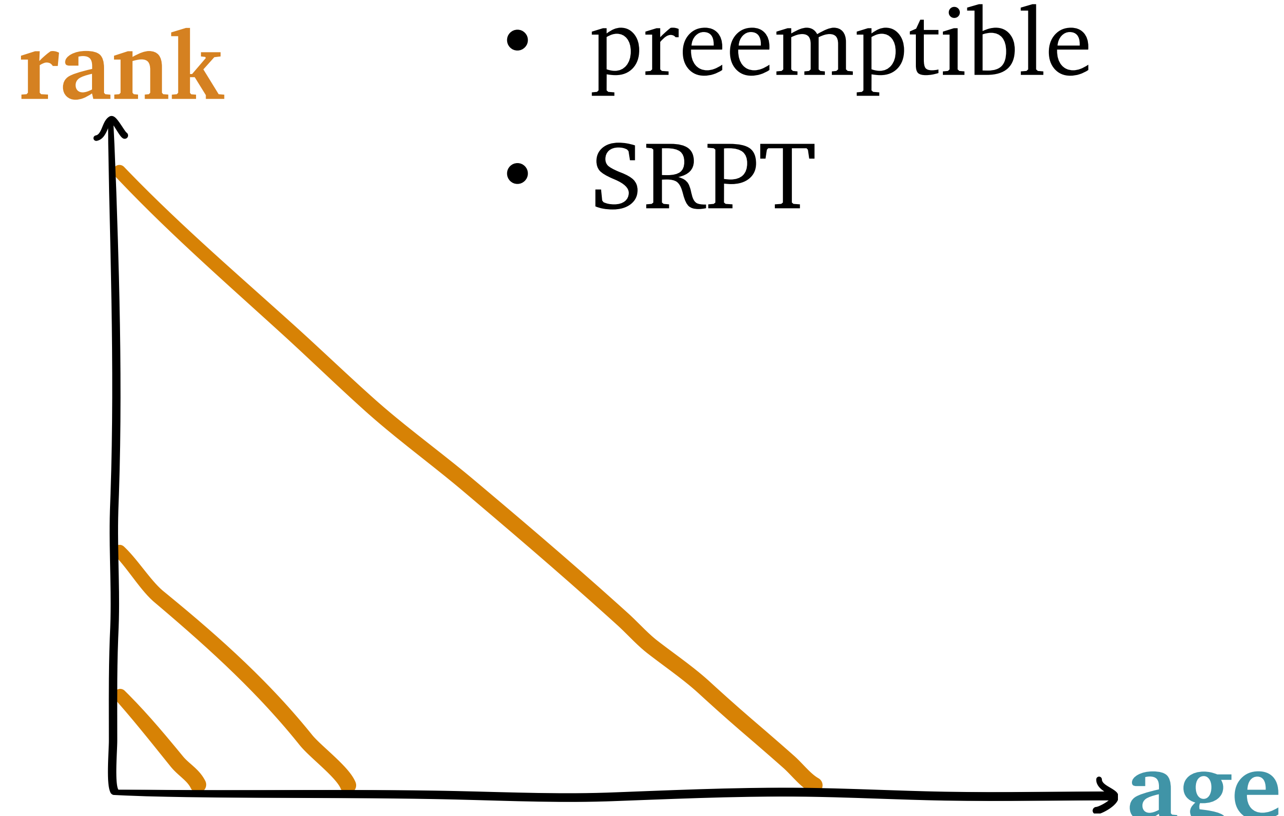
Humans

- unknown size
- nonpreemptible
- FCFS



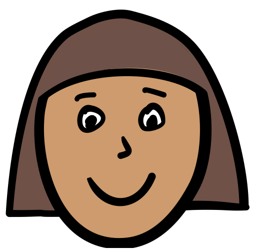
Robots

- known size
- preemptible
- SRPT



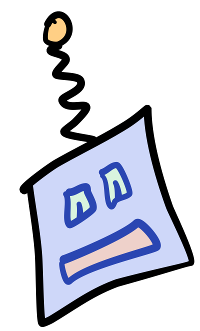
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



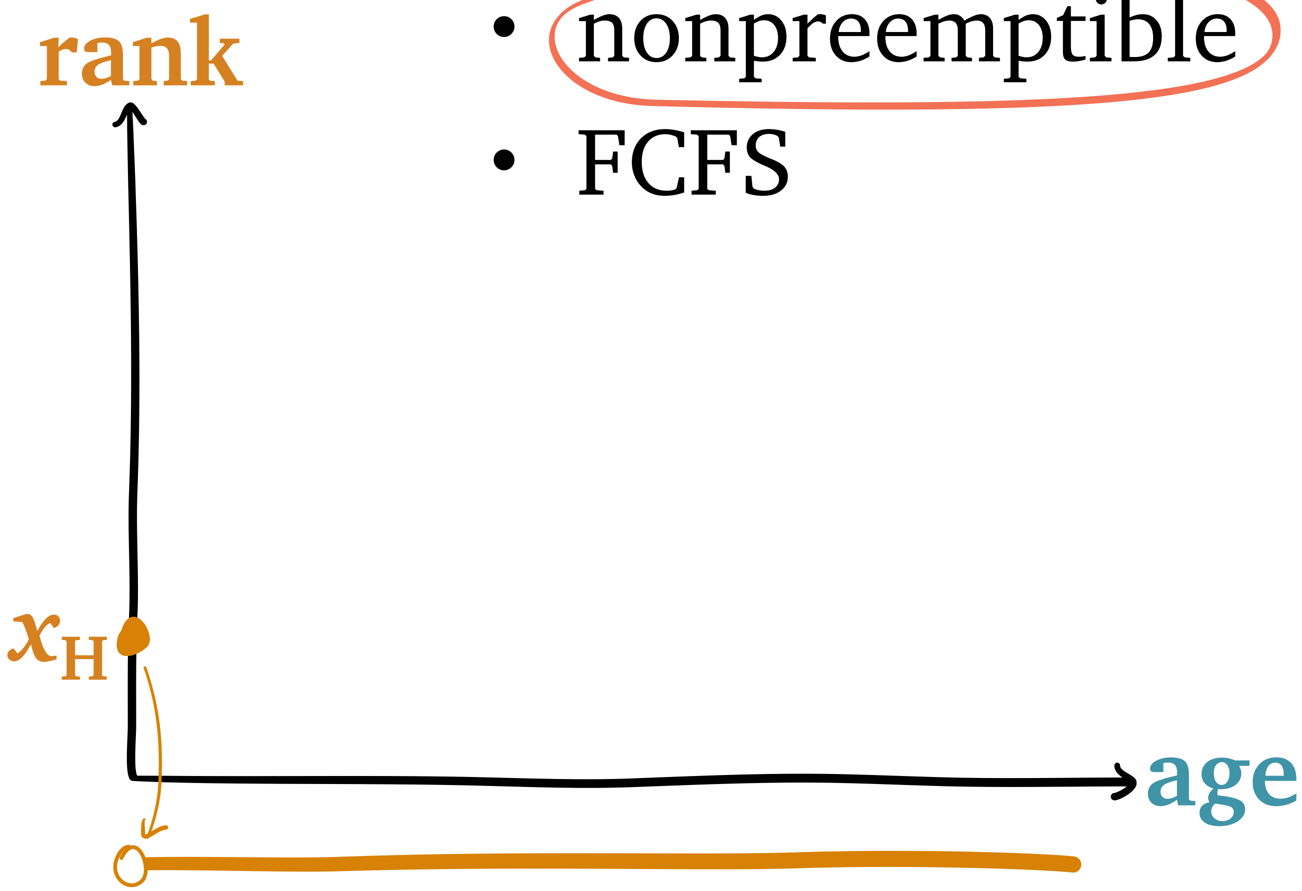
Humans

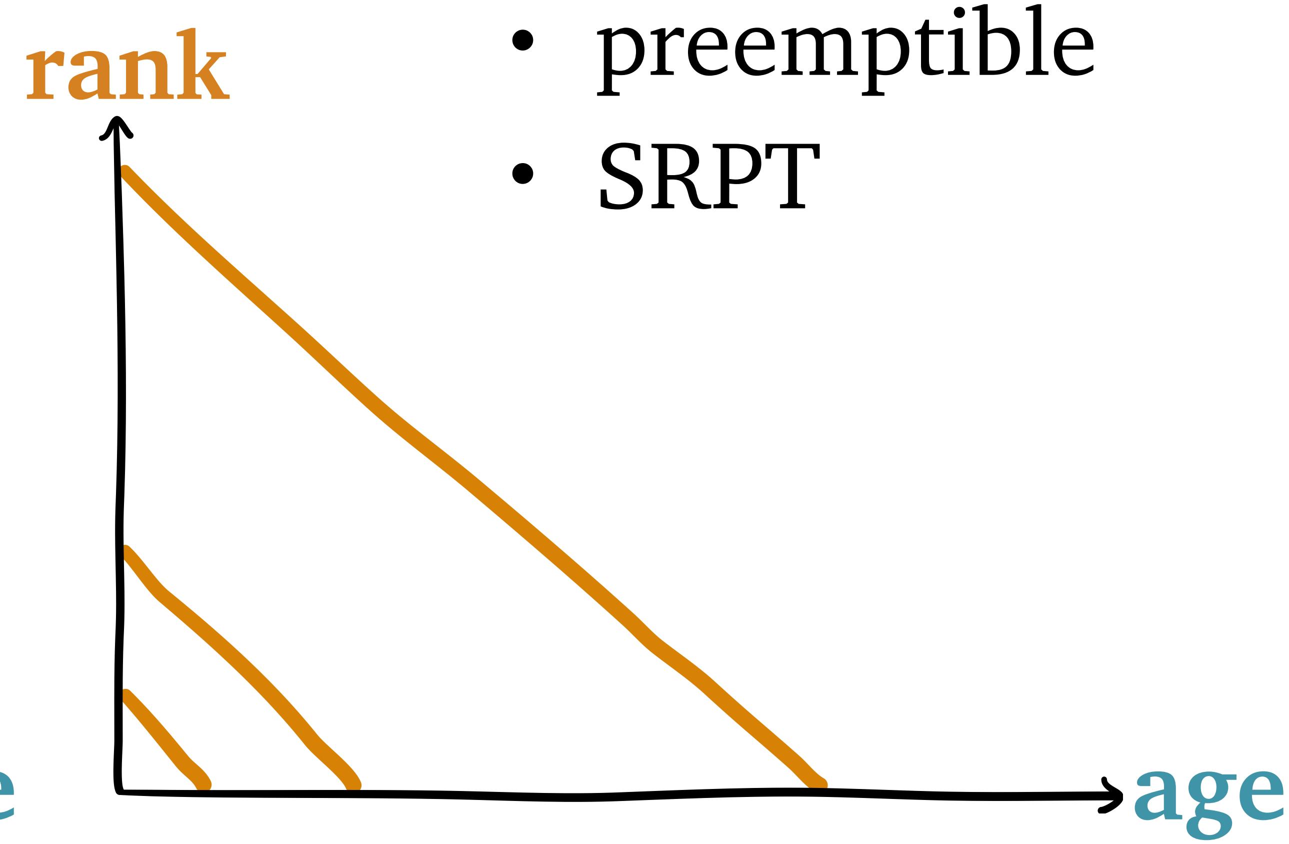
- unknown size
- nonpreemptible
- FCFS



Robots

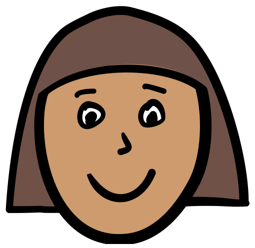
- known size
- preemptible
- SRPT





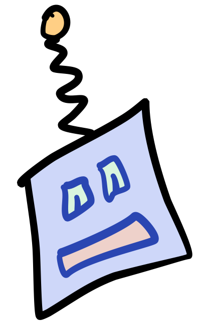
SOAP Policy: Mixture

Two customer classes: *humans* and *robots*



Humans

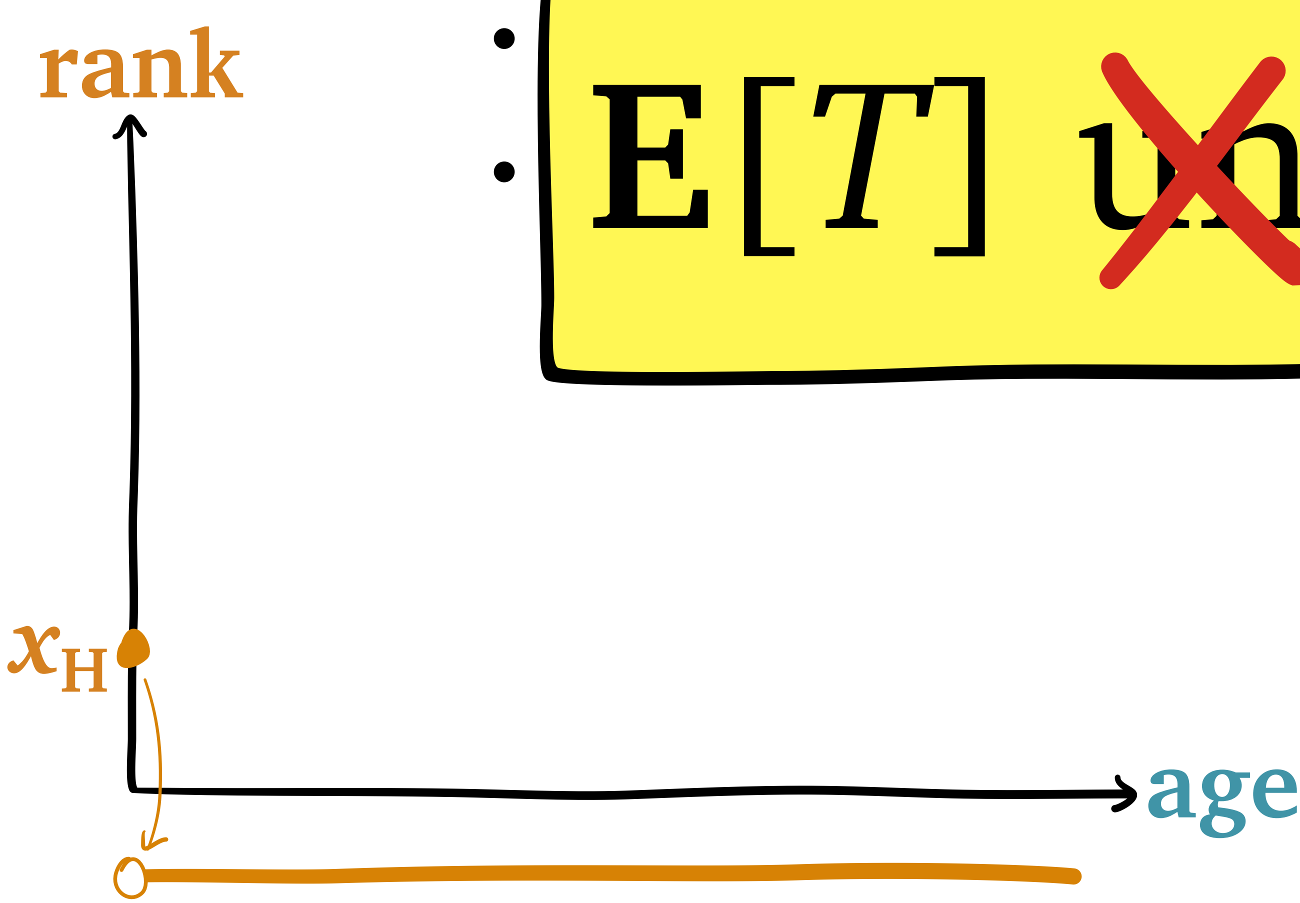
- unknown size
-
-

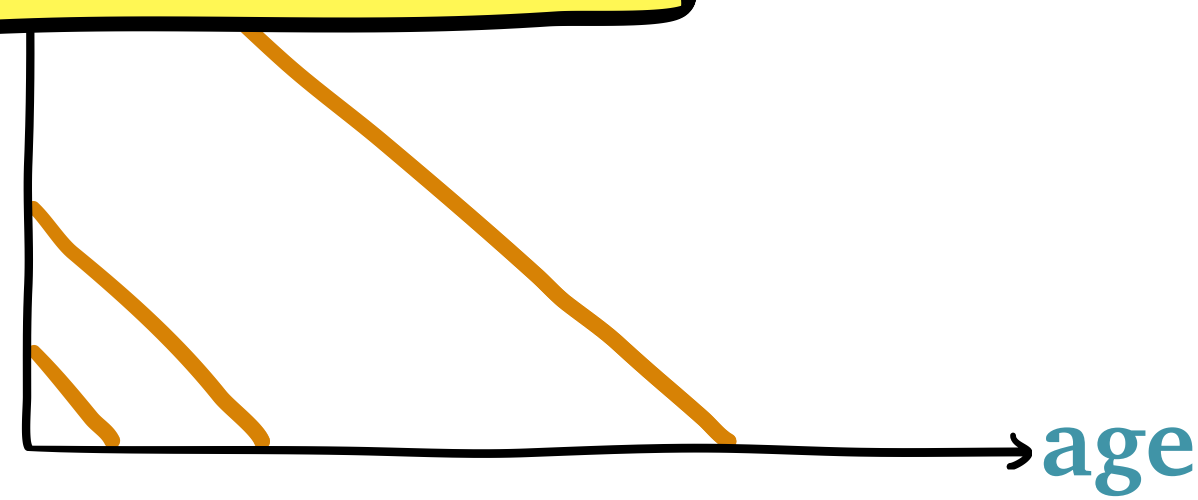


Robots

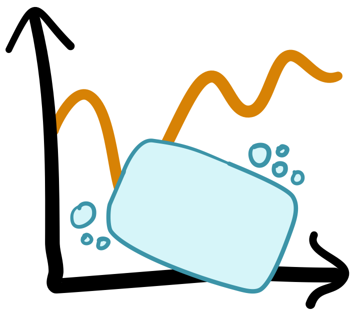
- known size
- mptible
- T

E[*T*] ~~un~~known!

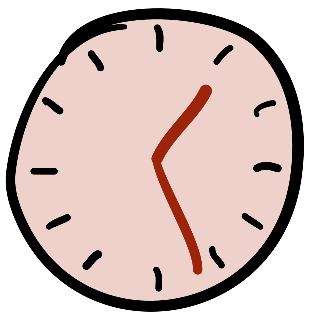




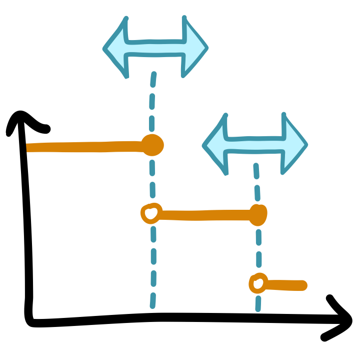
Outline



Part 1: *defining* **SOAP** policies



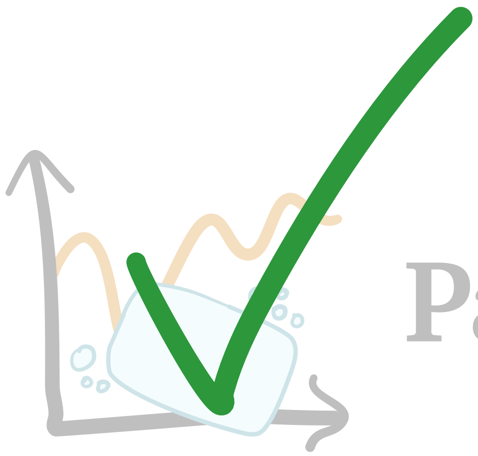
Part 2: *analyzing* **SOAP** policies



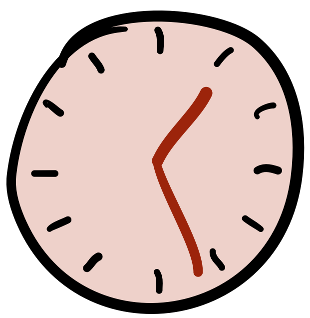
Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

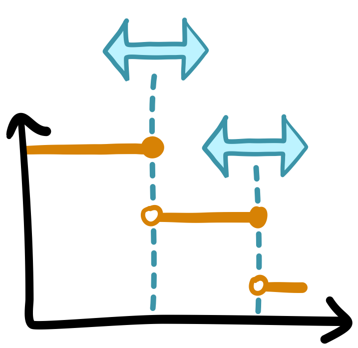
Outline



Part 1: *defining* SOAP policies

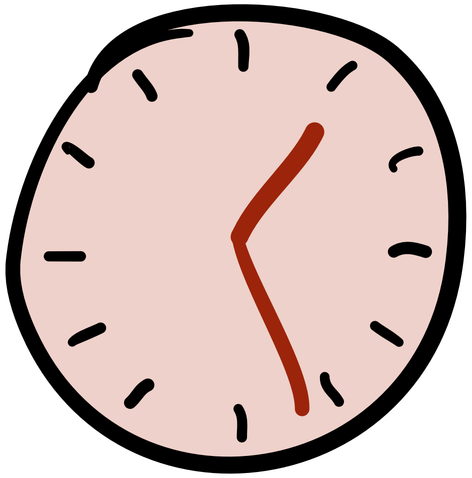


Part 2: *analyzing* **SOAP** policies



Part 3: *policy design* with **SOAP**

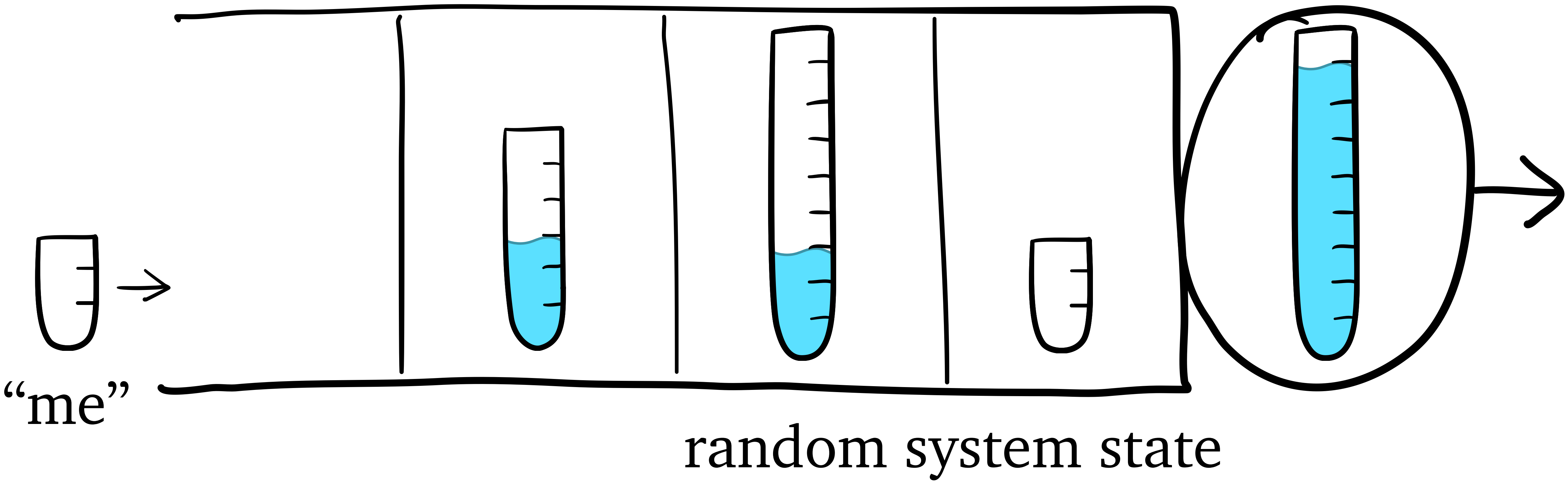
Part 4: *optimality proofs* with **SOAP**



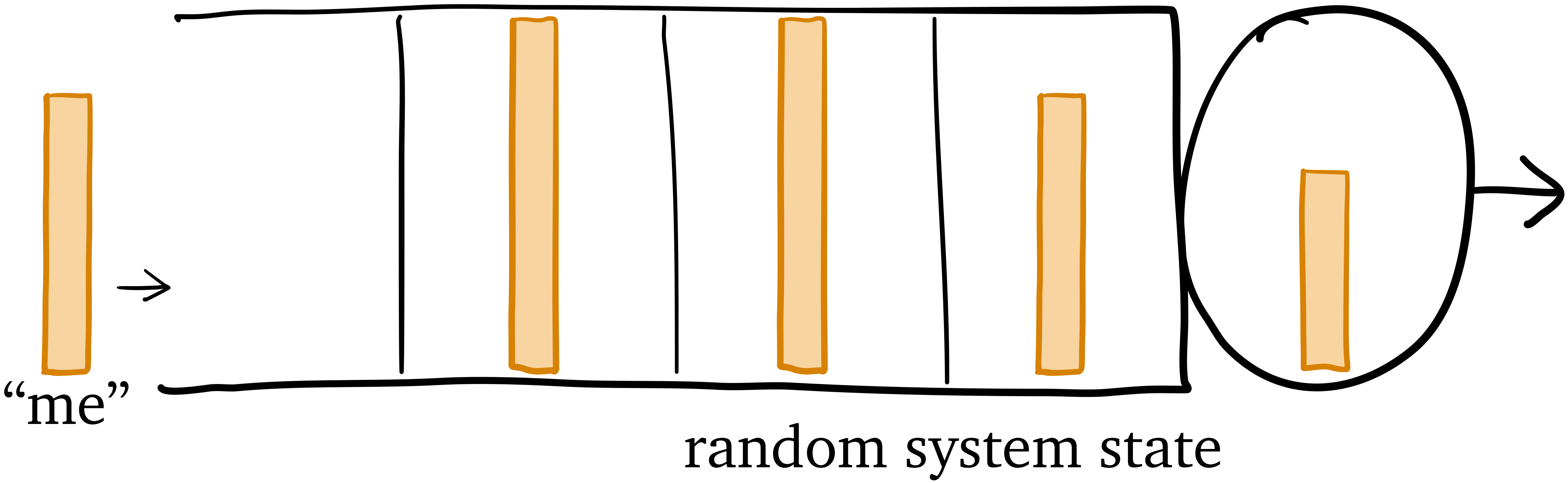
Part 2:

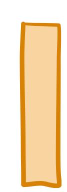
analyzing **SOAP** policies

Tagged Job Analysis

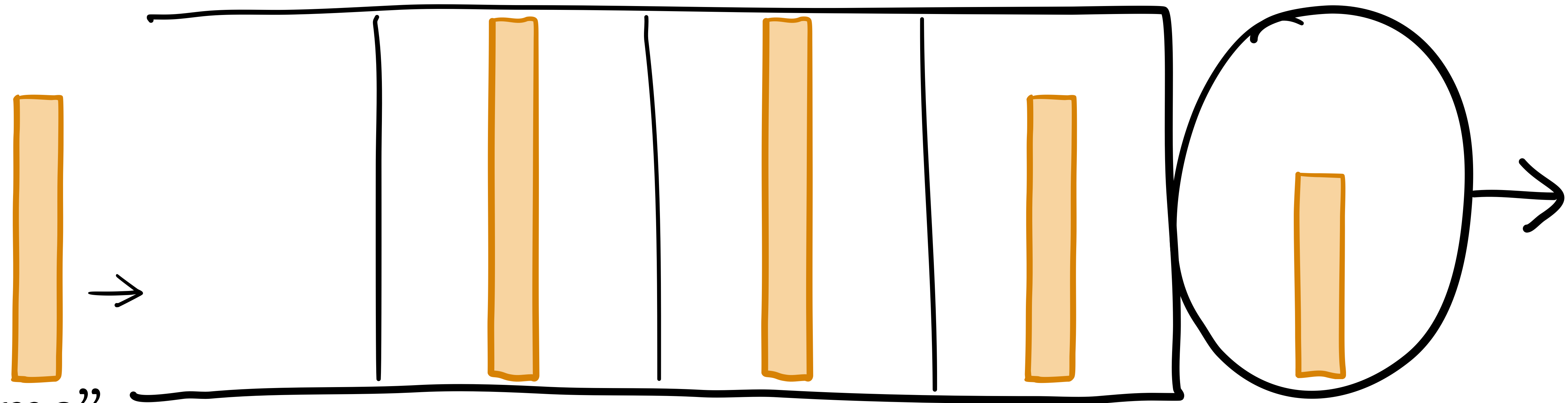


Tagged Job Analysis



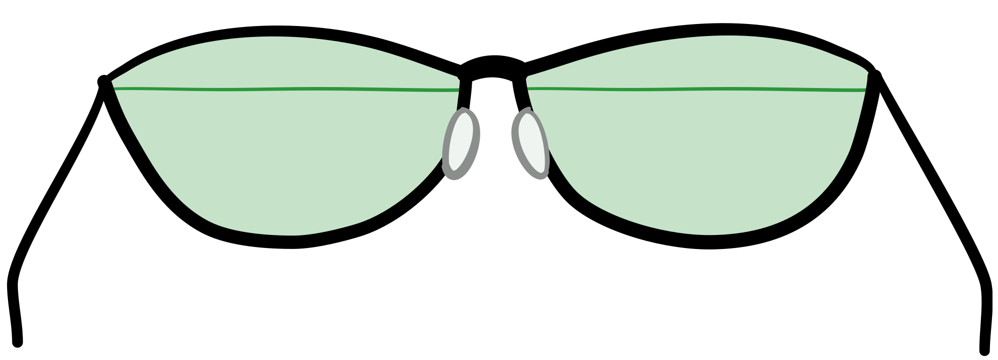
 = **rank**

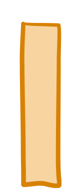
Tagged Job Analysis



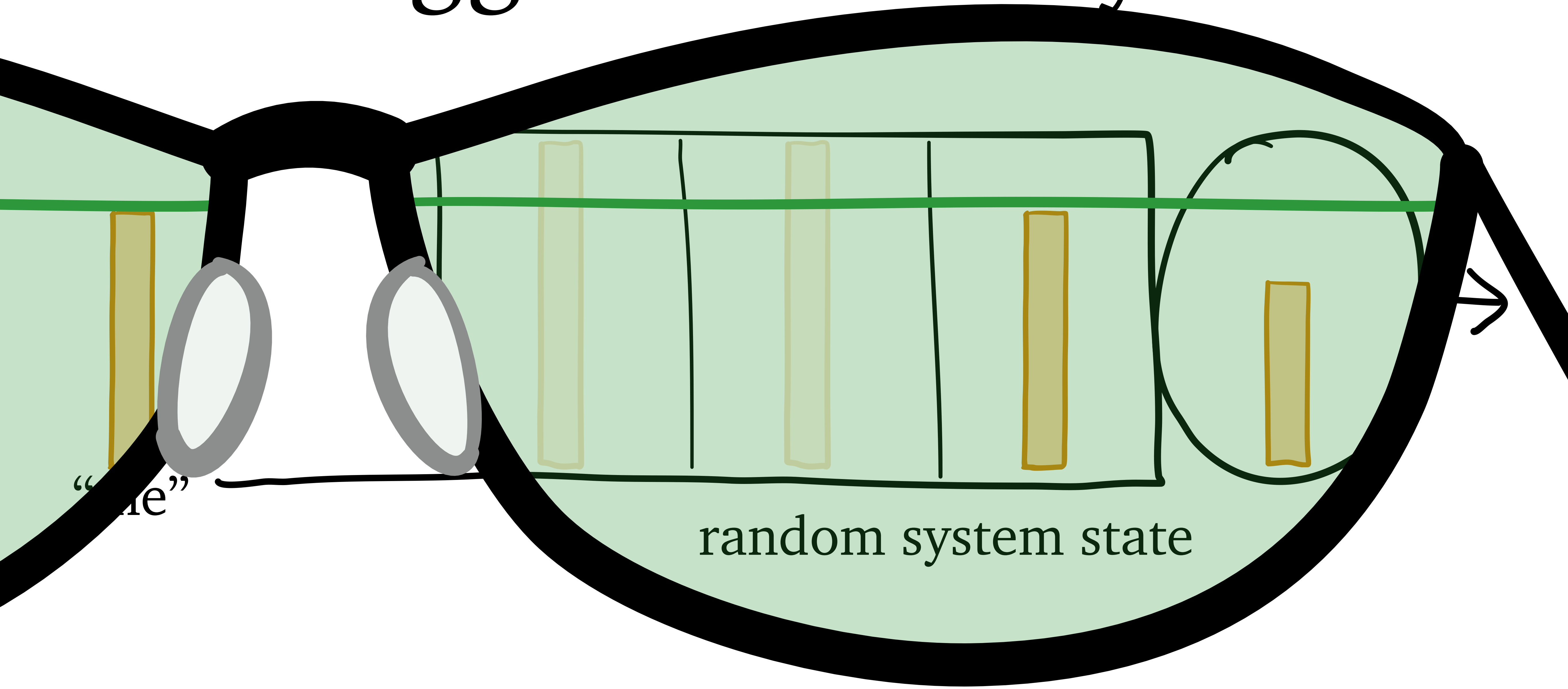
"me"

random system state



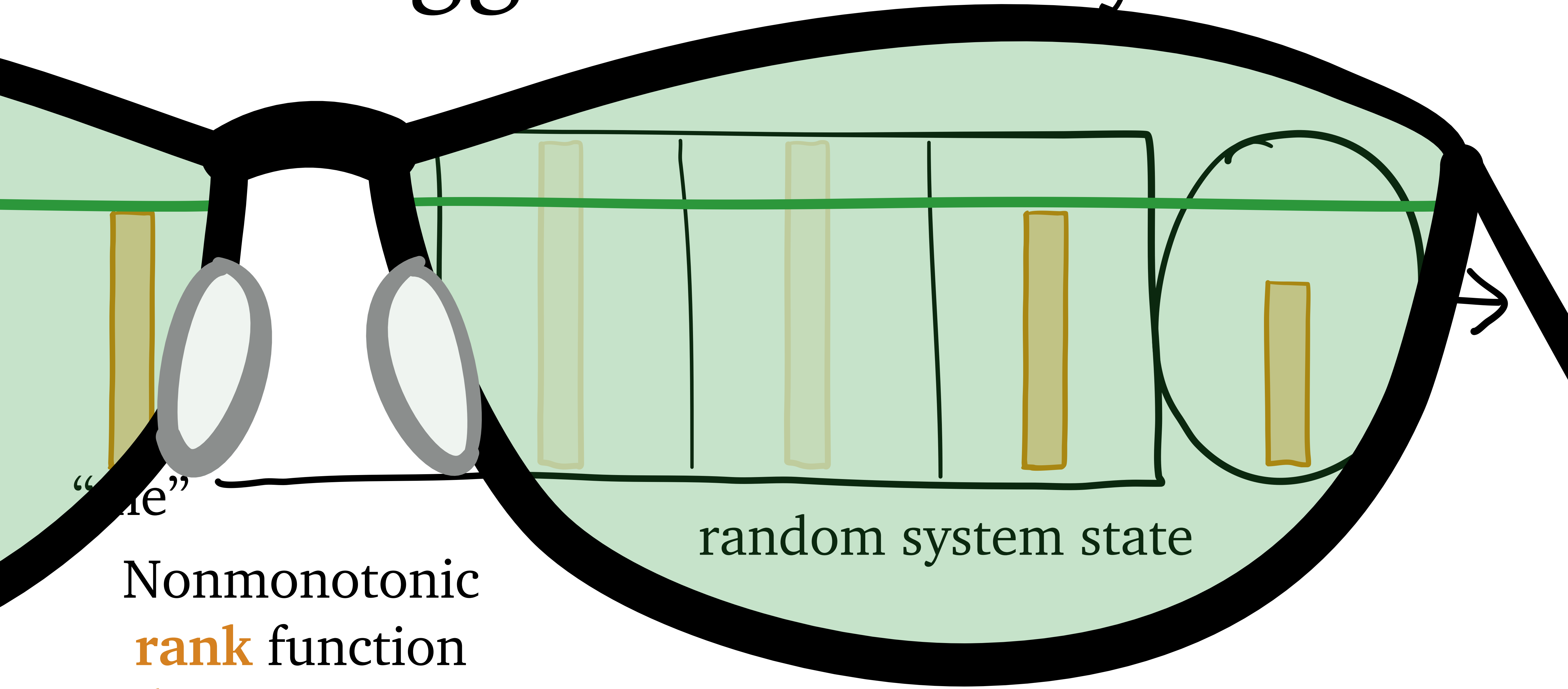
 = **rank**

Tagged Job Analysis

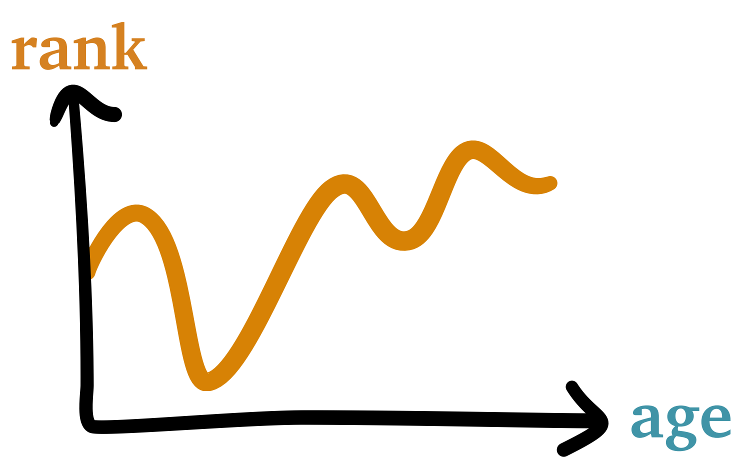


= **rank**

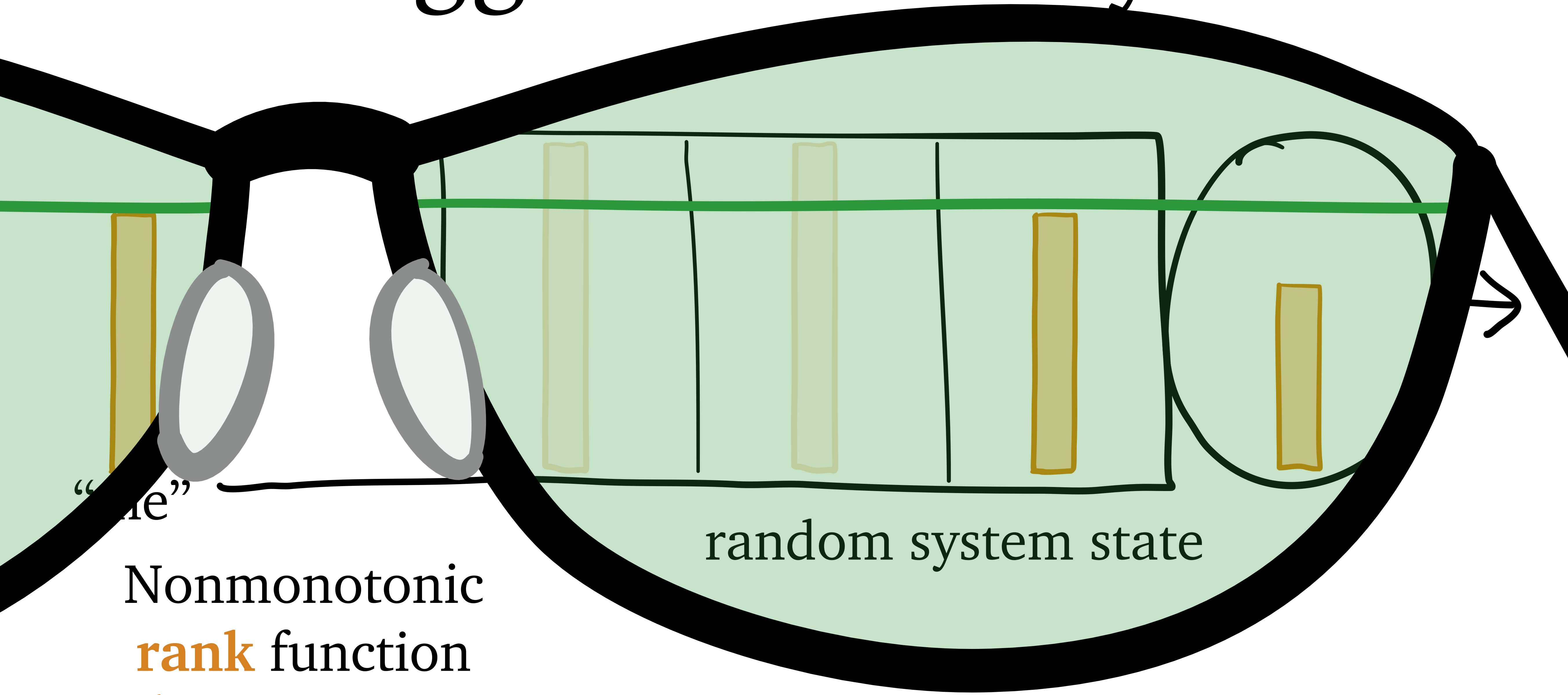
Tagged Job Analysis



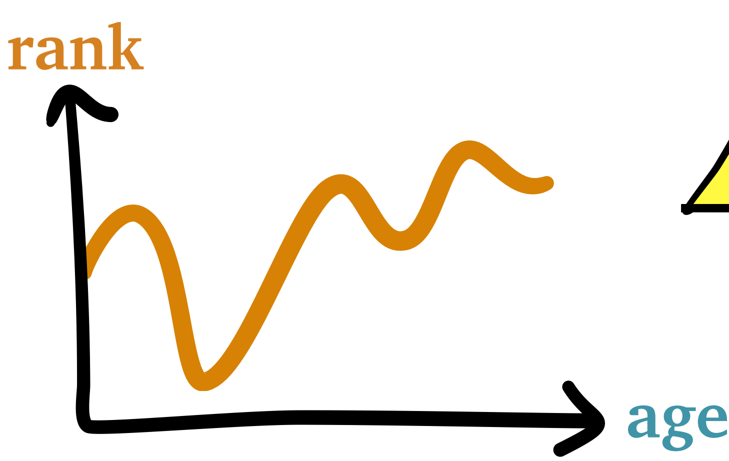
Nonmonotonic
rank function



Tagged Job Analysis

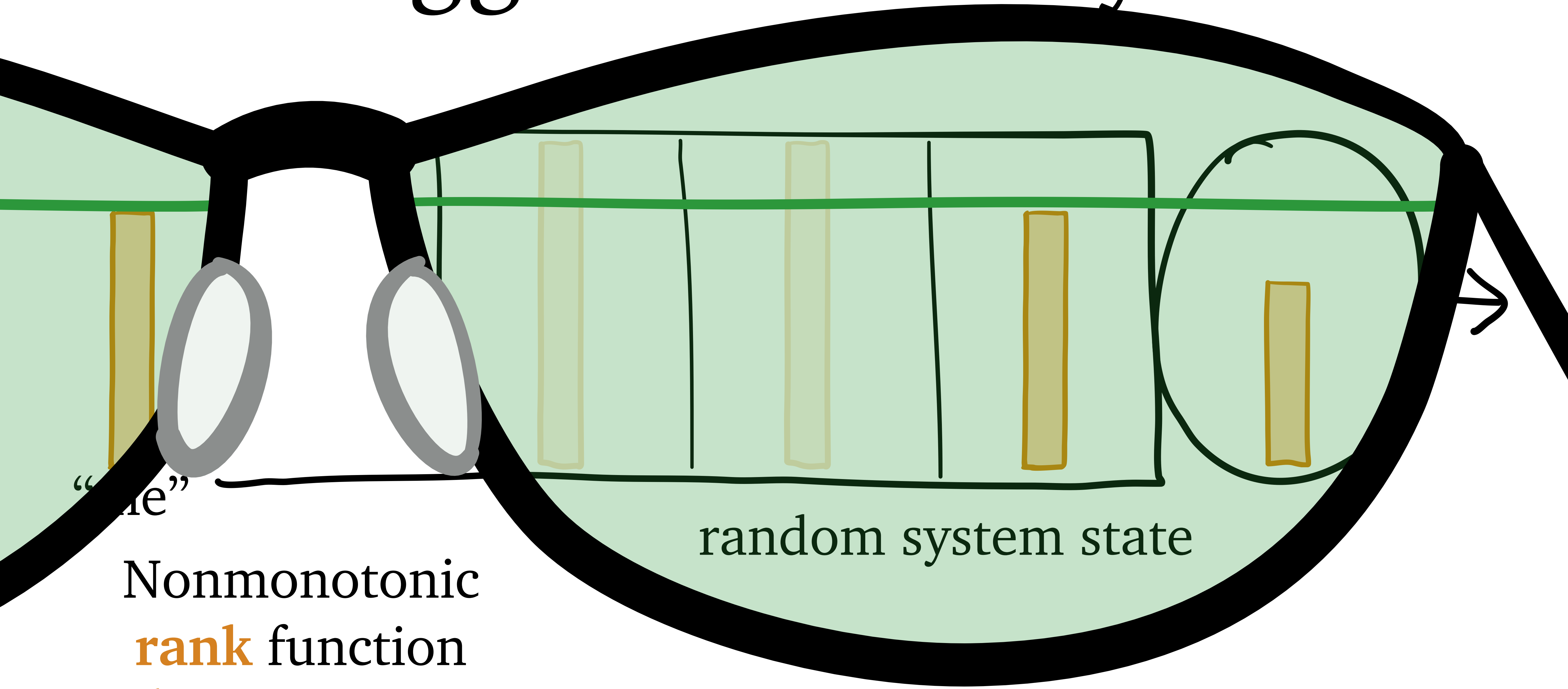


Nonmonotonic **rank** function

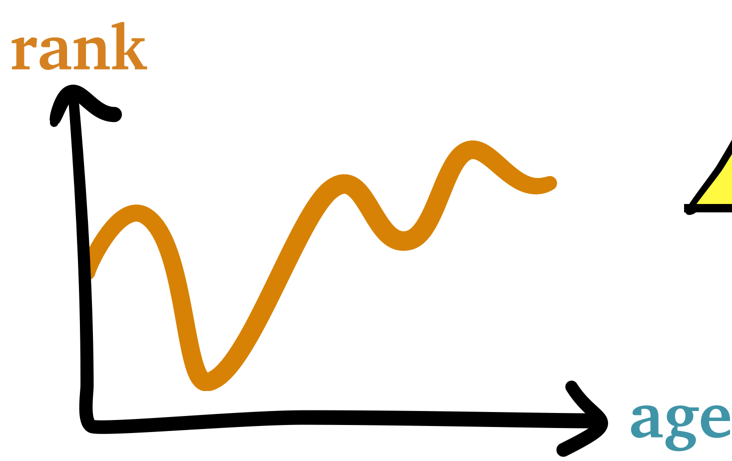


Two obstacles:

Tagged Job Analysis



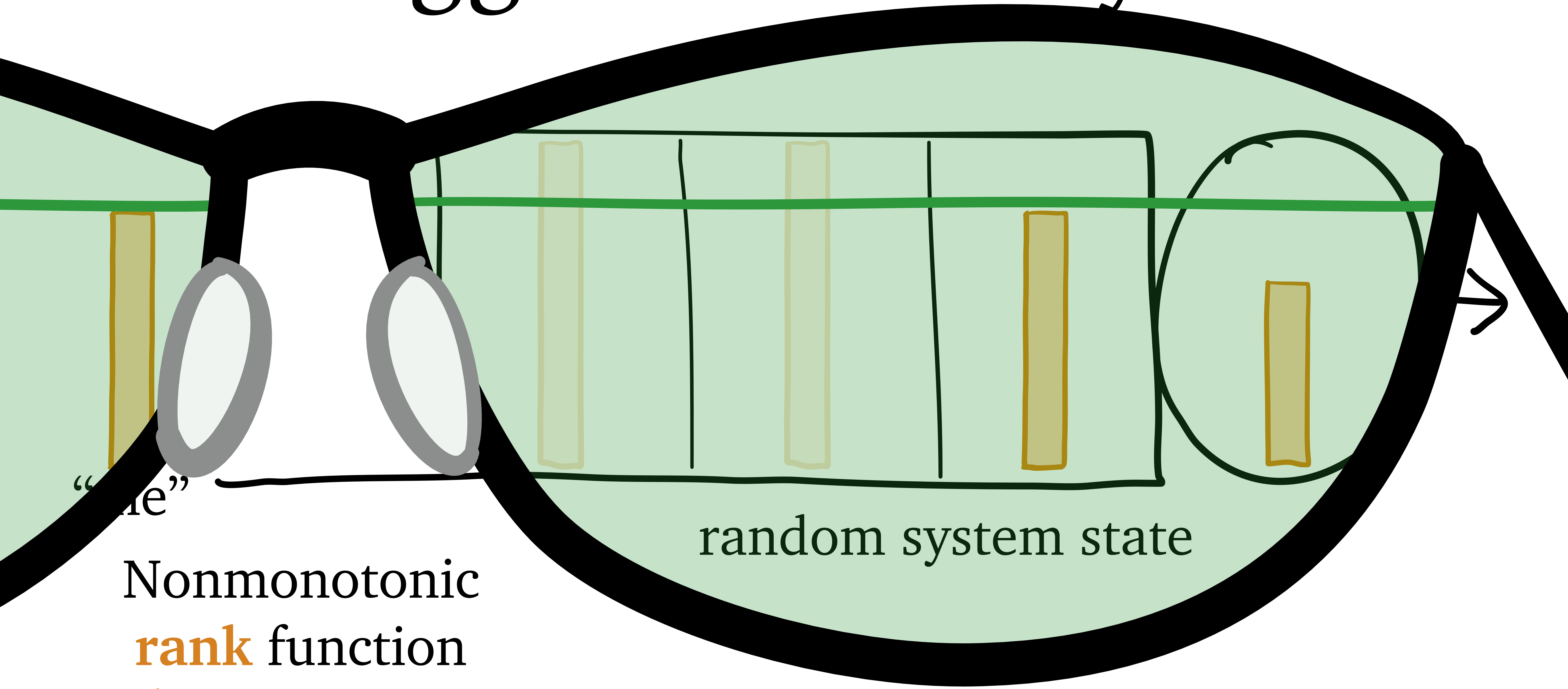
Nonmonotonic **rank** function



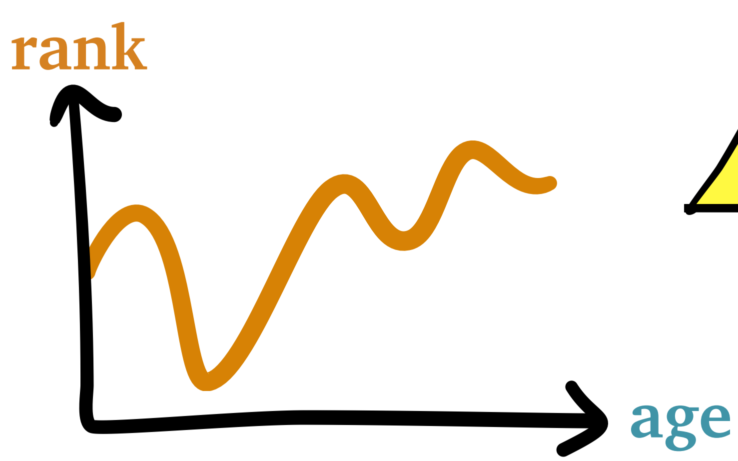
Two obstacles:

- *My rank* goes up and down

Tagged Job Analysis



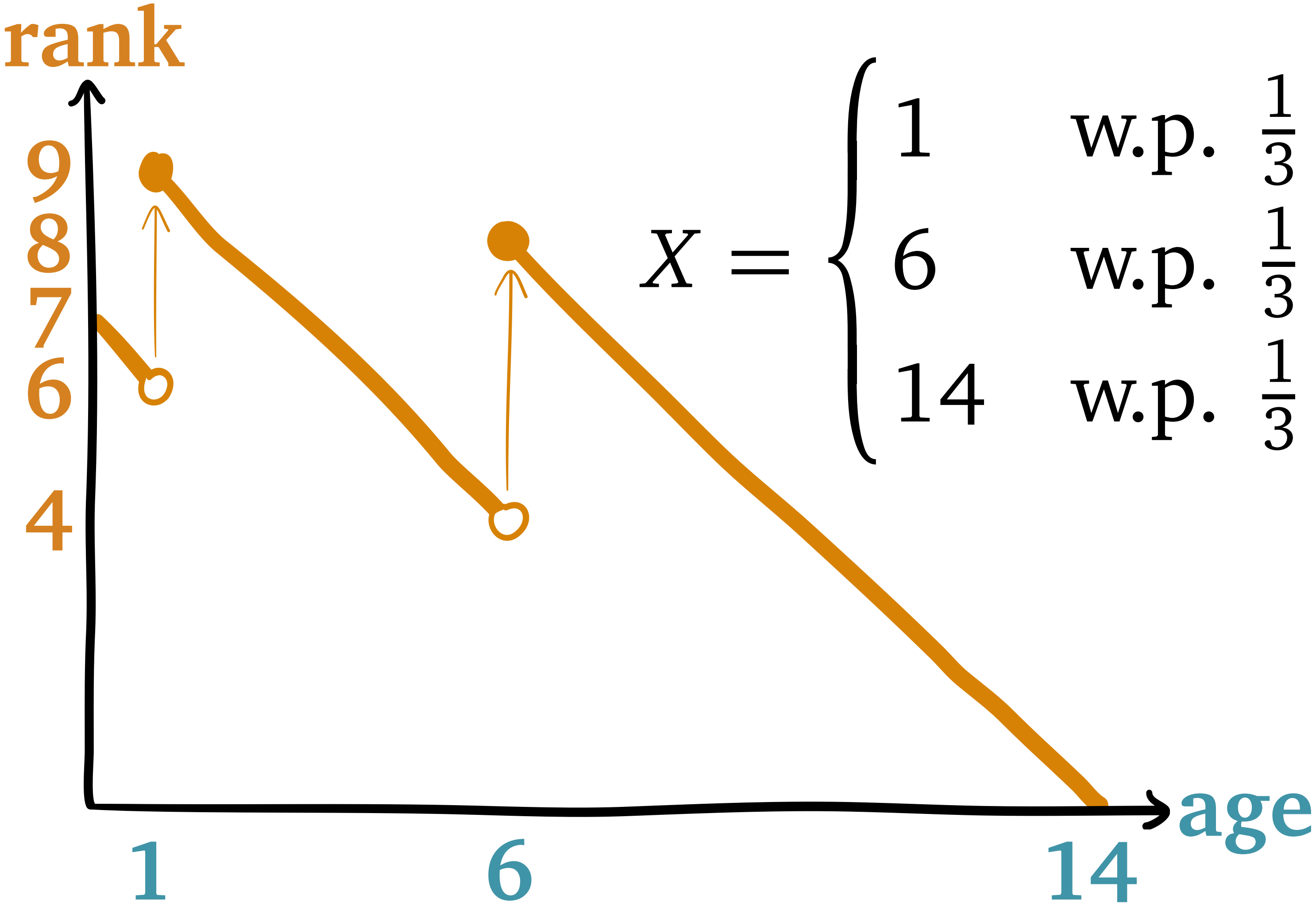
Nonmonotonic **rank** function



Two obstacles:

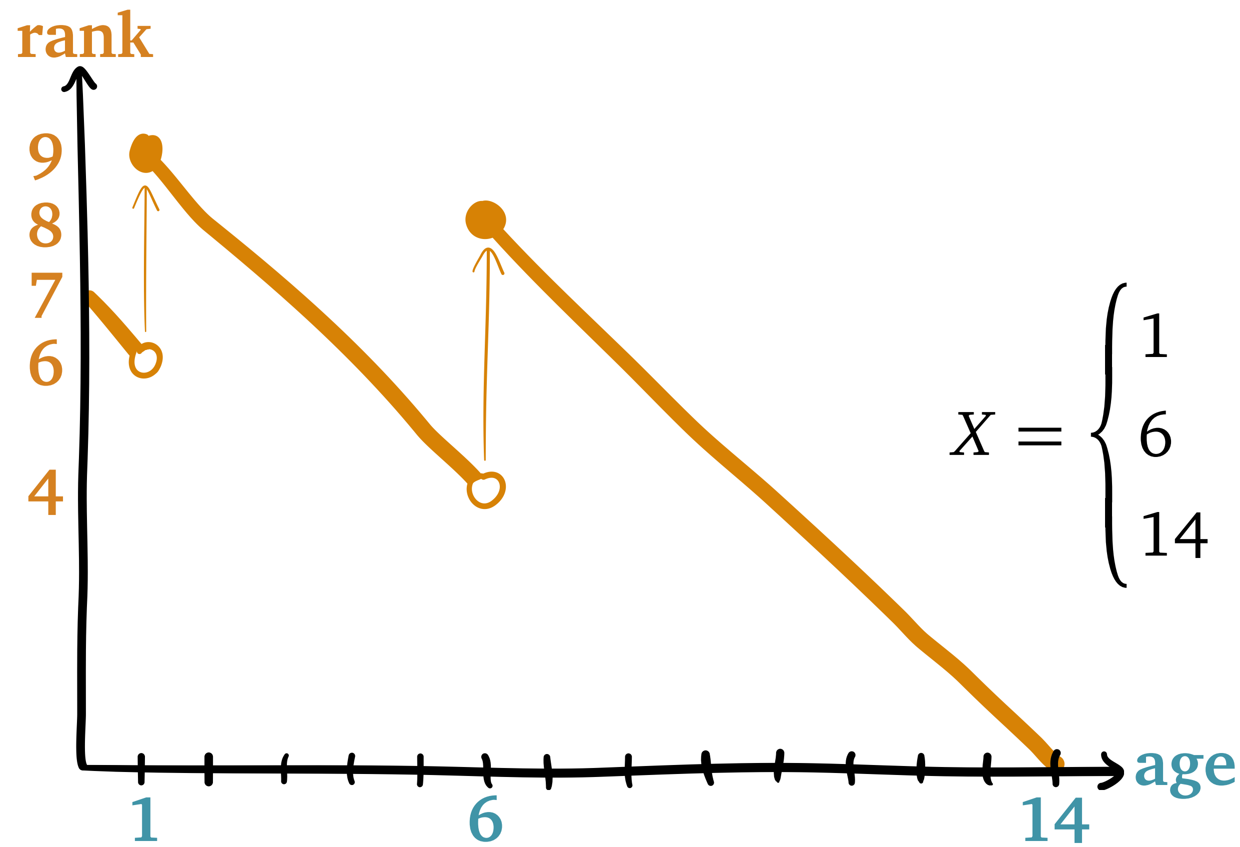
- *My rank* goes up and down
- *Others' ranks* go up and down too

Running example:
SERPT



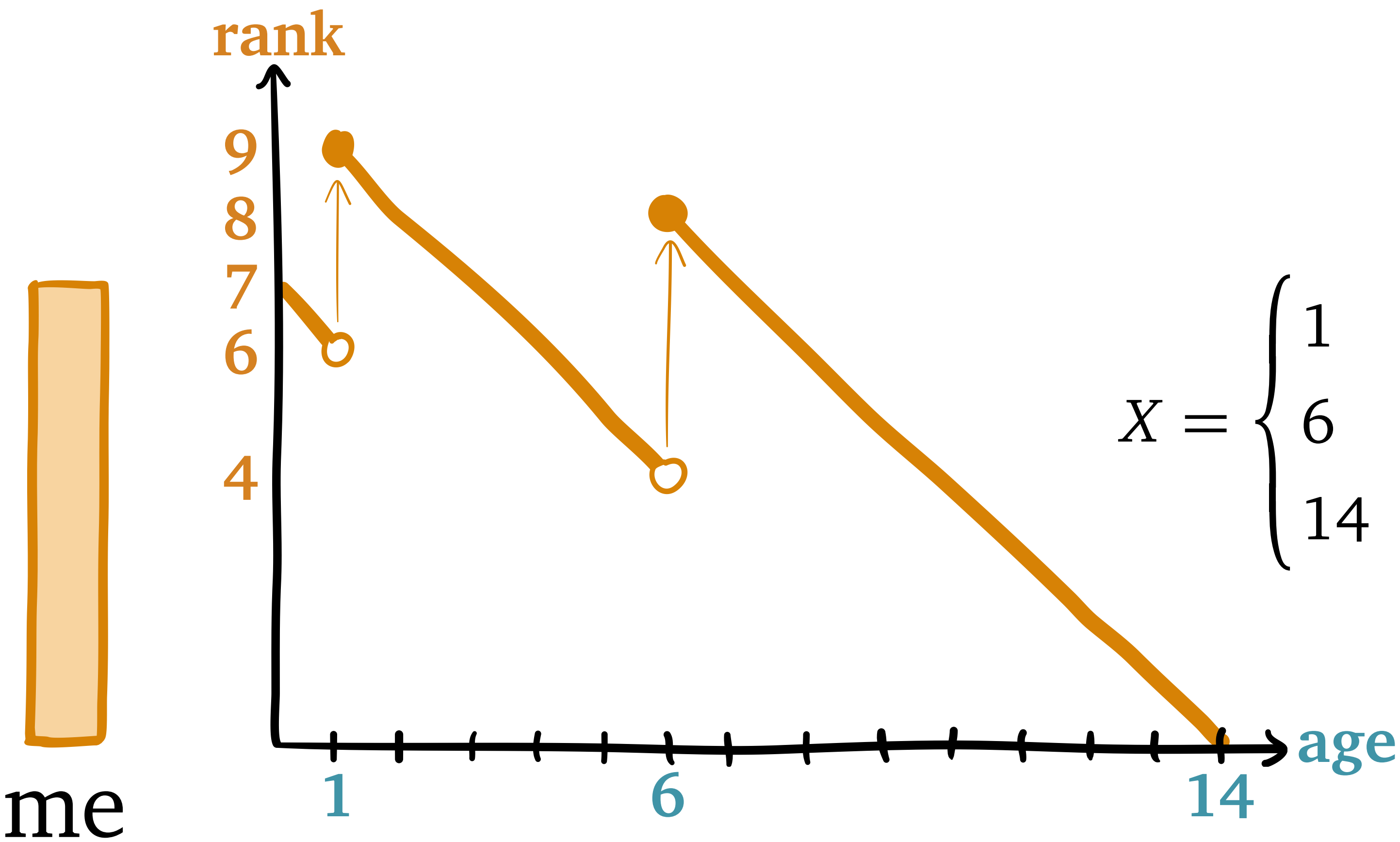
$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

Warmup: Empty System



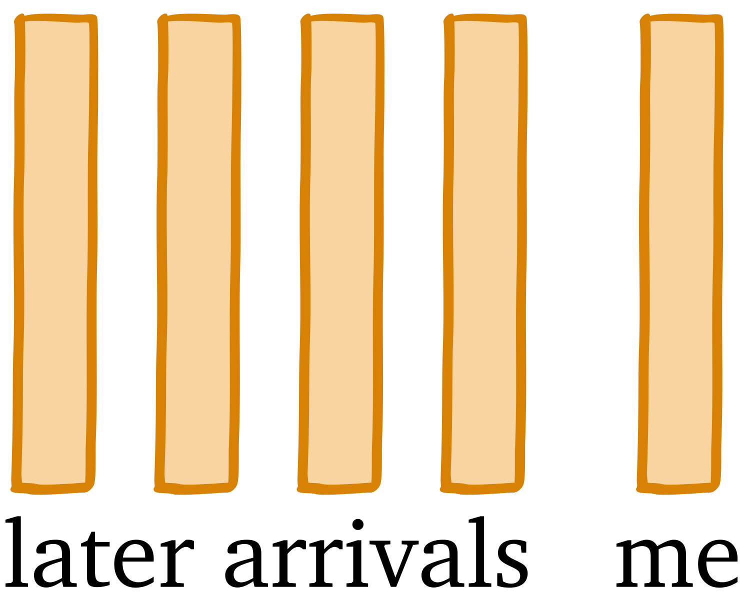
$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

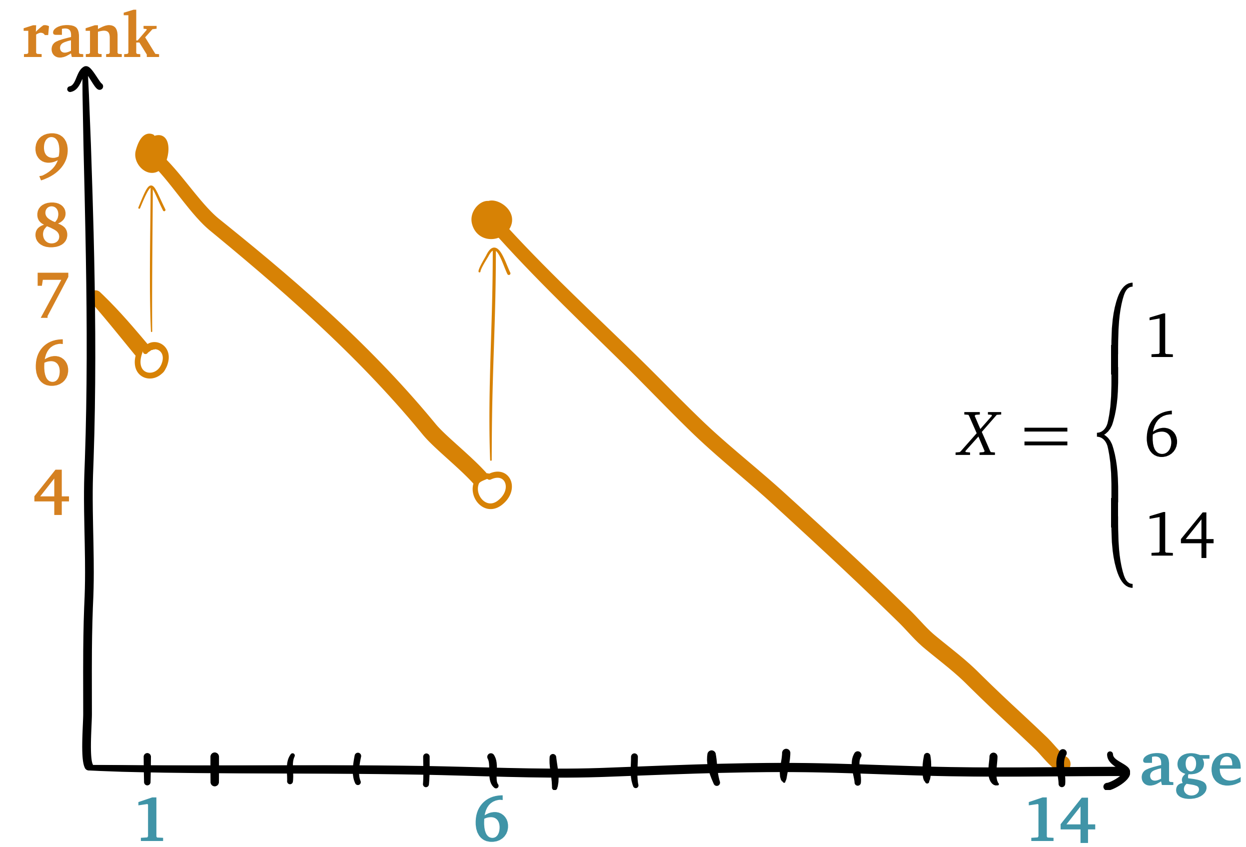
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

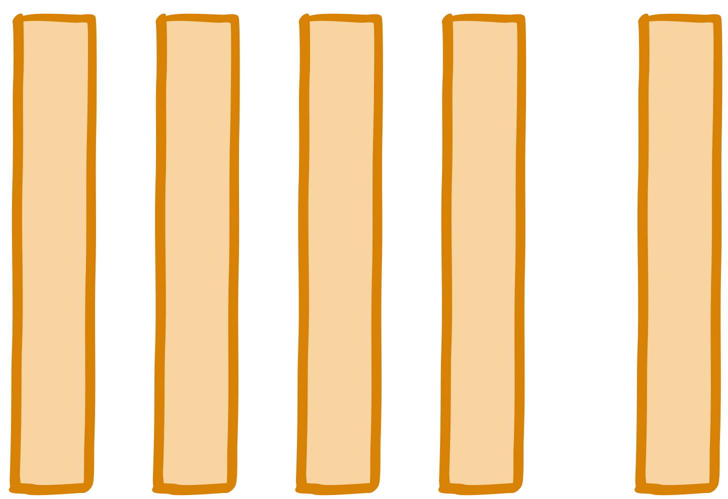
Warmup: Empty System

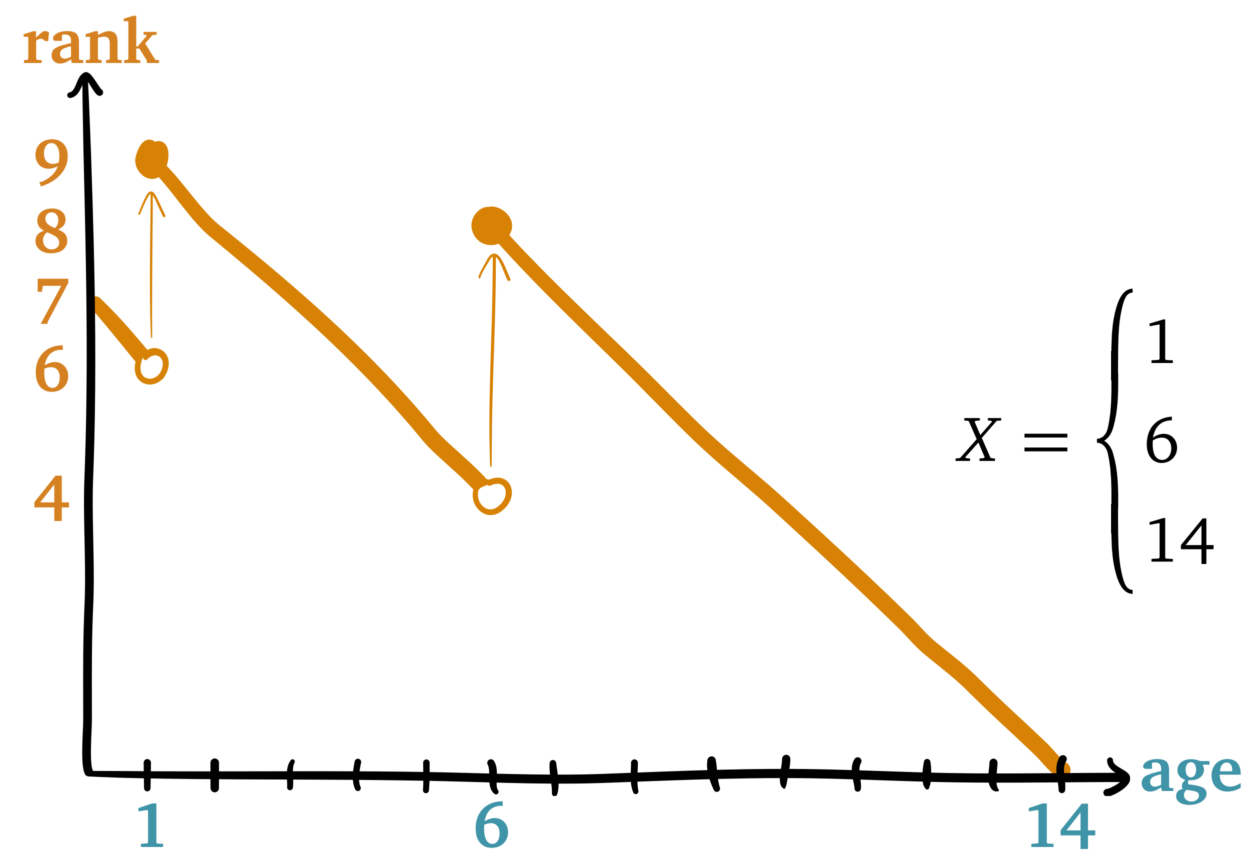




$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

Warmup: Empty System

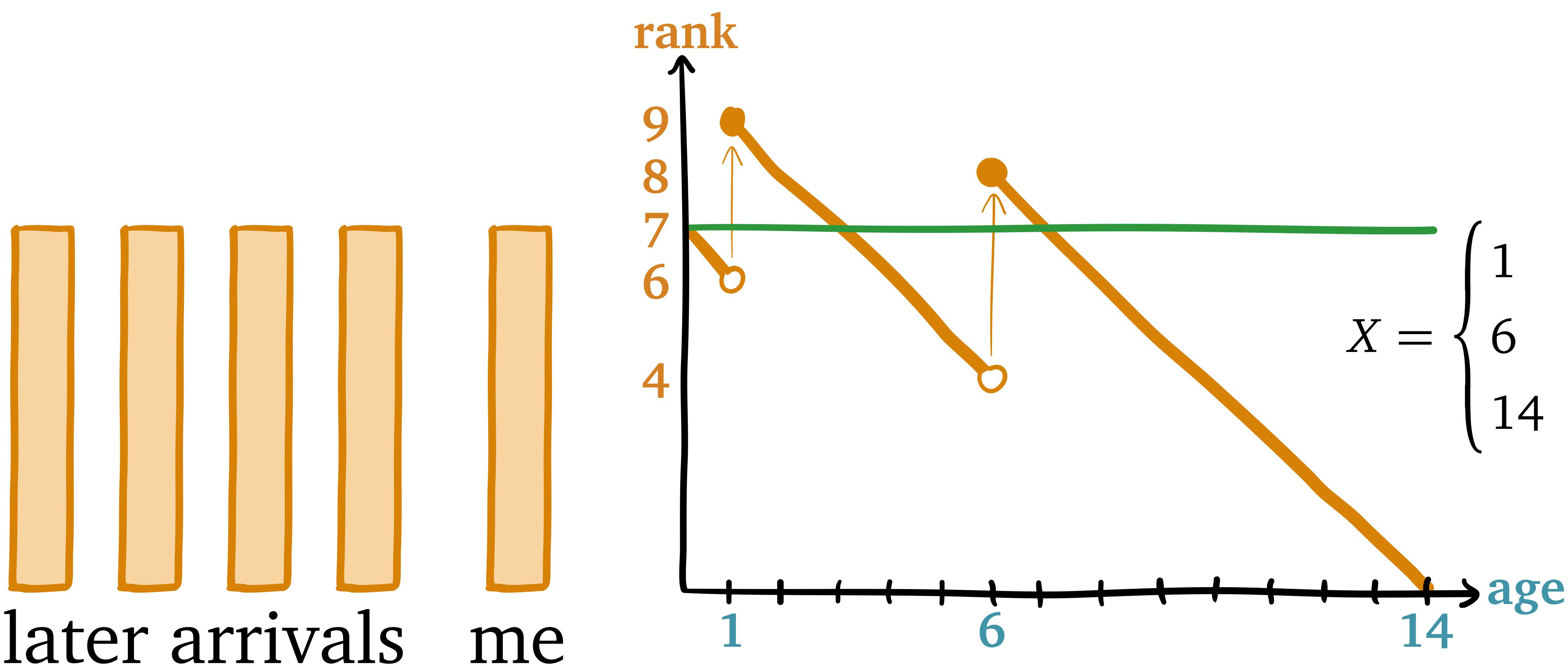




$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1		
6		
14		

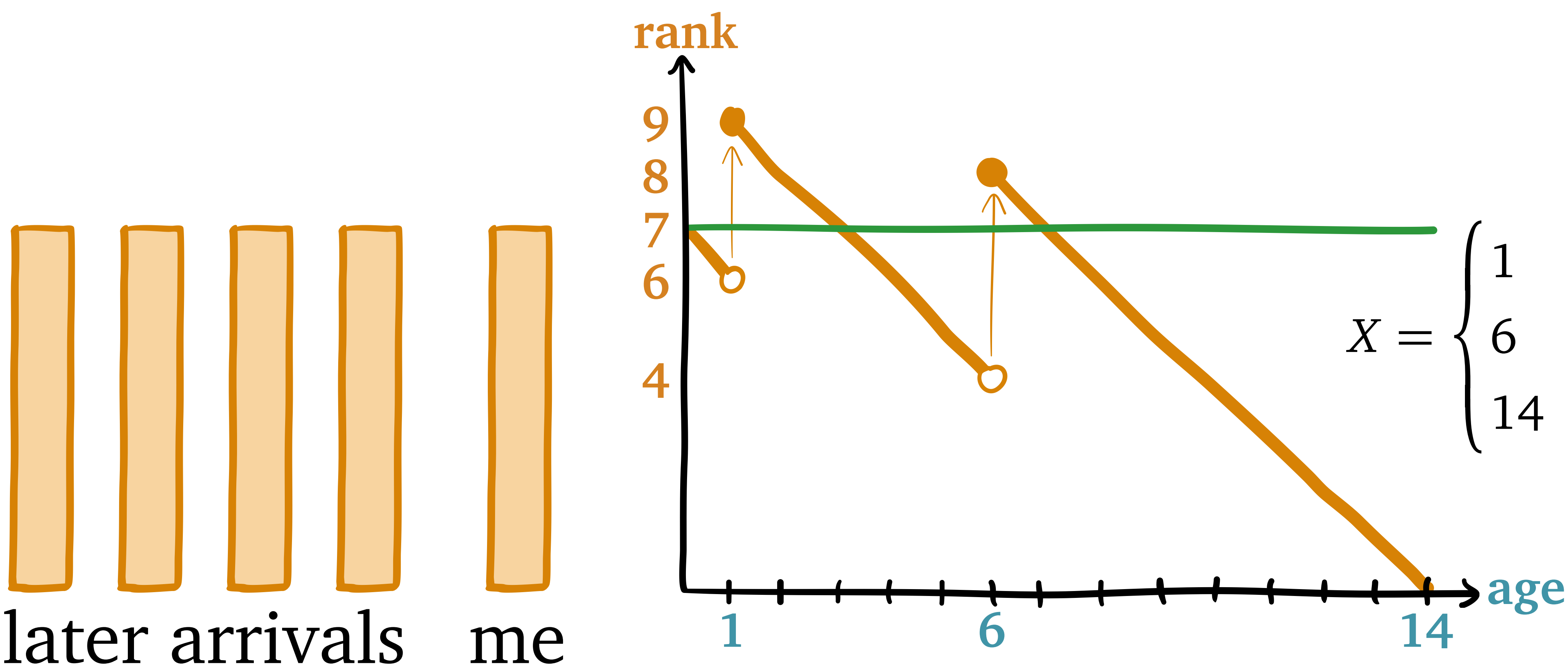
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1		
6		
14		

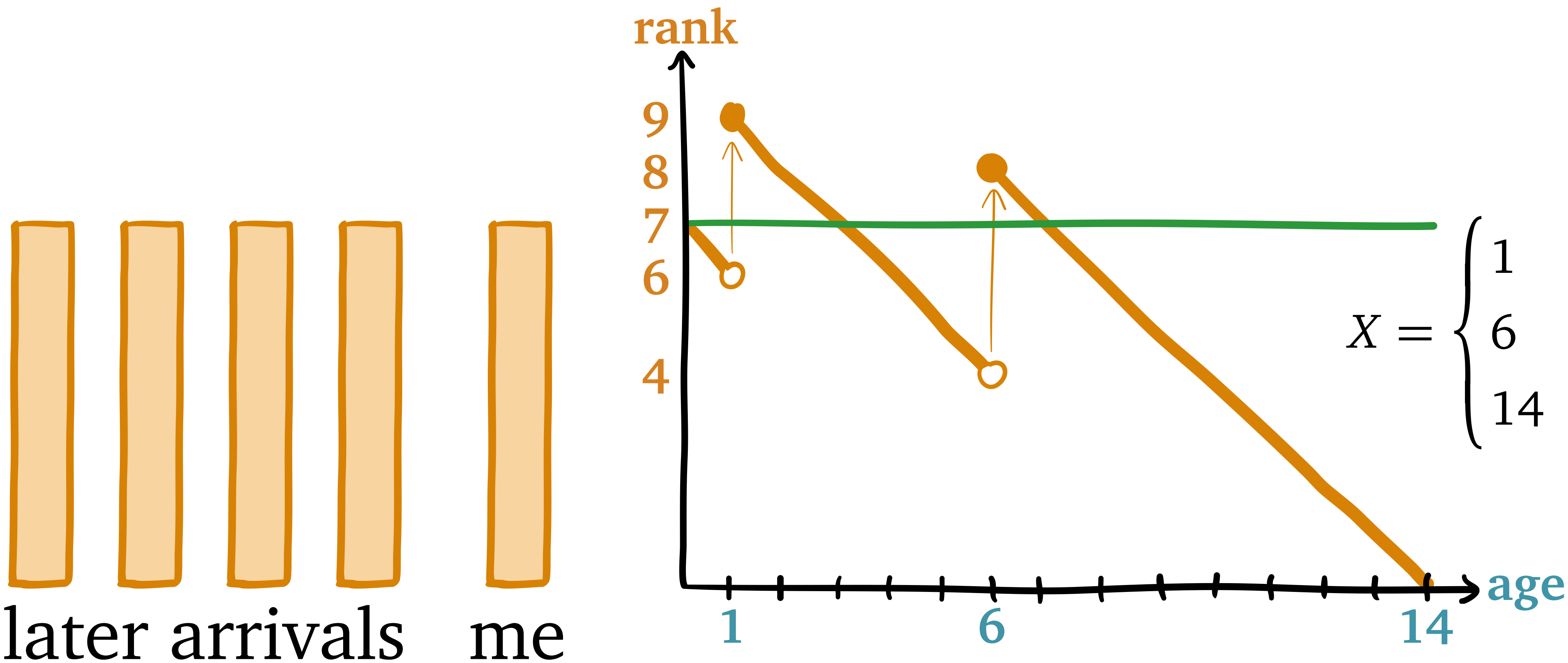
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	
6		
14		

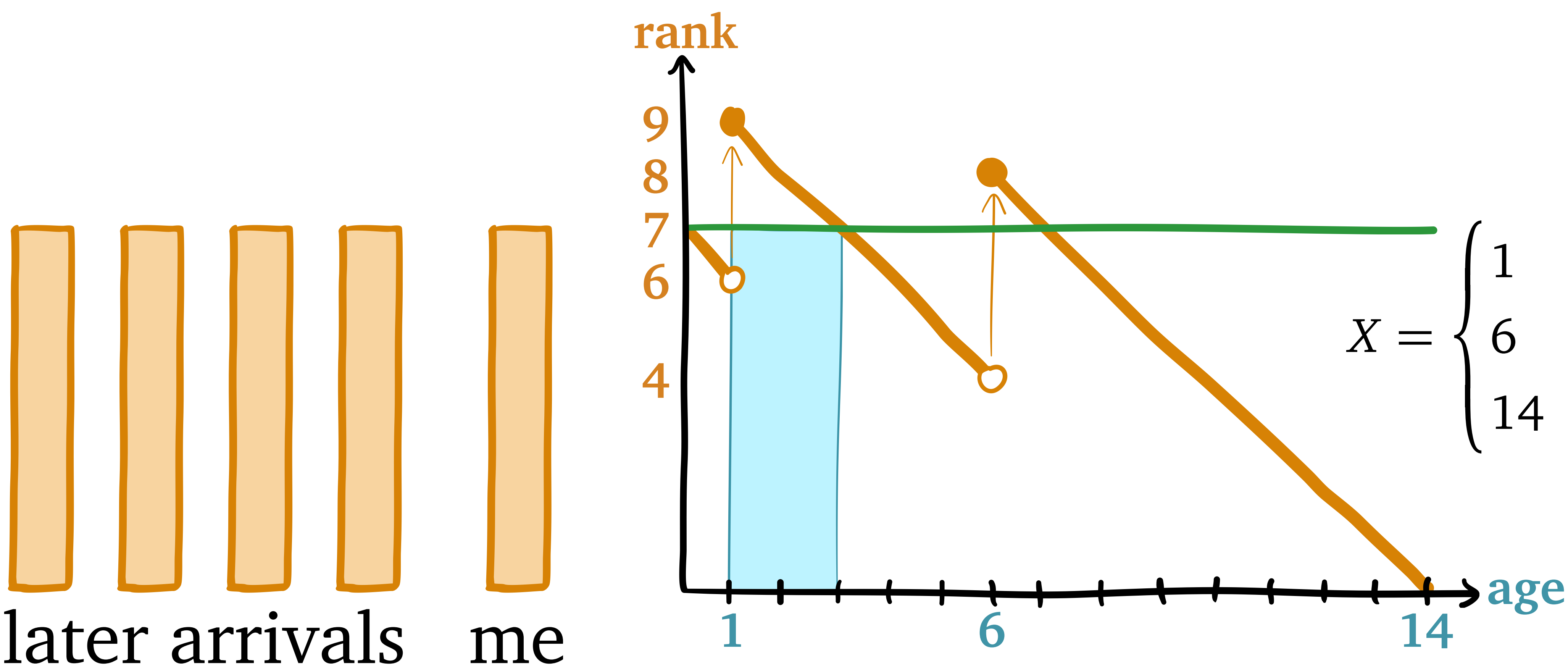
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6		
14		

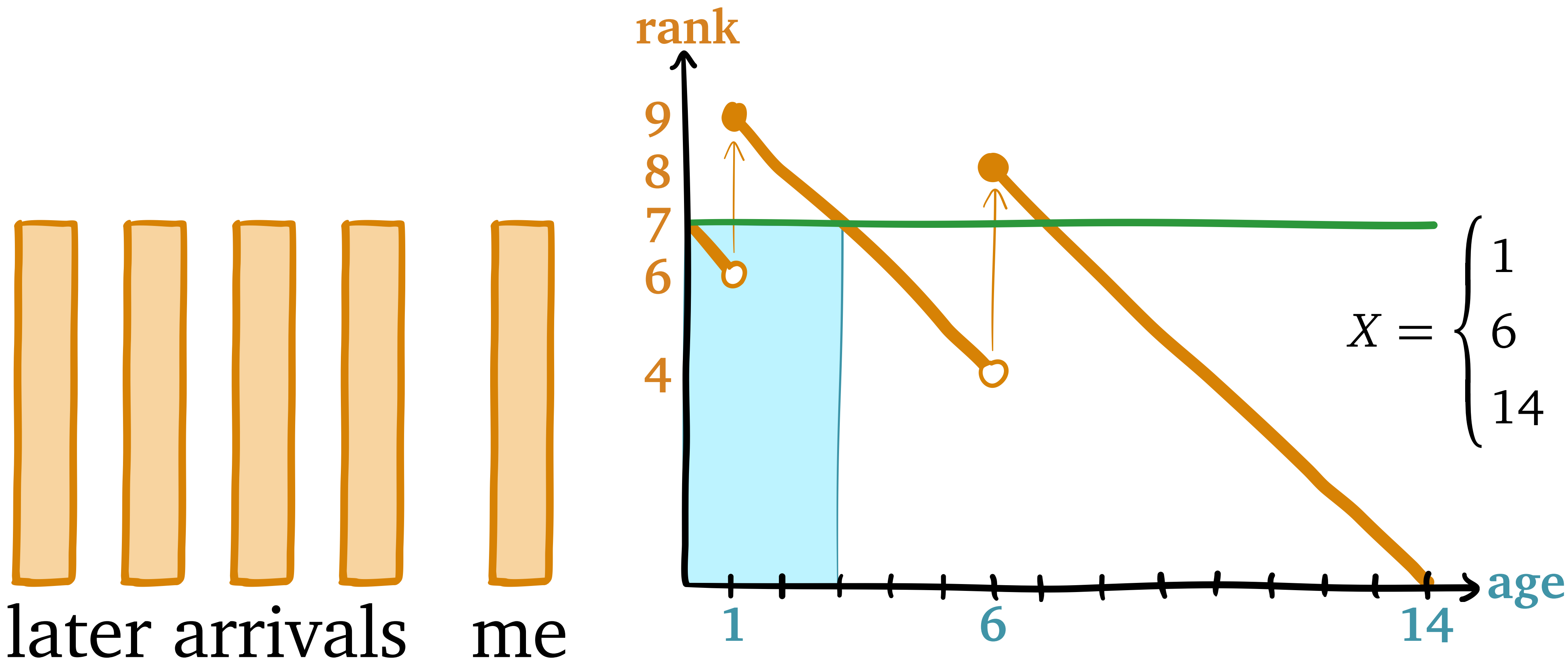
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6		
14		

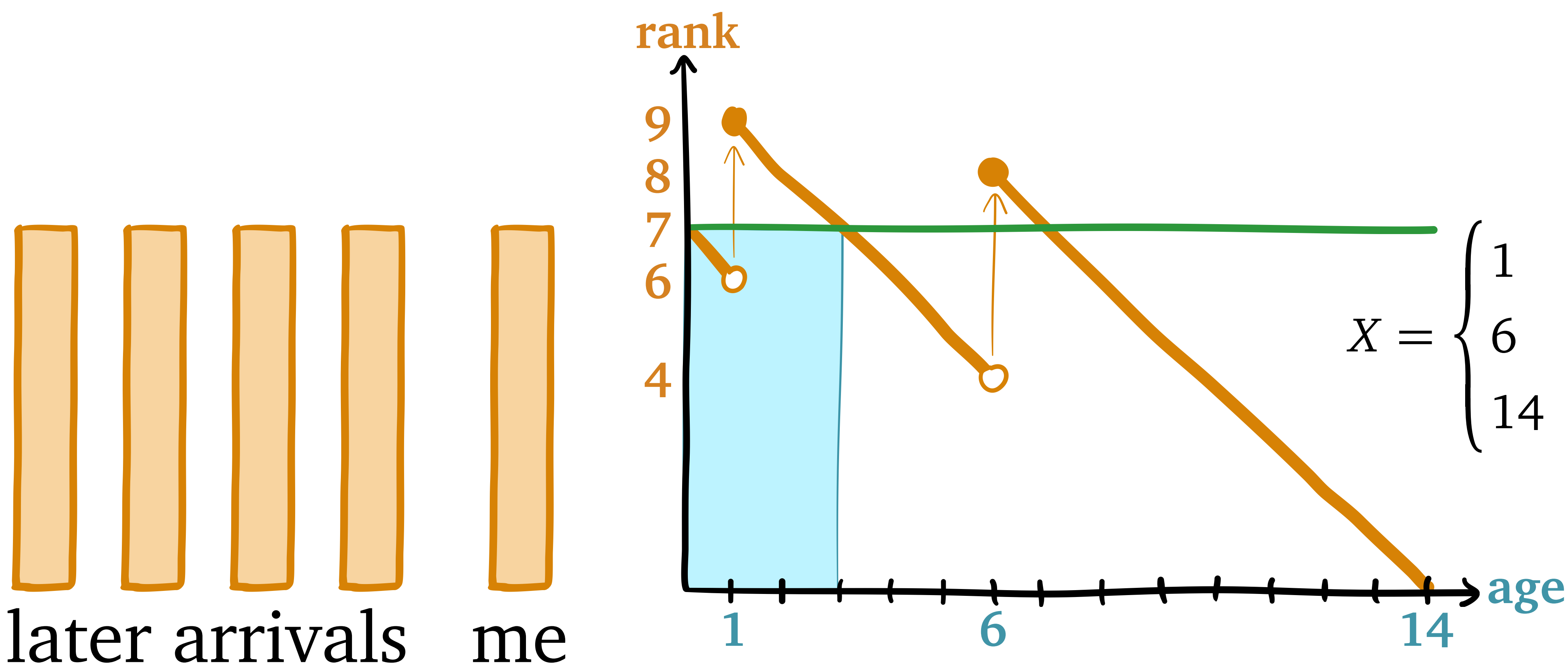
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6		
14		

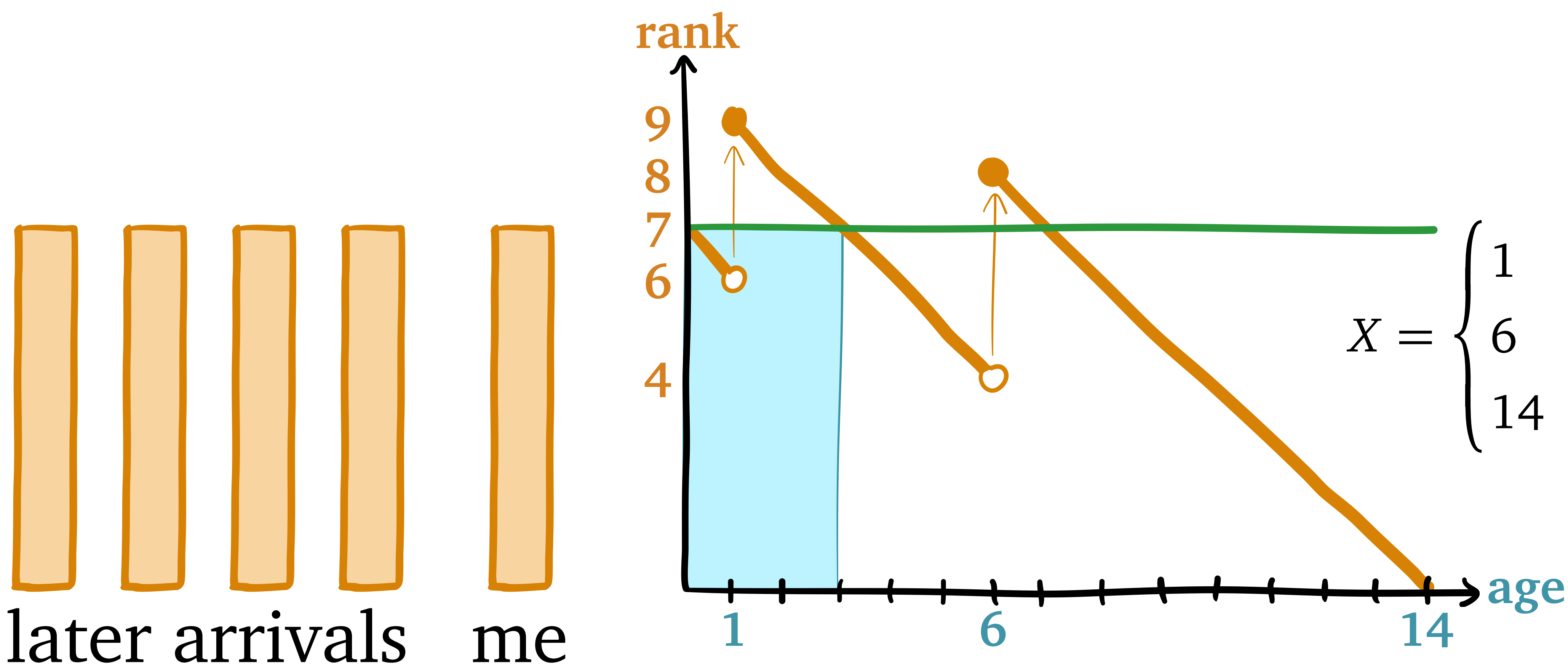
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \leq$ my age < 3	
14		

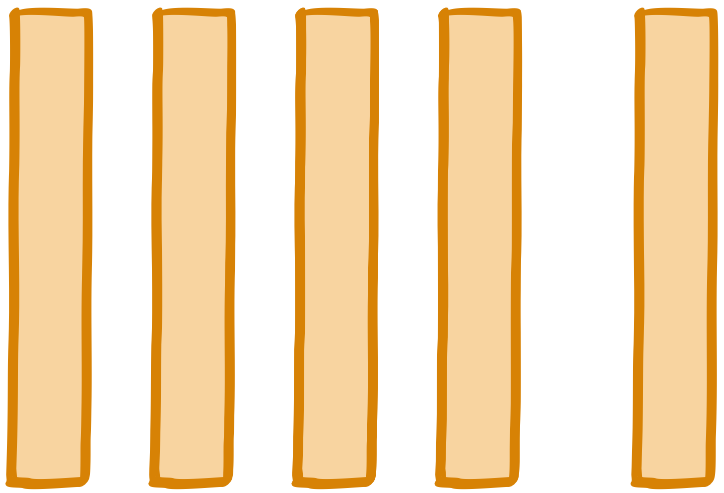
Warmup: Empty System



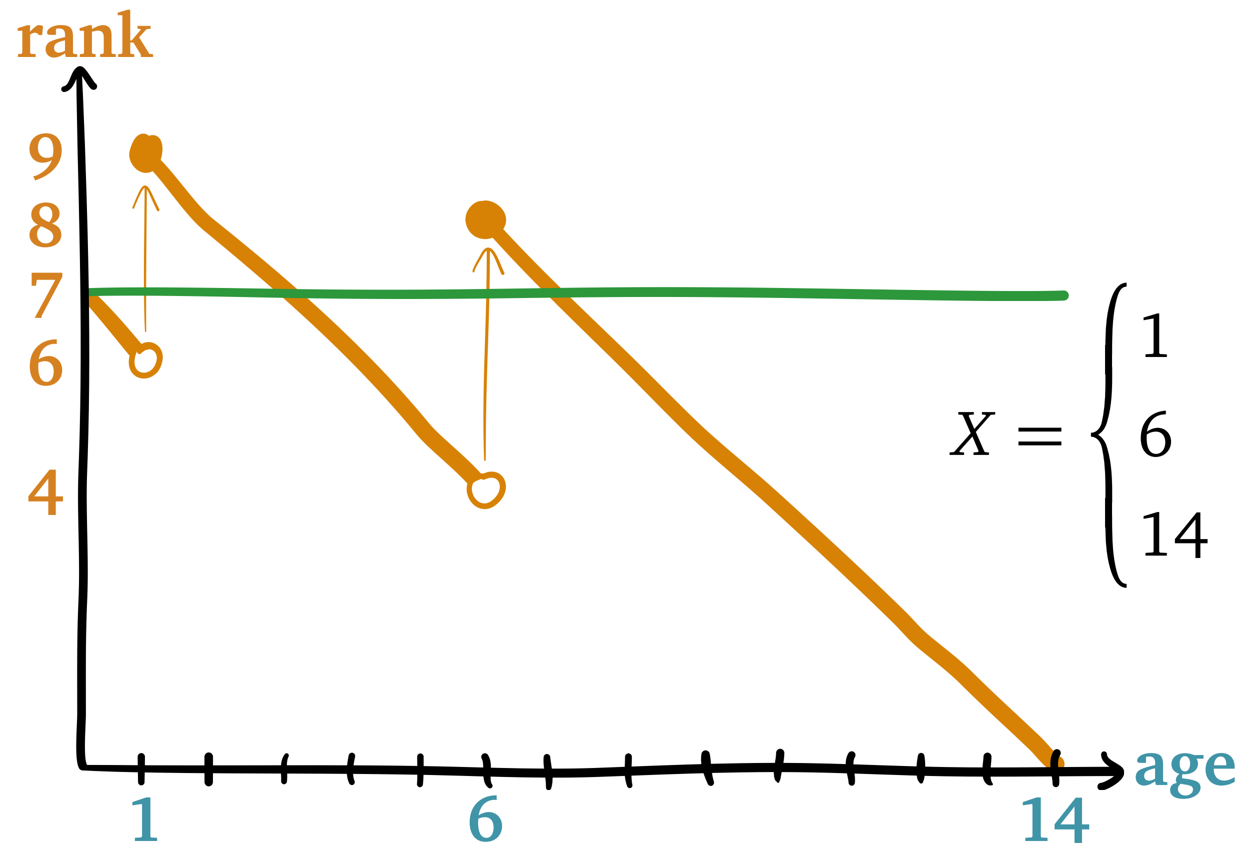
$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \leq$ my age < 3	1
14		

Warmup: Empty System



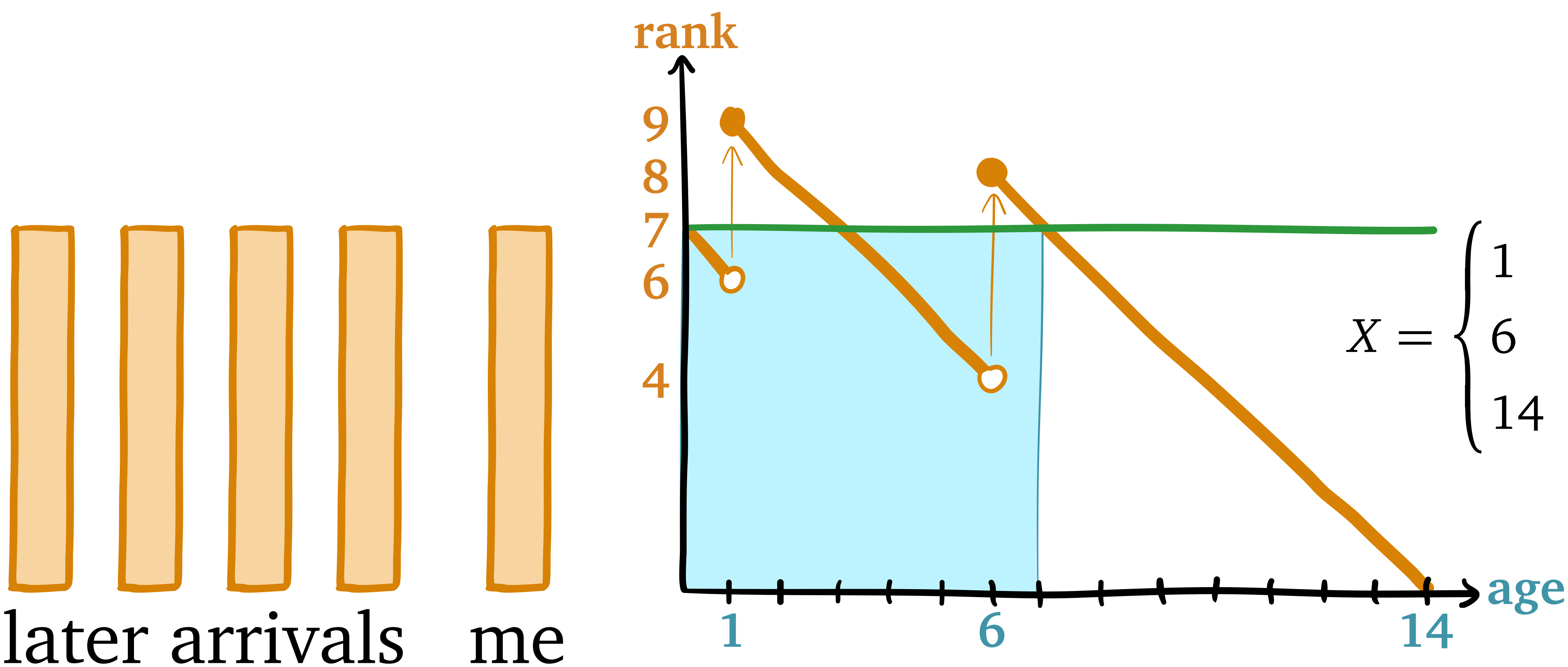
later arrivals me



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \leq$ my age < 3	1
14		

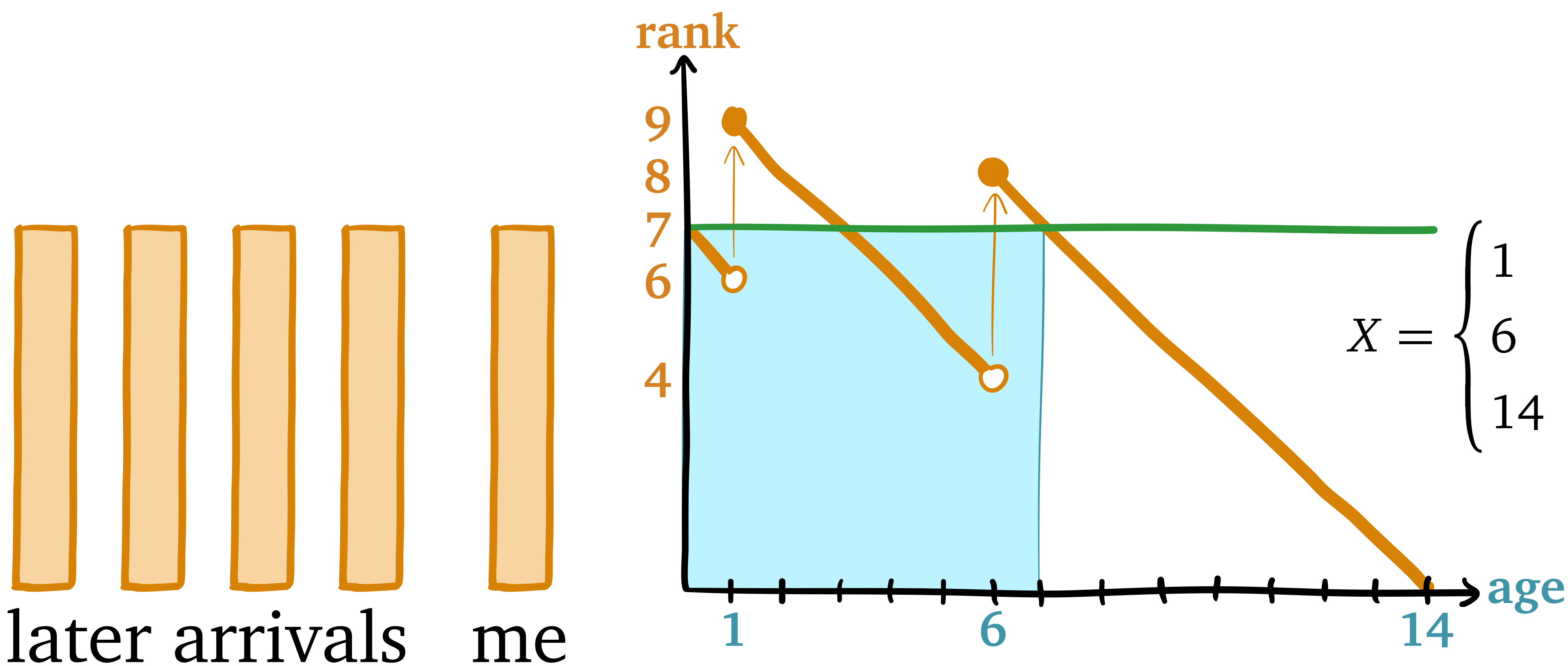
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \leq$ my age < 3	1
14		

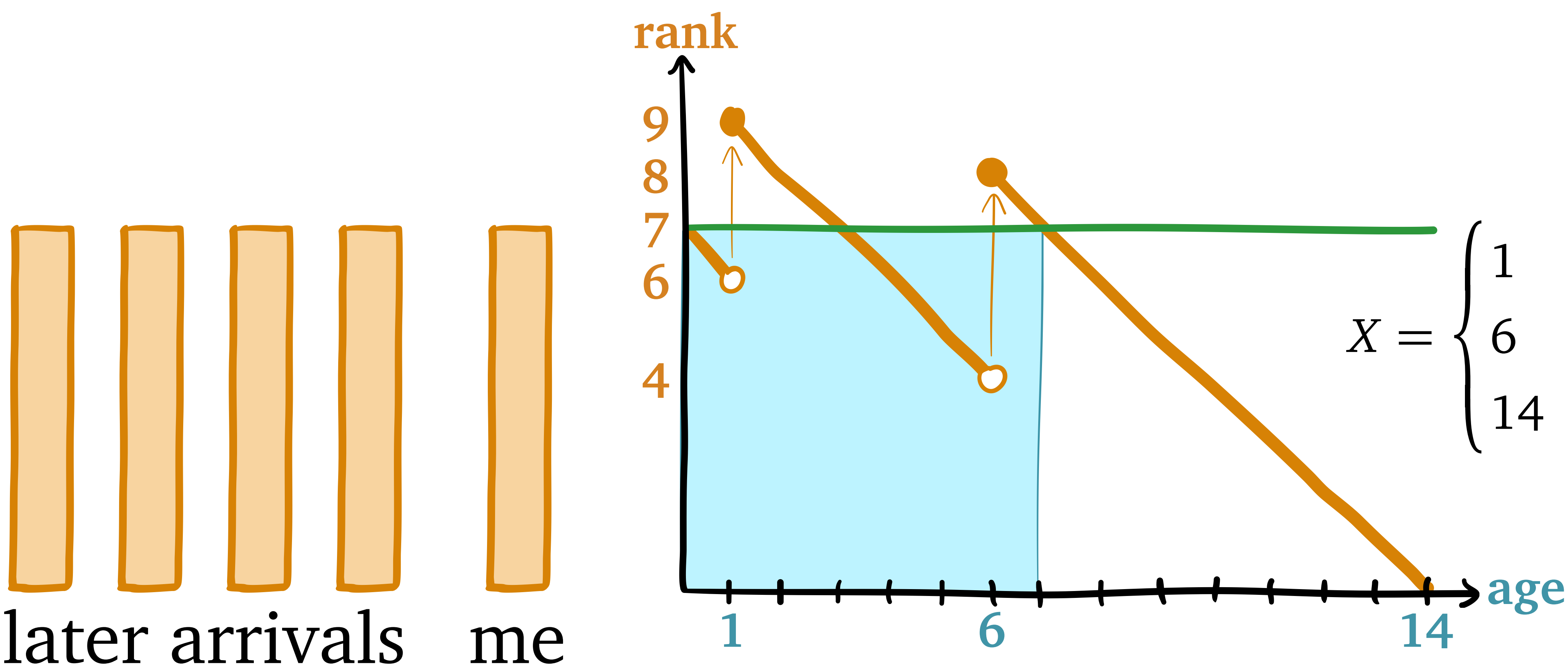
Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \le$ my age < 3	1
14	when $0 \le$ my age < 7	

Warmup: Empty System



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

My size	Which arrivals delay me?	By how much?
1	none	n/a
6	when $0 \leq$ my age < 3	1
14	when $0 \leq$ my age < 7	1

SOAP Insight #1:

Pessimism Principle

Replace my **rank** with my **worst** future rank

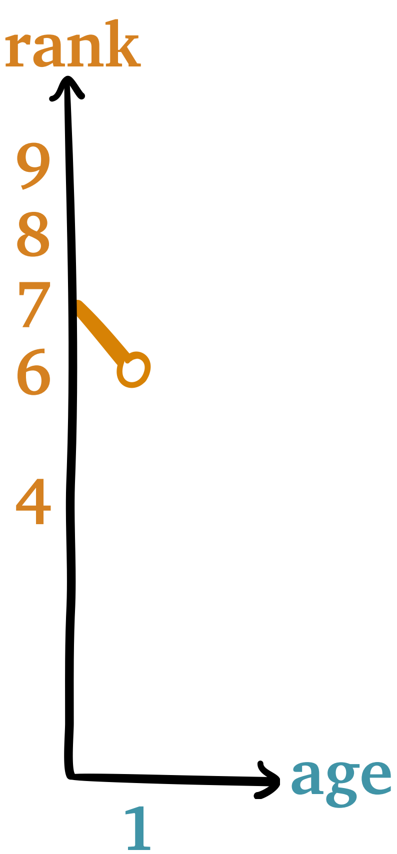
Pessimism Principle

Replace my **rank** with my **worst** future rank

Pessimism Principle

Replace my **rank** with my **worst** future rank

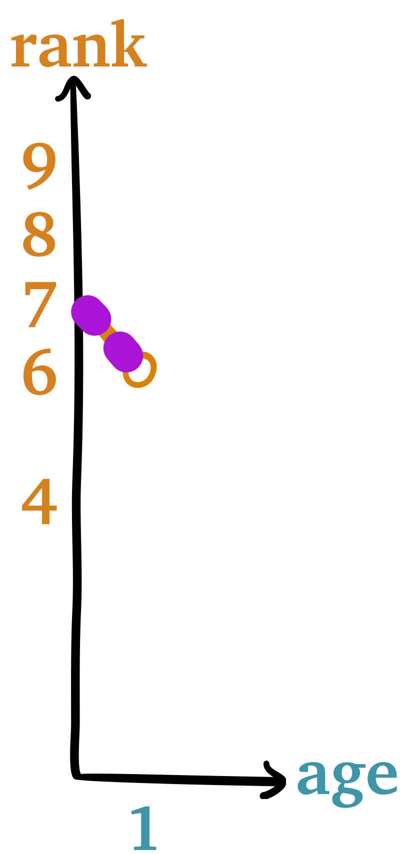
my size = 1



Pessimism Principle

Replace my **rank** with my **worst** future rank

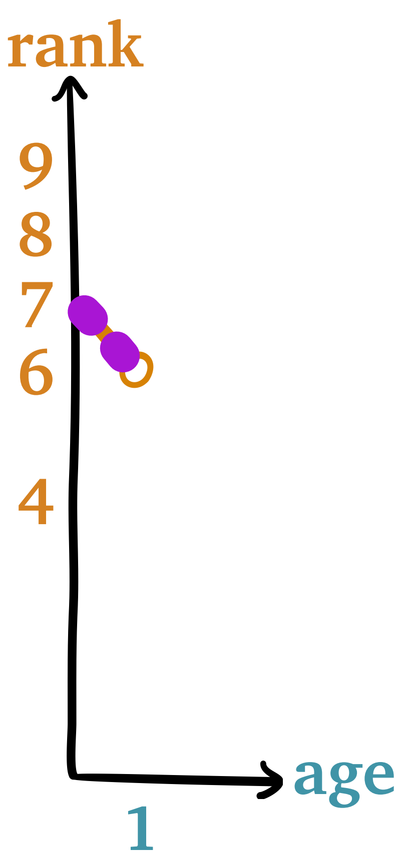
my size = 1



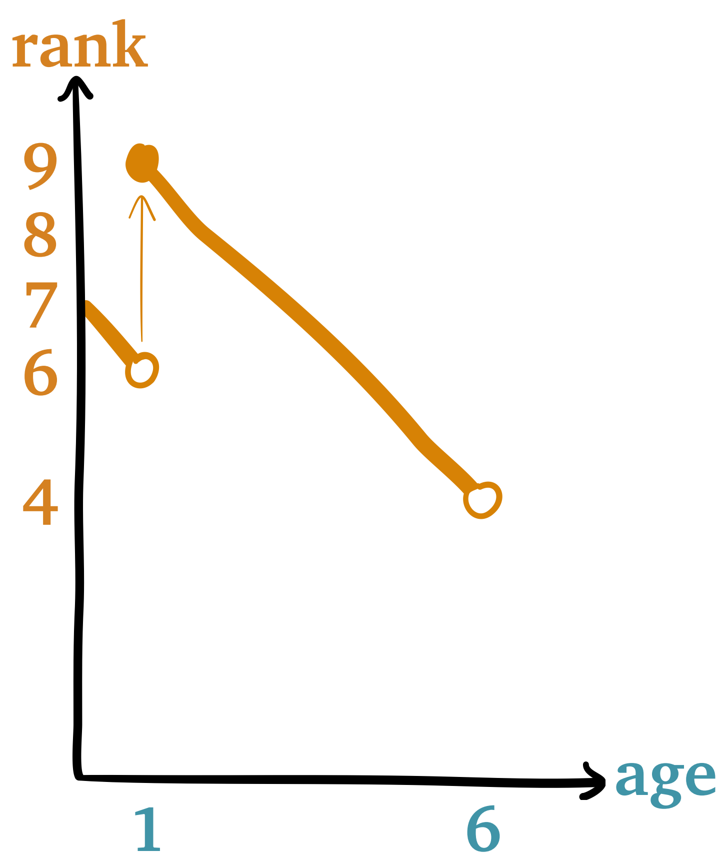
Pessimism Principle

Replace my **rank** with my **worst** future rank

my size = 1



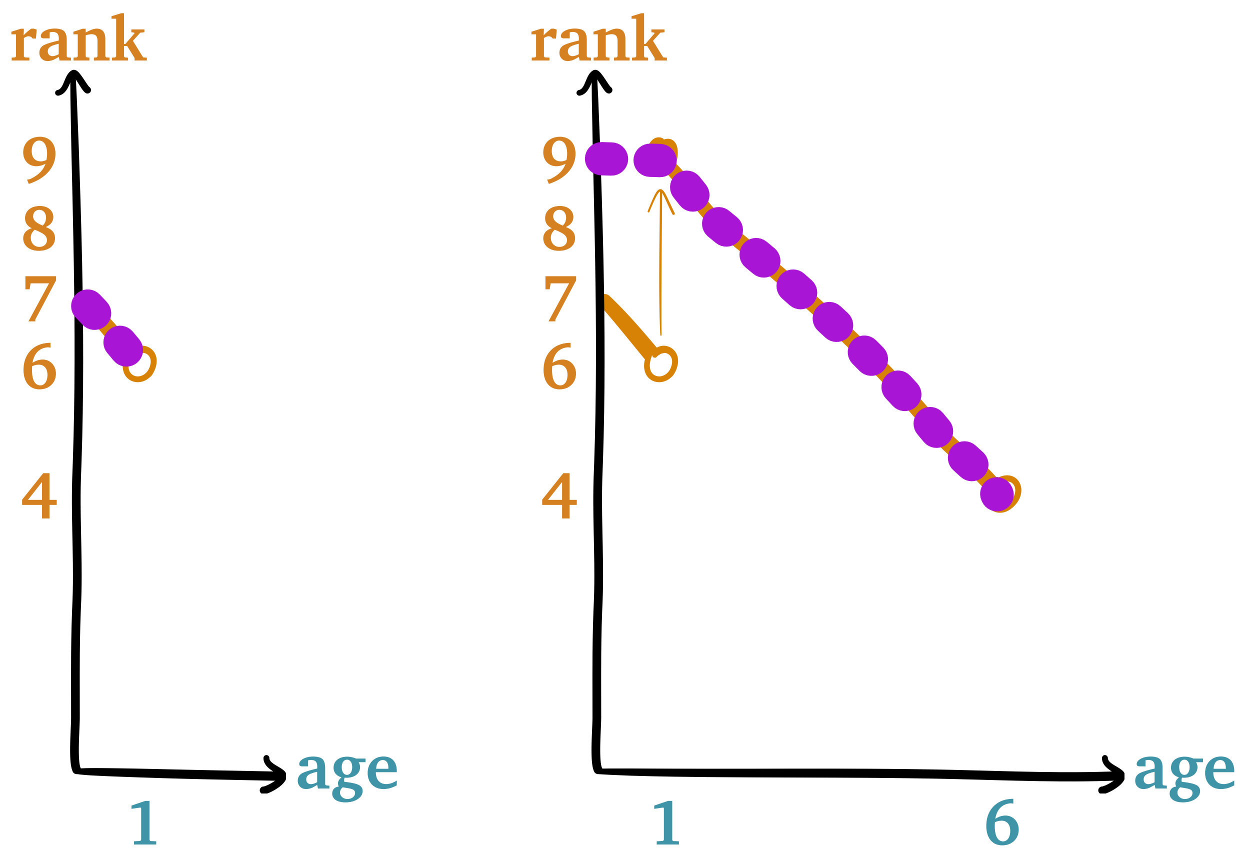
my size = 6



Pessimism Principle

Replace my **rank** with my **worst** future rank

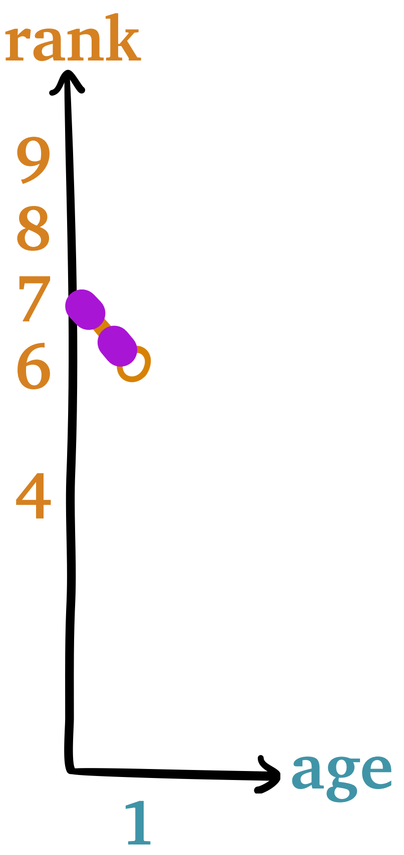
my size = 1 my size = 6



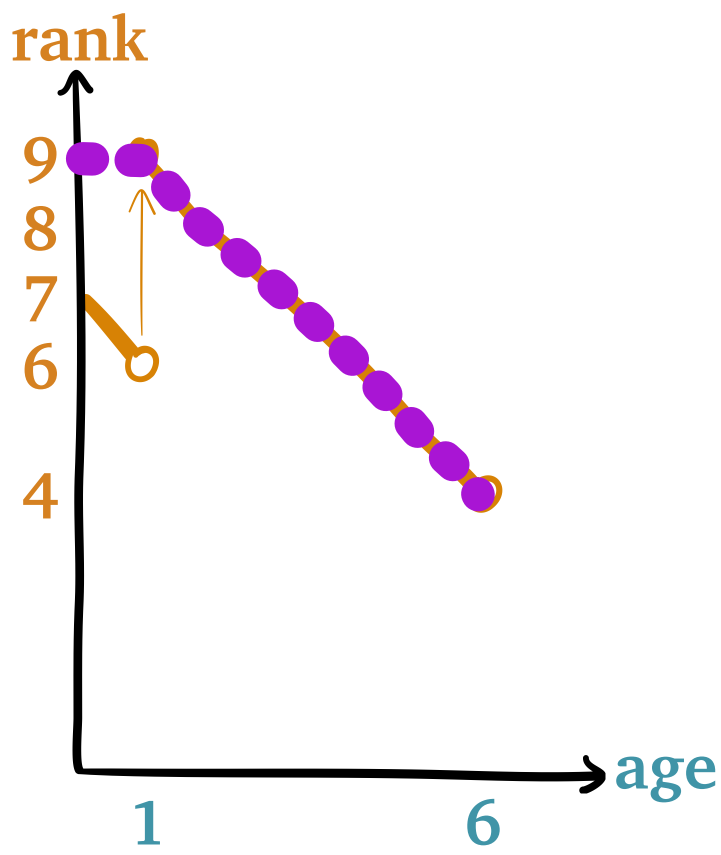
Pessimism Principle

Replace my **rank** with my **worst** future rank

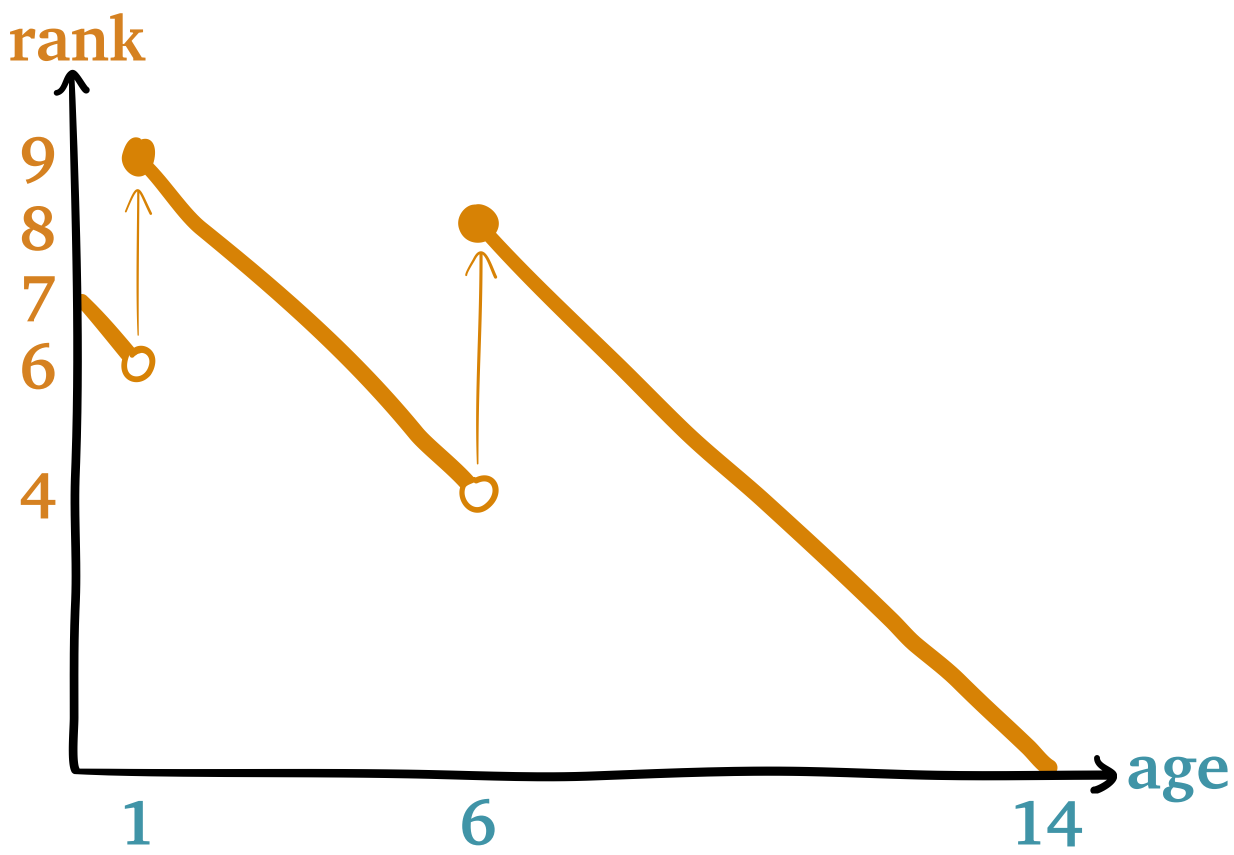
my size = 1



my size = 6

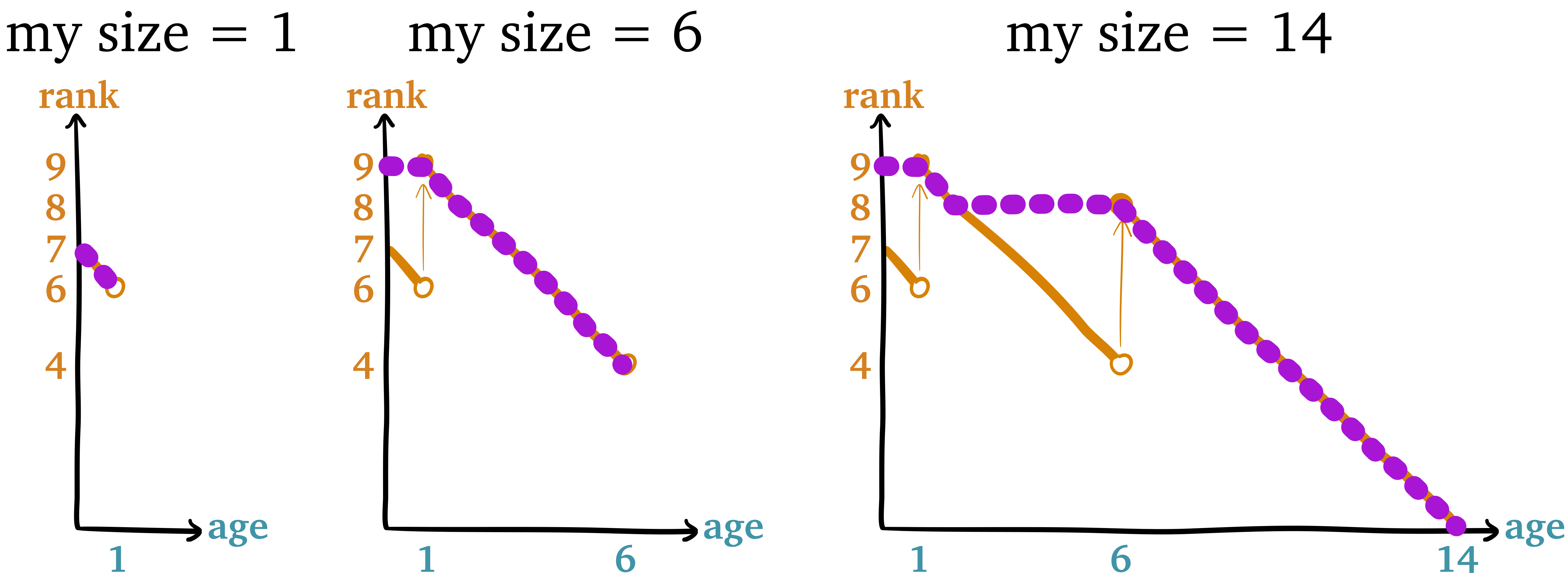


my size = 14



Pessimism Principle

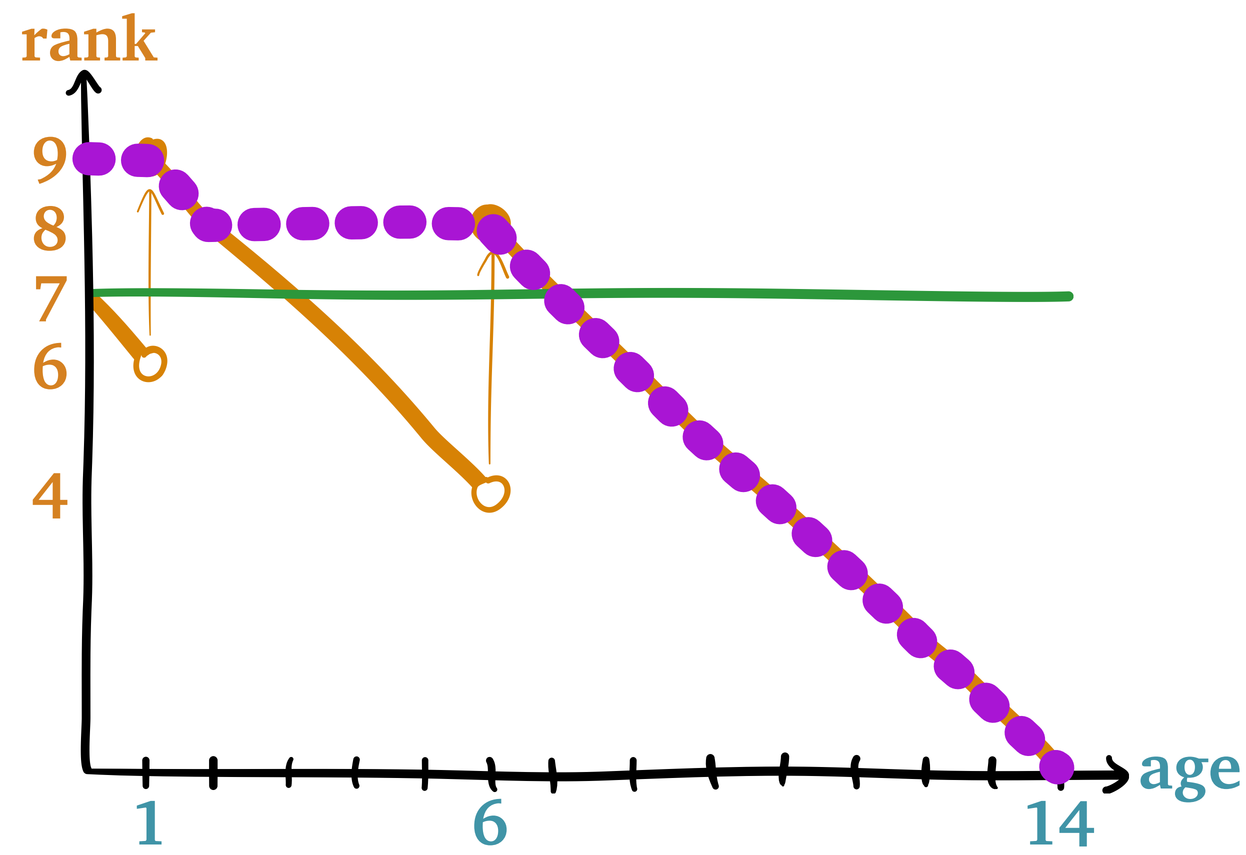
Replace my **rank** with my **worst** future rank



Pessimism Principle

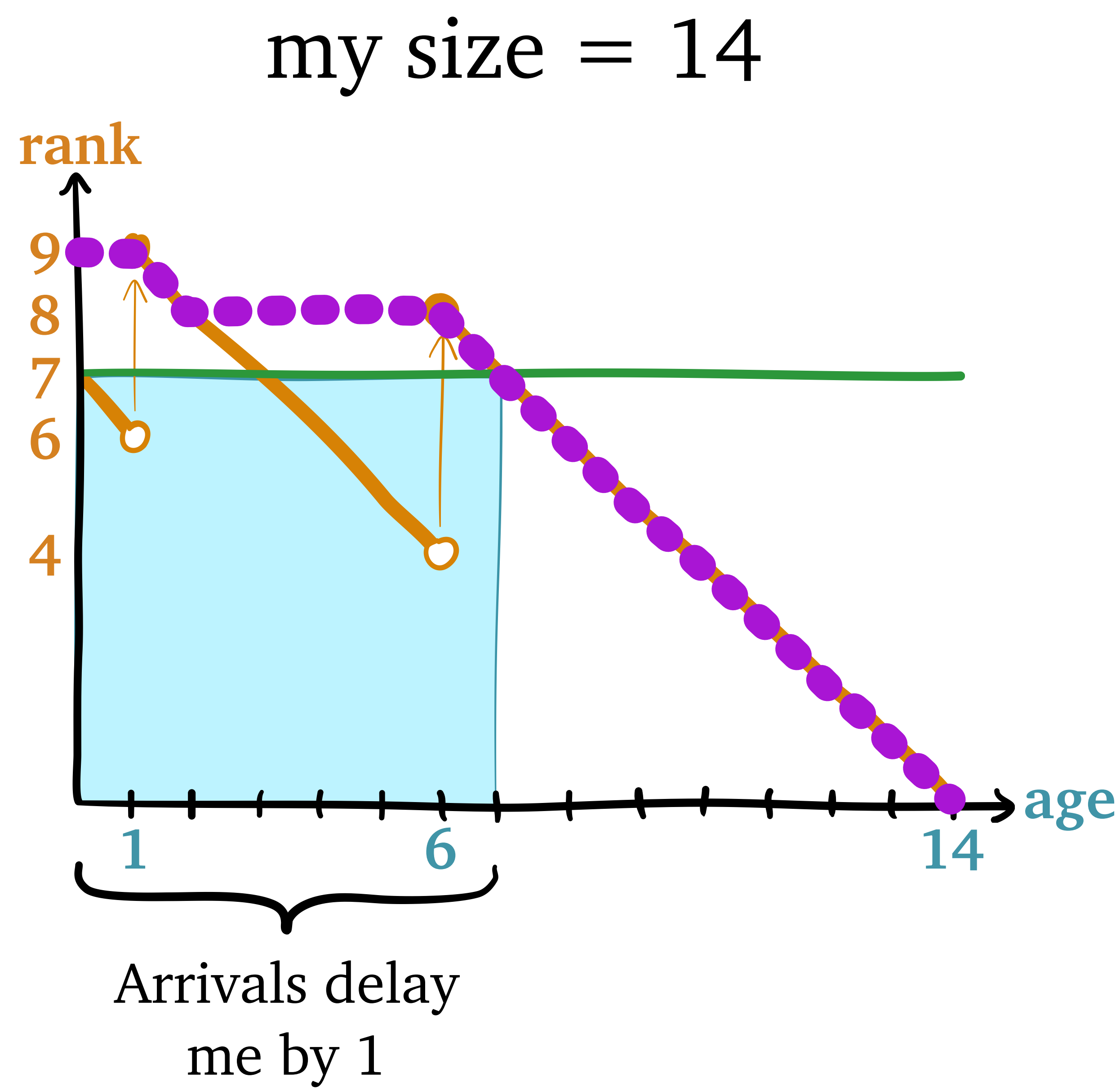
Replace my **rank** with my **worst** future rank

my size = 14



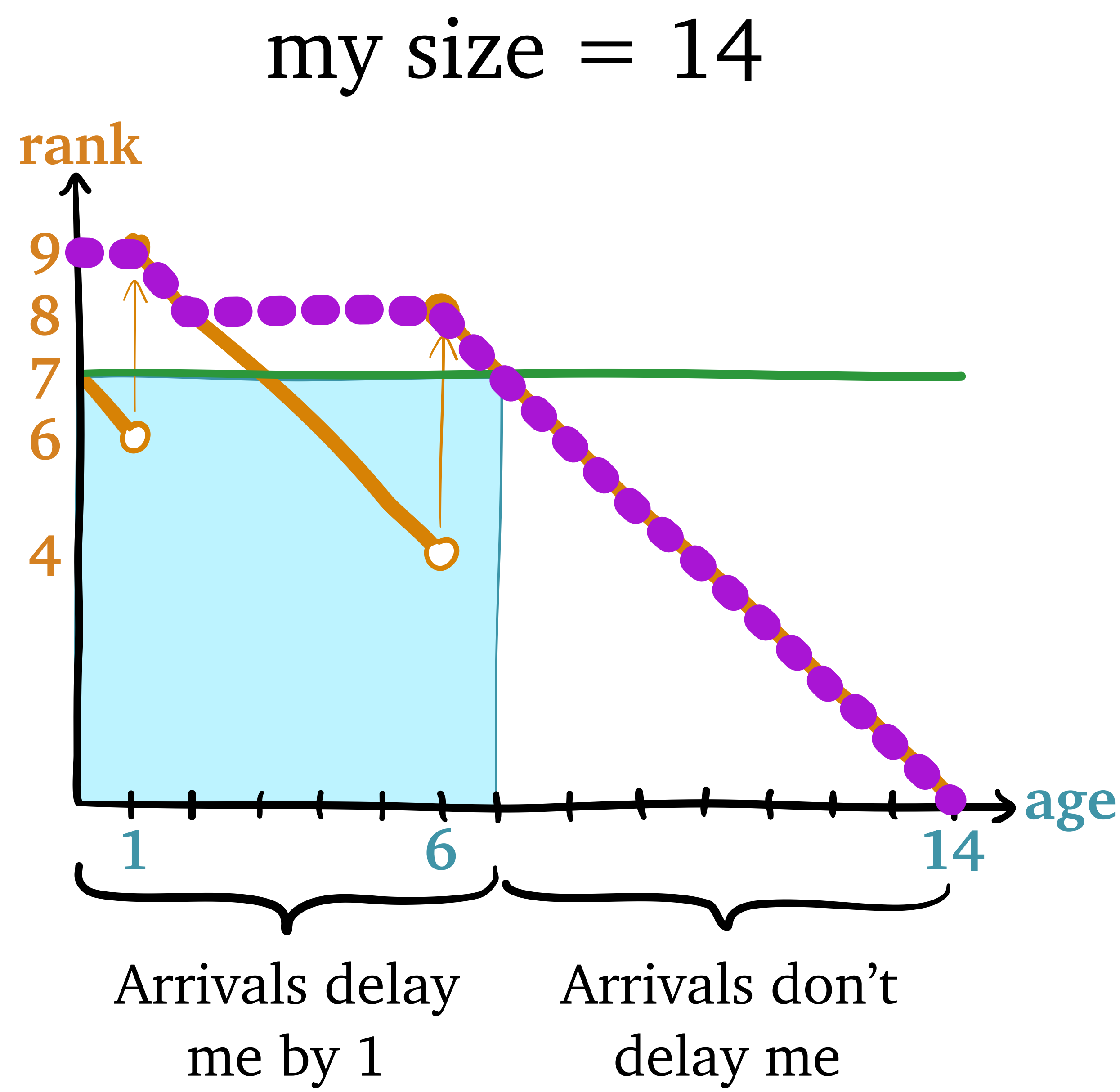
Pessimism Principle

Replace my **rank** with my **worst** future rank



Pessimism Principle

Replace my **rank** with my **worst** future rank

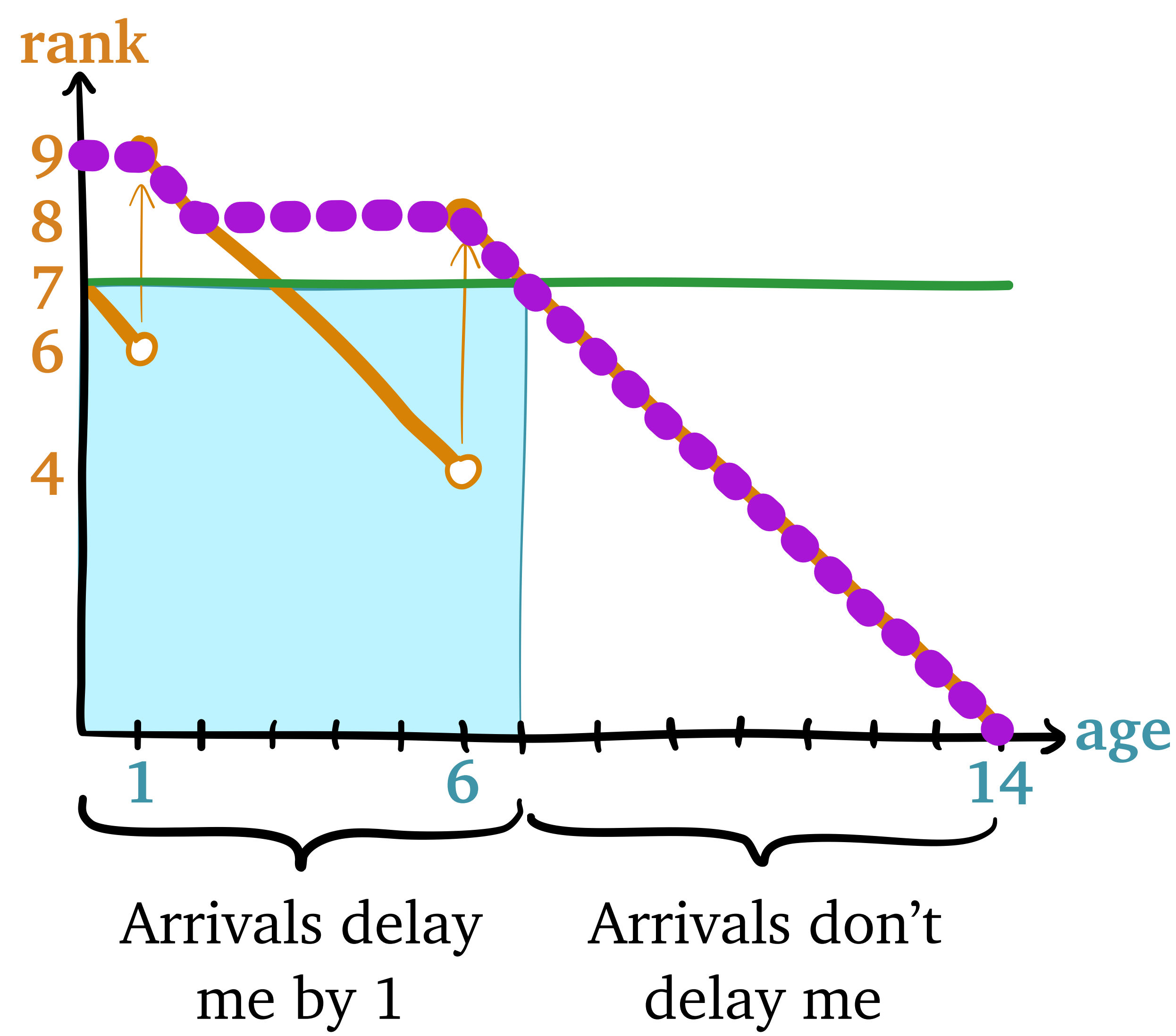


Pessimism Principle

Replace my **rank** with my **worst** future rank

$$\rho_{\text{new}}(a) = \begin{cases} \lambda \cdot 1 & 0 \le a < 7 \\ \lambda \cdot 0 & 7 \le a < 14 \end{cases}$$

my size $= 14$



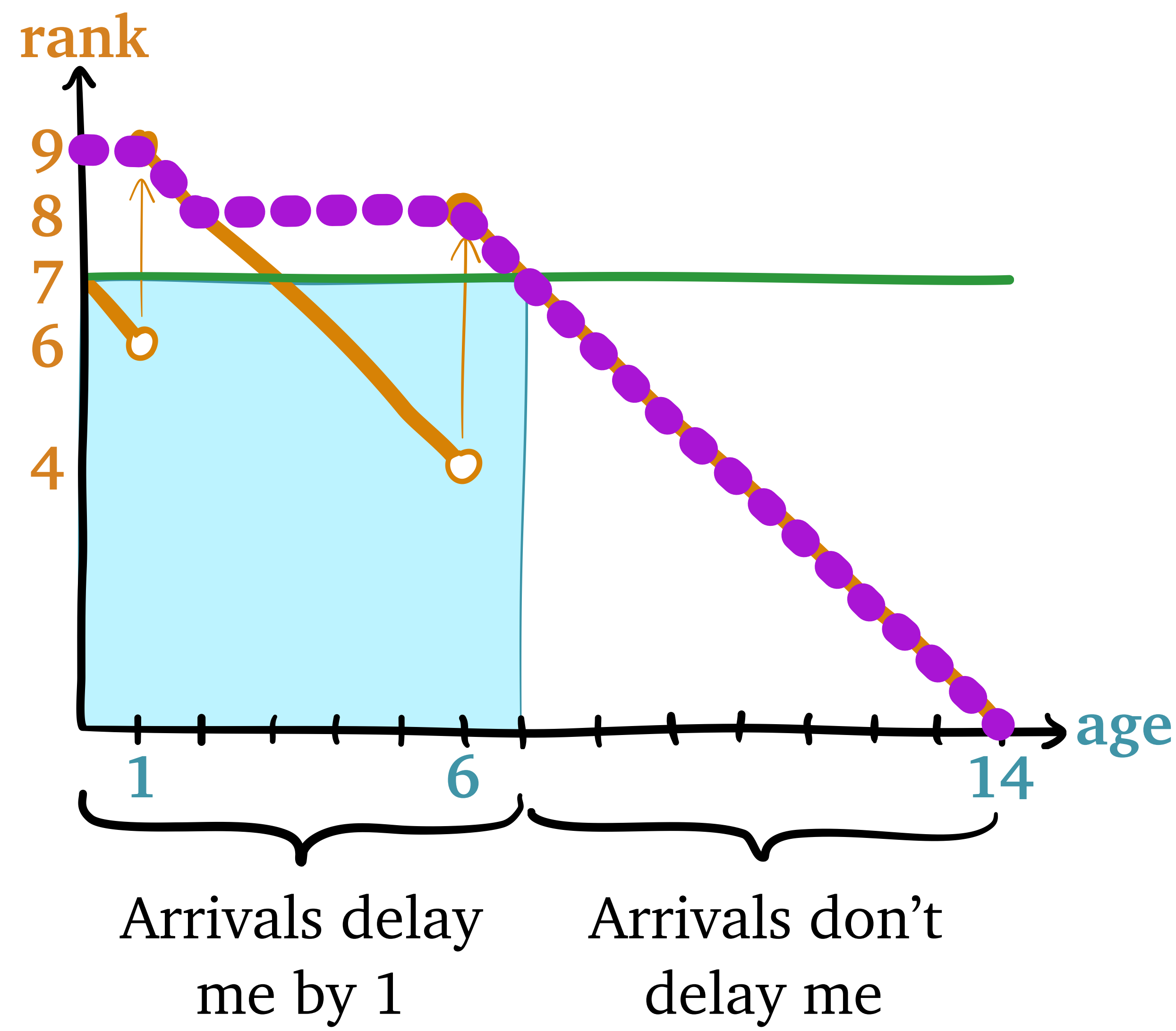
Pessimism Principle

Replace my **rank** with my **worst** future rank

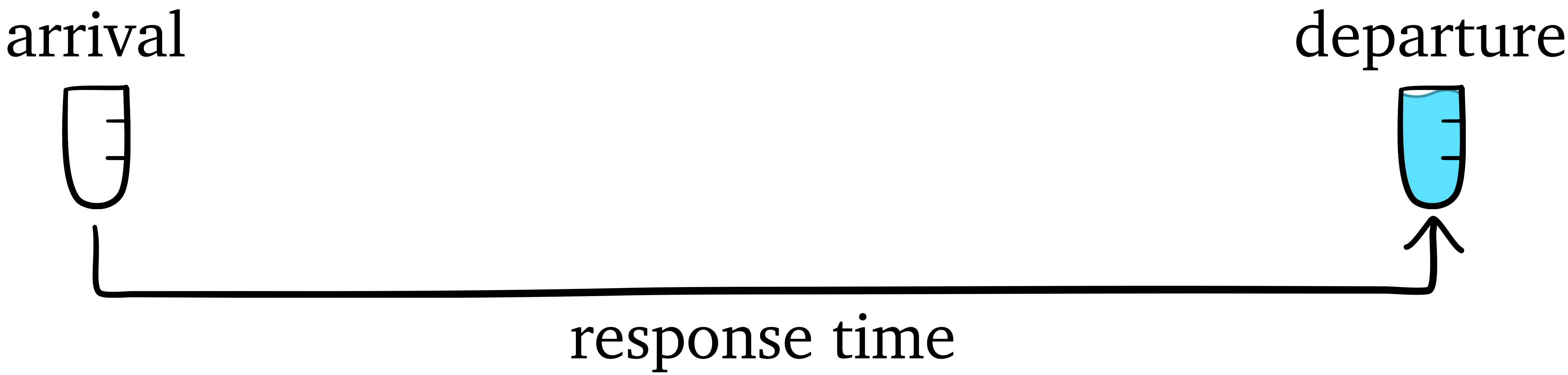
$$\rho_{\text{new}}(a) = \begin{cases} \lambda \cdot 1 & 0 \le a < 7 \\ \lambda \cdot 0 & 7 \le a < 14 \end{cases}$$

$$\mathbf{E}[T_{14} \mid \text{empty}] = \int_0^{14} \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)}$$

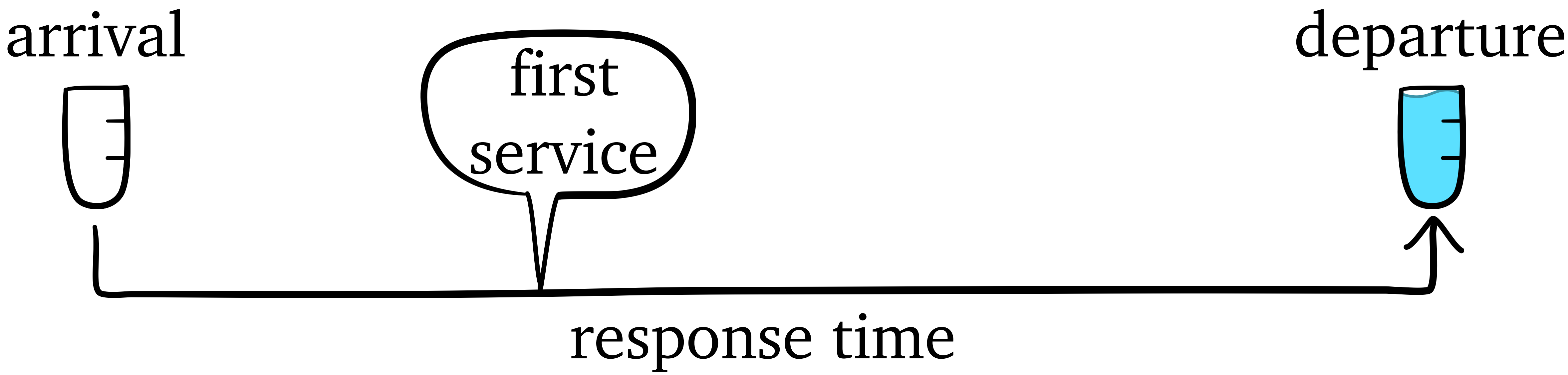
my size $= 14$



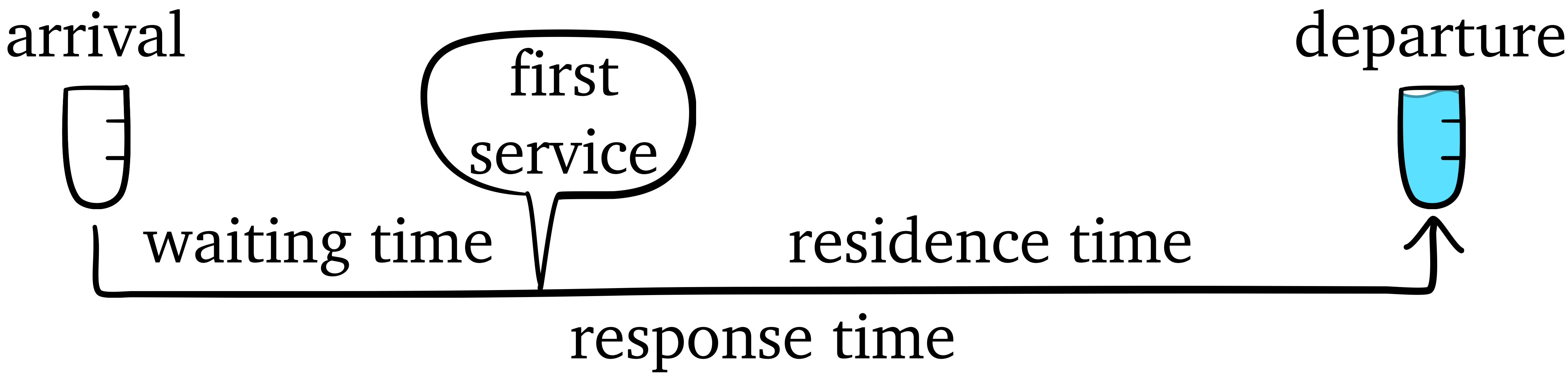
Response Time Analysis



Response Time Analysis



Response Time Analysis



Residence Time

Residence Time

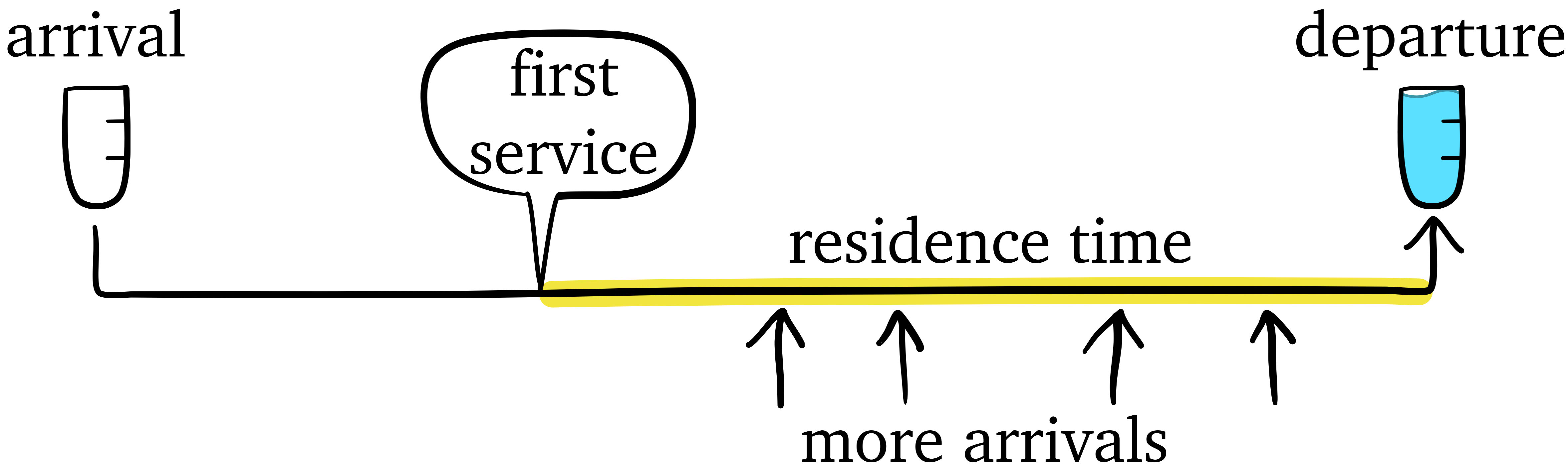
Question: is residence time…

Residence Time

Question: is residence time…

- my size?

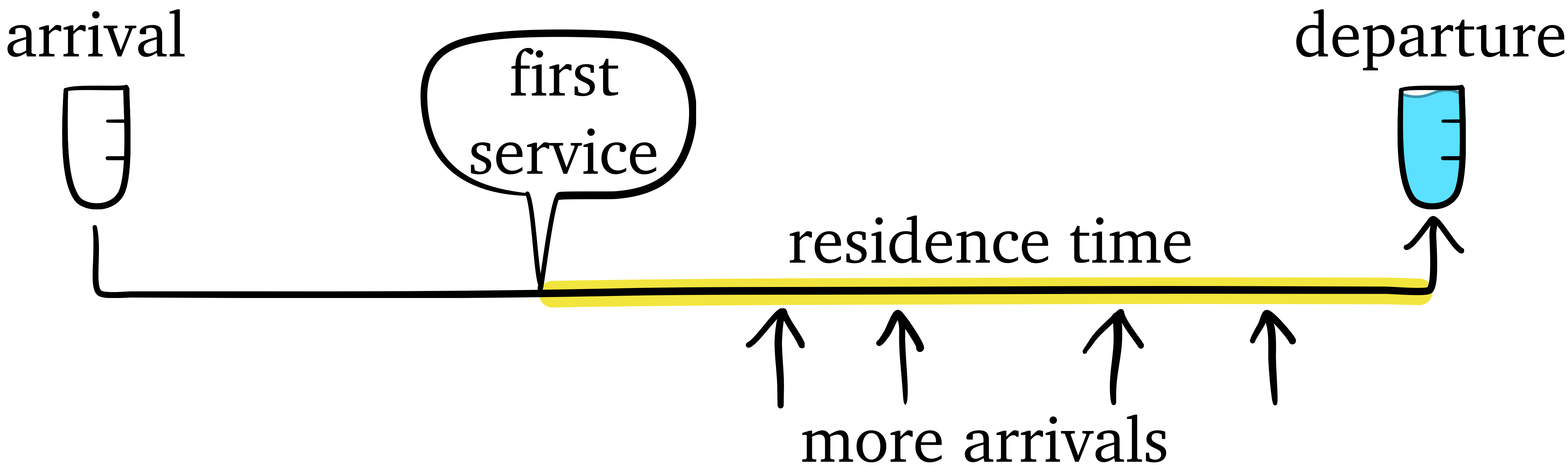
Residence Time



Question: is residence time…

- my size?

Residence Time



Question: is residence time…

- my size? ❌

Residence Time

Question: is residence time…

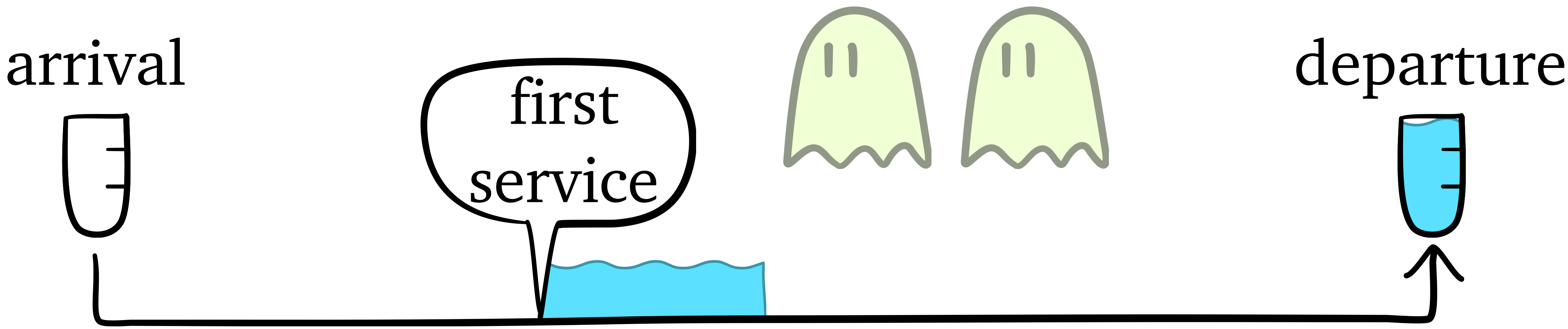
- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Residence Time

Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

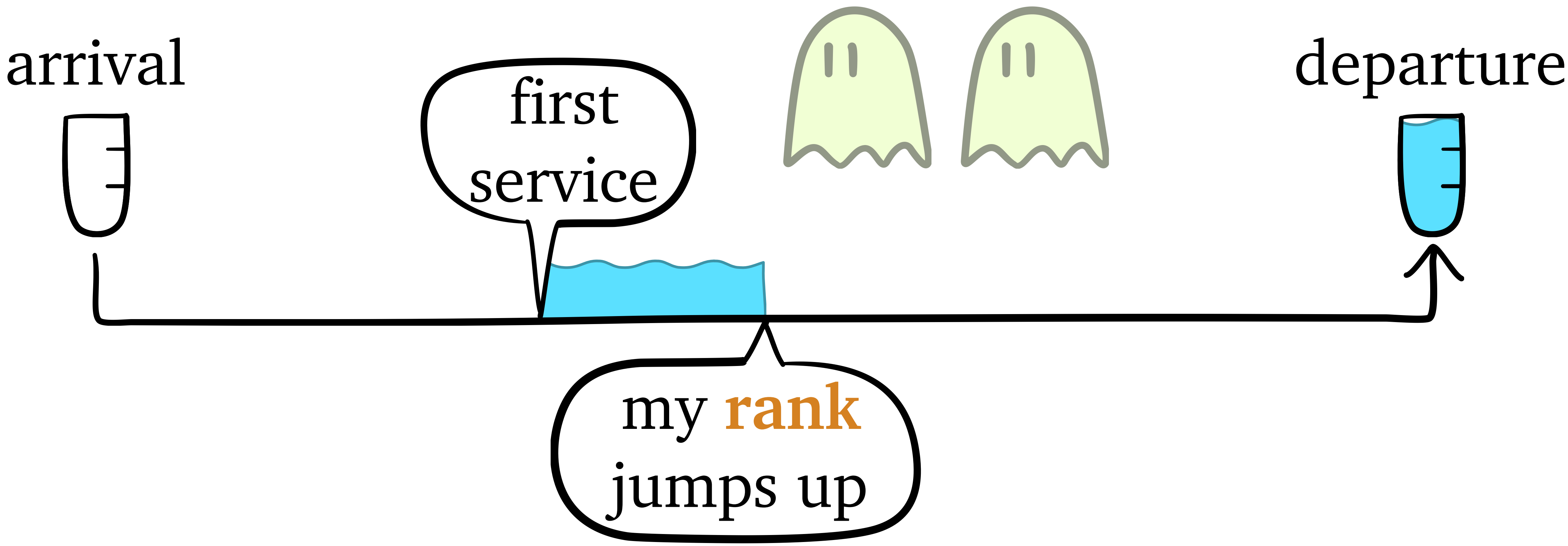
Residence Time



Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

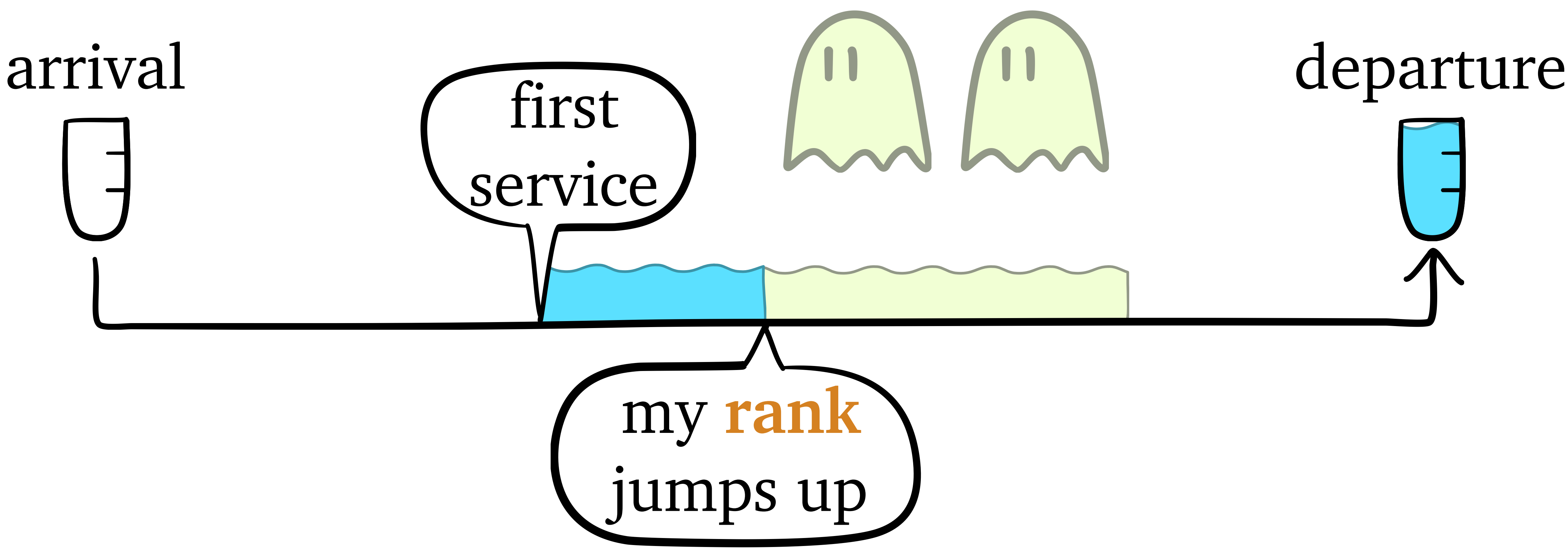
Residence Time



Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

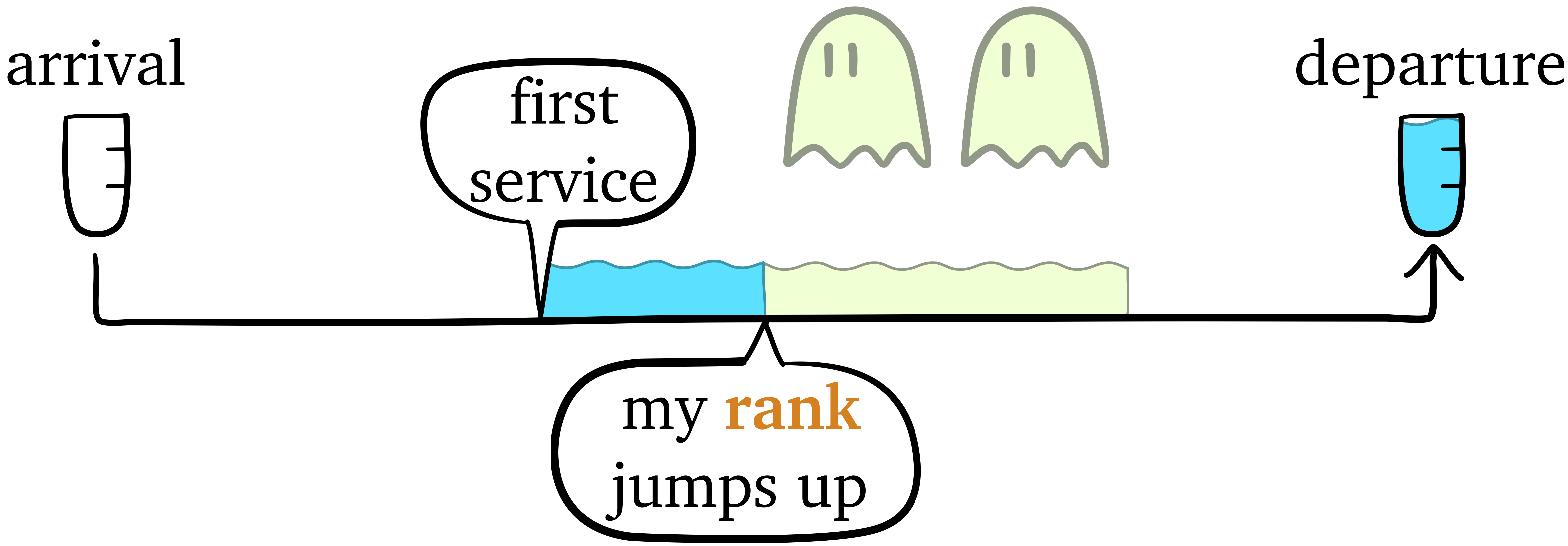
Residence Time



Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Residence Time

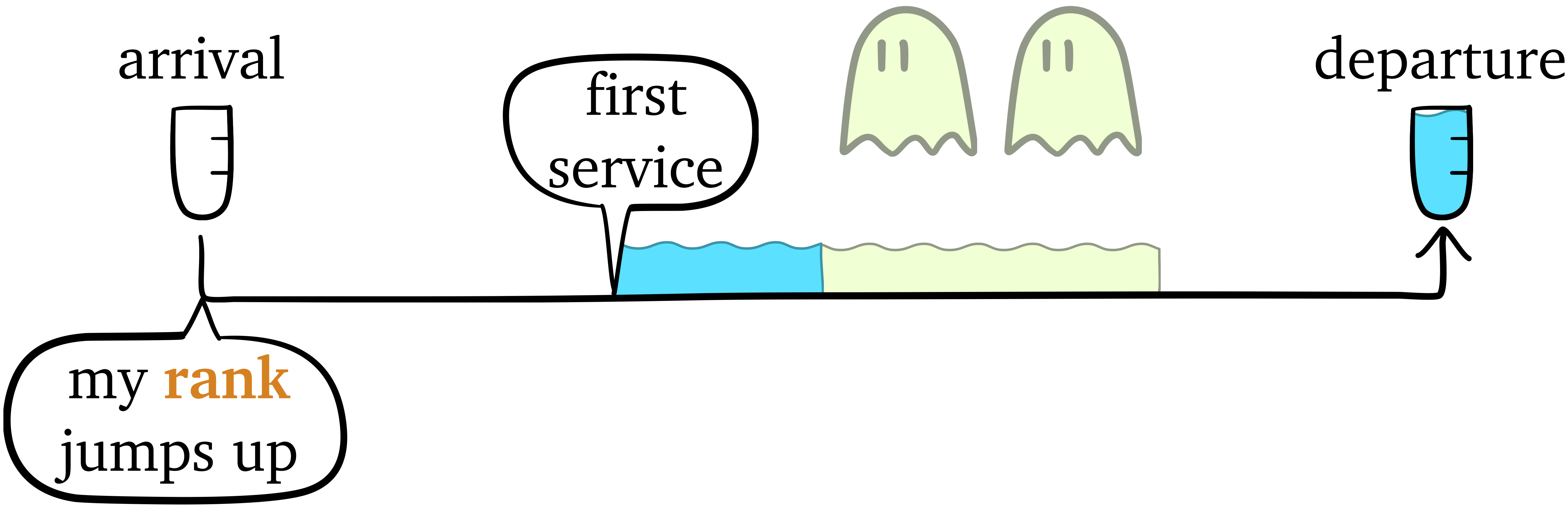


Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Pessimism Principle: replace my **rank** with my **worst** future rank

Residence Time

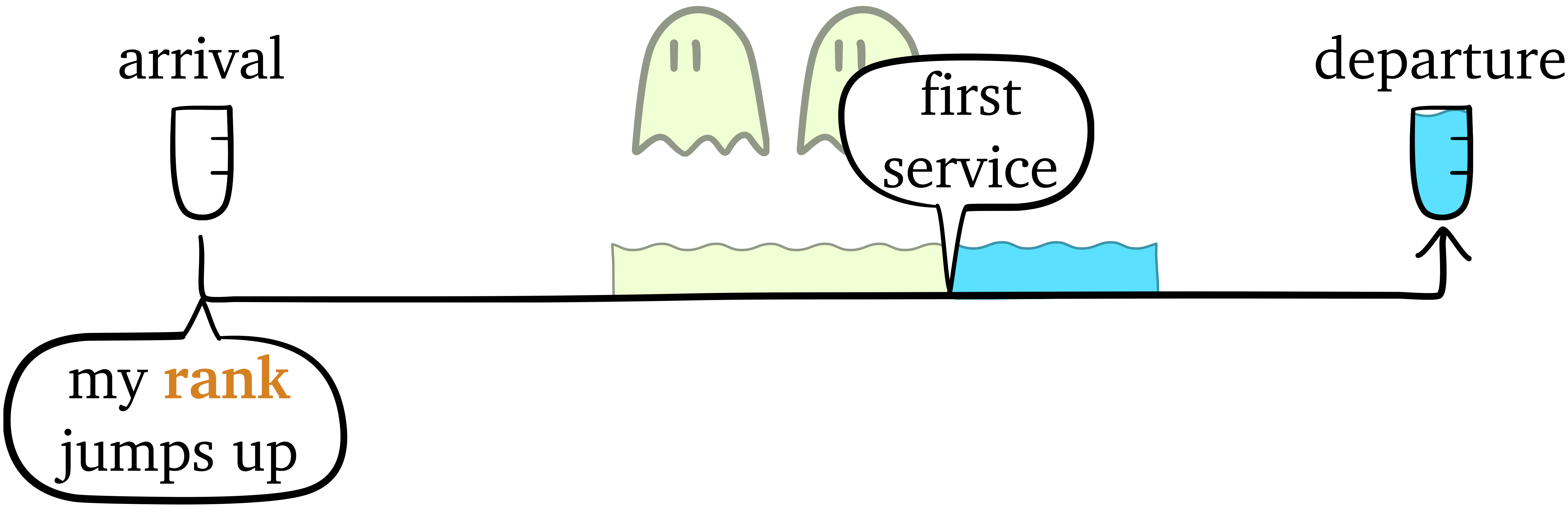


Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Pessimism Principle:
replace my **rank** with
my **worst** future rank

Residence Time

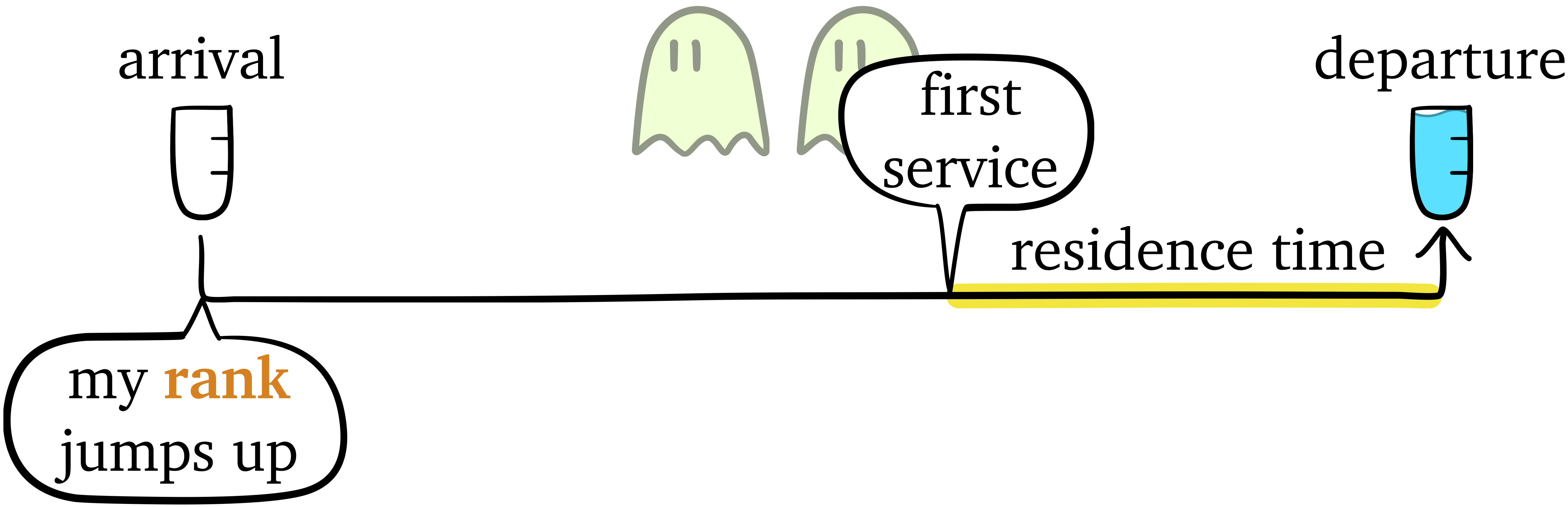


Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Pessimism Principle:
replace my **rank** with
my **worst** future rank

Residence Time

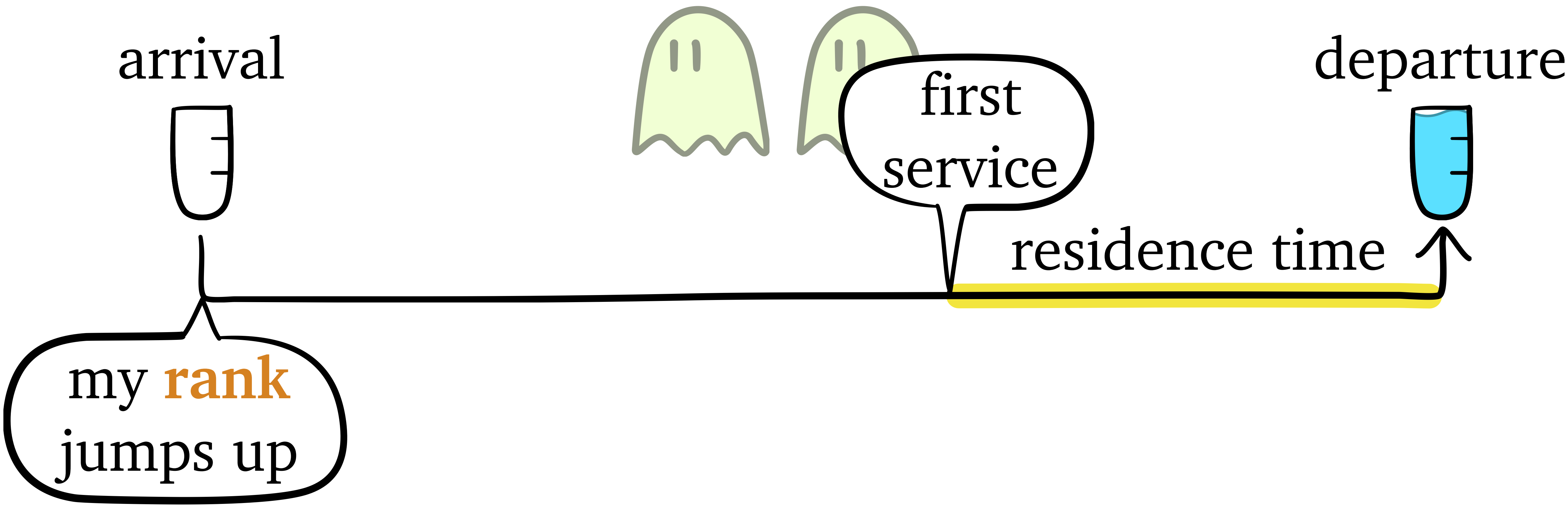


Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$?

Pessimism Principle:
replace my **rank** with my **worst** future rank

Residence Time

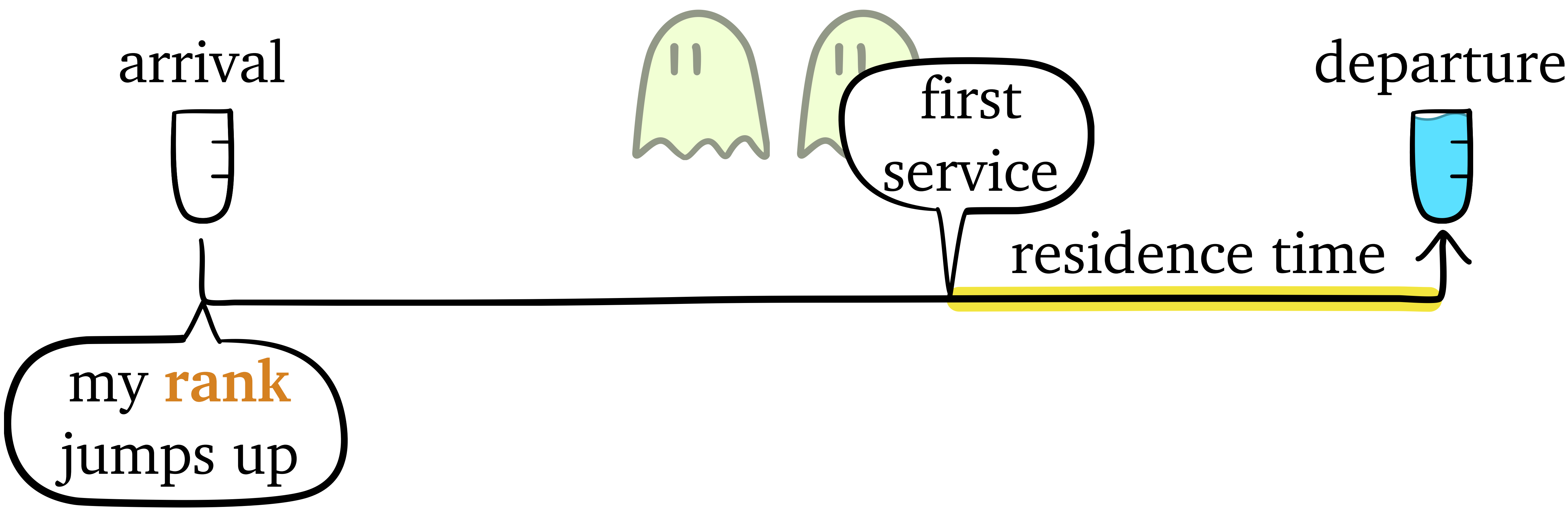


Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$? ✓

Pessimism Principle: replace my **rank** with my **worst** future rank

Residence Time



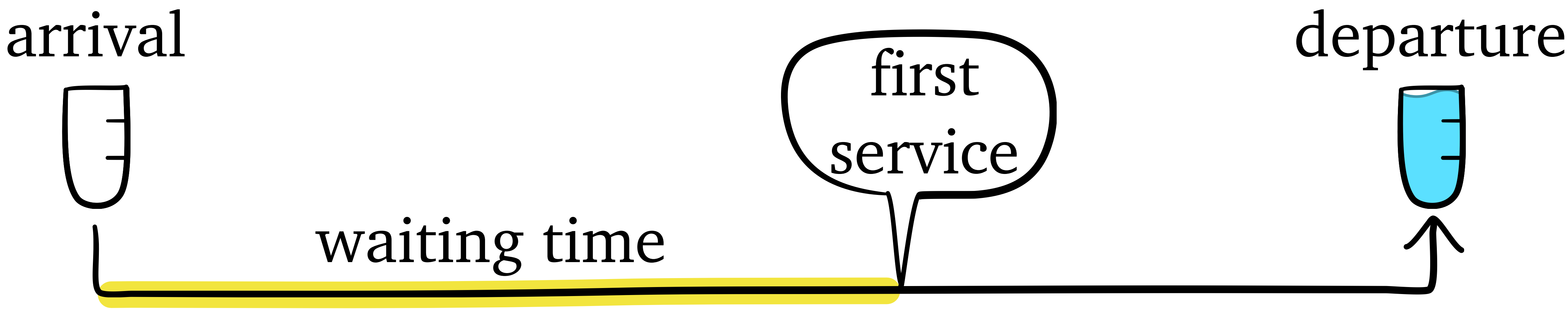
Question: is residence time…

- my size? ✗
- $\mathbf{E}[T \mid \text{empty}]$? ✓

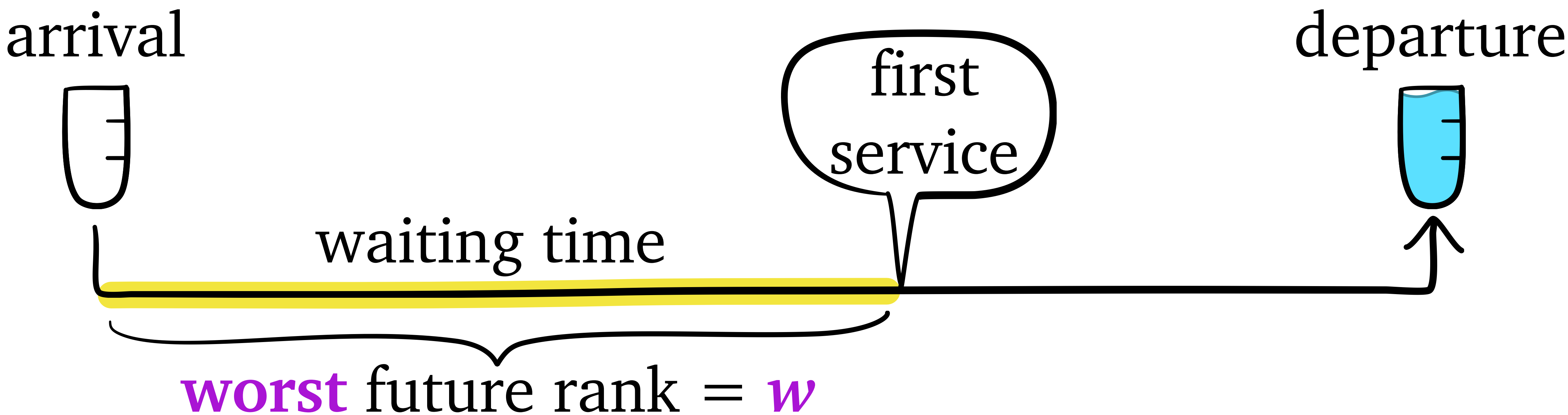
Pessimism Principle: replace my **rank** with my **worst** future rank

e.g. $$\mathbf{E}[R_{14}] = \mathbf{E}[T_{14} \mid \text{empty}] = \int_0^{14} \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)}$$

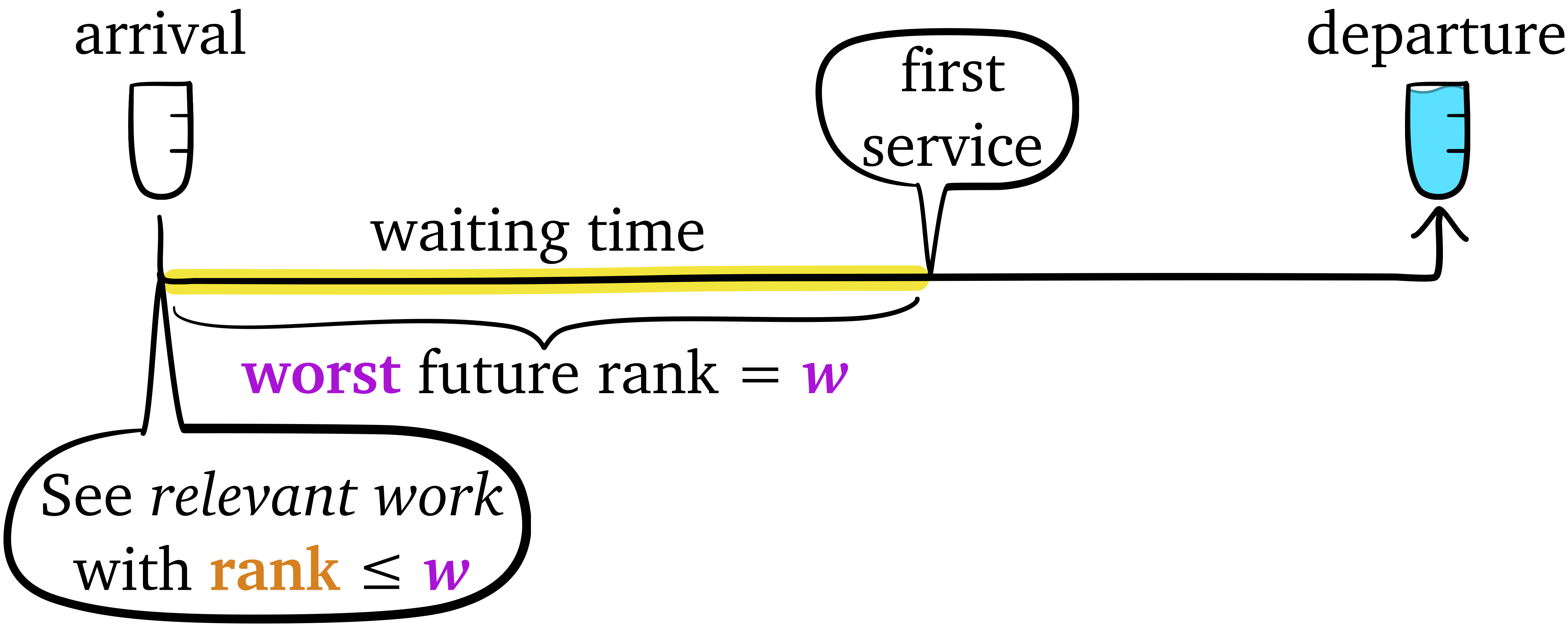
Waiting Time



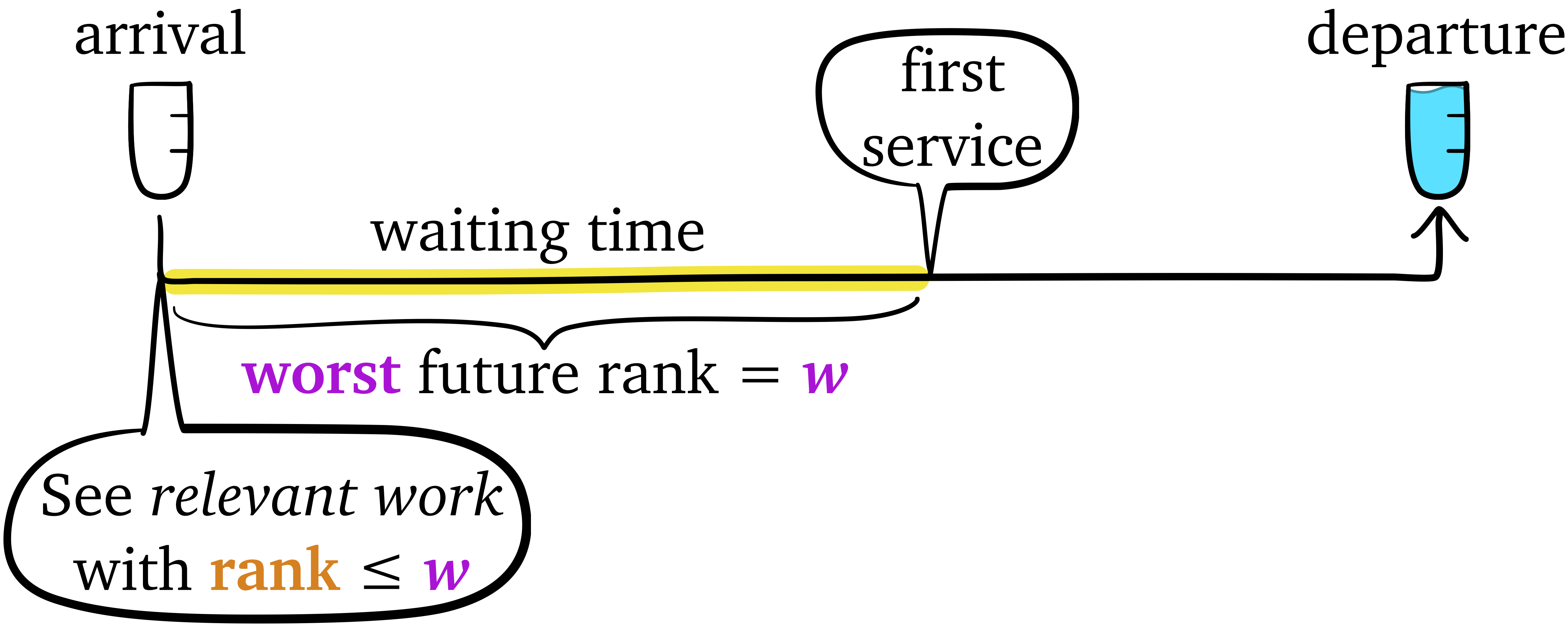
Waiting Time



Waiting Time

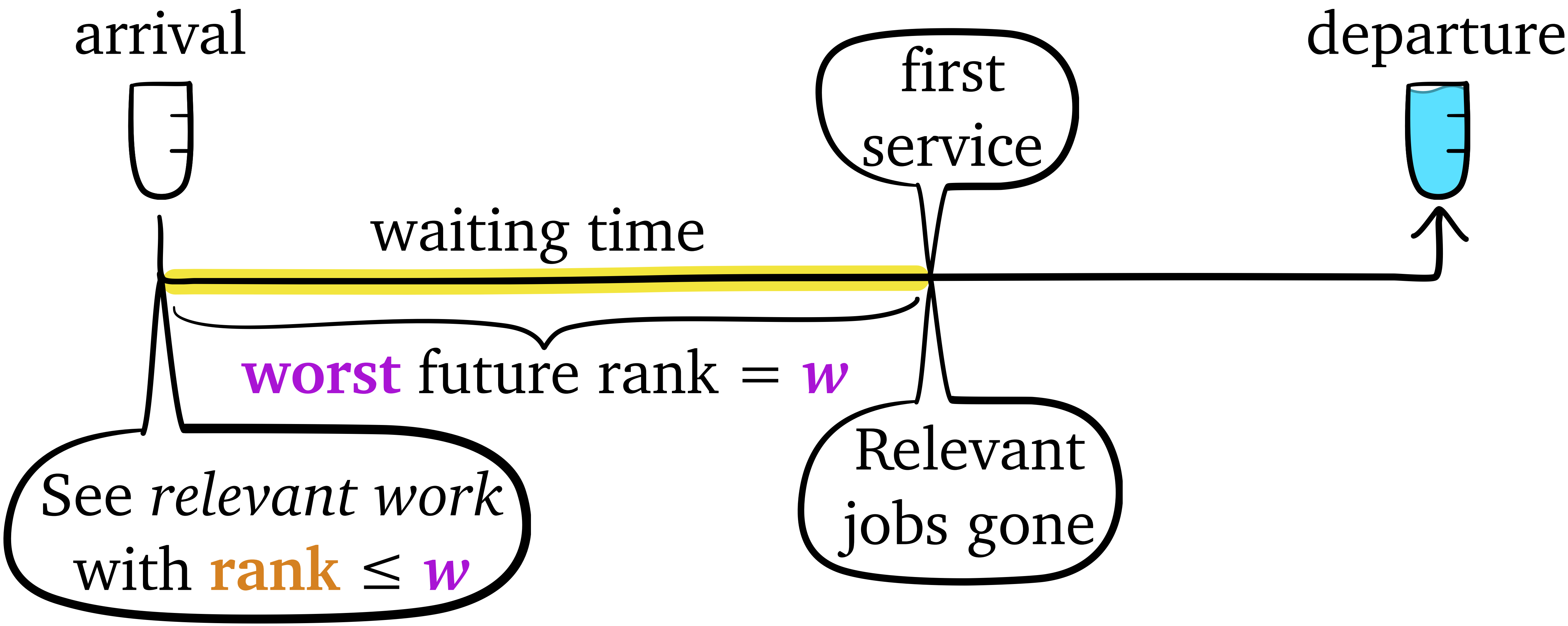


Waiting Time



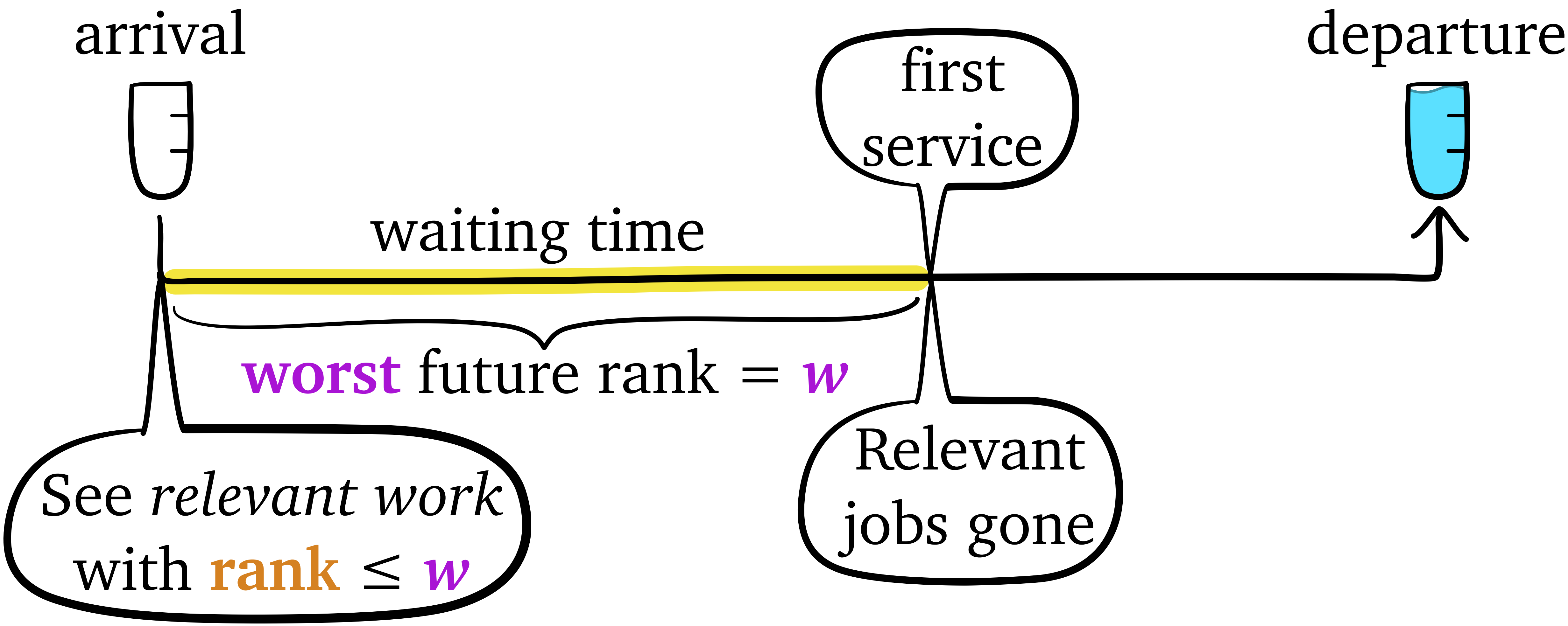
$U[w]$ = *relevant work*

Waiting Time



$U[w]$ = *relevant work*

Waiting Time

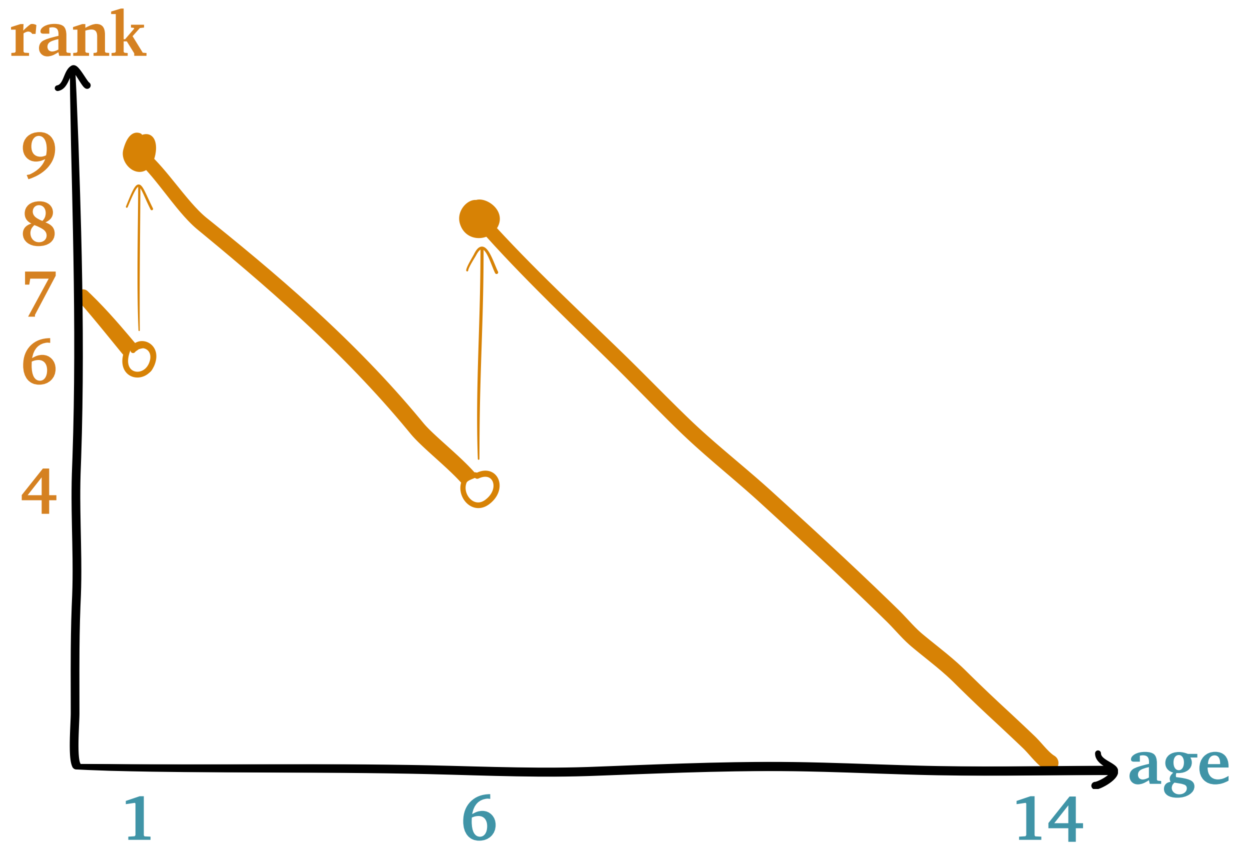


$U[w]$ = *relevant work*

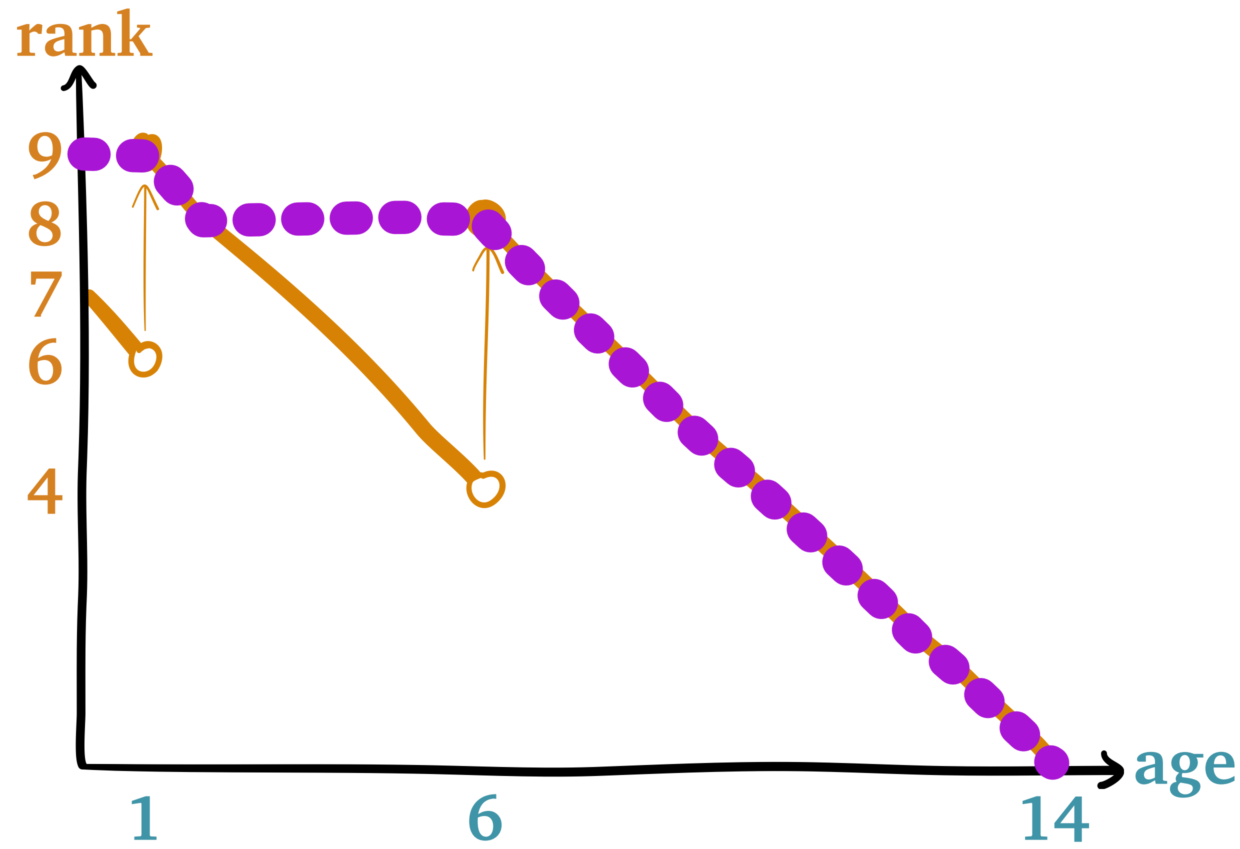
Waiting time is *busy period* started by $U[w]$

Response Time: Size 14

Response Time: Size 14

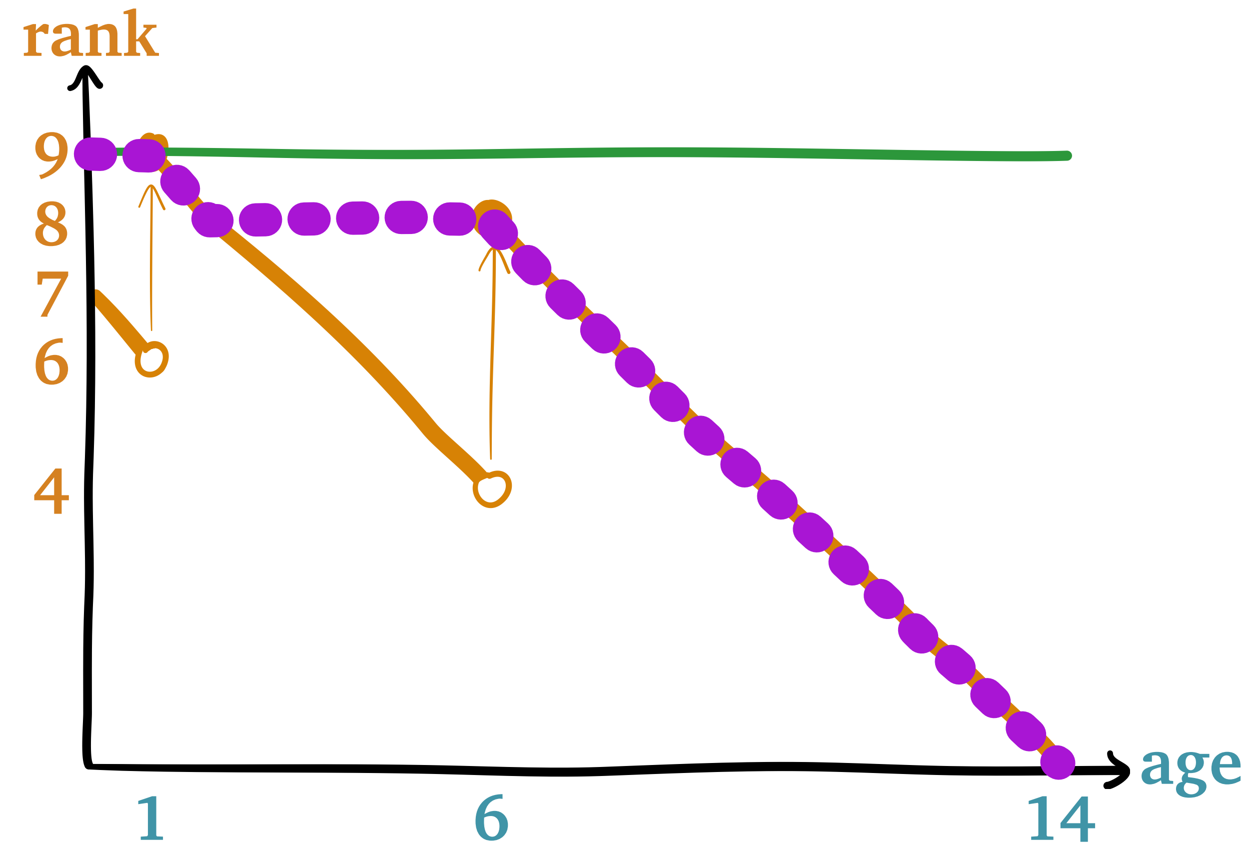


Response Time: Size 14



Response Time: Size 14

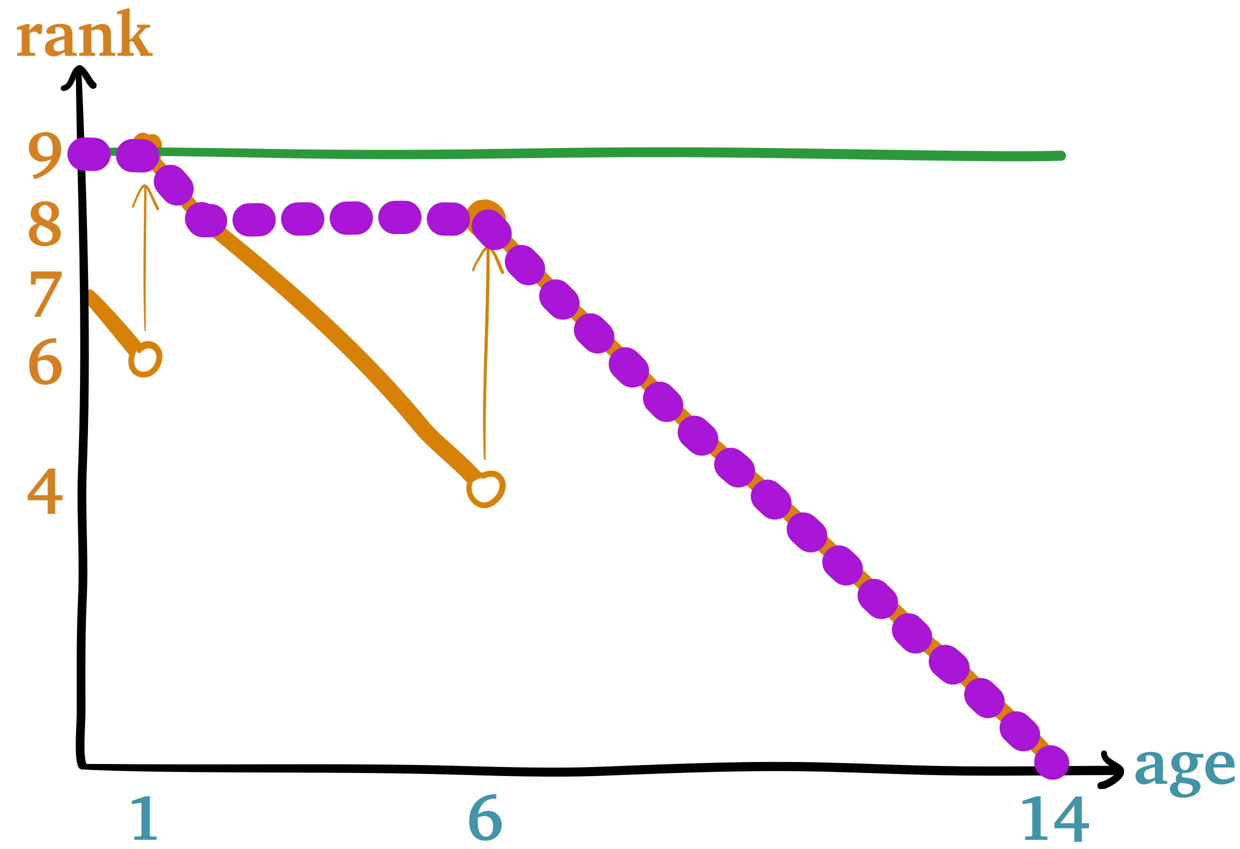
Relevant work ($w = 9$):



Response Time: Size 14

Relevant work ($w = 9$):

$$\mathbf{E}[U[9]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X^2]}{1-\rho}$$



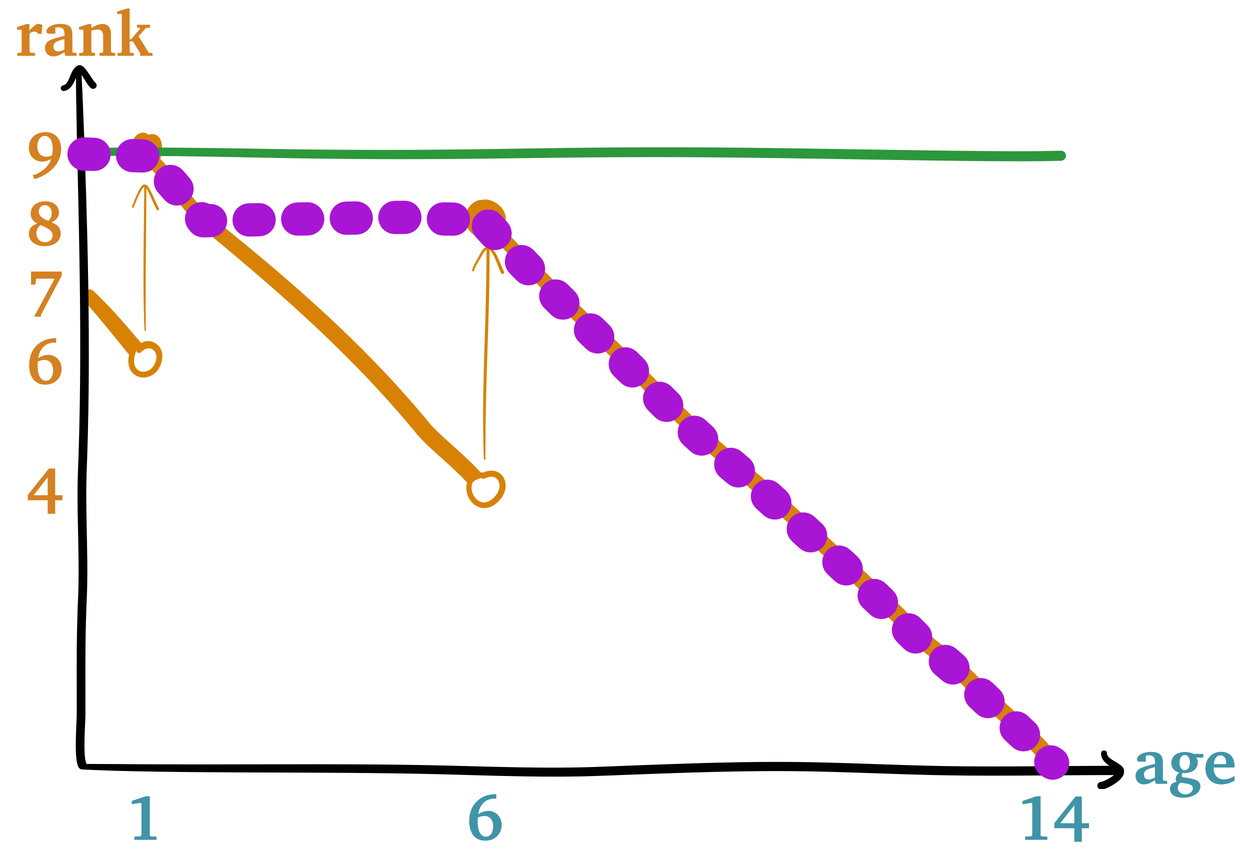
Response Time: Size 14

Relevant work ($w = 9$):

$$\mathbf{E}[U[9]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X^2]}{1-\rho}$$

Waiting time:

$$\mathbf{E}[Q_{14}] = \frac{\mathbf{E}[U[9]]}{1-\rho_{\text{new}}(0)}$$



Response Time: Size 14

Relevant work ($w = 9$):

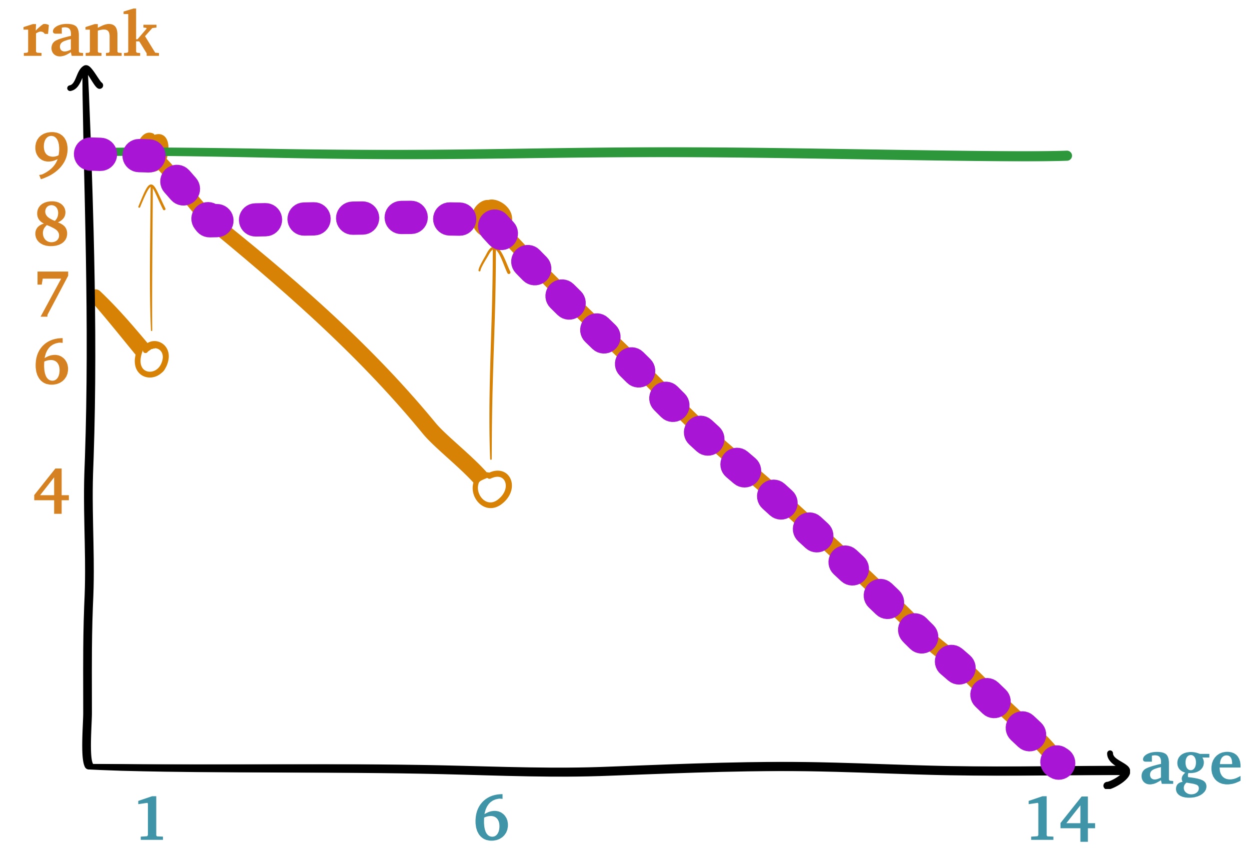
$$\mathbf{E}[U[9]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X^2]}{1-\rho}$$

Waiting time:

$$\mathbf{E}[Q_{14}] = \frac{\mathbf{E}[U[9]]}{1-\rho_{\text{new}}(0)}$$

Residence time:

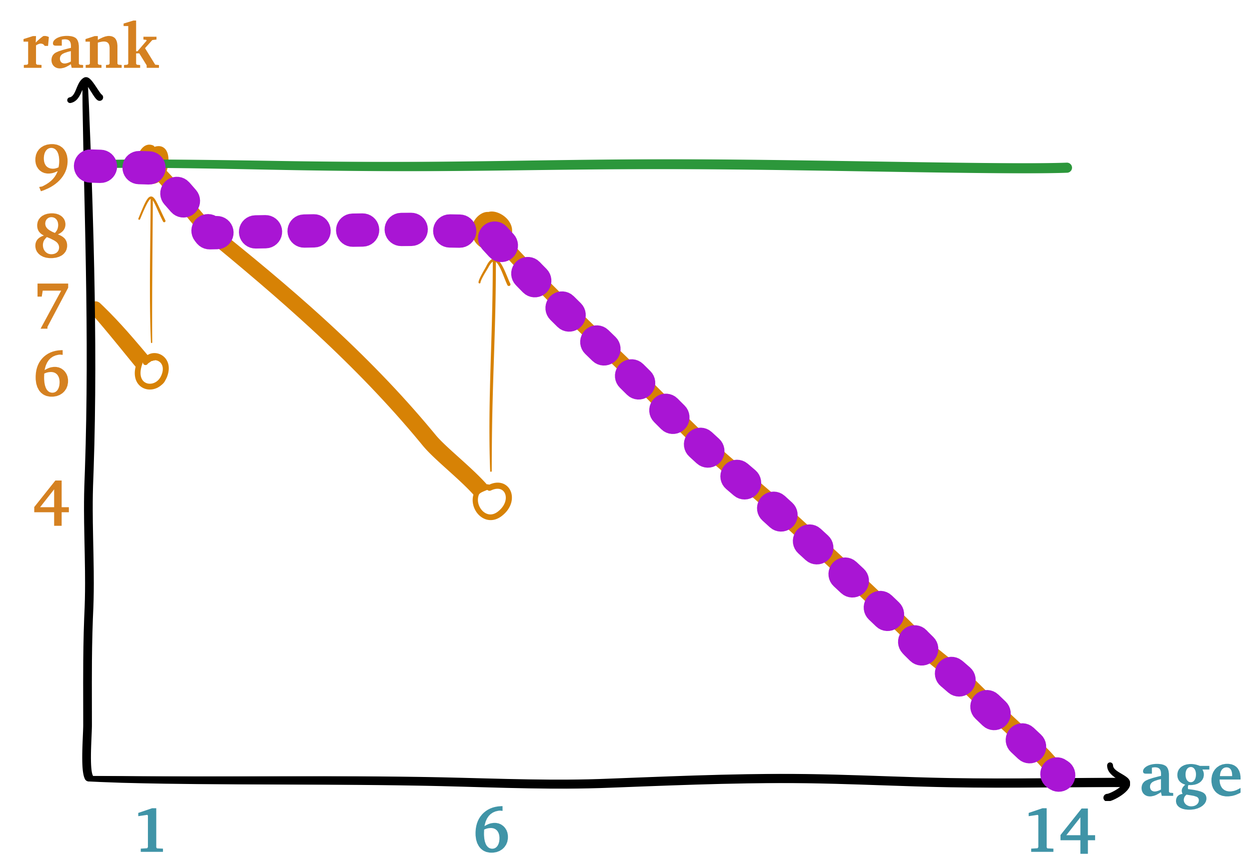
$$\mathbf{E}[R_{14}] = \int_0^{14} \frac{\mathrm{d}a}{1-\rho_{\text{new}}(a)}$$



Response Time: Size 14

Relevant work ($w = 9$):

$$\mathbf{E}[U[9]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X^2]}{1-\rho}$$



Waiting time:

$$\mathbf{E}[Q_{14}] = \frac{\mathbf{E}[U[9]]}{1-\rho_{\text{new}}(0)}$$

Residence time:

$$\mathbf{E}[R_{14}] = \int_0^{14} \frac{\mathrm{d}a}{1-\rho_{\text{new}}(a)}$$

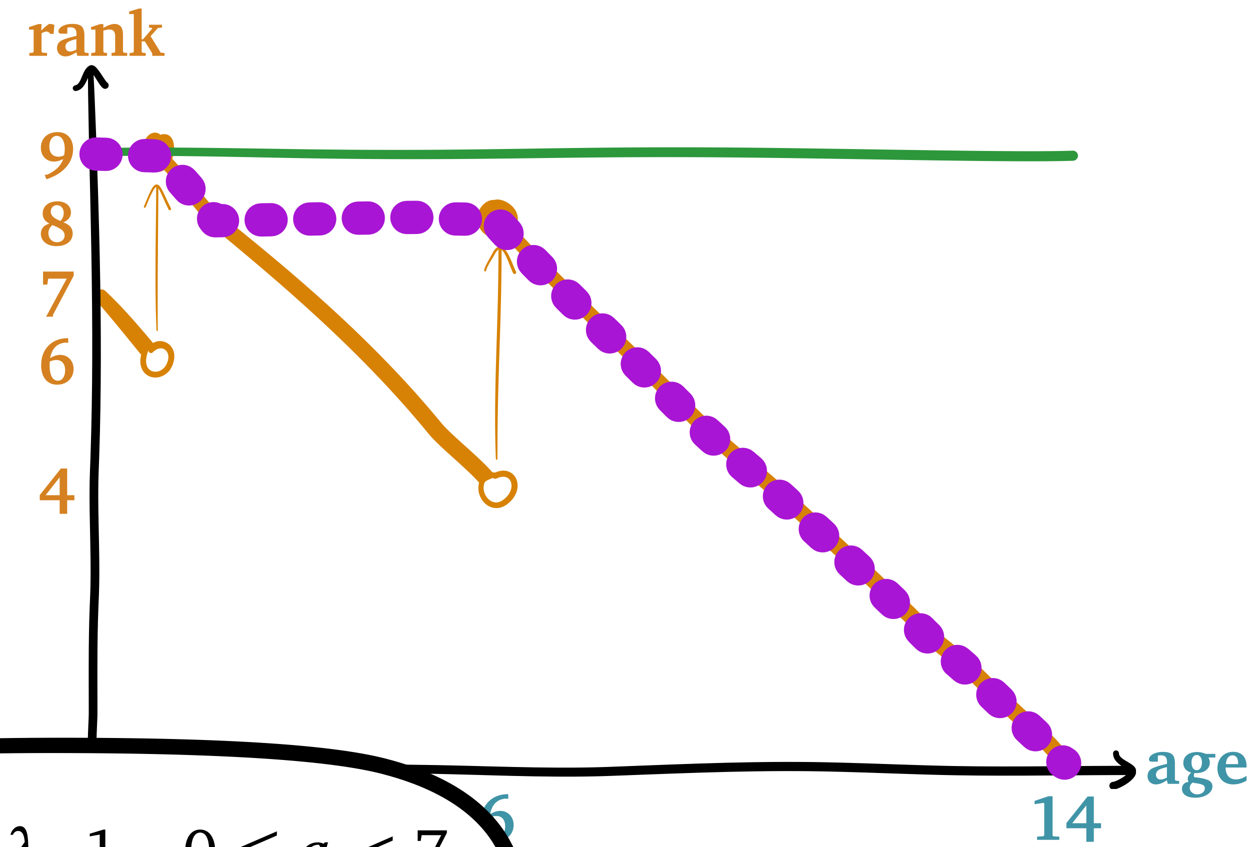
Response time:

$$\mathbf{E}[T_{14}] = \mathbf{E}[Q_{14}] + \mathbf{E}[R_{14}]$$

Response Time: Size 14

Relevant work ($w = 9$):

$$\mathbf{E}[U[9]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X^2]}{1-\rho}$$



Waiting time:

$$\mathbf{E}[Q_{14}] = \frac{\mathbf{E}[U[9]]}{1-\rho_{\text{new}}(0)}$$

$$\rho_{\text{new}}(a) = \begin{cases} \lambda \cdot 1 & 0 \le a < 7 \\ \lambda \cdot 0 & 7 \le a < 14 \end{cases}$$

Residence time:

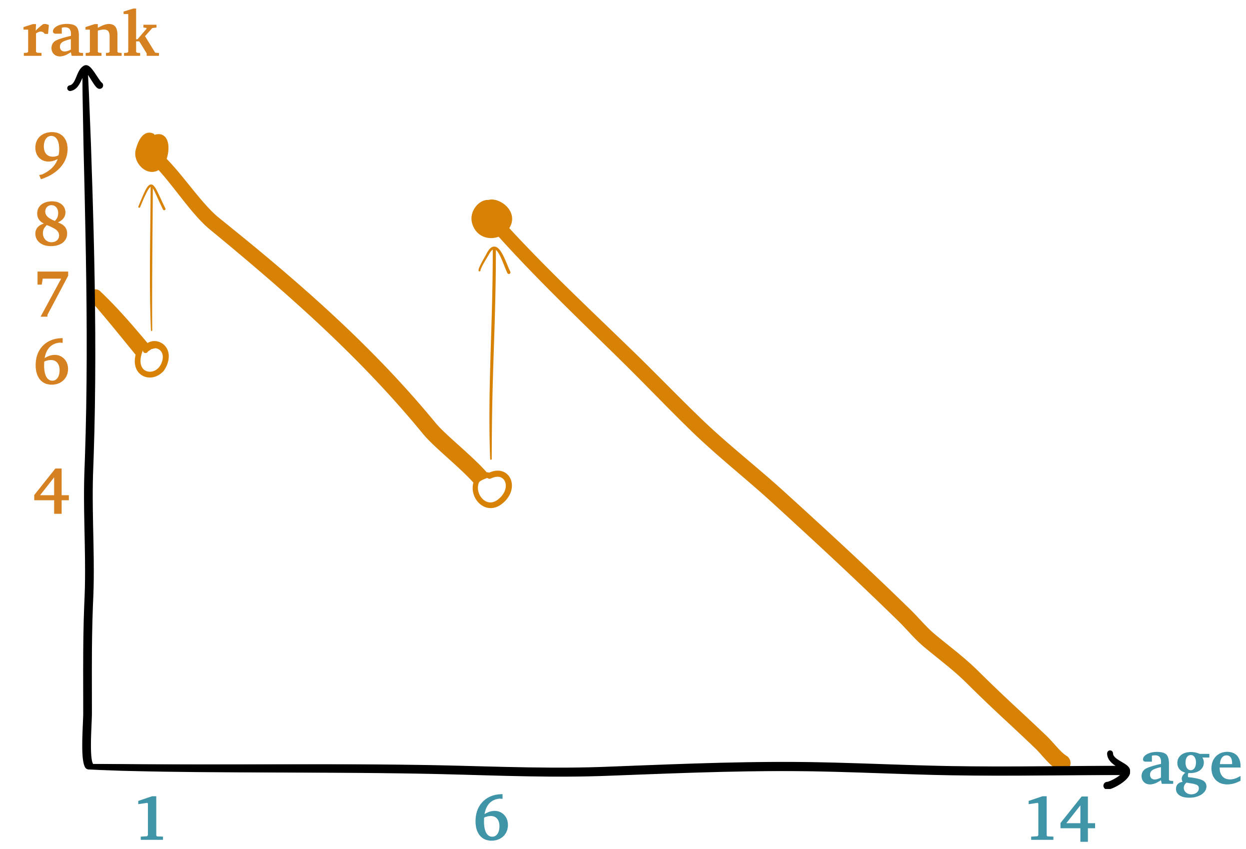
$$\mathbf{E}[R_{14}] = \int_0^{14} \frac{\mathrm{d}a}{1-\rho_{\text{new}}(a)}$$

time:

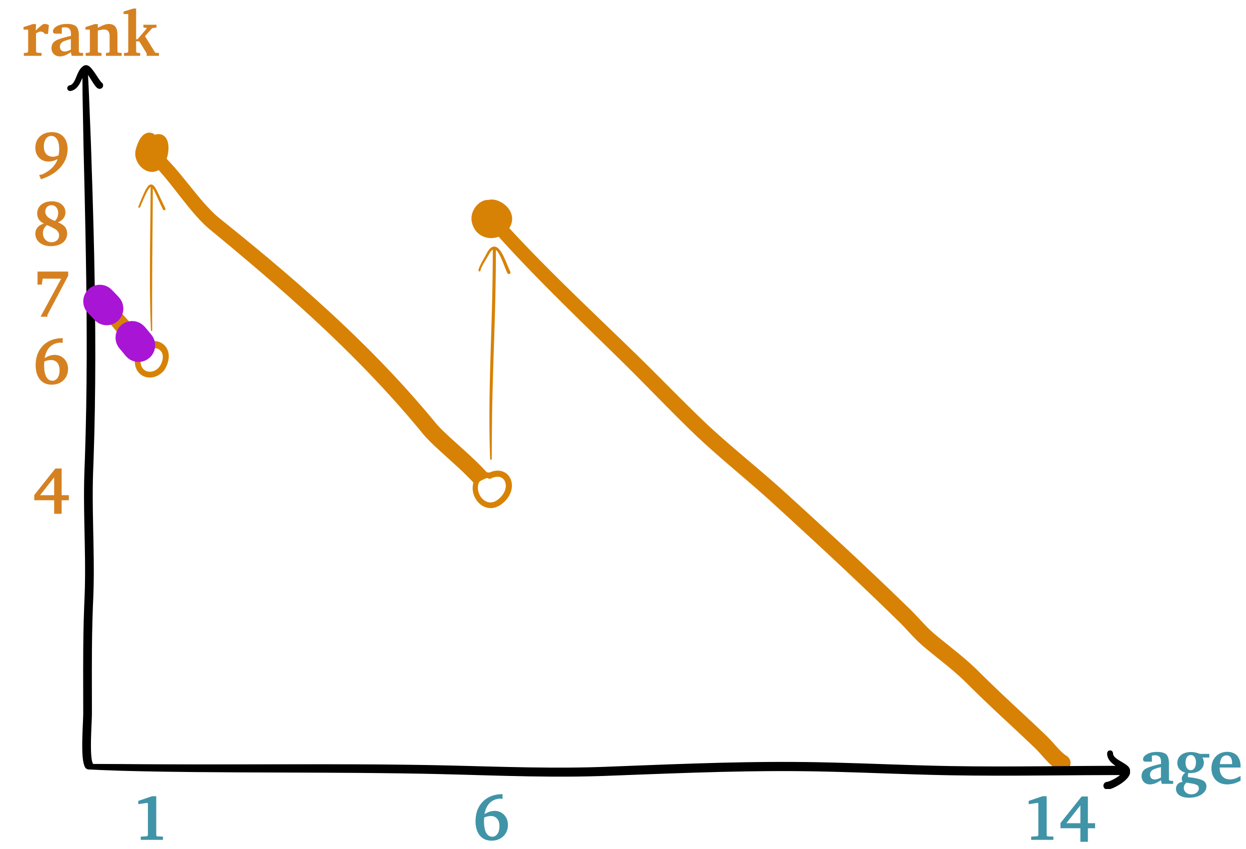
$$\mathbf{E}[T_{14}] = \mathbf{E}[Q_{14}] + \mathbf{E}[R_{14}]$$

Response Time: Size 1

Response Time: Size 1

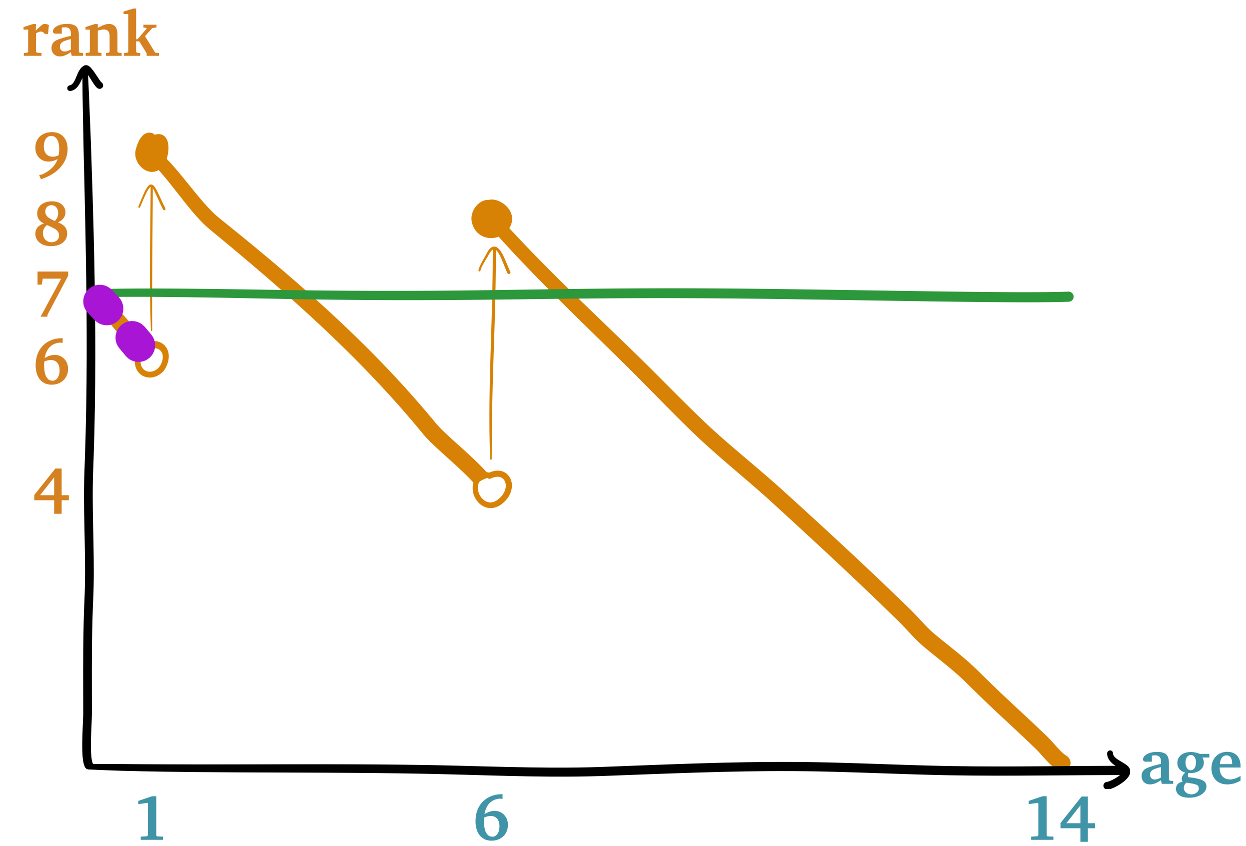


Response Time: Size 1



Response Time: Size 1

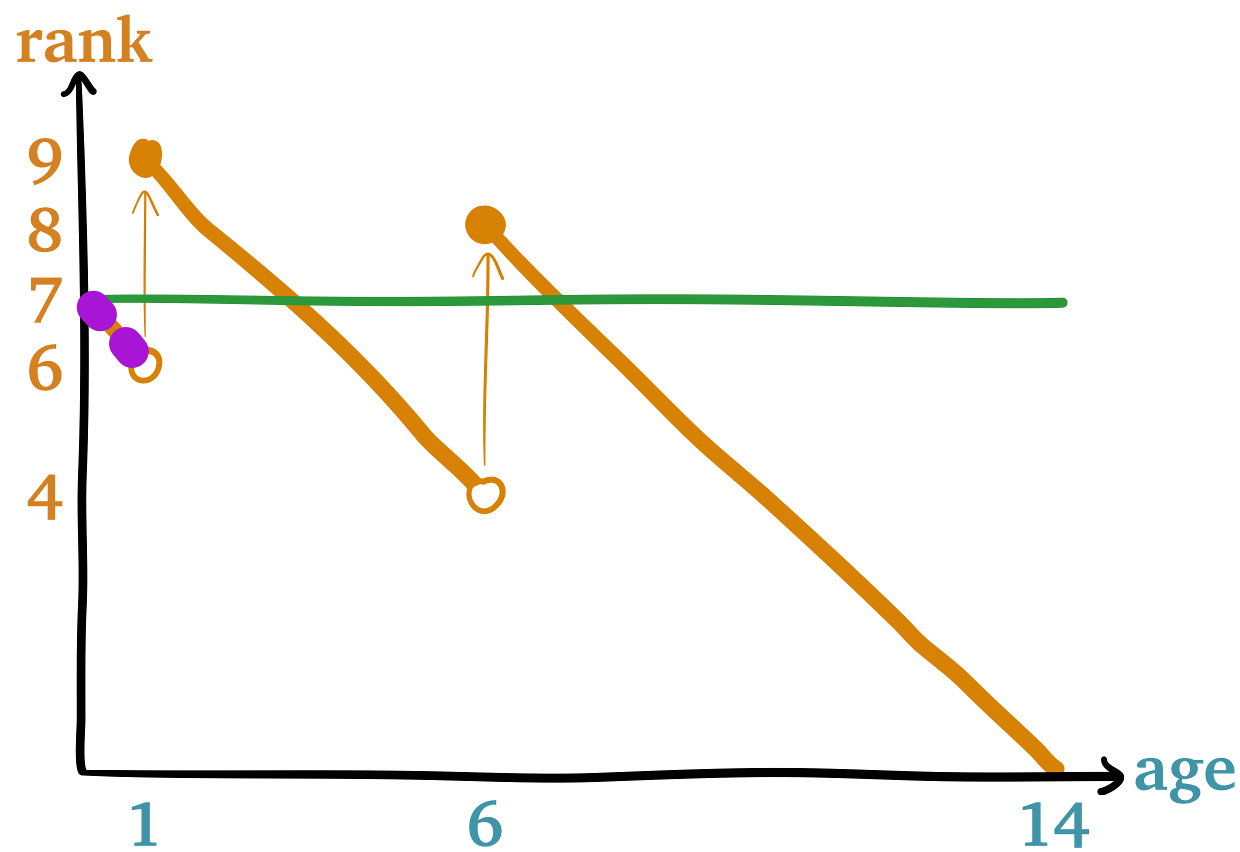
Relevant work ($w = 7$):



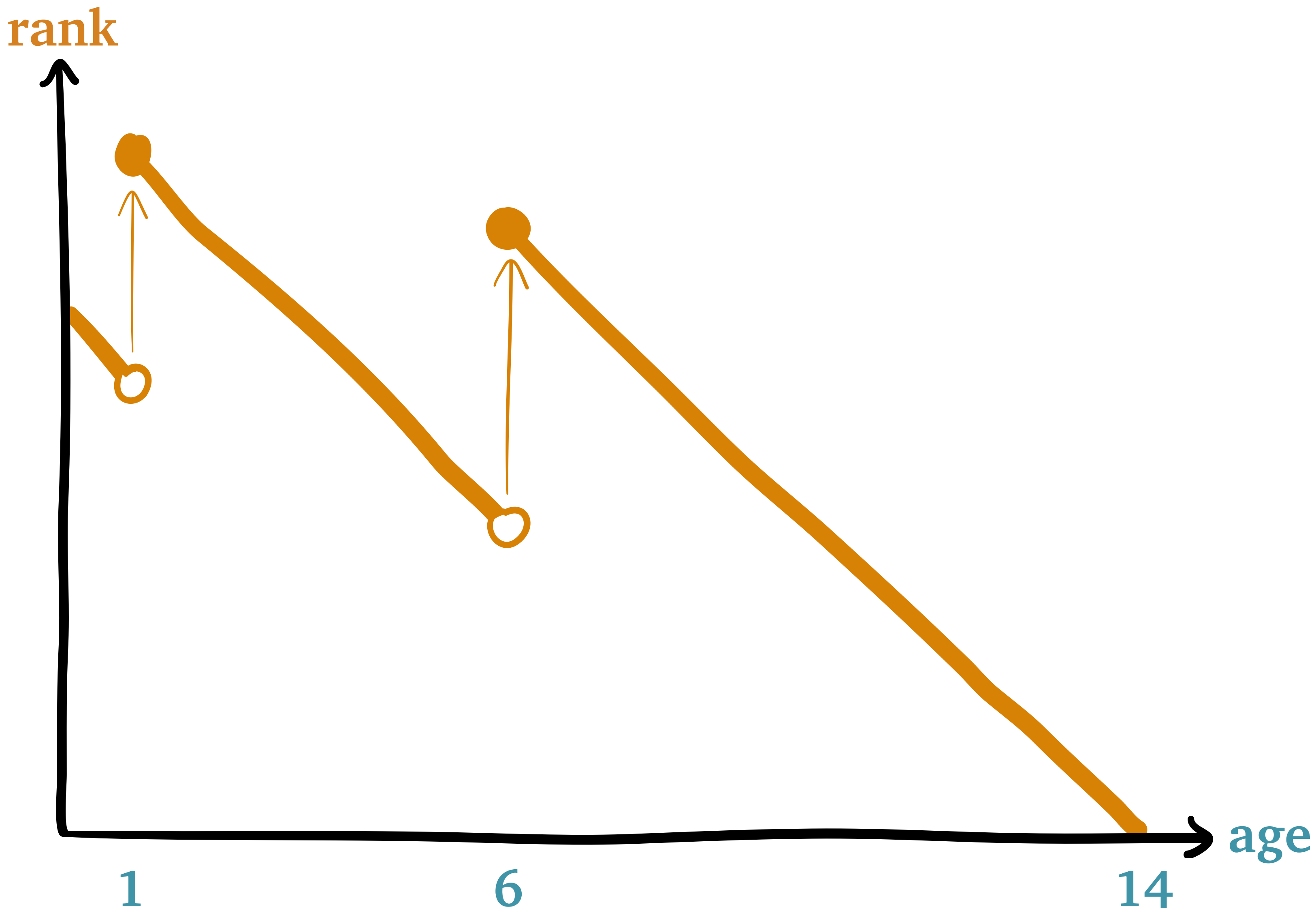
Response Time: Size 1

Relevant work ($\color{purple}{w} = 7$):

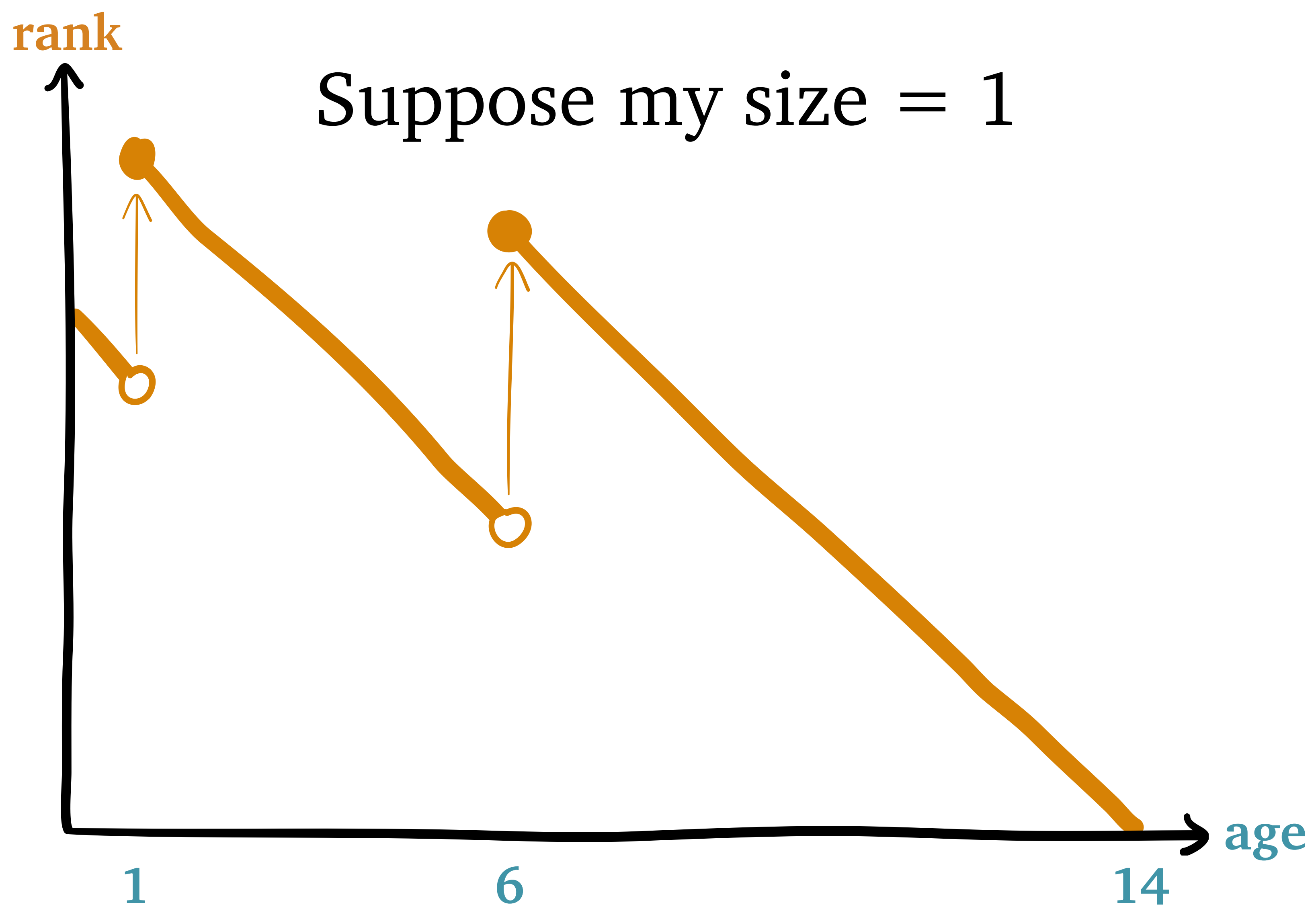
$$\mathbf{E}[U[7]] = ???$$



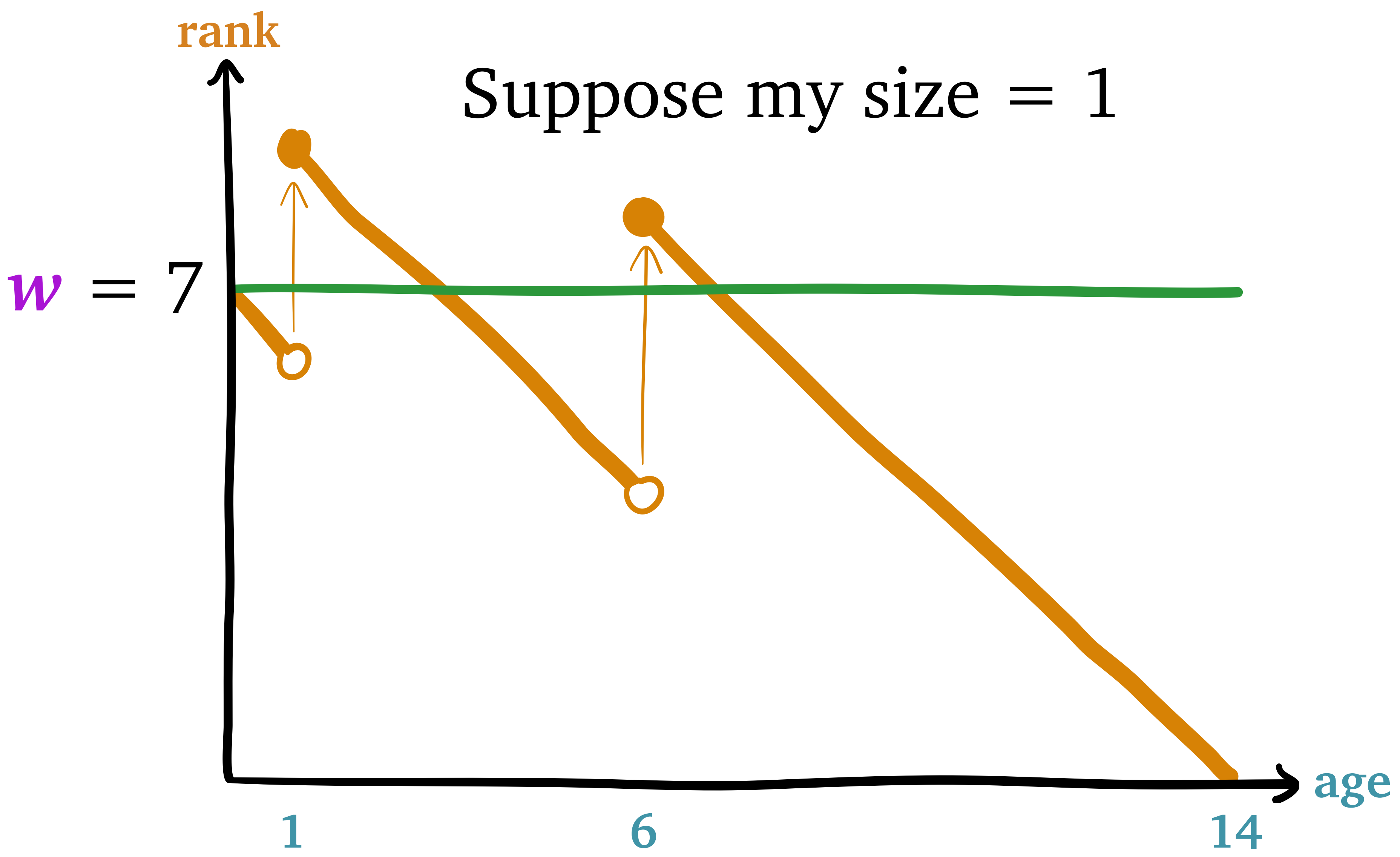
Relevant Work



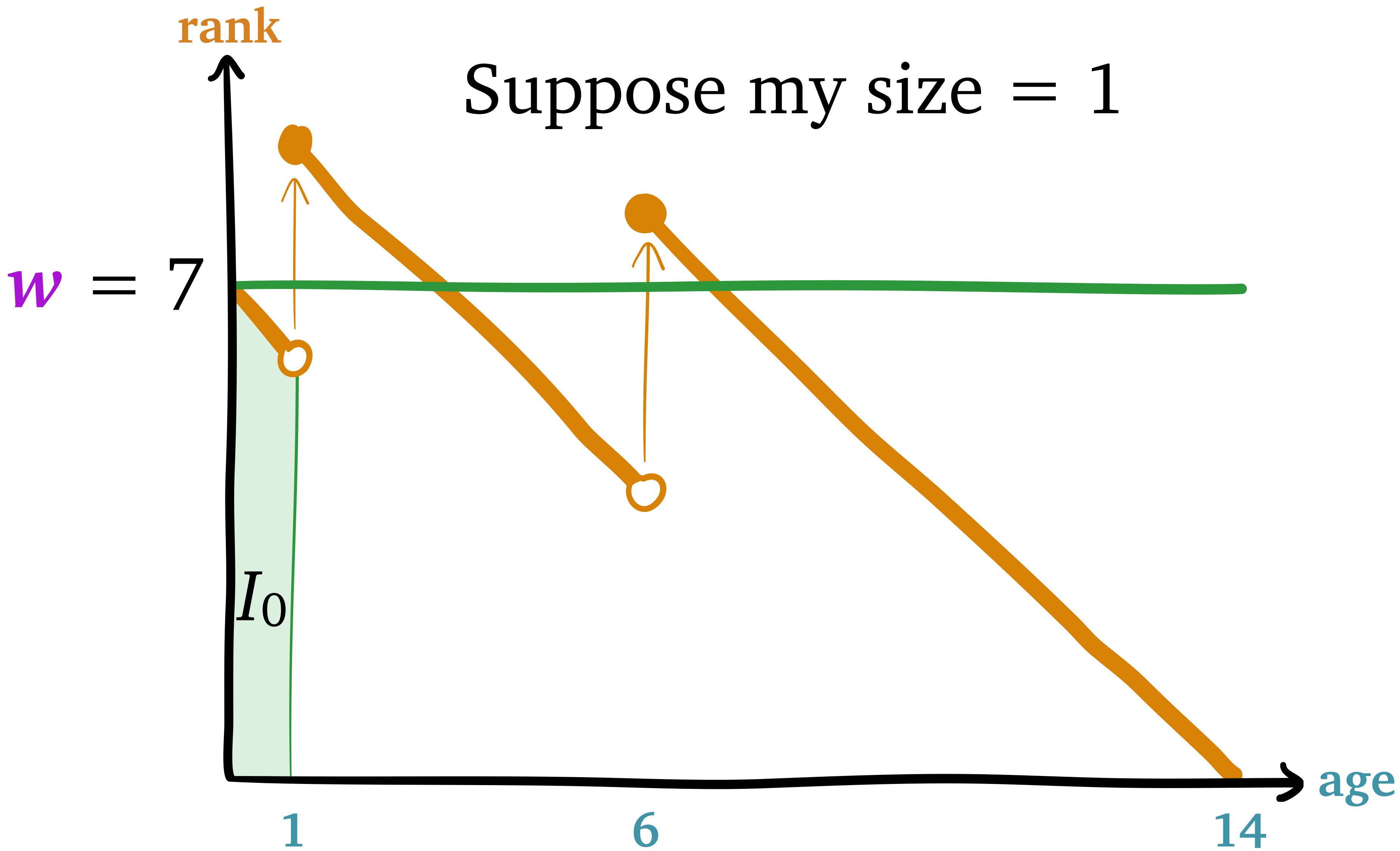
Relevant Work



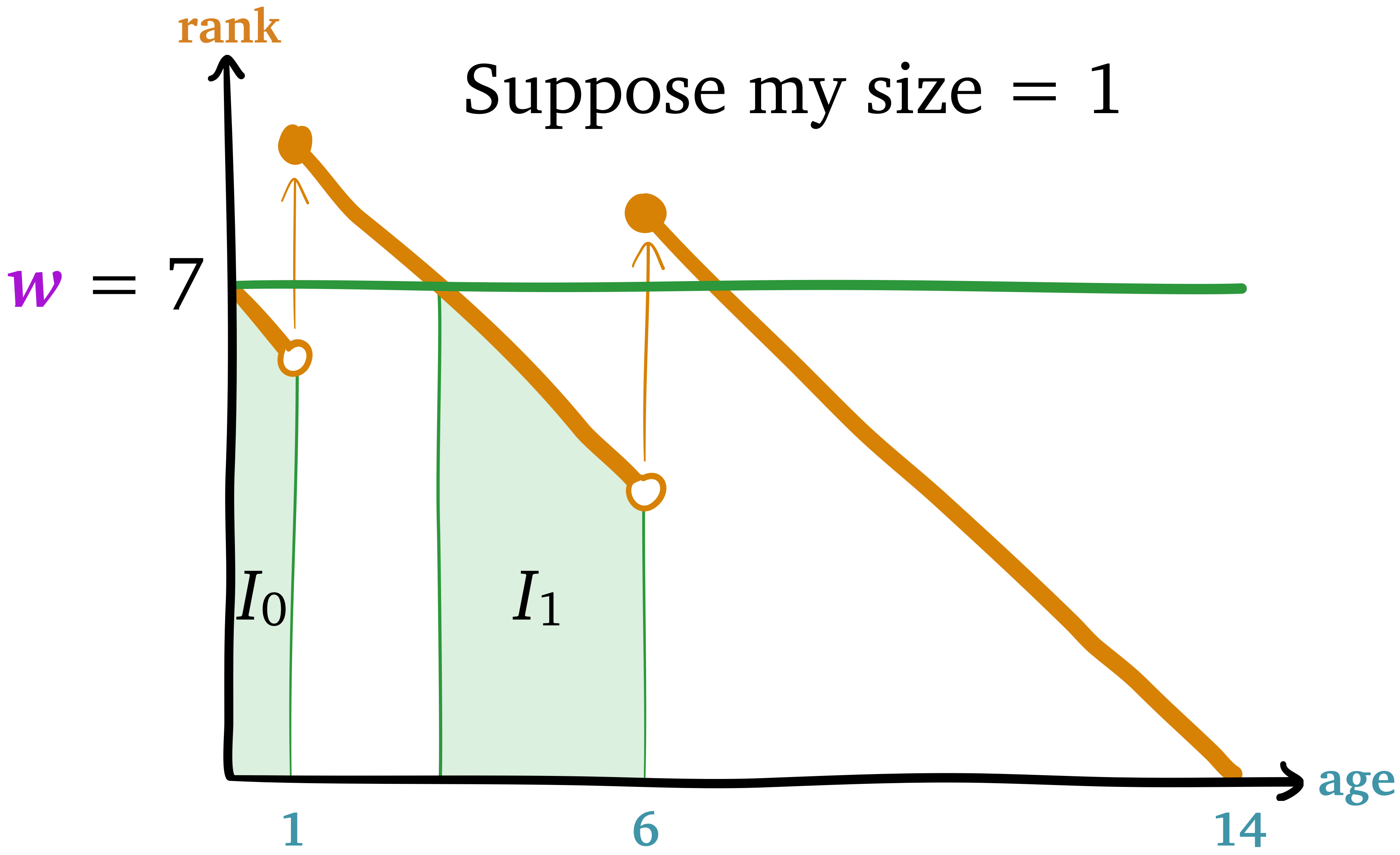
Relevant Work



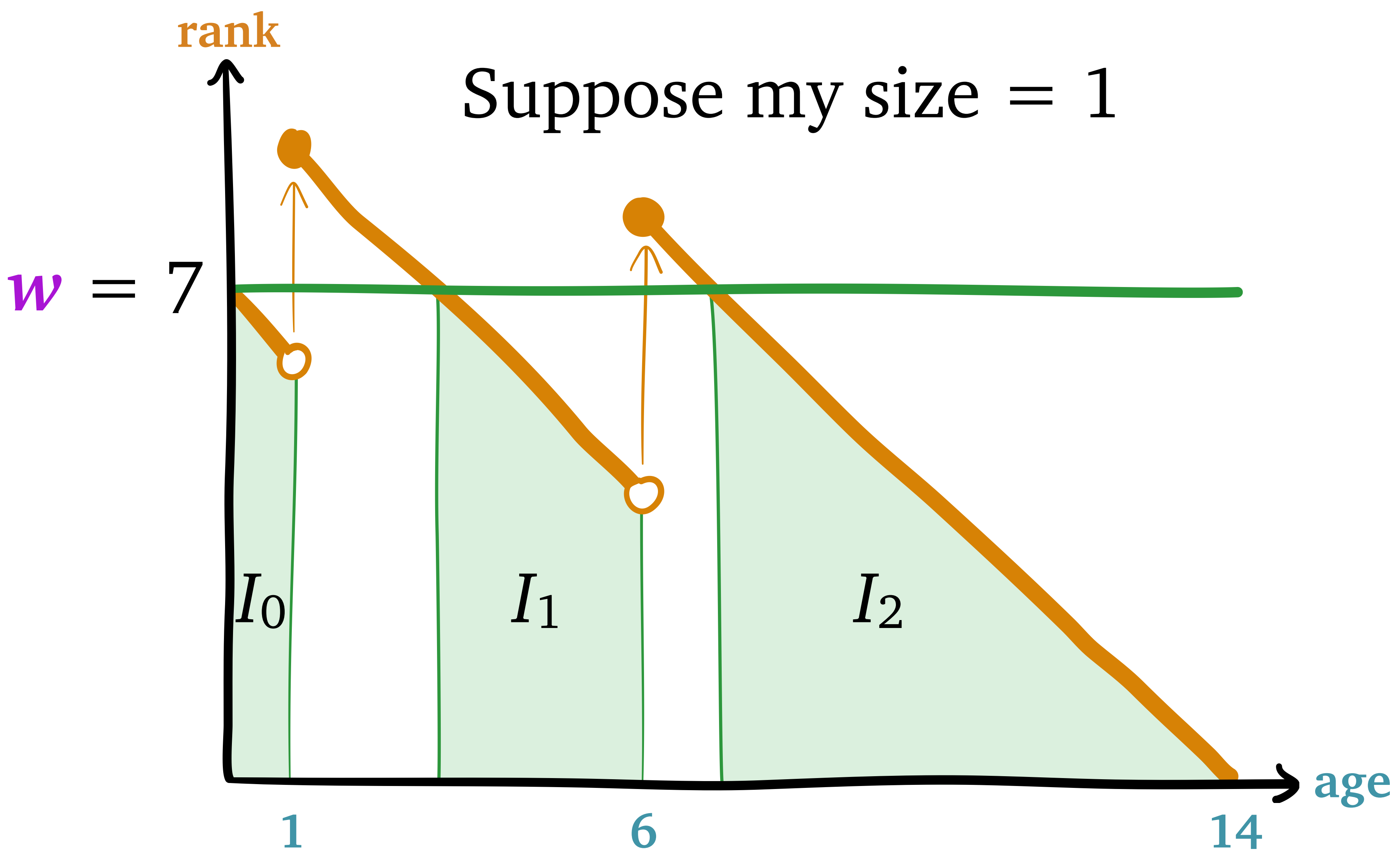
Relevant Work



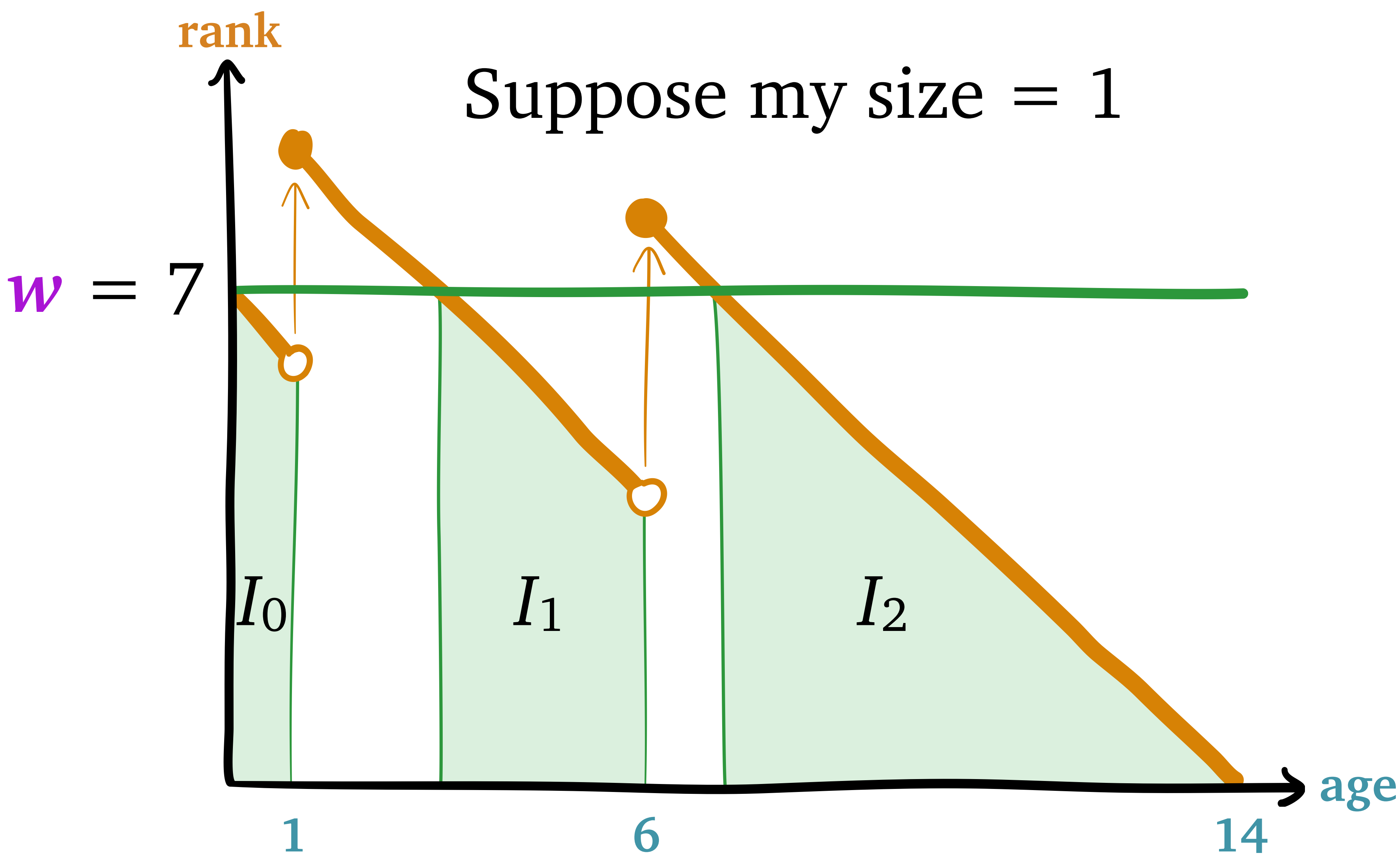
Relevant Work



Relevant Work

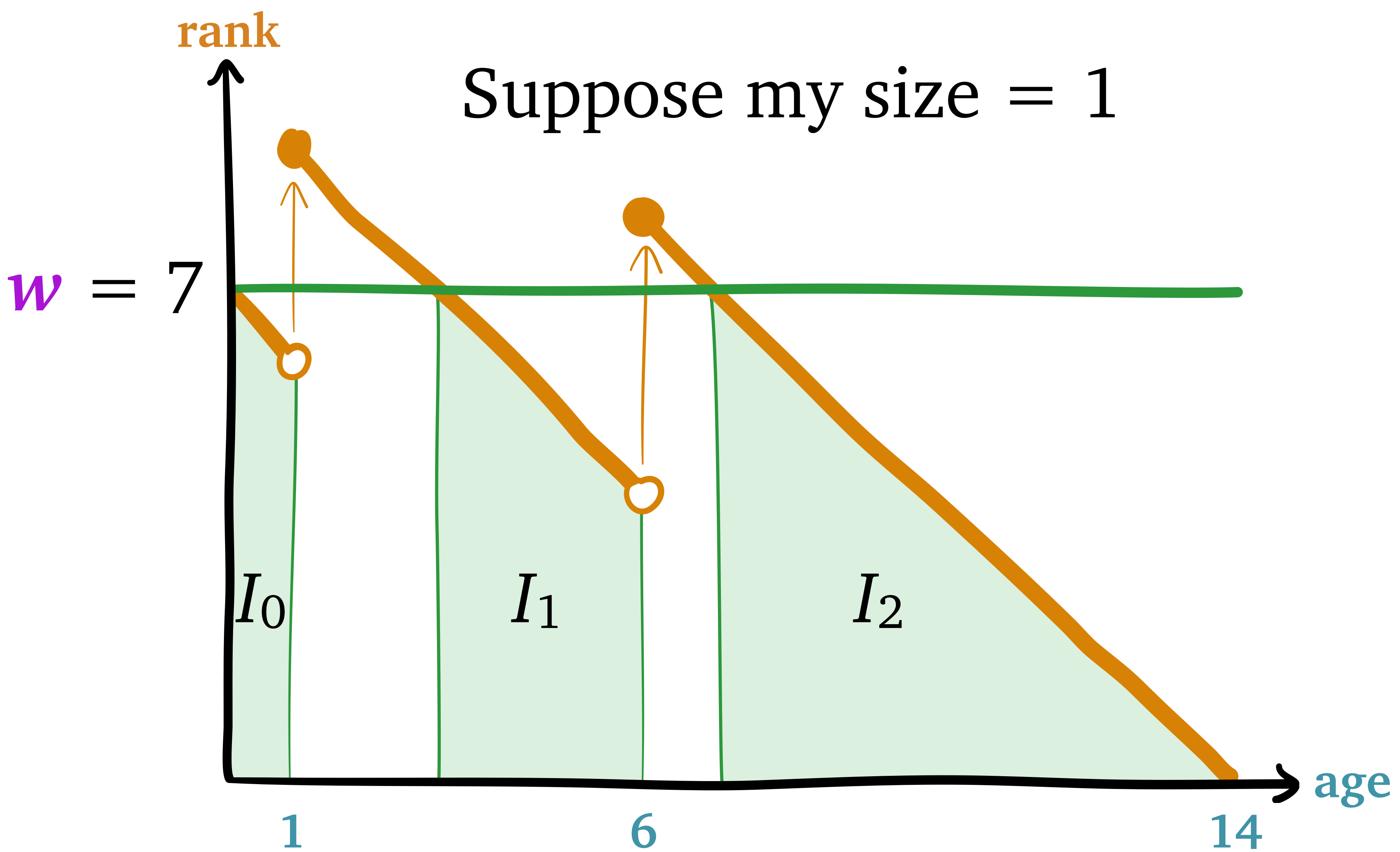


Relevant Work



Two causes of relevant work:

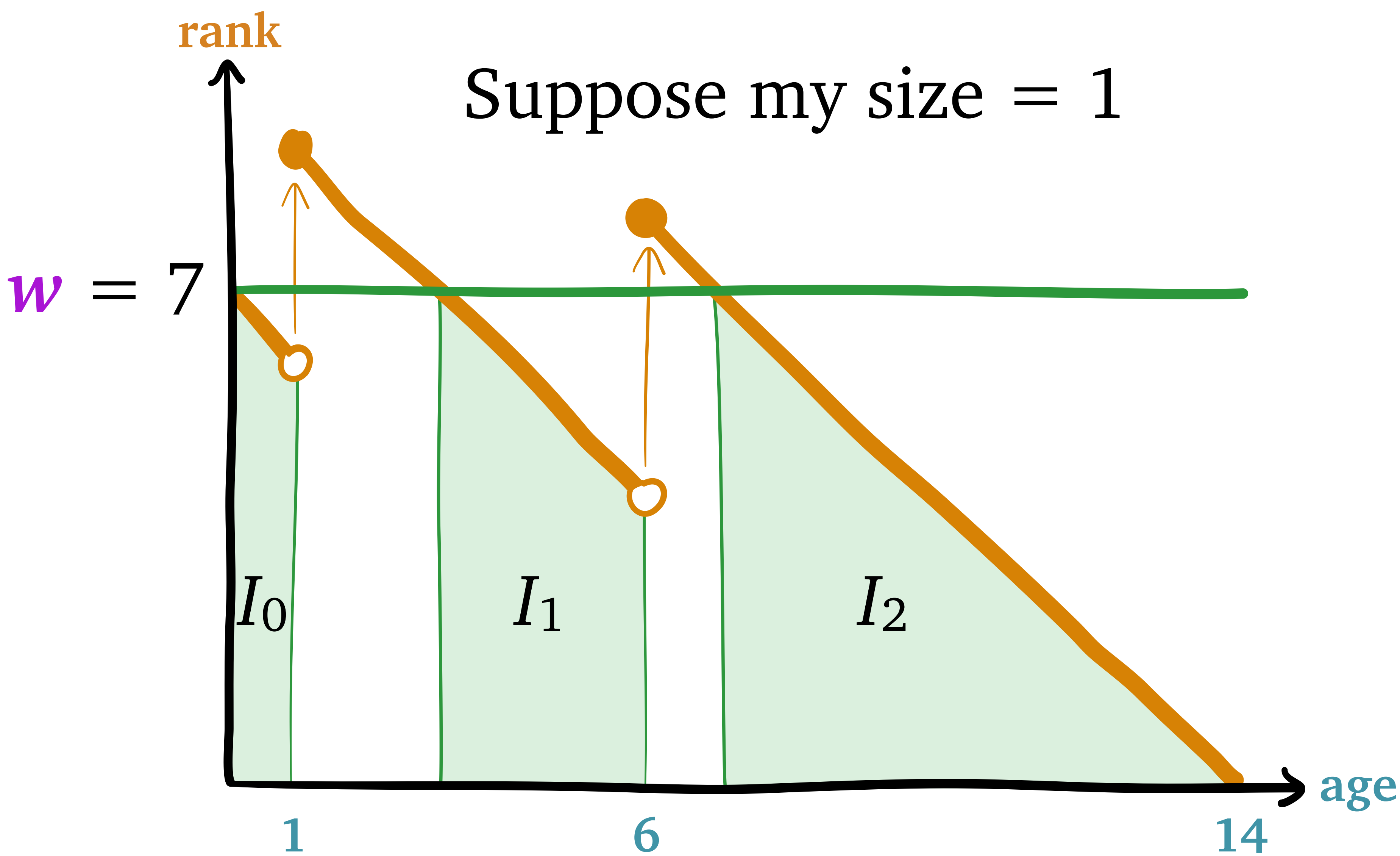
Relevant Work



Two causes of relevant work:

- I_0: arrivals

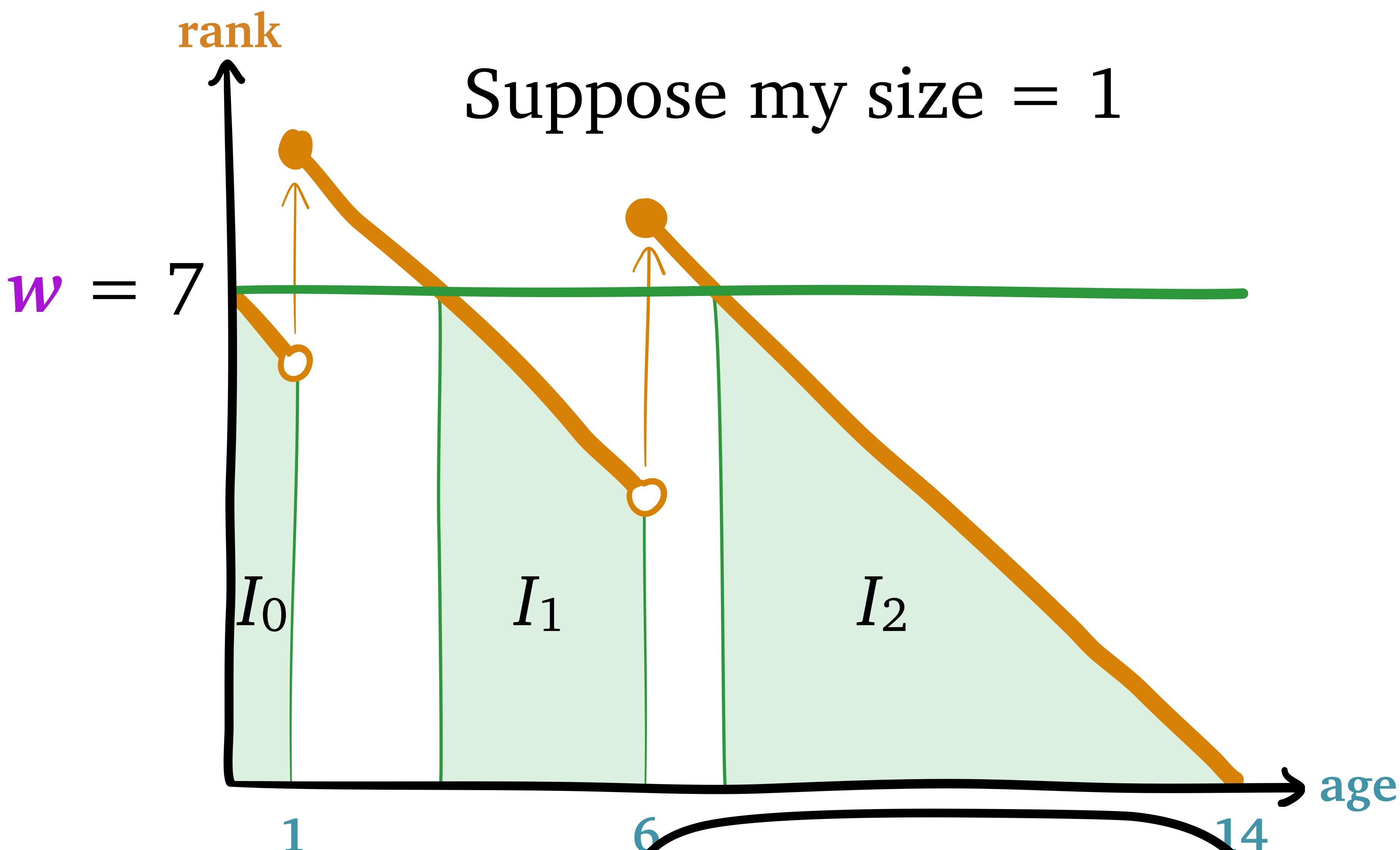
Relevant Work



Two causes of relevant work:

- I_0: arrivals
- I_1, I_2: **recyclings**

Relevant Work

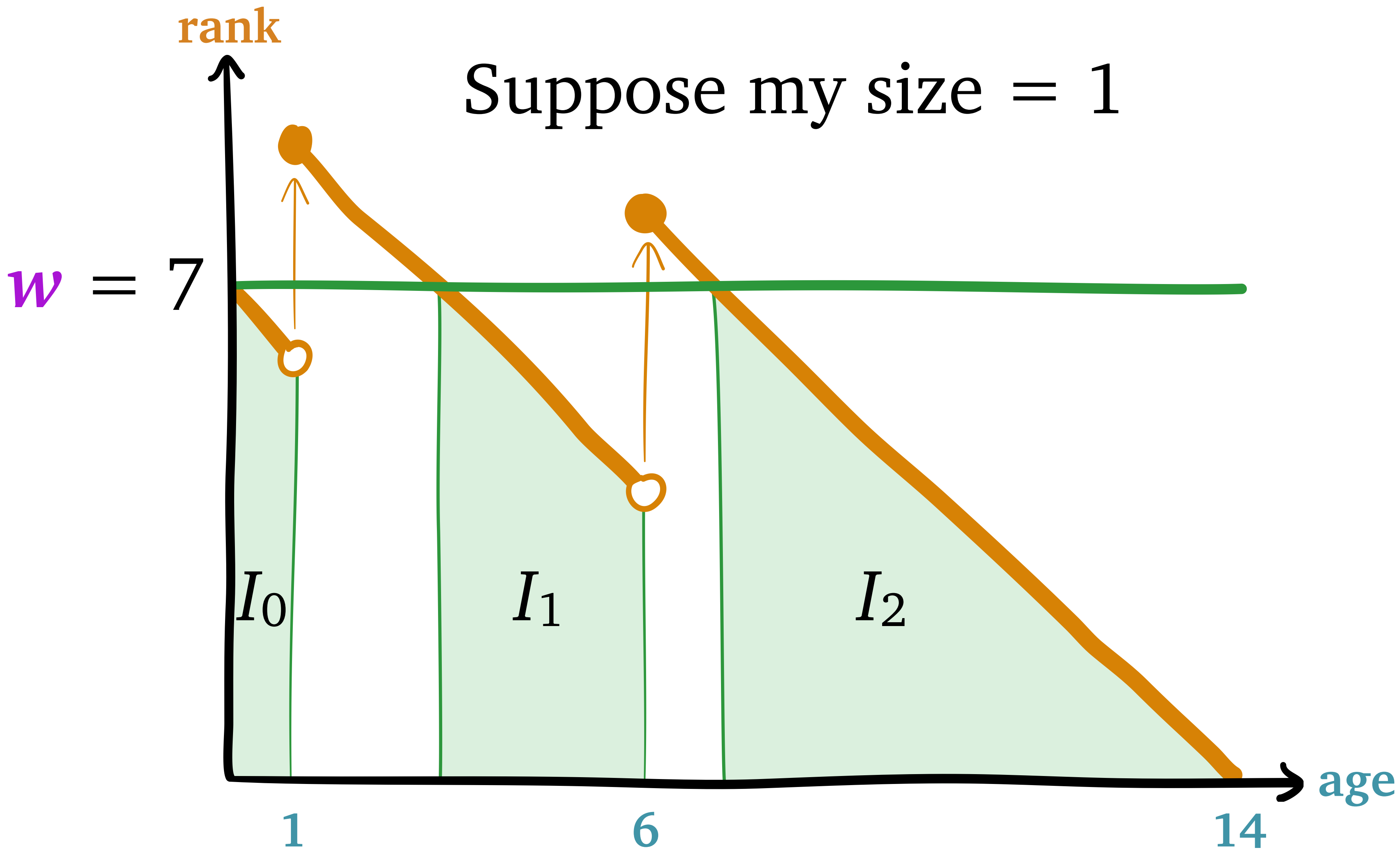


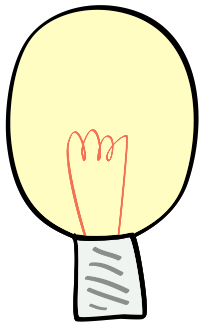
Two caus

go from **rank** $> w$
to **rank** $\leq w$

- I_0: arrivals
- I_1, I_2: **recyclings**

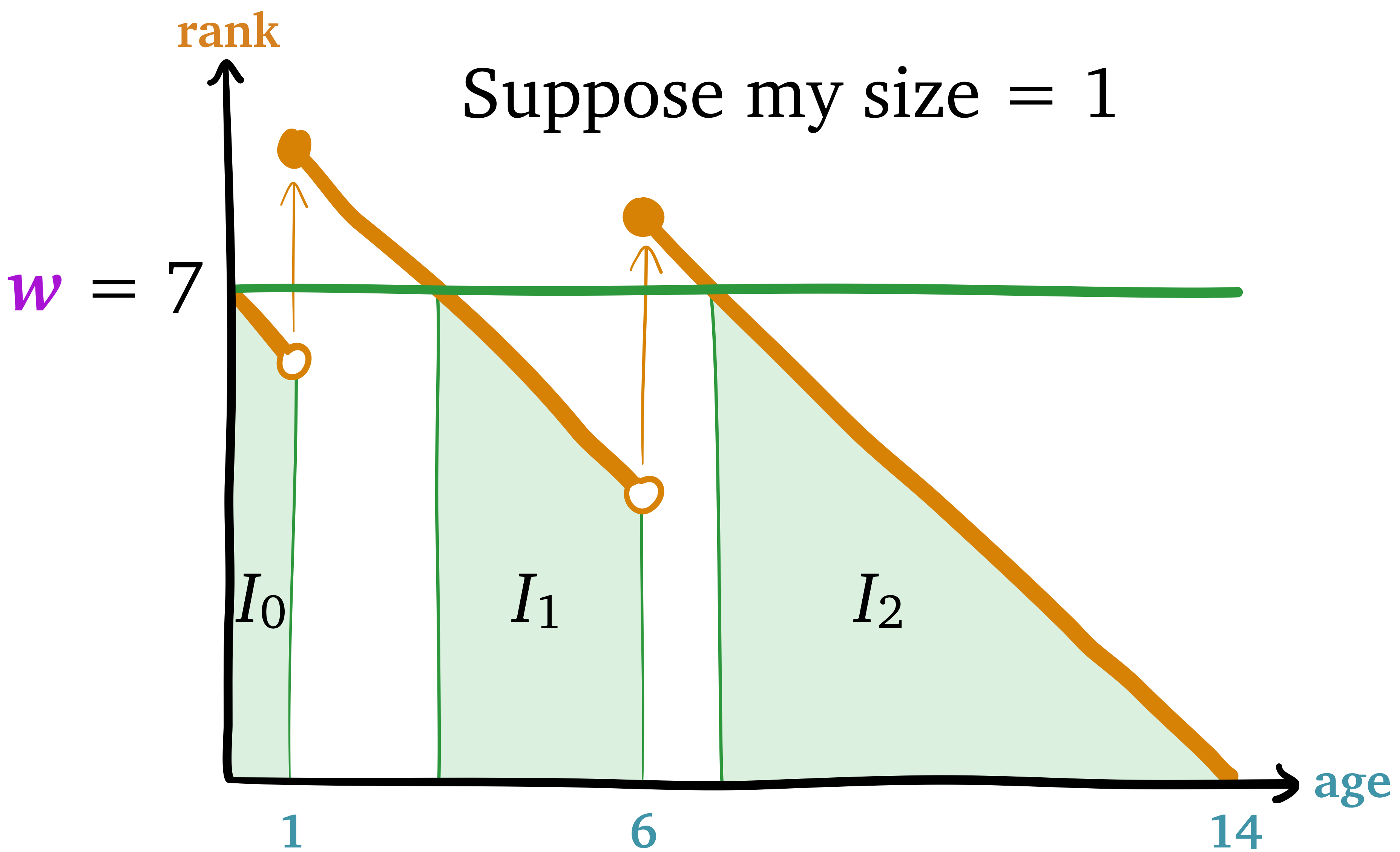
Relevant Work

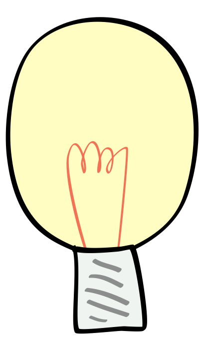




Observations:

Relevant Work

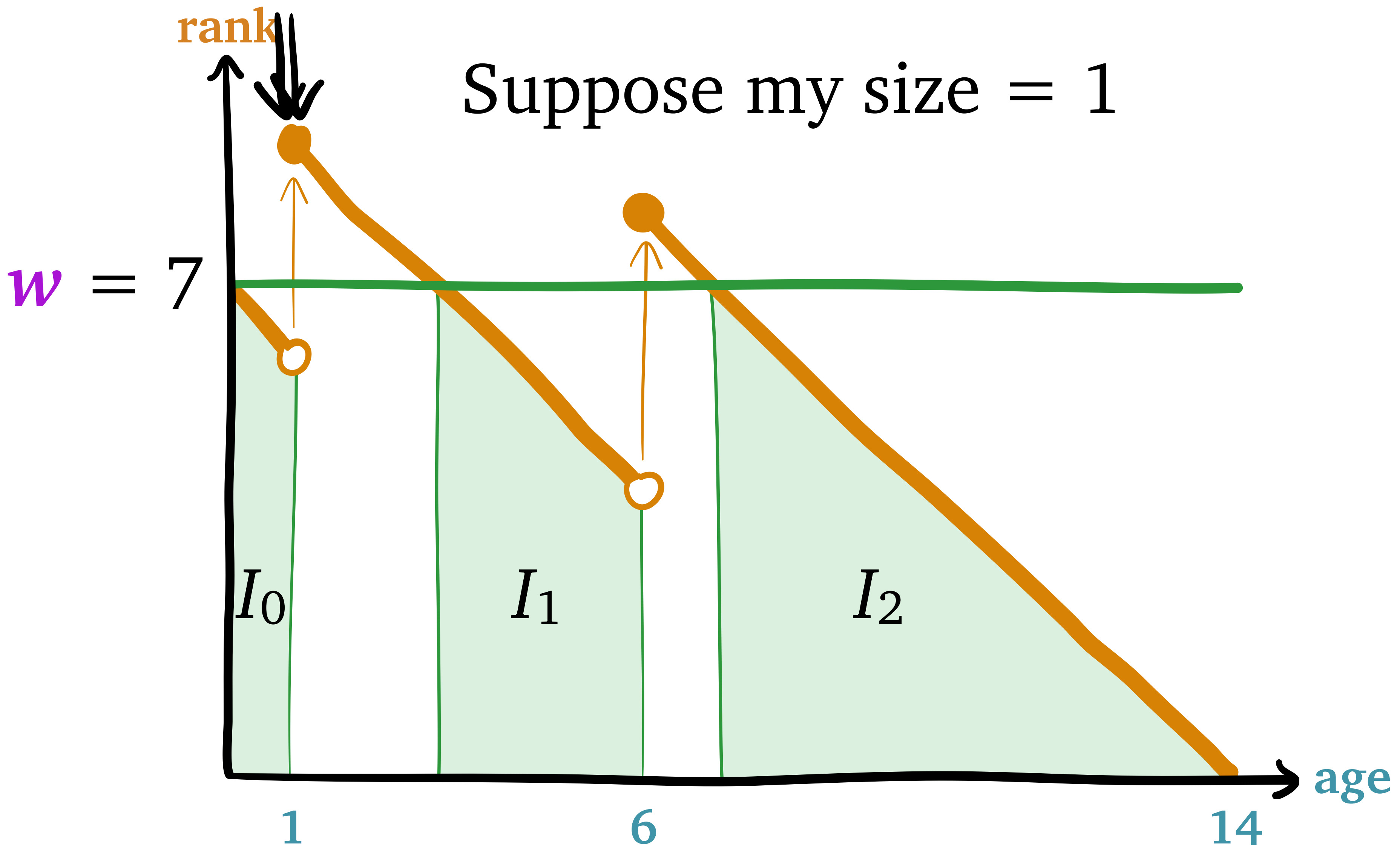


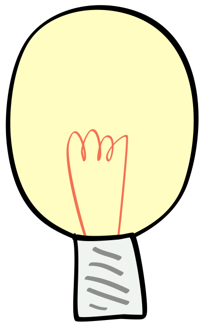


Observations:

- at most one **recycled** job at a time

Relevant Work

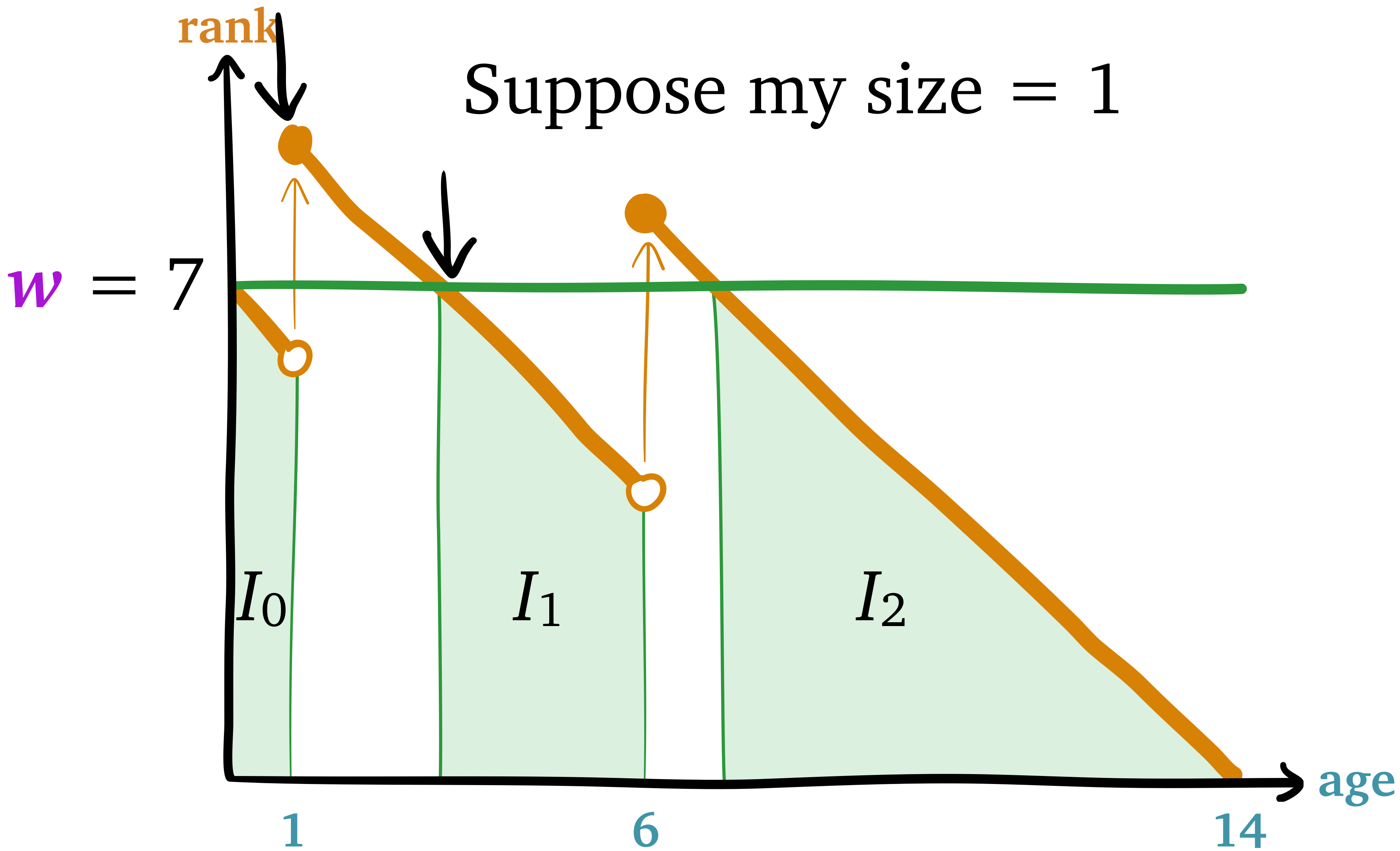


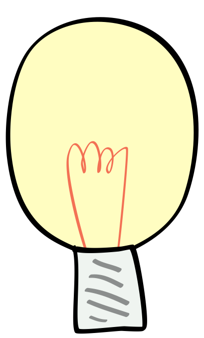


Observations:

- at most one **recycled** job at a time

Relevant Work

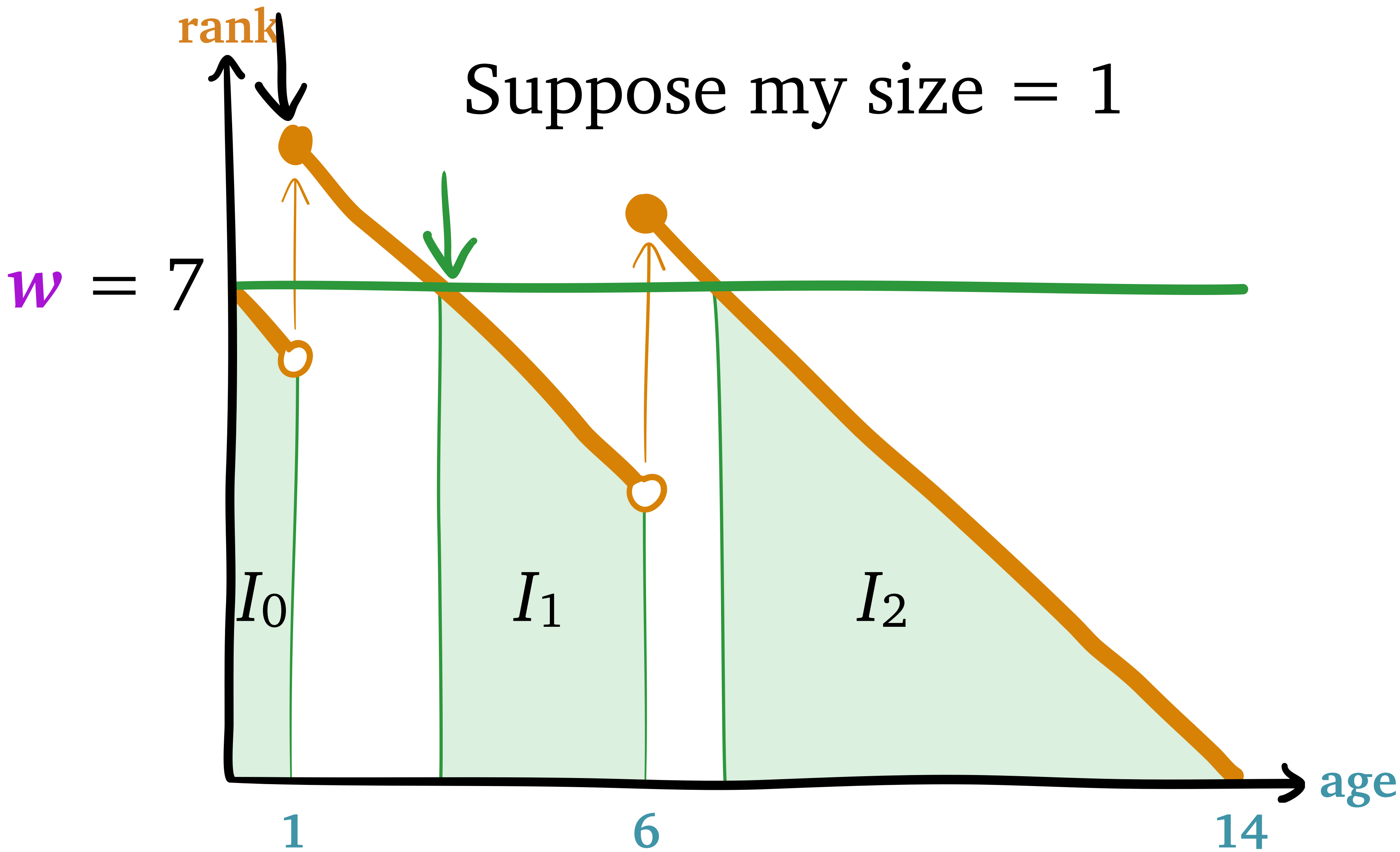


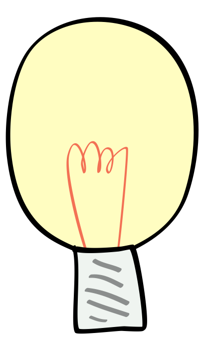


Observations:

- at most one **recycled** job at a time

Relevant Work

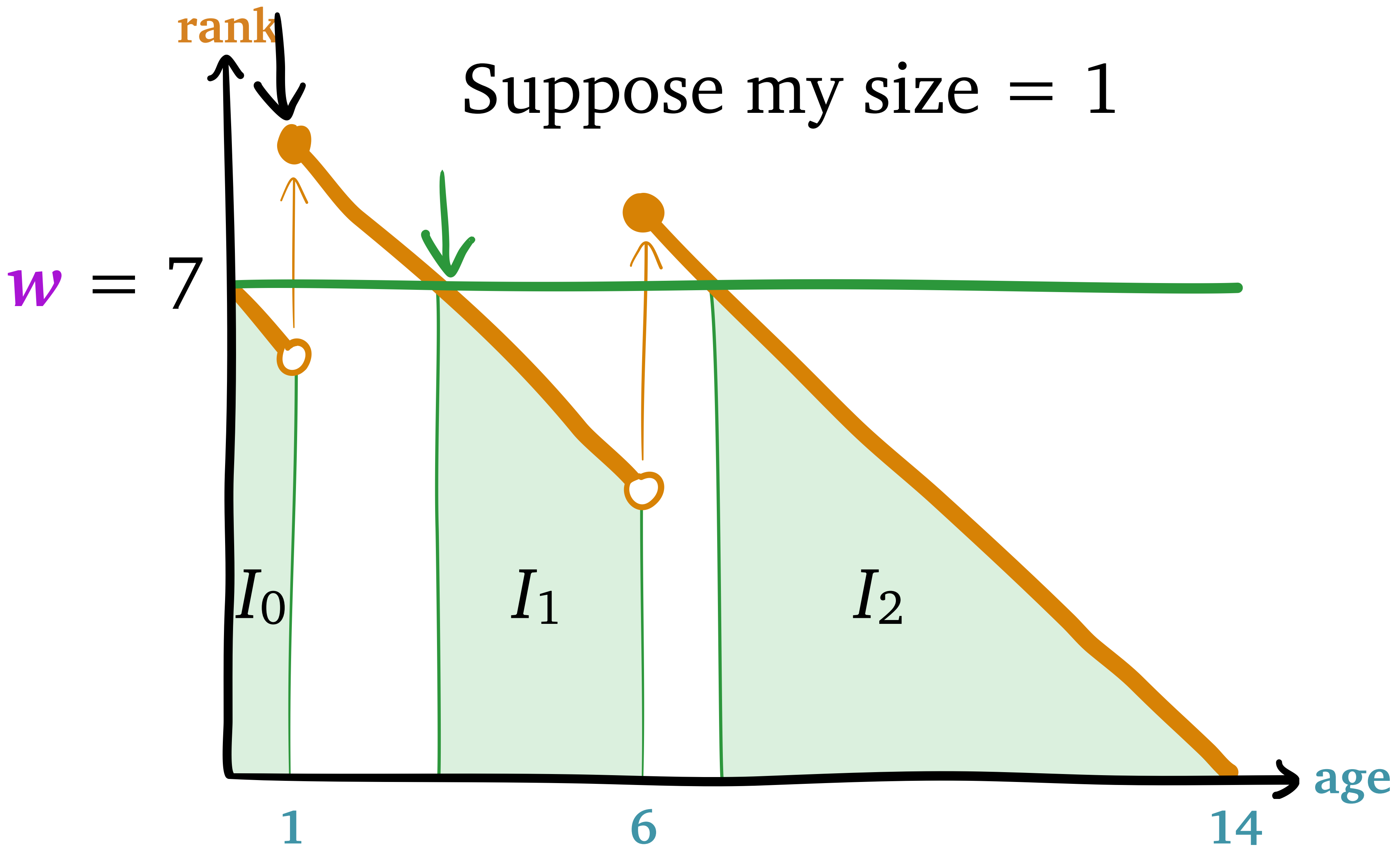


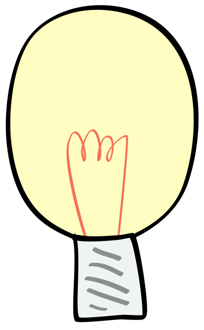


Observations:

- at most one **recycled** job at a time

Relevant Work





Observations:

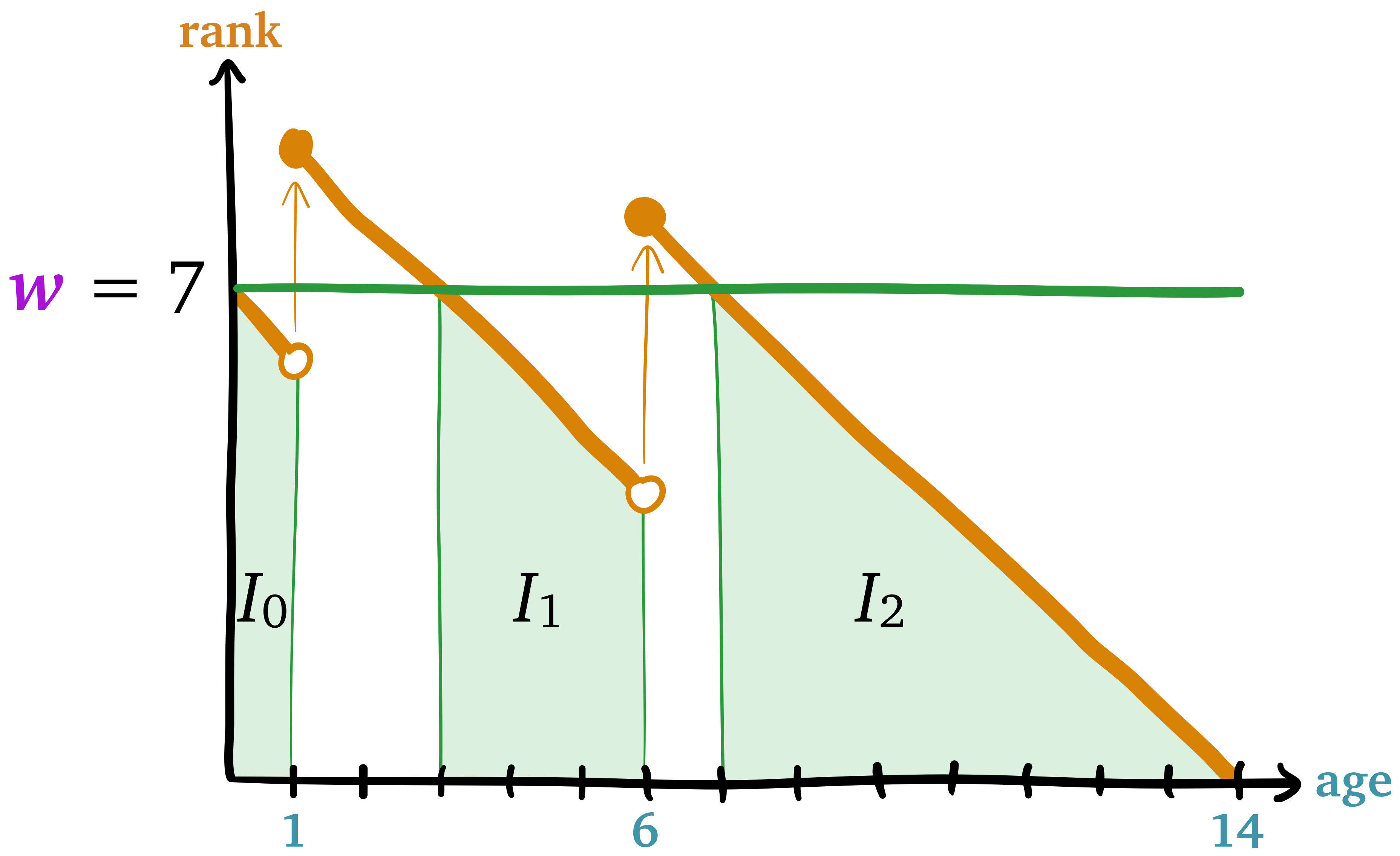
- at most one **recycled** job at a time
- **recyclings** occur only when no relevant work

SOAP Insight #2:

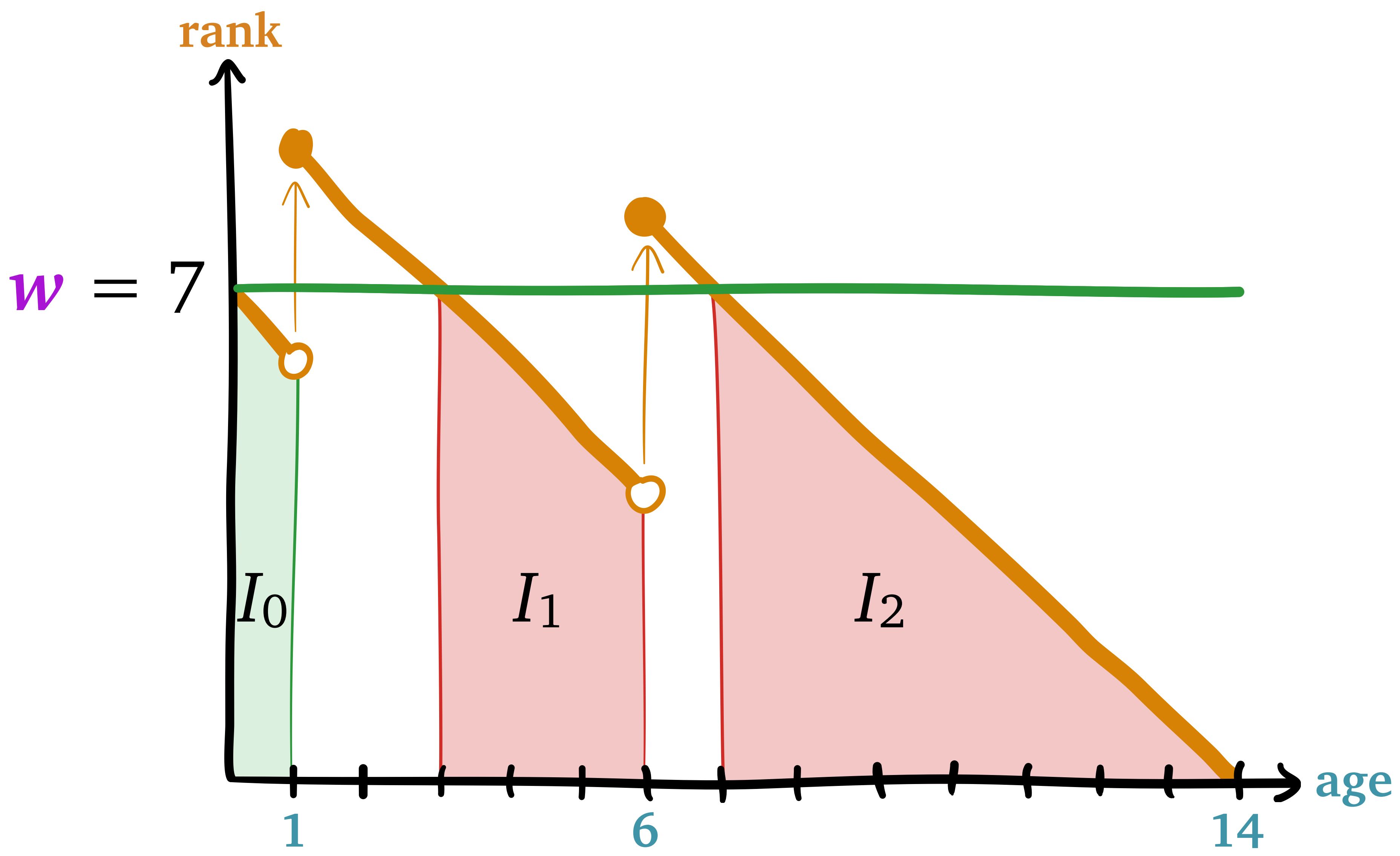
Vacation Transformation

Replace **recycled** jobs with server **vacations**

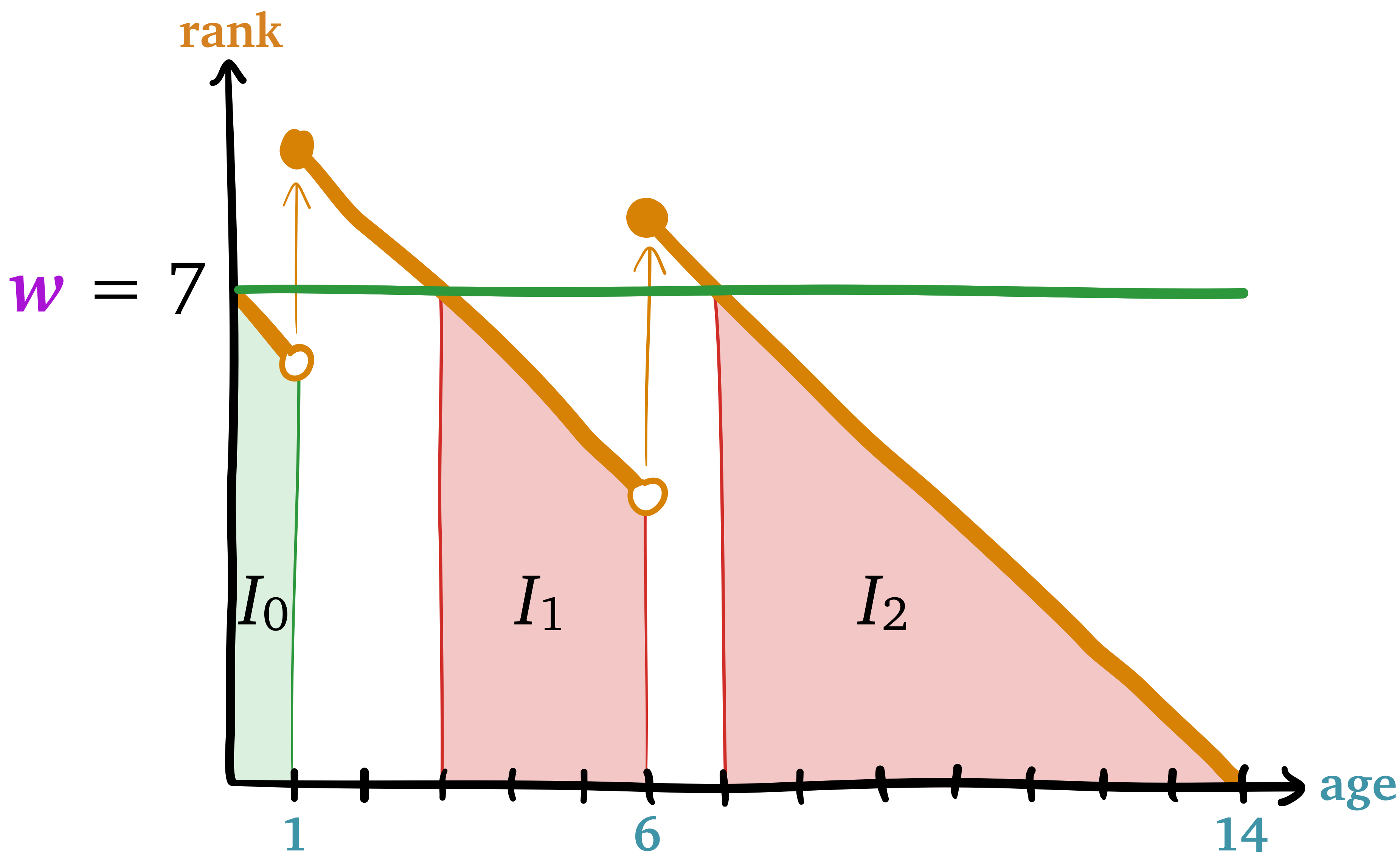
Vacation Transformation



Vacation Transformation



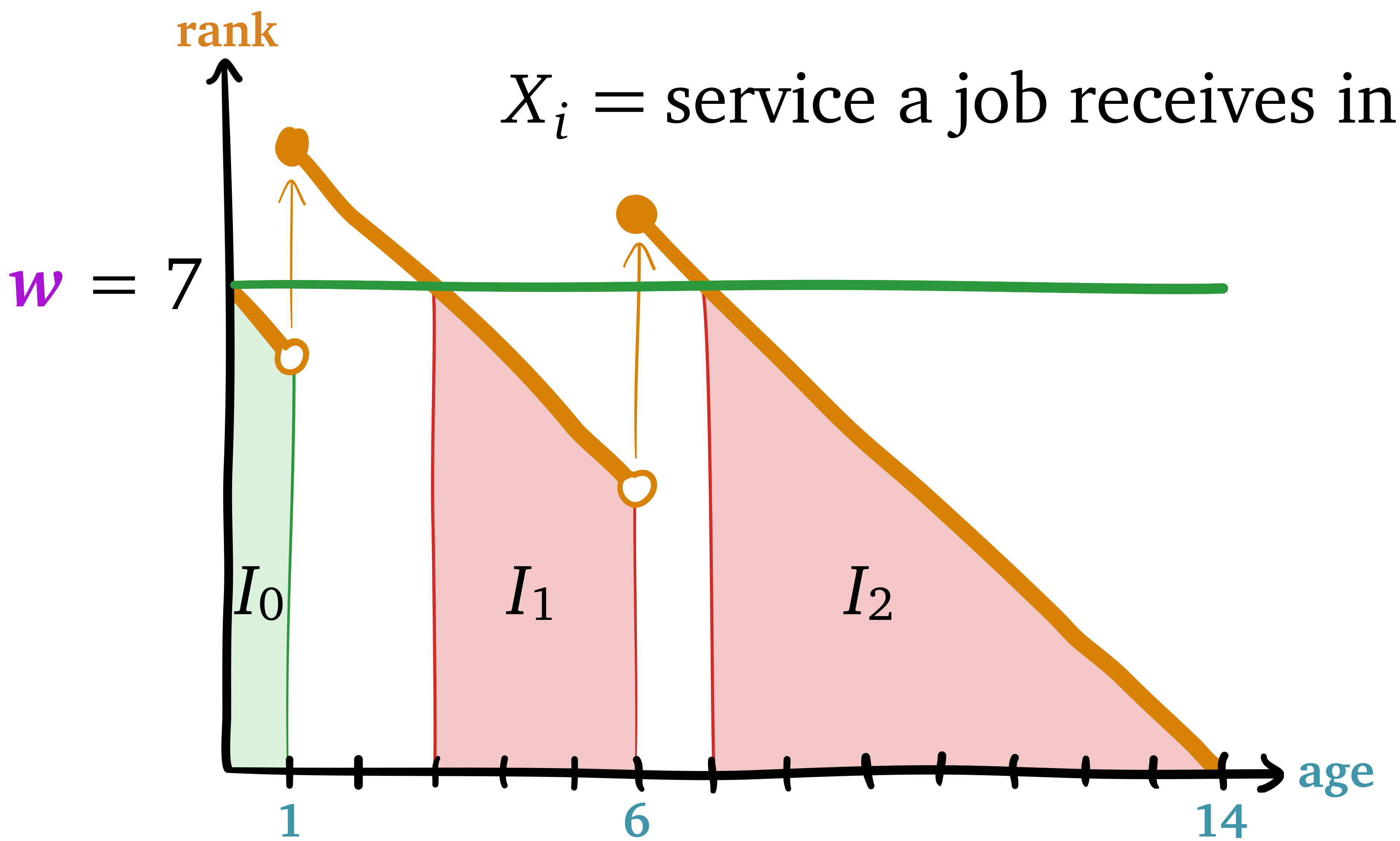
Vacation Transformation



$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

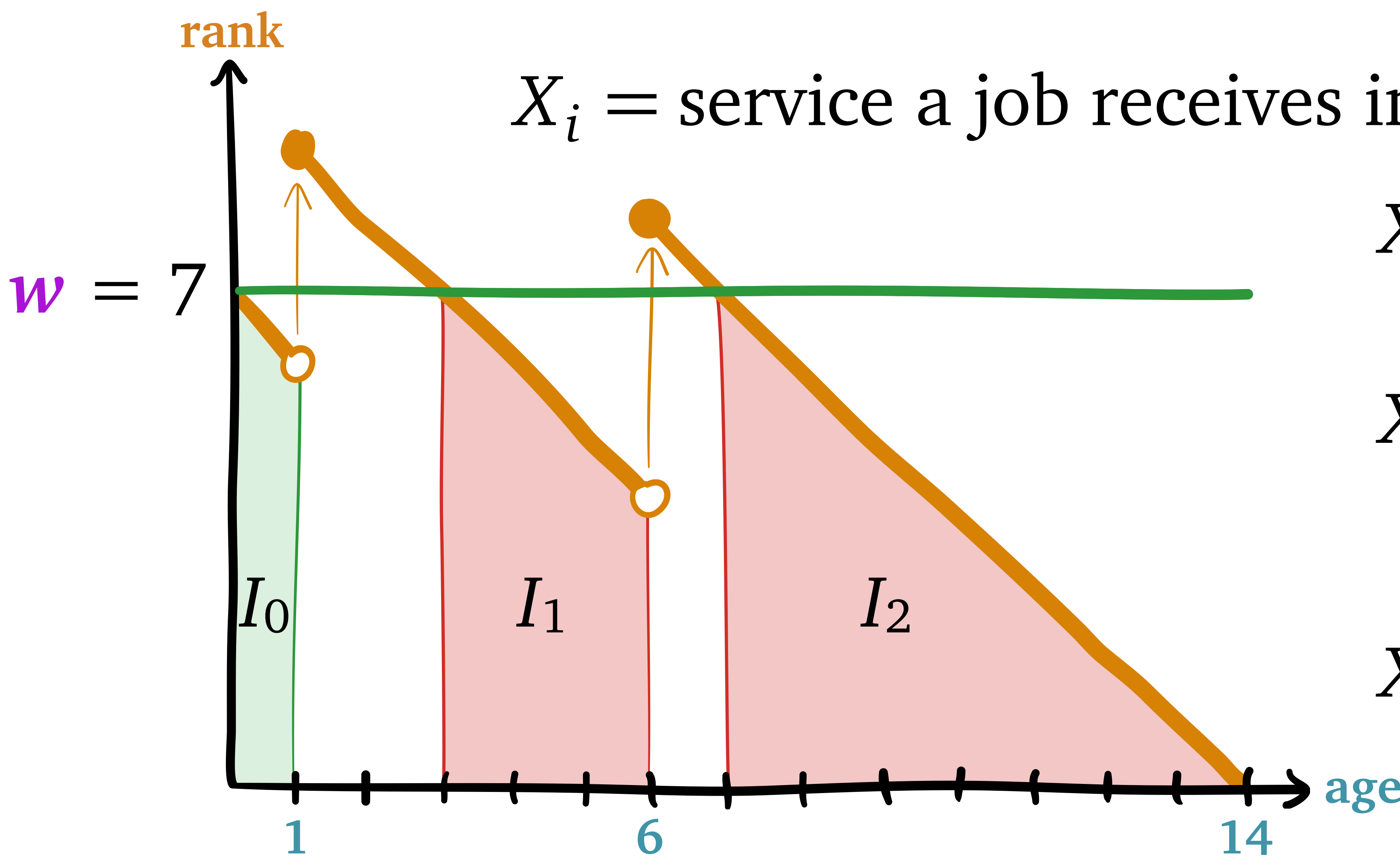
Vacation Transformation



$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

Vacation Transformation



X_i = service a job receives in I_i

$X_0 =$

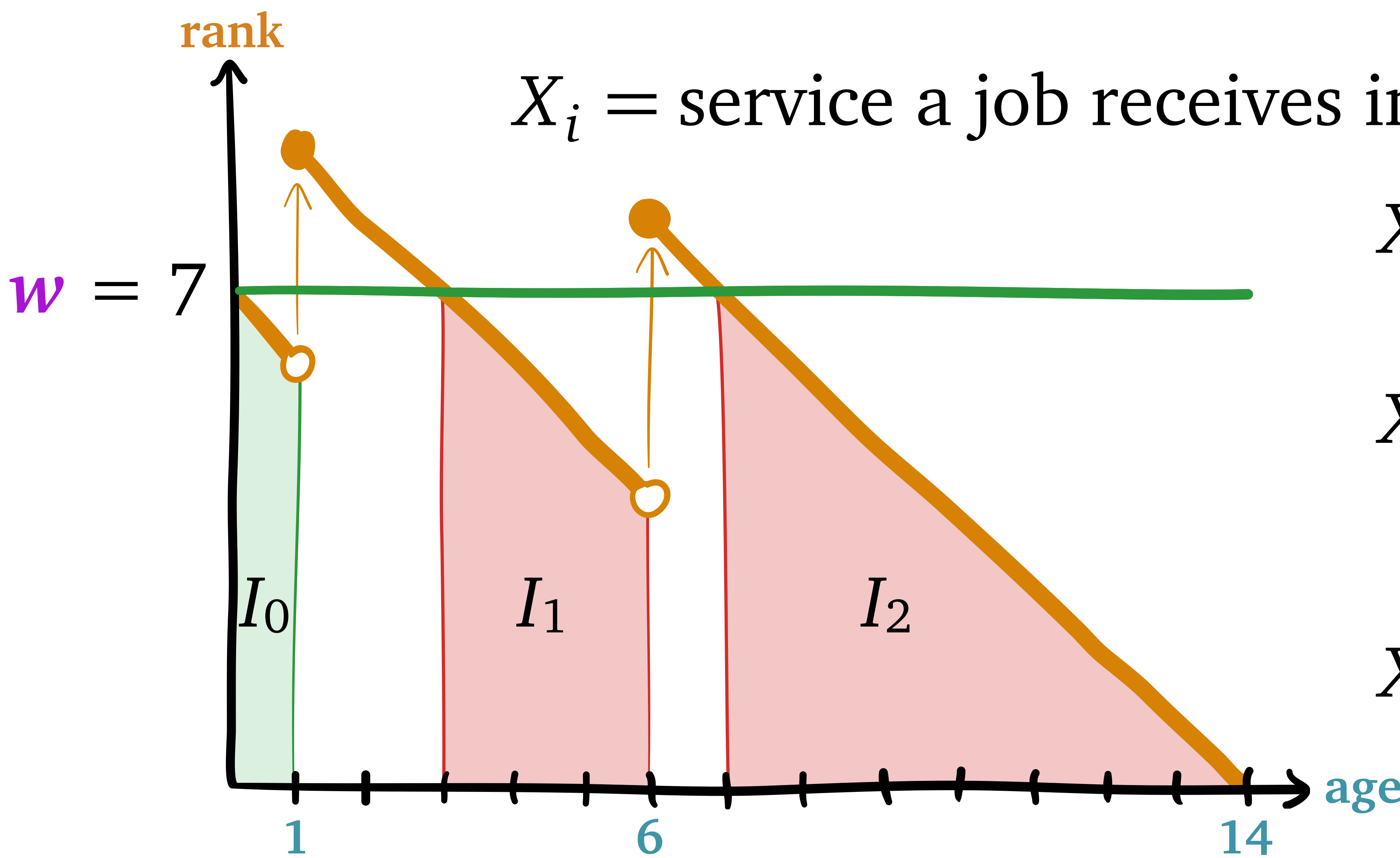
$X_1 =$

$X_2 =$

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

Vacation Transformation



X_i = service a job receives in I_i

$X_0 = 1$

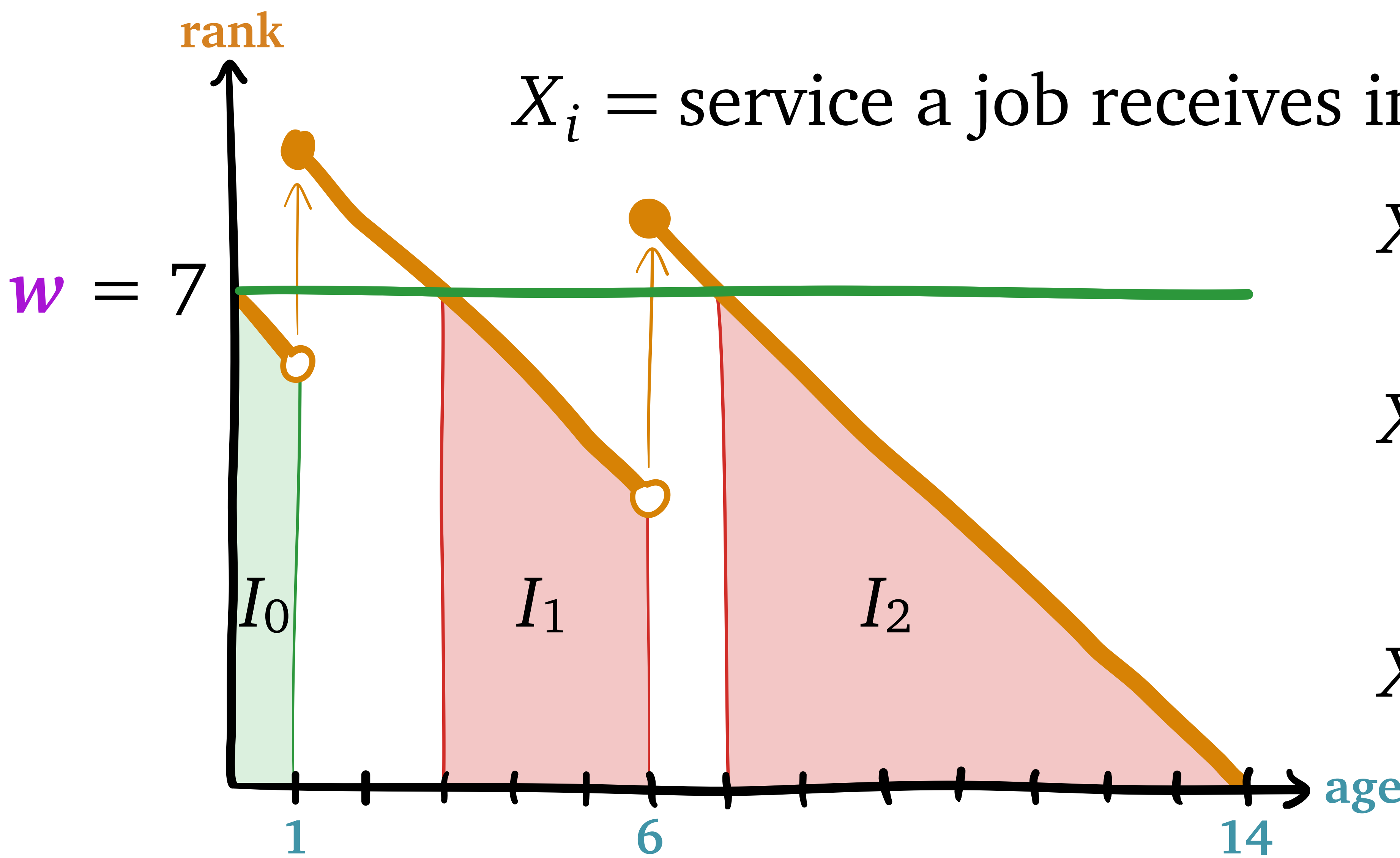
$X_1 =$

$X_2 =$

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

Vacation Transformation



X_i = service a job receives in I_i

$X_0 = 1$

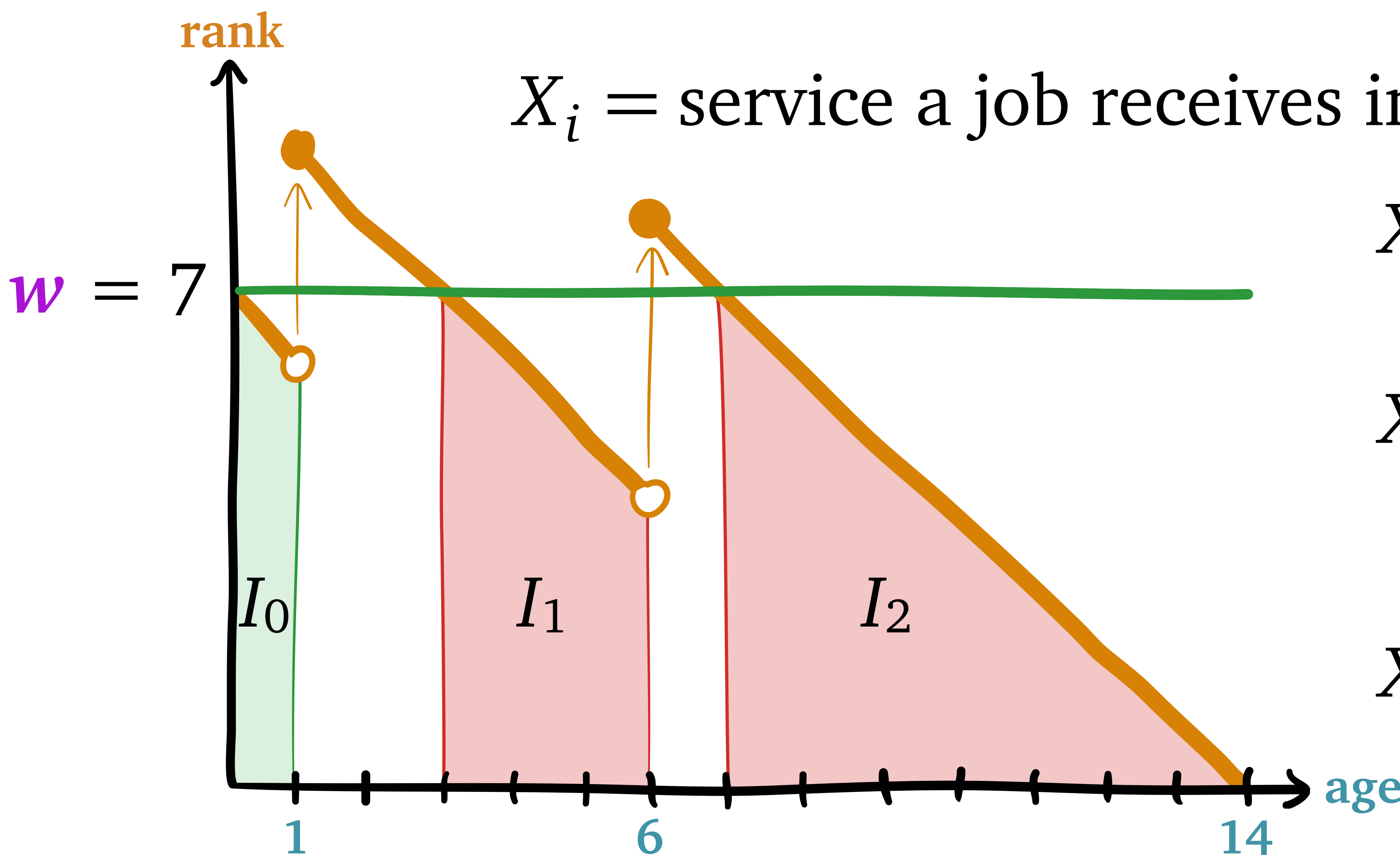
$$X_1 = \begin{cases} 0 & \text{w.p. } \frac{1}{3} \\ 3 & \text{w.p. } \frac{2}{3} \end{cases}$$

$X_2 =$

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

Vacation Transformation



X_i = service a job receives in I_i

$$X_0 = 1$$

$$X_1 = \begin{cases} 0 & \text{w.p. } \frac{1}{3} \\ 3 & \text{w.p. } \frac{2}{3} \end{cases}$$

$$X_2 = \begin{cases} 0 & \text{w.p. } \frac{2}{3} \\ 7 & \text{w.p. } \frac{1}{3} \end{cases}$$

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

(Fuhrmann and Cooper, 1985)

Response Time: Size 1

Relevant work ($\boldsymbol{w} = 7$):

$$\mathbf{E}[U[7]] = ???$$

Response Time: Size 1

Relevant work (w = 7):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Response Time: Size 1

Relevant work ($\boldsymbol{w} = 7$):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Waiting time:

$$\mathbf{E}[Q_1] = \frac{\mathbf{E}[U[7]]}{1 - \rho_{\text{new}}(0)}$$

Response Time: Size 1

Relevant work (w = 7):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Waiting time:

$$\mathbf{E}[Q_1] = \frac{\mathbf{E}[U[7]]}{1 - \rho_{\text{new}}(0)}$$

Residence time:

$$\mathbf{E}[R_1] = \int_0^1 \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)}$$

Response Time: Size 1

Relevant work ($w = 7$):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Waiting time:

$$\mathbf{E}[Q_1] = \frac{\mathbf{E}[U[7]]}{1 - \rho_{\text{new}}(0)}$$

Residence time:

$\rho_{\text{new}}(a) = \lambda \cdot 0$

$$\mathbf{E}[R_1] = \int_0^1 \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)}$$

Response Time: Size 1

Relevant work (***w*** = 7):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Waiting time:

$$\mathbf{E}[Q_1] = \frac{\mathbf{E}[U[7]]}{1 - \rho_{\text{new}}(0)} = \mathbf{E}[U[7]]$$

Residence time:

$\rho_{\text{new}}(a) = \lambda \cdot 0$

$$\mathbf{E}[R_1] = \int_0^1 \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)} = 1$$

Response Time: Size 1

Relevant work ($w = 7$):

$$\mathbf{E}[U[7]] = \frac{\lambda}{2} \cdot \frac{\mathbf{E}[X_0^2] + \mathbf{E}[X_1^2] + \mathbf{E}[X_2^2]}{1 - \lambda \mathbf{E}[X_0]}$$

Waiting time:

$$\mathbf{E}[Q_1] = \frac{\mathbf{E}[U[7]]}{1 - \rho_{\text{new}}(0)} = \mathbf{E}[U[7]]$$

Residence time:

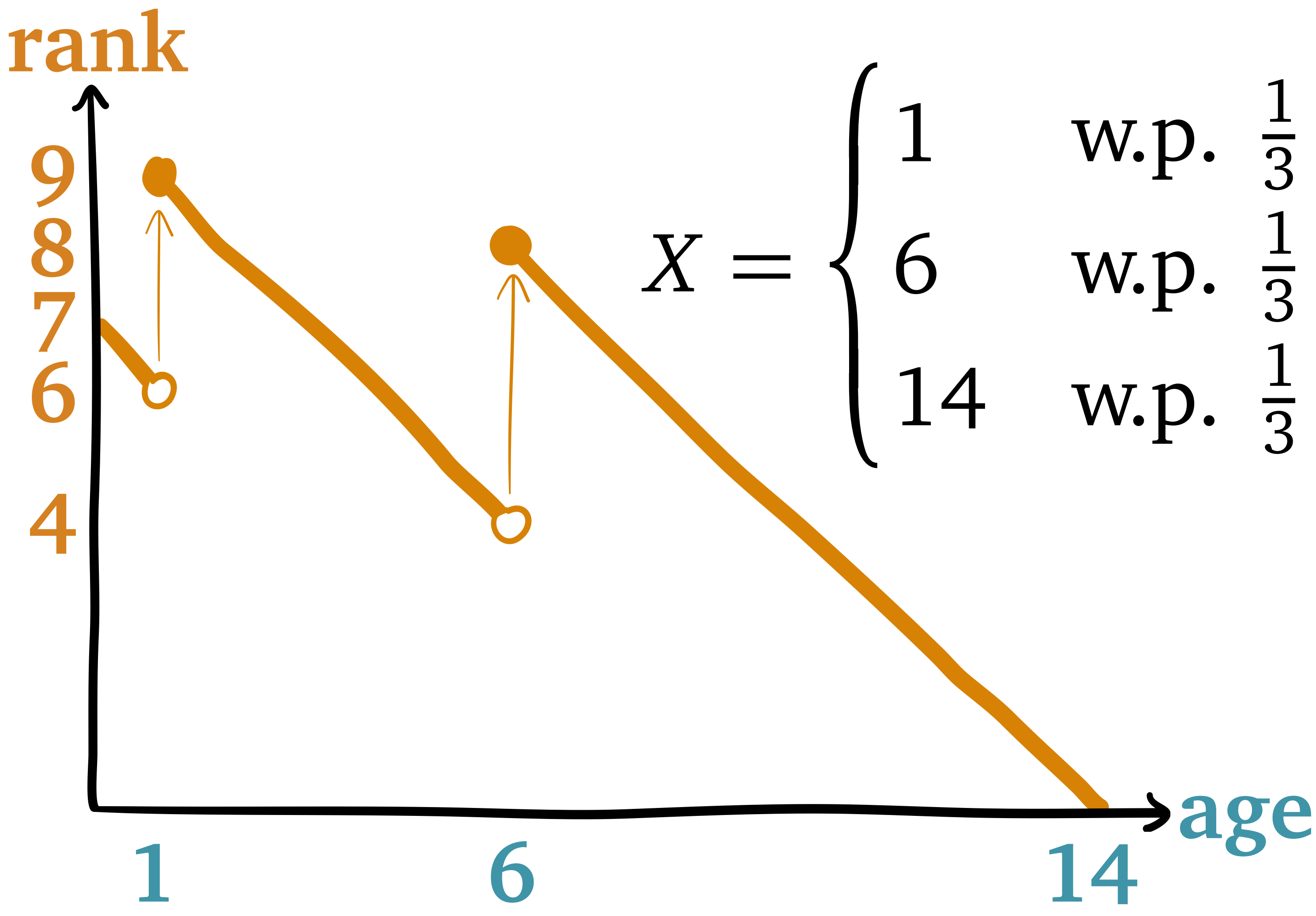
$\rho_{\text{new}}(a) = \lambda \cdot 0$

$$\mathbf{E}[R_1] = \int_0^1 \frac{\mathrm{d}a}{1 - \rho_{\text{new}}(a)} = 1$$

Response time:

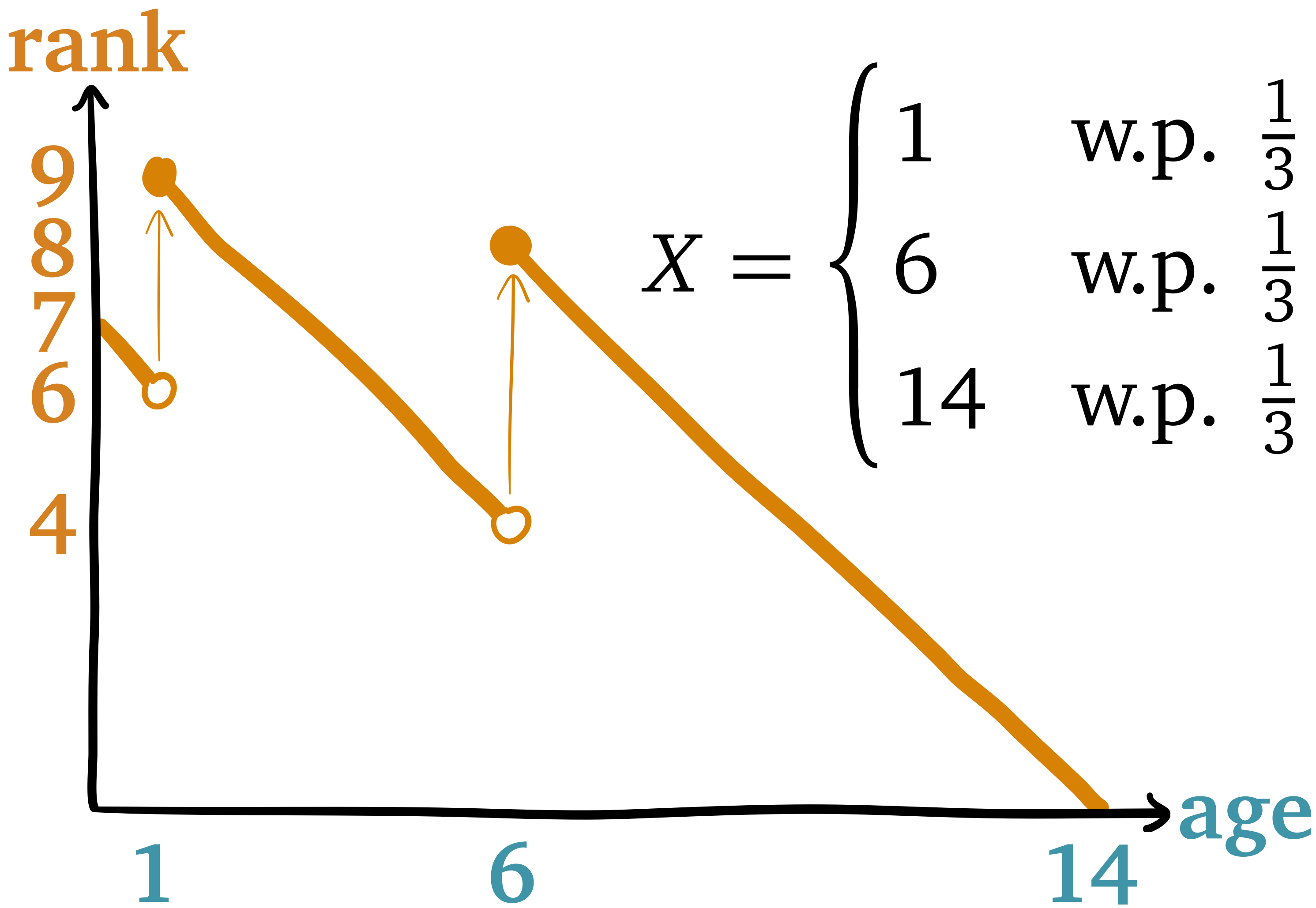
$$\mathbf{E}[T_1] = \mathbf{E}[Q_1] + \mathbf{E}[R_1]$$

Running example:
SERPT



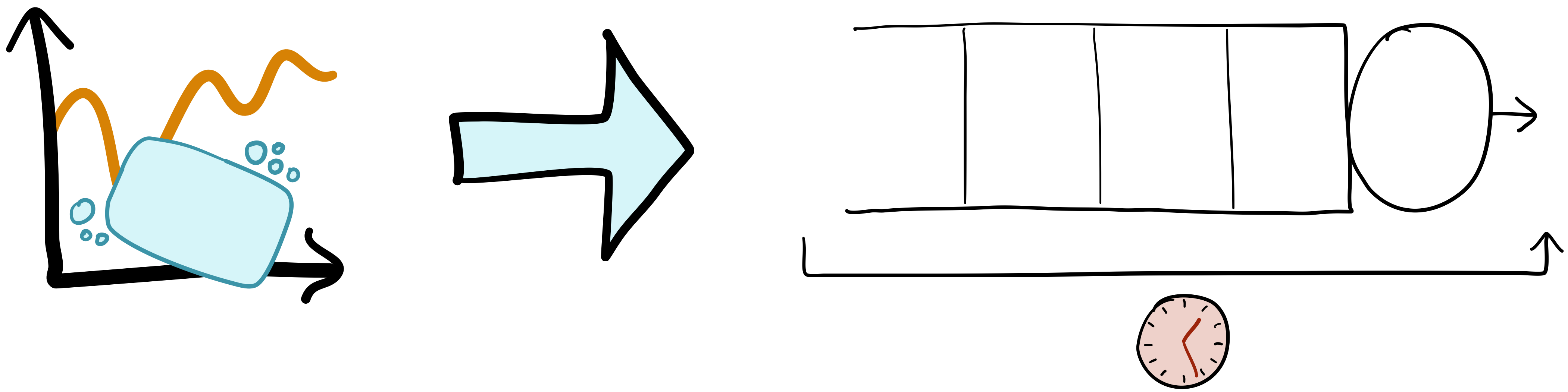
$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

Running example: **SERPT**

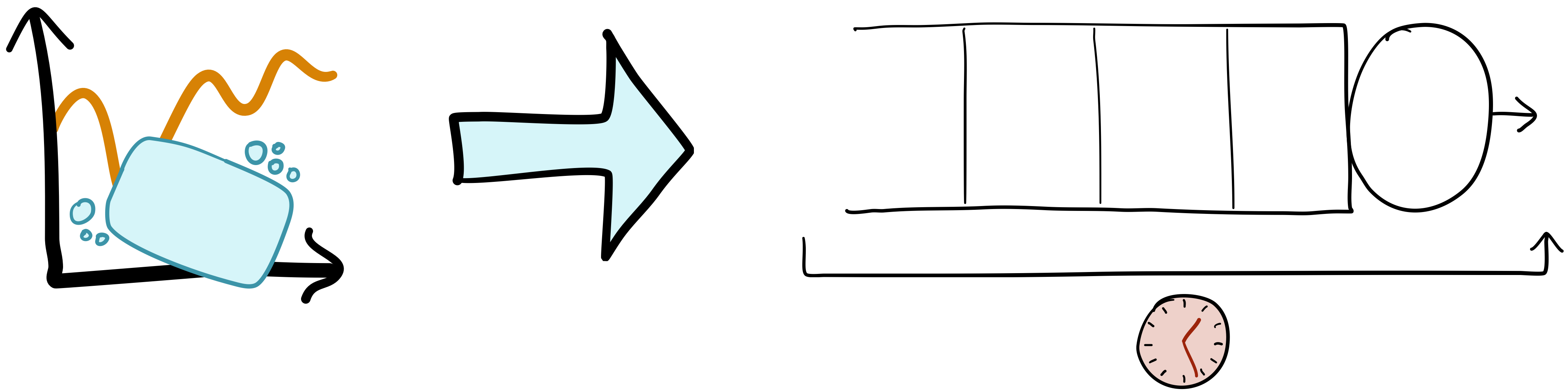


$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

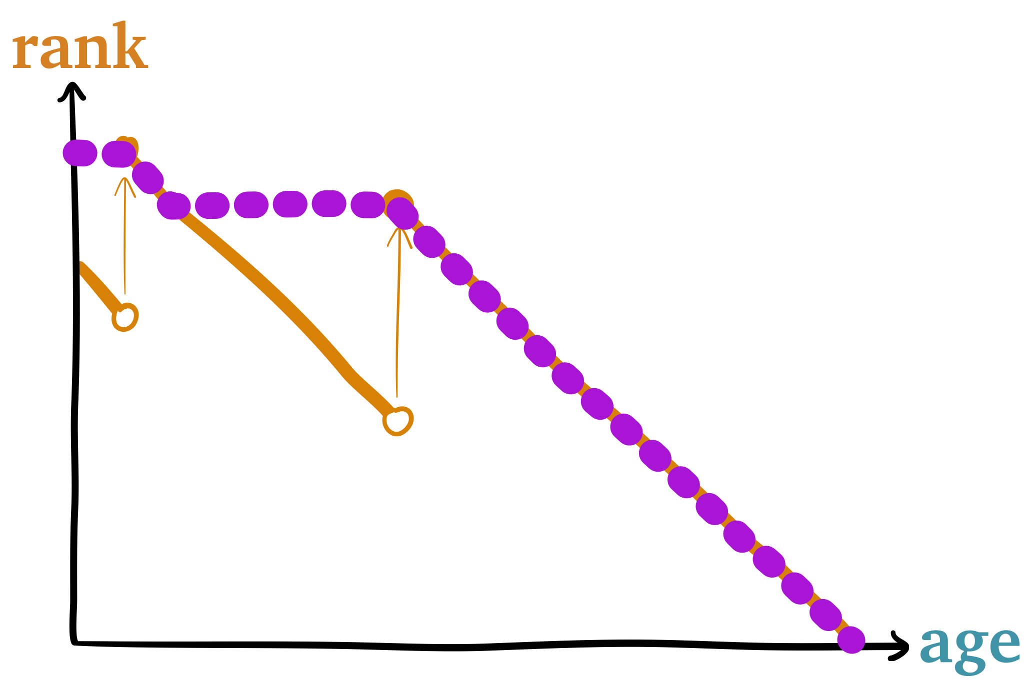
$\mathbf{E}[T]$ of *any* **SOAP** Policy



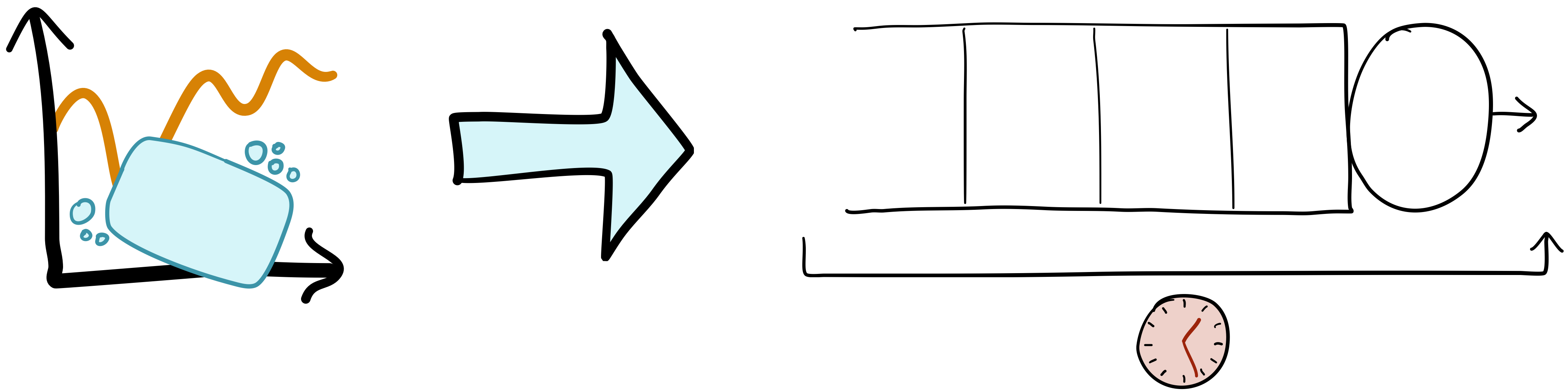
$\mathbf{E}[T]$ of *any* **SOAP** Policy



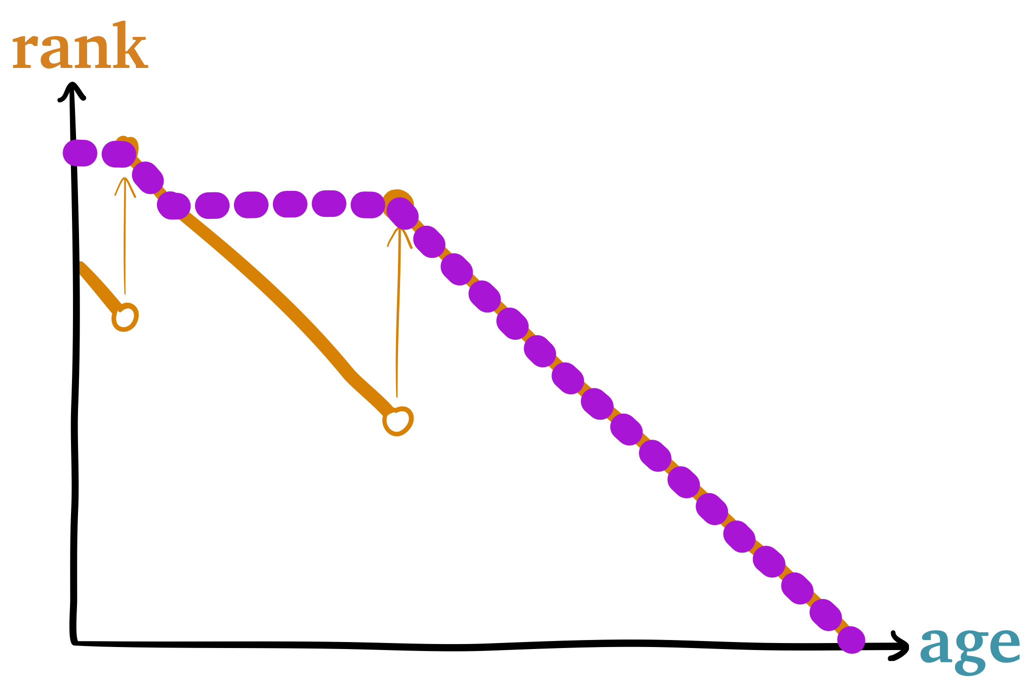
Worst Future Rank



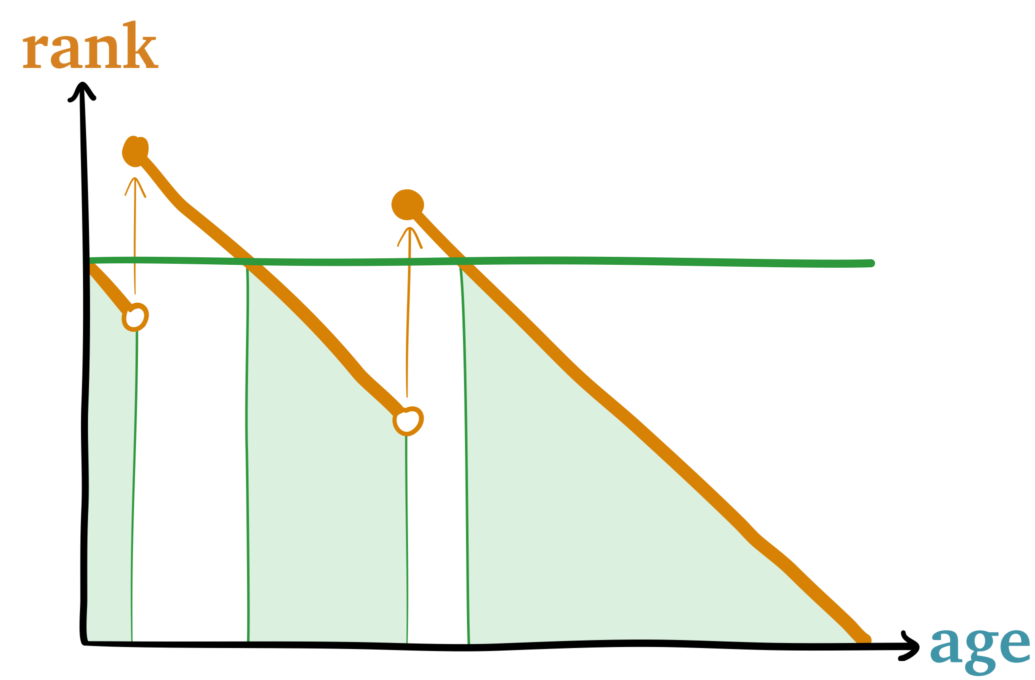
$\mathbf{E}[T]$ of *any* **SOAP** Policy



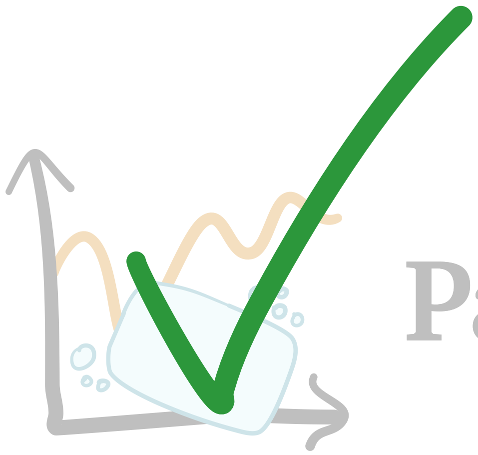
Worst Future Rank



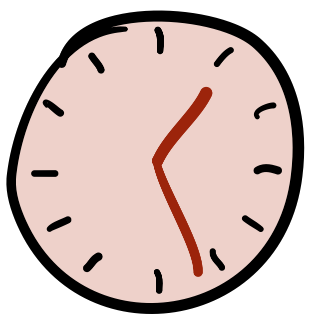
Relevant Intervals



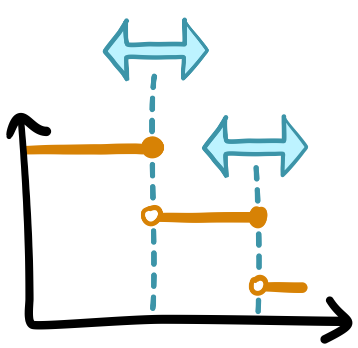
Outline



Part 1: *defining* **SOAP** policies



Part 2: *analyzing* **SOAP** policies



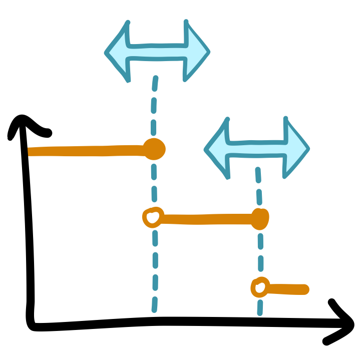
Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

Outline

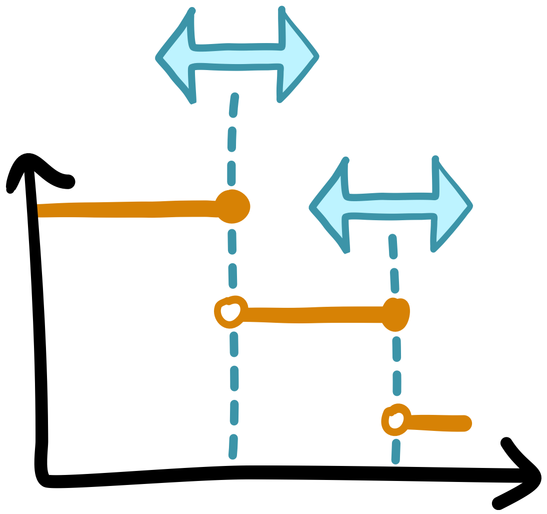
Part 1: *defining* **SOAP** policies

Part 2: *analyzing* **SOAP** policies



Part 3: *policy design* with **SOAP**

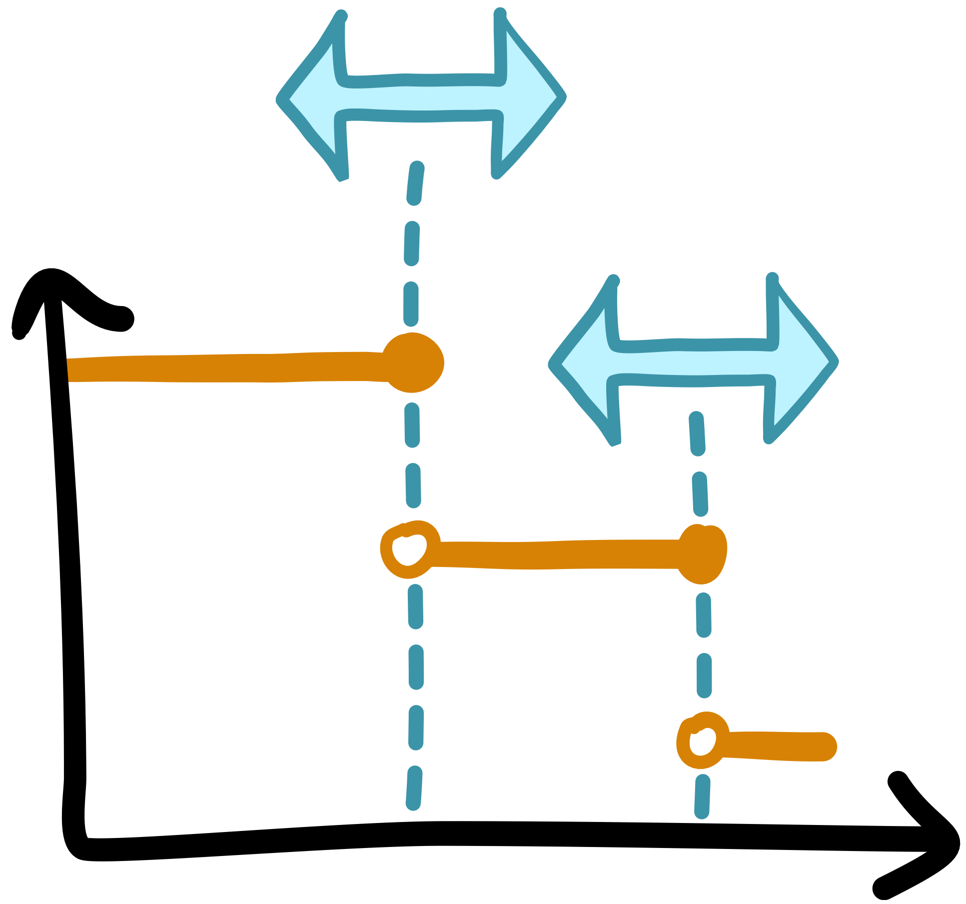
Part 4: *optimality proofs* with **SOAP**



Part 3:

policy design with **SOAP**

Bucketed SRPT



Question: given number of priority levels, which job sizes go in which size buckets?

Two Buckets

$X = $ bounded Pareto on $[1, 10^6]$ with $\alpha = 1$

Two Buckets

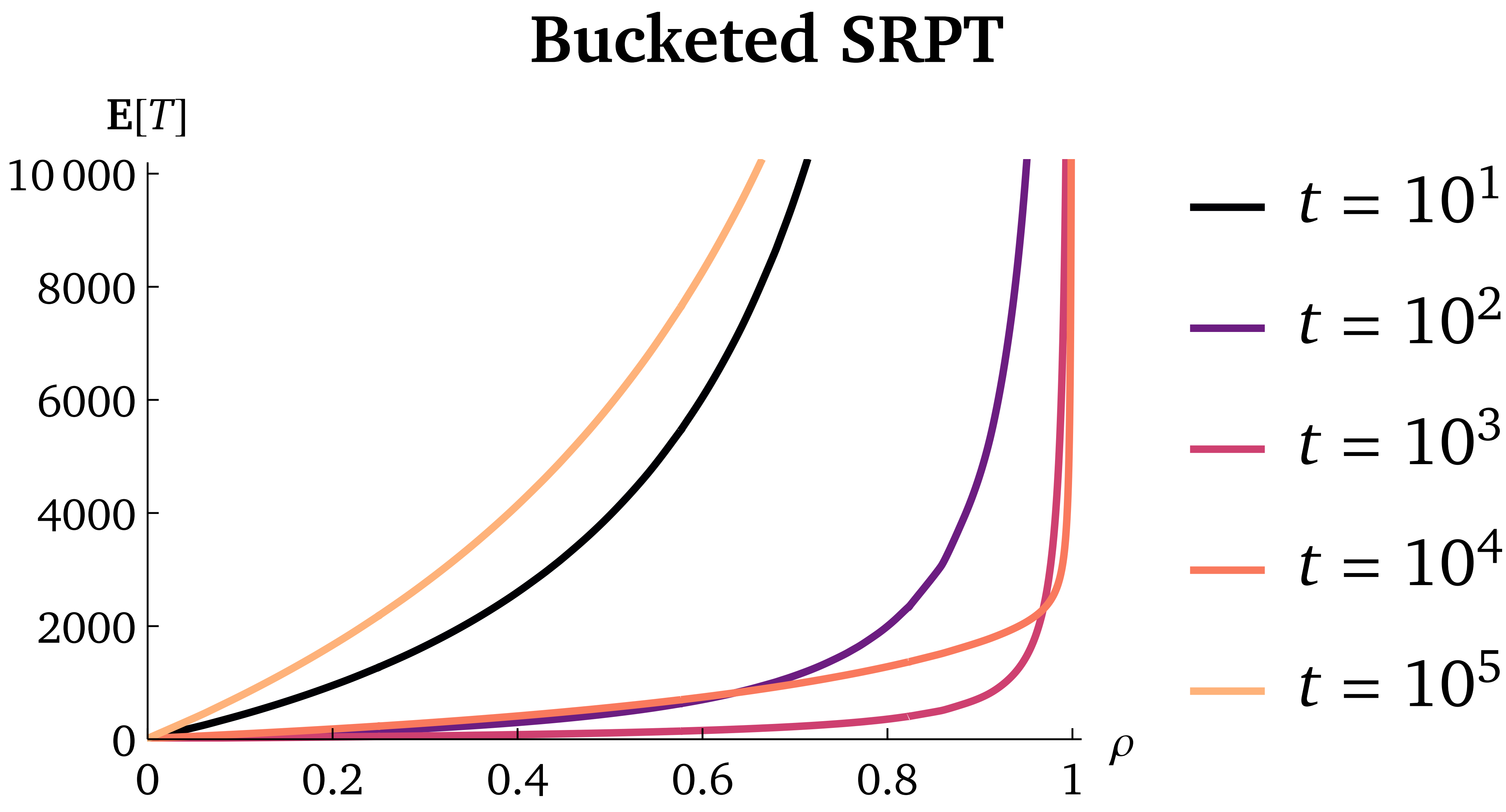
$X = $ bounded Pareto on $[1, 10^6]$ with $\alpha = 1$

$t = $ threshold between buckets

Two Buckets

X = bounded Pareto on $[1, 10^6]$ with $\alpha = 1$

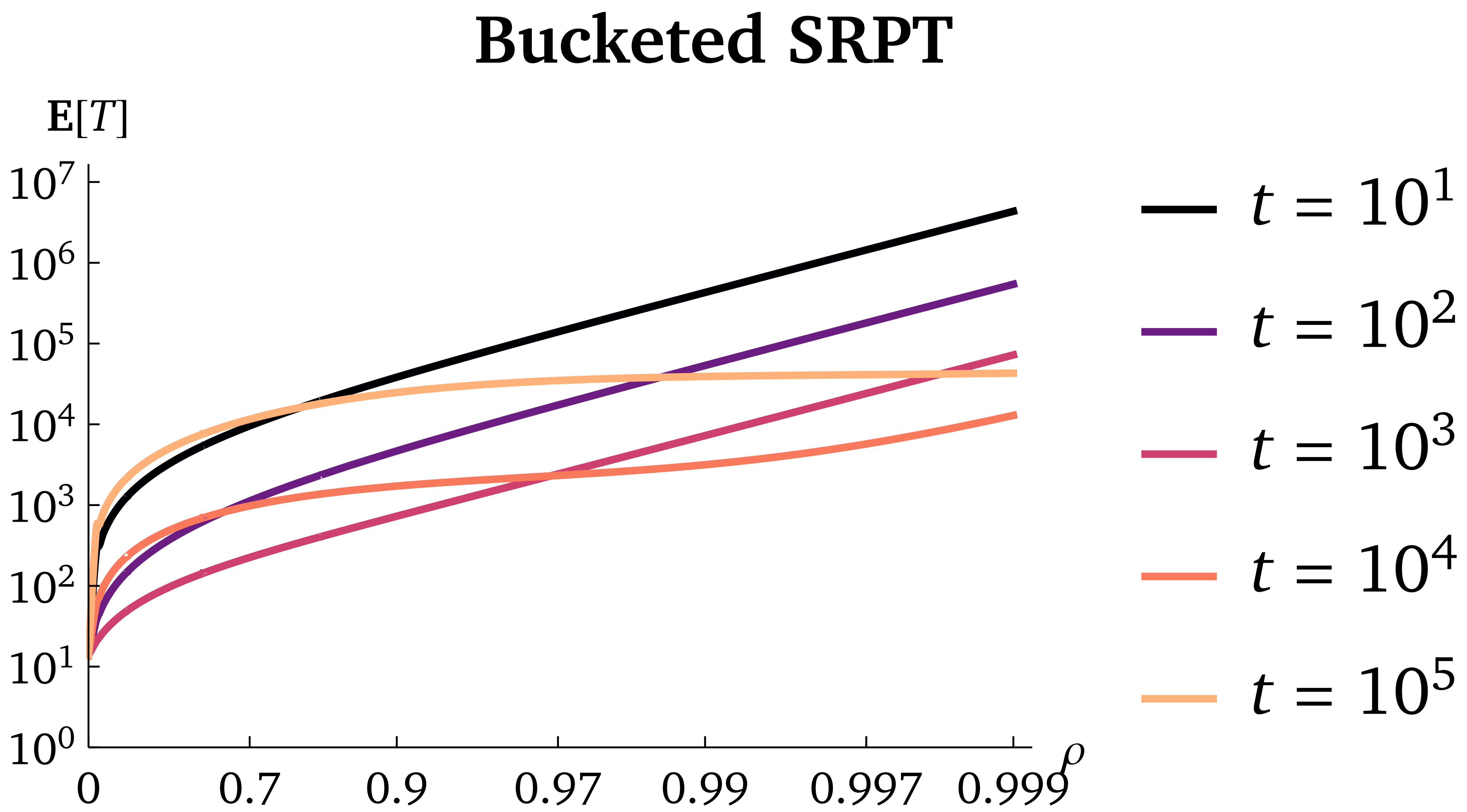
t = threshold between buckets



Two Buckets

$X =$ bounded Pareto on $[1, 10^6]$ with $\alpha = 1$

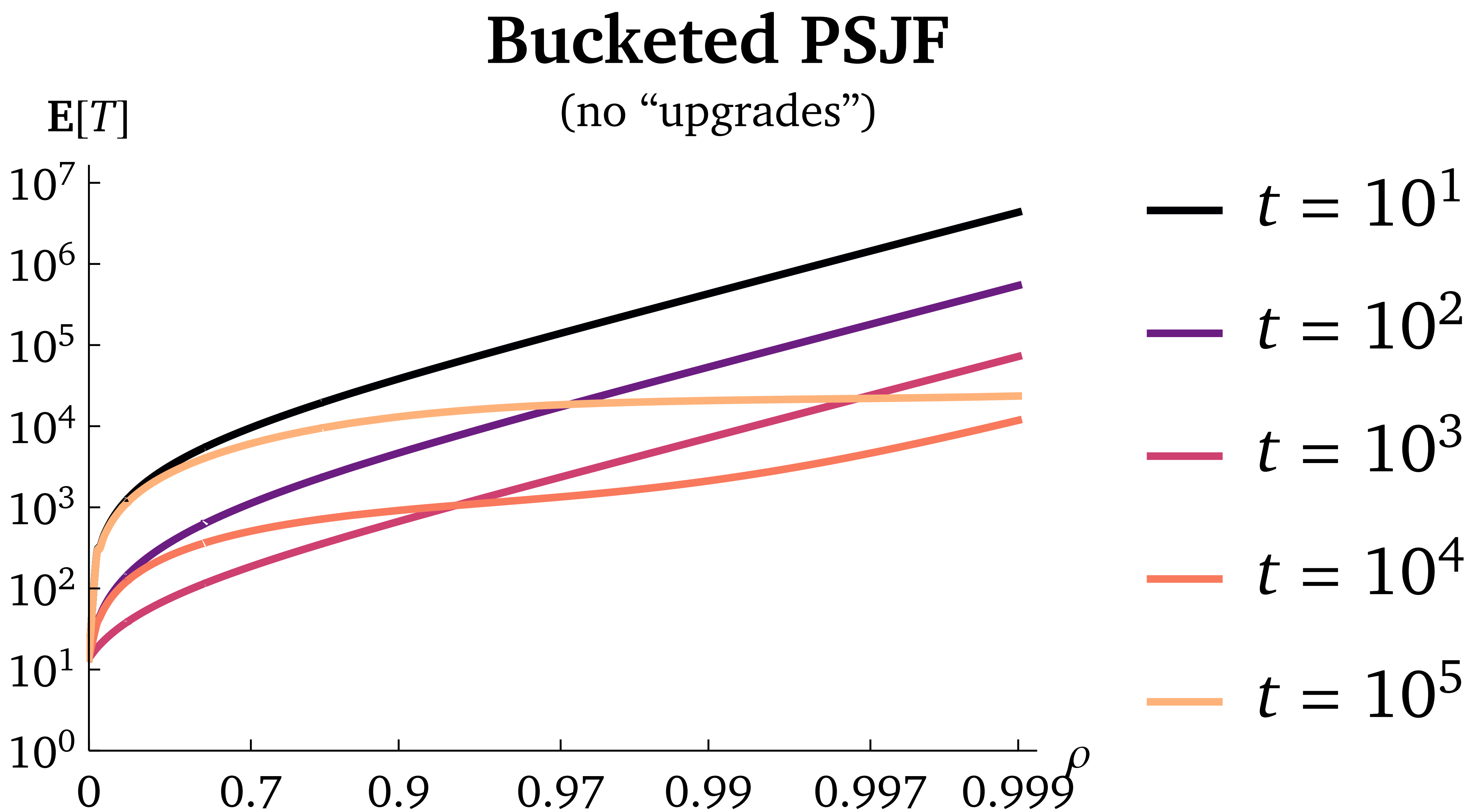
$t =$ threshold between buckets



Two Buckets

$X =$ bounded Pareto on $[1, 10^6]$ with $\alpha = 1$

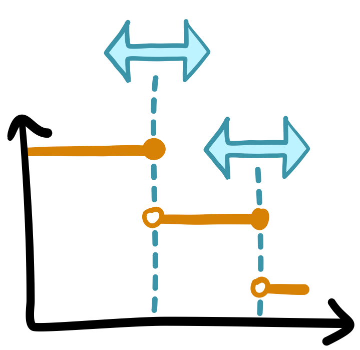
$t =$ threshold between buckets



Outline

Part 1: *defining* **SOAP** policies

Part 2: *analyzing* **SOAP** policies



Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

Outline

Part 1: *defining* **SOAP** policies

Part 2: *analyzing* **SOAP** policies

Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

Part 4:

optimality proofs with SOAP

Gittins vs. SERPT

Gittins vs. SERPT

Gittins

$$r(a) = \sup_{\Delta > 0} \frac{\mathbf{E}[\min\{X - a, \Delta\} \mid X > a]}{\mathbf{P}[X - a \leq \Delta \mid X > a]}$$

Gittins vs. SERPT

Gittins

$$r(a) = \sup_{\Delta > 0} \frac{\mathbf{E}[\min\{X - a, \Delta\} \mid X > a]}{\mathbf{P}[X - a \le \Delta \mid X > a]}$$

SERPT

$$r(a) = \mathbf{E}[X - a \mid X > a]$$

Gittins vs. SERPT

Gittins

$$r(a) = \sup_{\Delta > 0} \frac{\mathbf{E}[\min\{X - a, \Delta\} \mid X > a]}{\mathbf{P}[X - a \leq \Delta \mid X > a]}$$

Minimizes $\mathbf{E}[T]$, but can be intractable

SERPT

$$r(a) = \mathbf{E}[X - a \mid X > a]$$

Gittins vs. SERPT

Gittins

$$r(a) = \sup_{\Delta > 0} \frac{\mathbf{E}[\min\{X - a, \Delta\} \mid X > a]}{\mathbf{P}[X - a \leq \Delta \mid X > a]}$$

Minimizes $\mathbf{E}[T]$, but can be intractable

SERPT

$$r(a) = \mathbf{E}[X - a \mid X > a]$$

Simple, but no $\mathbf{E}[T]$ guarantee

Gittins vs. SERPT

Gittins

$$r(a) = \sup_{\Delta > 0} \frac{\mathbf{E}[\min\{X - a, \Delta\} \mid X > a]}{\mathbf{P}[X - a \leq \Delta \mid X > a]}$$

Minimizes $\mathbf{E}[T]$, but can be intractable

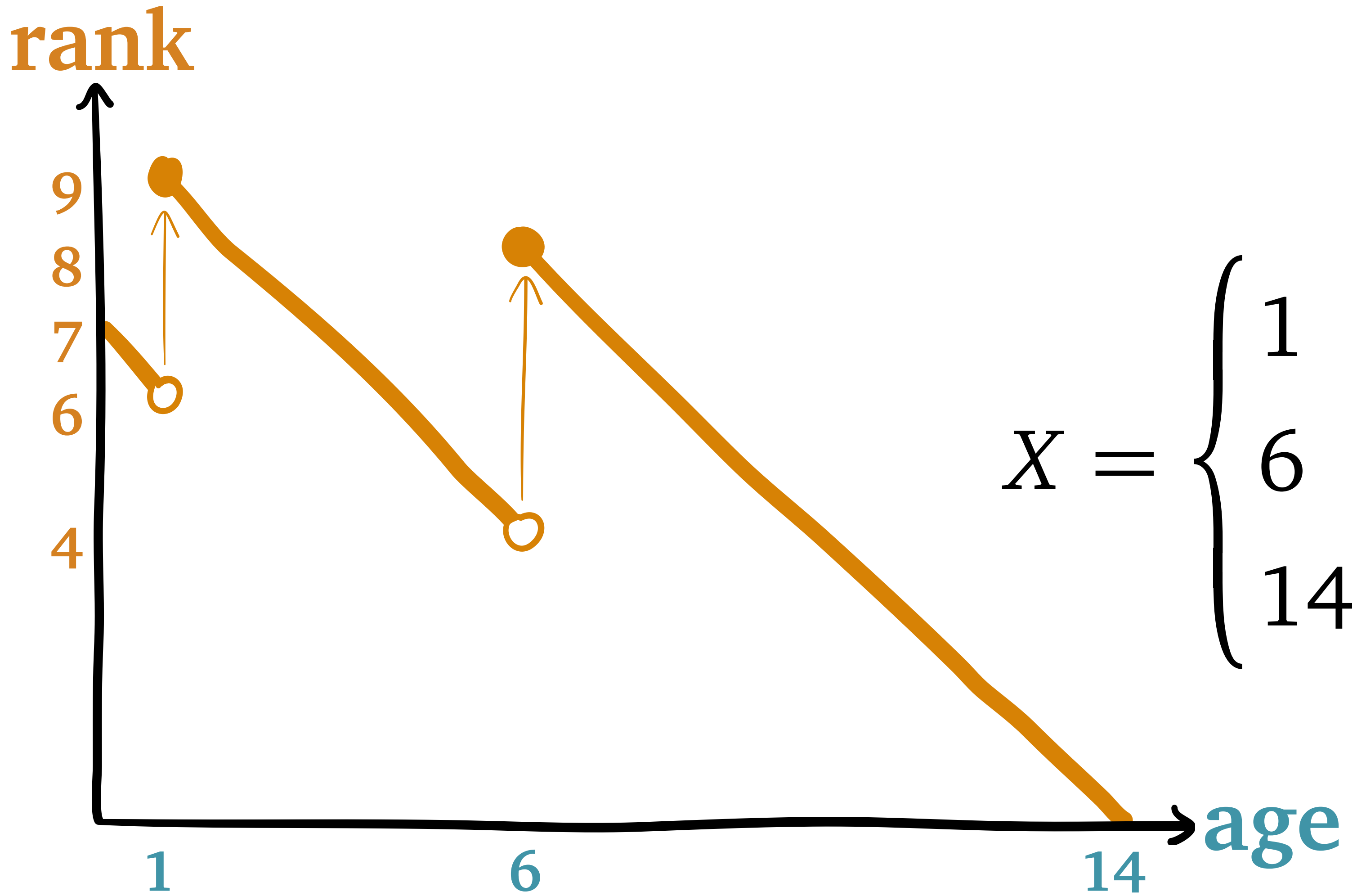
SERPT

$$r(a) = \mathbf{E}[X - a \mid X > a]$$

Simple, but no $\mathbf{E}[T]$ guarantee

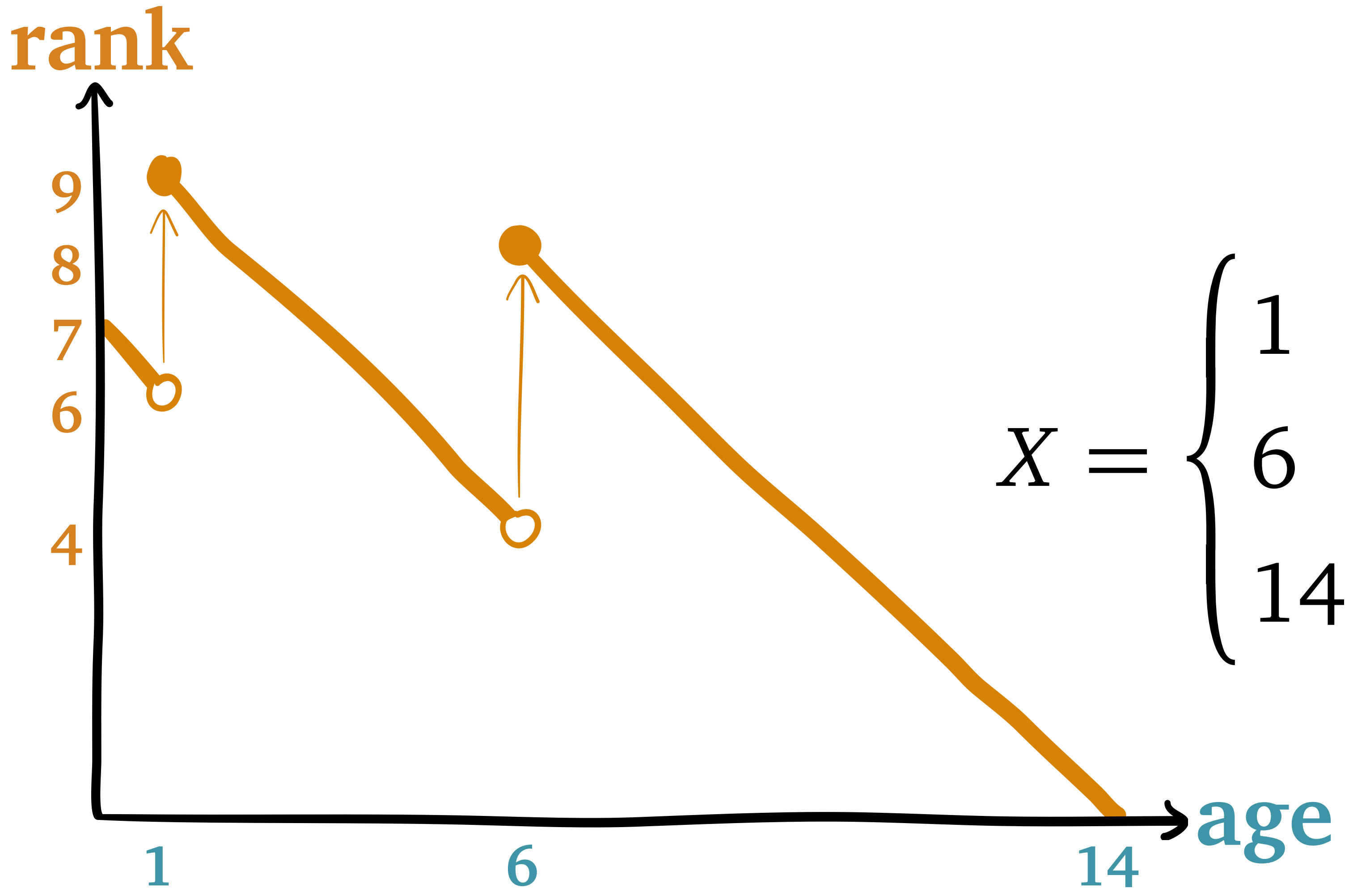
Question: is there a *simple* policy with *near-optimal* $\mathbf{E}[T]$?

Monotonic SERPT



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

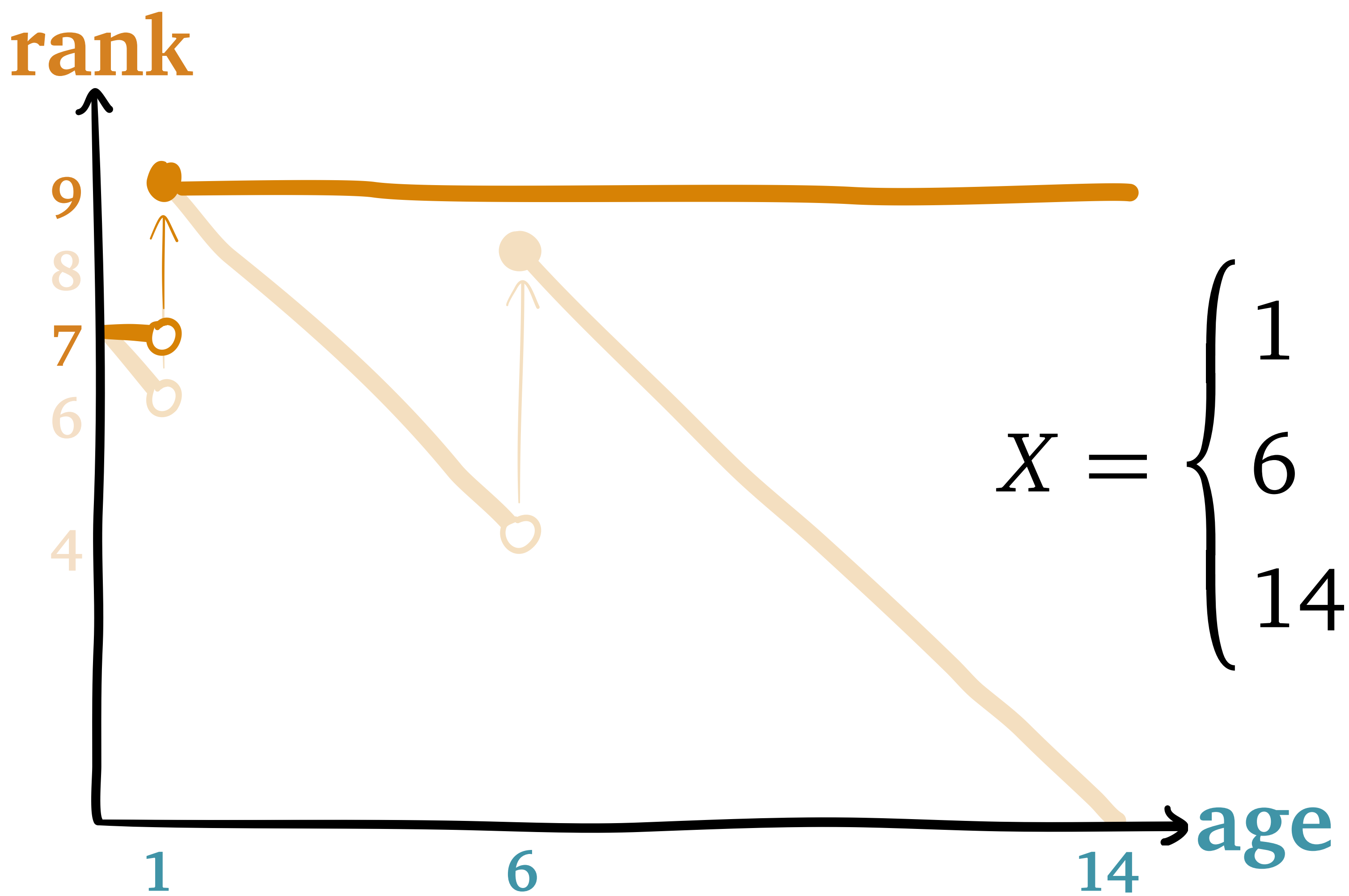
Monotonic SERPT



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

M-SERPT is like SERPT,
but **rank** *never goes down*

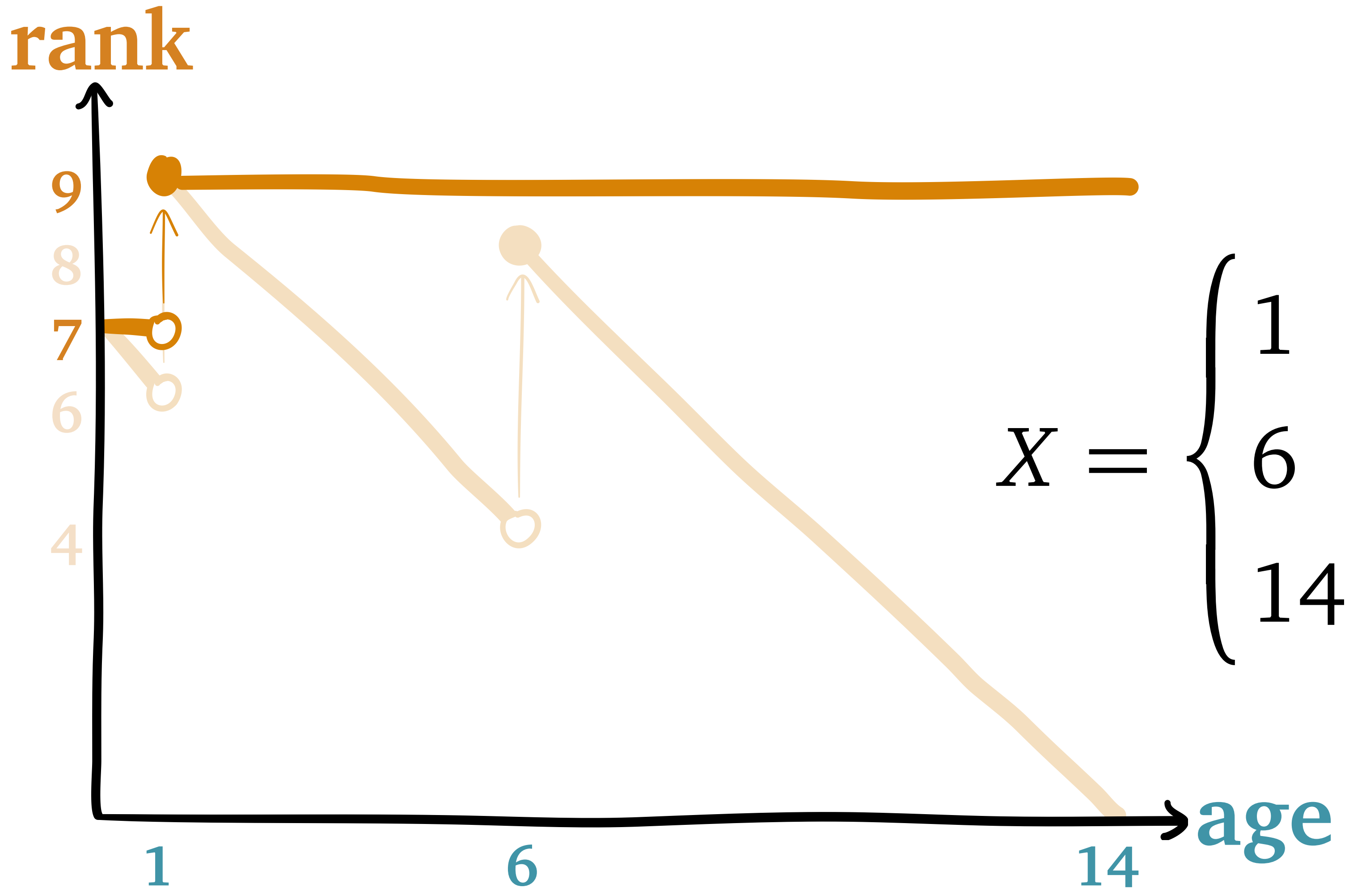
Monotonic SERPT



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

M-SERPT is like SERPT,
but ***rank*** *never goes down*

Monotonic SERPT



$$X = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

M-SERPT is like SERPT,
but ***rank*** *never goes down*

Theorem:

$$\frac{\mathbf{E}[T \text{ of M-SERPT}]}{\mathbf{E}[T \text{ of Gittins}]} \leq 5$$

Outline

Part 1: *defining* **SOAP** policies

Part 2: *analyzing* **SOAP** policies

Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

Outline

Part 1: *defining* **SOAP** policies

Part 2: *analyzing* **SOAP** policies

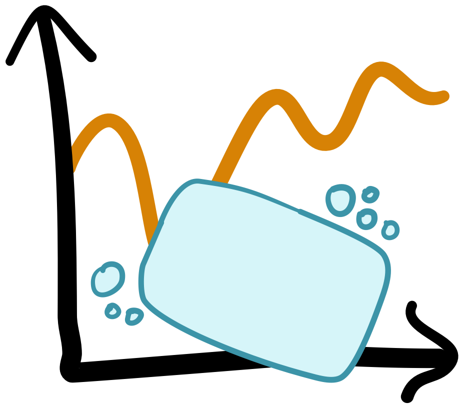
Part 3: *policy design* with **SOAP**

Part 4: *optimality proofs* with **SOAP**

SOAP Summary

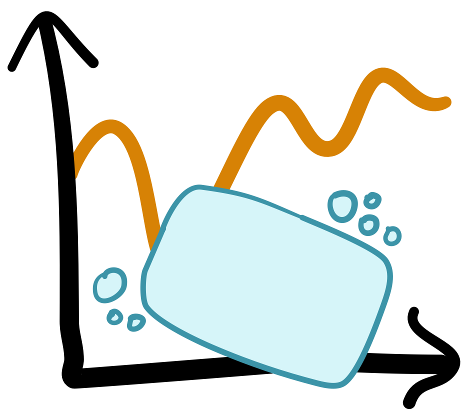
SOAP Summary

Idea: schedule with **rank** functions

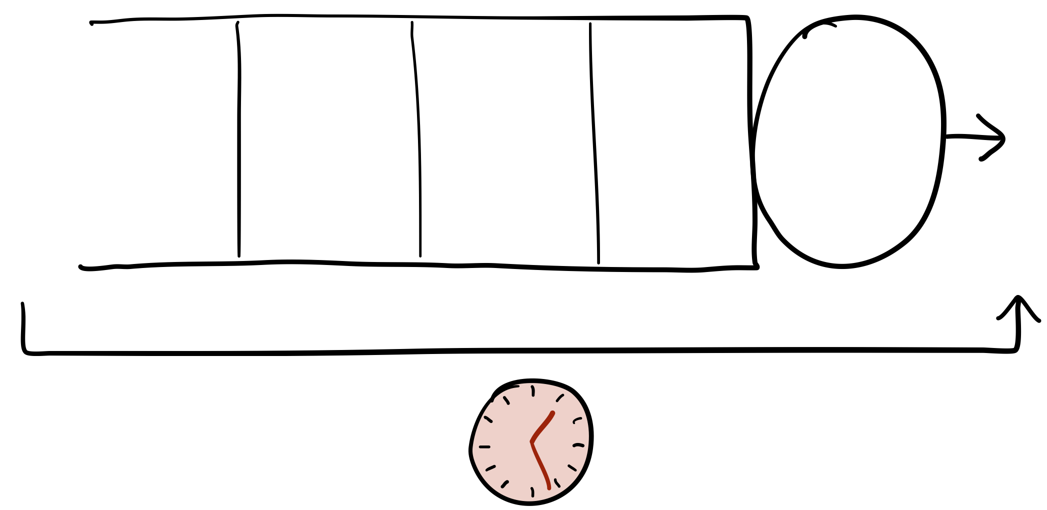


SOAP Summary

Idea: schedule with **rank** functions

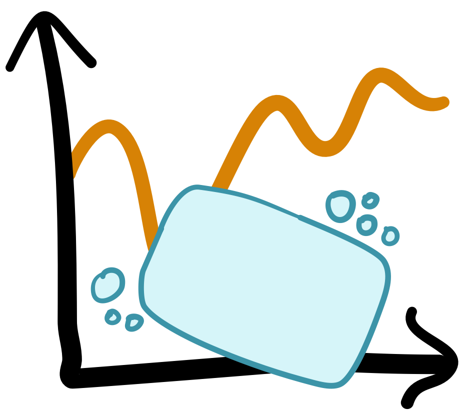


Result: universal response time analysis

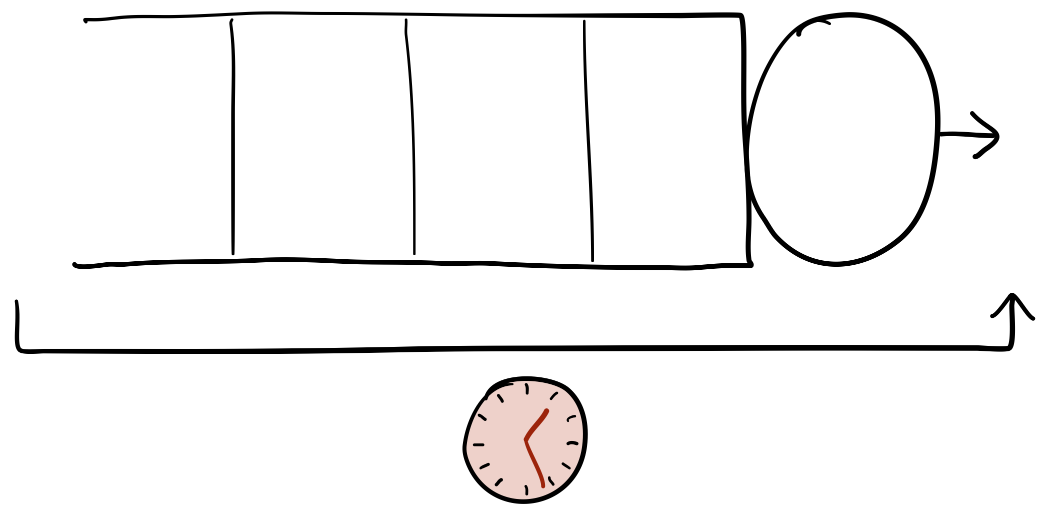


SOAP Summary

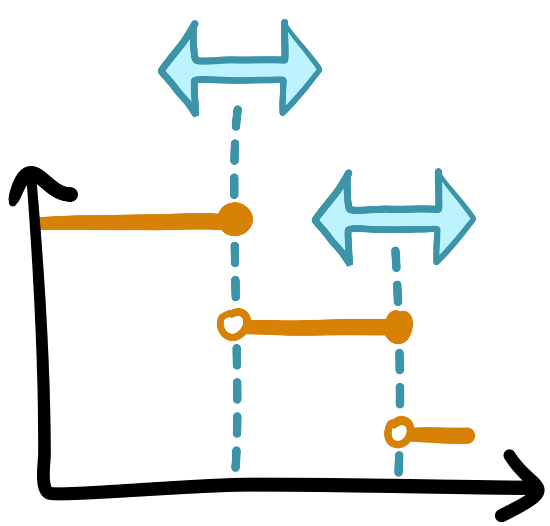
Idea: schedule with rank functions



Result: universal response time analysis



Impact: optimize and prove guarantees



References: SOAP

Z. Scully, M. Harchol-Balter, and A. Scheller-Wolf (2018). **SOAP: One Clean Analysis of All Age-Based Scheduling Policies**. *Proceedings of the ACM on Measurement and Analysis of Computing Systems (POMACS)*, 2(1), 16. Presented at SIGMETRICS 2018.

Z. Scully and M. Harchol-Balter (2018). **SOAP Bubbles: Robust Scheduling Under Adversarial Noise**. In *56th Annual Allerton Conference on Communication, Control, and Computing* (pp. 144–154). IEEE.

Z. Scully, M. Harchol-Balter, and A. Scheller-Wolf (2019). **Simple Near-Optimal Scheduling for the M/G/1**. *ACM SIGMETRICS Performance Evaluation Review*, to appear. Presenting at MAMA 2019 this Friday!

References: Analyzing $\mathbf{E}[T]$

L. Kleinrock and R. R. Muntz (1972). **Processor sharing queueing models of mixed scheduling disciplines for time shared system**. *Journal of the ACM (JACM)*, 19(3), 464–482.

S. W. Furhmann and R. B. Cooper (1985). **Stochastic Decompositions in the M/G/1 Queue with Generalized Vacations**. *Operations Research*, 33(5), 1117–1129.

M. Harchol-Balter (2013). ***Performance Modeling and Design of Computer Systems: Queueing Theory in Action***. Cambridge University Press.

References: Possible Applications

M. Harchol-Balter, Schroeder, B., Bansal, N., and Agrawal, M. (2003). **Size-based scheduling to improve web performance**. *ACM Transactions on Computer Systems (TOCS)*, 21(2), 207–233.

B. Montazeri, Y. Li, M. Alizadeh, and J. Ousterhout (2018). **Homa: A receiver-driven low-latency transport protocol using network priorities**. In *Proceedings of the 2018 Conference of the ACM Special Interest Group on Data Communication* (pp. 221–235). ACM.

S. Emadi, R. Ibrahim, and S. Kesavan (2019). **Can "very noisy" information go a long way? An exploratory analysis of personalized scheduling in service systems**. Working paper.

M. Mitzenmacher (2019). **Scheduling with Predictions and the Price of Misprediction**. Preprint, *arXiv:1902.00732*.

B. Kamphorst (2018). ***Heavy-traffic behaviour of scheduling policies in queues*** (Doctoral dissertation, Technische Universiteit Eindhoven).

Y. Chen and J. Dong (2019). **The Power of Two in Queue Scheduling**. Working paper.

Bonus Slides

Full **SOAP** Definition

A ***SOAP*** *policy* is any policy expressible by a ***rank*** *function* of the form:

Full SOAP Definition

A ***SOAP*** *policy* is any policy expressible by a ***rank*** *function* of the form:

$$\textbf{descriptor} \times \textbf{age} \to \textbf{rank}$$

Full SOAP Definition

A ***SOAP*** *policy* is any policy expressible by a ***rank*** *function* of the form:

size, class, etc.

$$\textbf{descriptor} \times \textbf{age} \to \textbf{rank}$$

Full SOAP Definition

A ***SOAP** policy* is any policy expressible by a ***rank** function* of the form:

size, class, etc.

$$\textbf{descriptor} \times \textbf{age} \to \textbf{rank}$$

FB

$$r_{\varnothing}(a) = a$$

Full SOAP Definition

A ***SOAP*** *policy* is any policy expressible by a ***rank*** *function* of the form:

size, class, etc.

$$\textbf{descriptor} \times \textbf{age} \rightarrow \textbf{rank}$$

FB

$r_{\varnothing}(a) = a$

SRPT

$r_x(a) = x - a$

Full SOAP Definition

A ***SOAP*** *policy* is any policy expressible by a ***rank*** *function* of the form:

size, class, etc.

$$\textbf{descriptor} \times \textbf{age} \to \textbf{rank}$$

FB

$r_\varnothing(a) = a$

SRPT

$r_x(a) = x - a$

Descriptor can be anything that:

- does not change while a job is in the system
- is i.i.d. for each job

FAQ:
What *isn't* a **SOAP** policy?

FAQ:
What *isn't* a **SOAP** policy?

- **Rank** changes when not in service

FAQ:
What *isn't* a **SOAP** policy?

- **Rank** changes when not in service
- **Rank** depends on system-wide state

FAQ:
What *isn't* a **SOAP** policy?

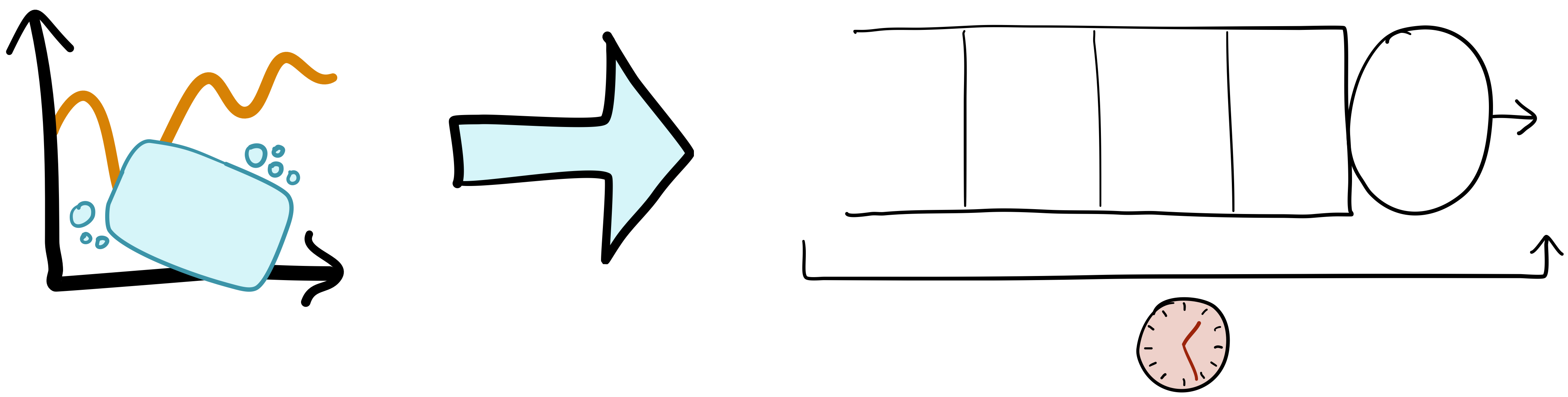
- **Rank** changes when not in service
- **Rank** depends on system-wide state
- Non-FCFS tiebreaking

FAQ:
What *isn't* a **SOAP** policy?

- **Rank** changes when not in service
- **Rank** depends on system-wide state
- Non-FCFS tiebreaking

Excludes: EDF, accumulating priority, PS

$\mathbf{E}[T]$ of *any* **SOAP** Policy



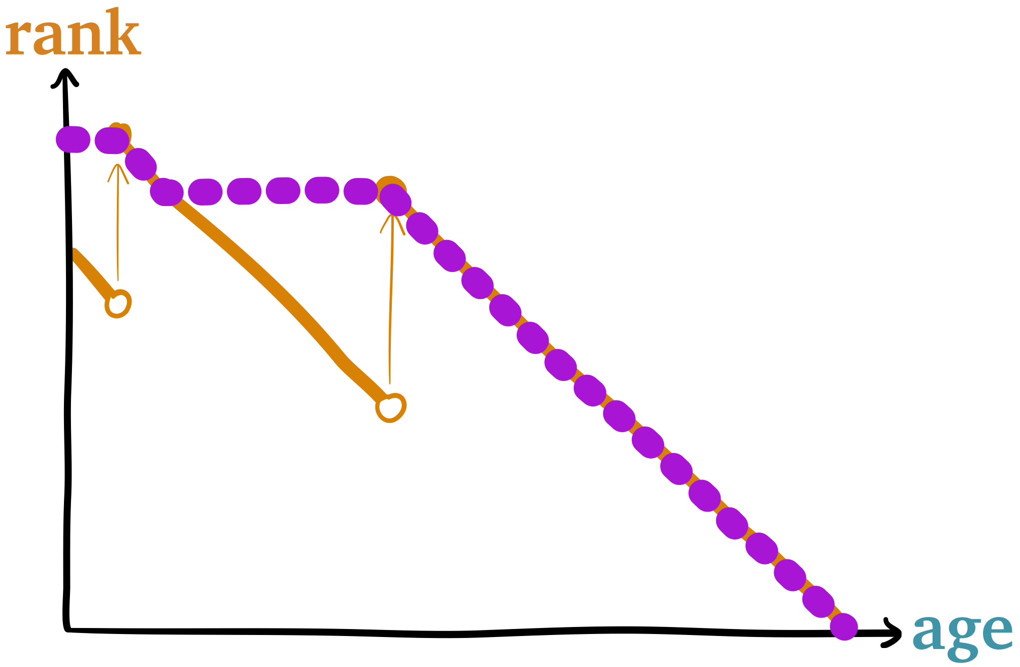
Worst Future Rank

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$



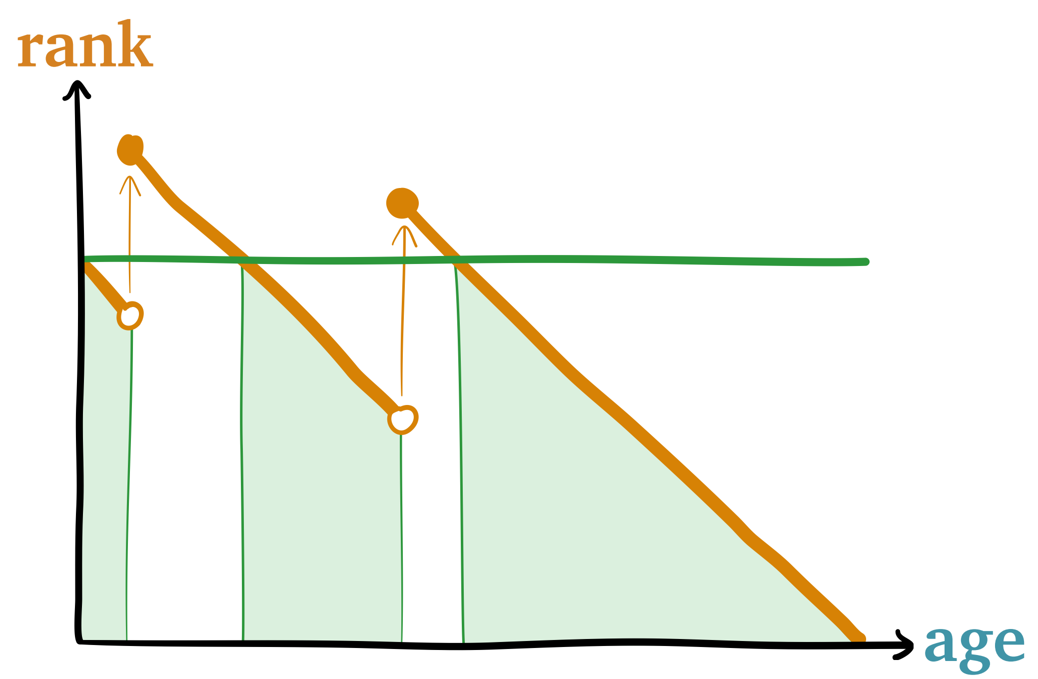
Relevant Intervals

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \leq w$$

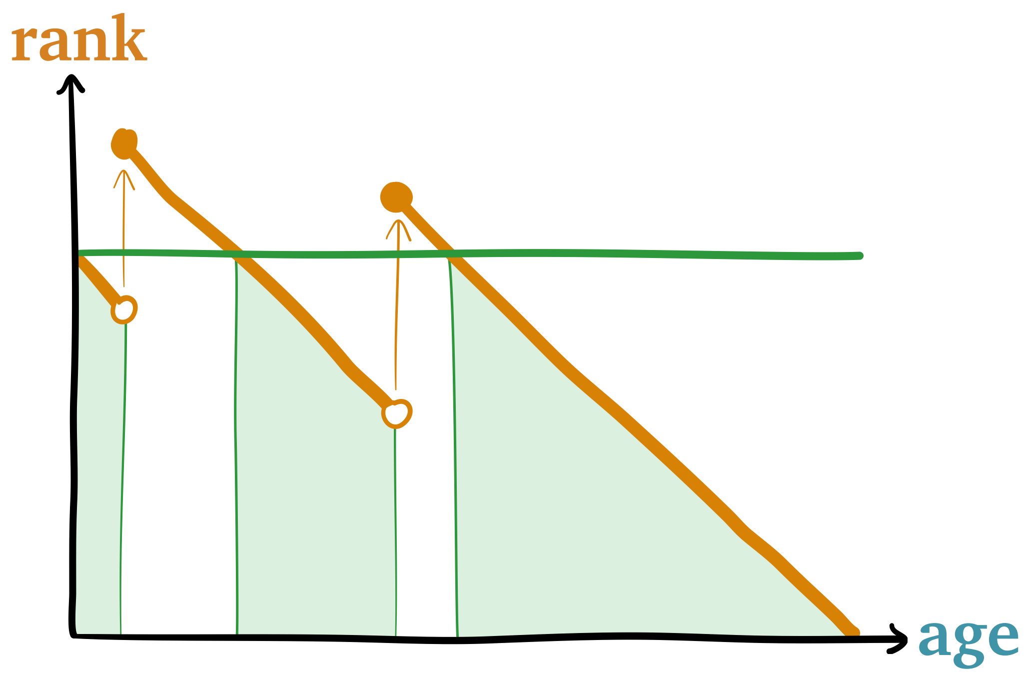
Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \leq w$$



Relevant Intervals

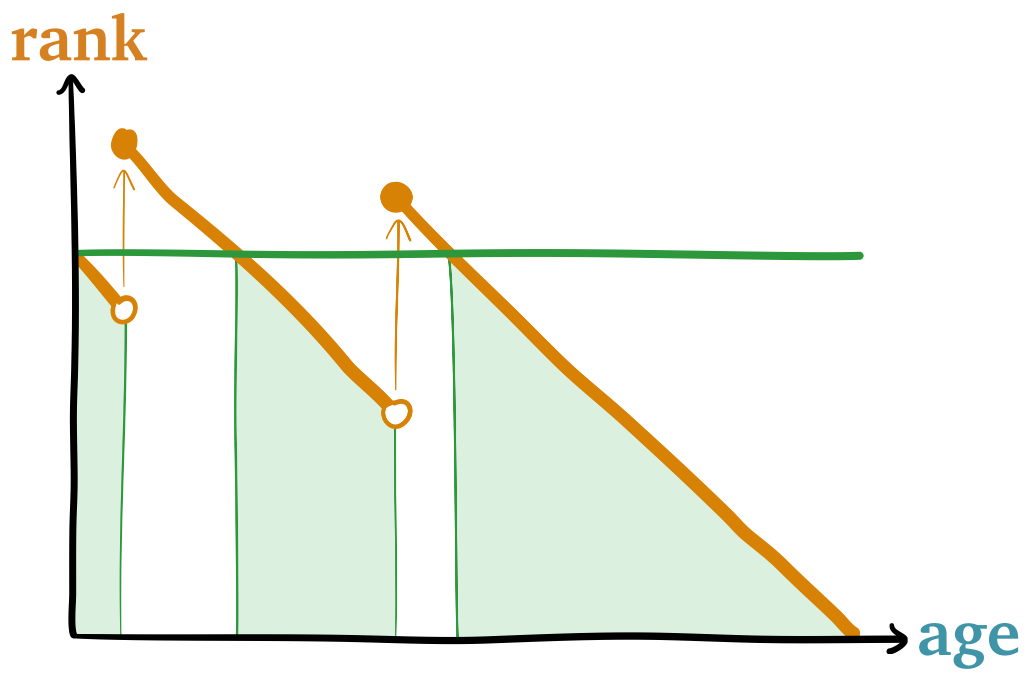
$$I_i[w] = i\text{th interval when } r(a) \leq w$$



Detail: start with $i = 0$ iff first interval contains age 0, else start with $i = 1$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \leq w$$



Detail: start with $i = 0$ iff first interval contains age 0, else start with $i = 1$

Detail: interval can be empty

SOAP Analysis: One Descriptor

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

$$\boxed{\mathbf{E}[T_x] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty} \mathbf{E}[X_i[w_x]^2]}{(1-\rho_0[w_x])(1-\rho_{\text{new}}[w_x])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\text{new}}[w_x(a)]}}$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

$$w_x = w_x(0)$$

$$\boxed{\mathbf{E}[T_x] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty}\mathbf{E}[X_i[w_x]^2]}{(1-\rho_0[w_x])(1-\rho_{\text{new}}[w_x])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\text{new}}[w_x(a)]}}$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

$$w_x = w_x(0)$$

$$\boxed{\mathbf{E}[T_x] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty} \mathbf{E}[X_i[w_x]^2]}{(1-\rho_0[w_x])(1-\rho_{\mathrm{new}}[w_x])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\mathrm{new}}[w_x(a)]}}$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

$$X_i[w] = \text{service a job receives in } I_i[w]$$

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

$$w_x = w_x(0)$$

$$\boxed{\mathbf{E}[T_x] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty} \mathbf{E}[X_i[w_x]^2]}{(1-\rho_0[w_x])(1-\rho_{\mathrm{new}}[w_x])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\mathrm{new}}[w_x(a)]}}$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

$$X_i[w] = \text{service a job receives in } I_i[w]$$

$$\rho_0[w] = \lambda \mathbf{E}[X_0[w]]$$

SOAP Analysis: One Descriptor

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

$$w_x = w_x(0)$$

$$\boxed{\mathbf{E}[T_x] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty} \mathbf{E}[X_i[w_x]^2]}{(1-\rho_0[w_x])(1-\rho_{\text{new}}[w_x])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\text{new}}[w_x(a)]}}$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

$$X_i[w] = \text{service a job receives in } I_i[w]$$

$$\rho_0[w] = \lambda \mathbf{E}[X_0[w]]$$

$$\rho_{\text{new}}[w] = \lambda \mathbf{E}[X_0[w-]]$$

SOAP Analysis: Complete

Worst Future Rank

$$w_x(a) = \sup_{a \le b < x} r(b)$$

Relevant Intervals

$$I_i[w] = i\text{th interval when } r(a) \le w$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

Relevant Intervals

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

Relevant Intervals

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

Relevant Intervals

X_d = size distribution for descriptor d

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

Relevant Intervals

X_d = size distribution for descriptor d

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

$$X_i[w] = X_{i,D}[w]$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

Relevant Intervals

X_d = size distribution for descriptor d

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

$$X_i[w] = X_{i,D}[w]$$

D = descriptor distribution

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

$$w_{d,x} = w_{d,x}(0)$$

Relevant Intervals

X_d = size distribution for descriptor d

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

$$X_i[w] = X_{i,D}[w]$$

D = descriptor distribution

$$\rho_0[w] = \lambda \mathbf{E}[X_0[w]]$$

$$\rho_{\text{new}}[w] = \lambda \mathbf{E}[X_0[w-]]$$

SOAP Analysis: Complete

Worst Future Rank

$$w_{d,x}(a) = \sup_{a \le b < x} r_d(b)$$

$$w_{d,x} = w_{d,x}(0)$$

$$\boxed{\mathbf{E}[T_{d,x}] = \frac{\frac{\lambda}{2}\sum_{i=0}^{\infty}\mathbf{E}[X_i[w_{d,x}]^2]}{(1-\rho_0[w_{d,x}])(1-\rho_{\mathrm{new}}[w_{d,x}])} + \int_0^x \frac{\mathrm{d}a}{1-\rho_{\mathrm{new}}[w_{d,x}(a)]}}$$

Relevant Intervals

X_d = size distribution for descriptor d

$$I_{i,d}[w] = i\text{th interval when } r_d(a) \le w$$

$$X_{i,d}[w] = \text{service a job of descriptor } d \text{ receives in } I_{i,d}[w]$$

$$X_i[w] = X_{i,D}[w]$$

D = descriptor distribution

$$\rho_0[w] = \lambda\mathbf{E}[X_0[w]]$$

$$\rho_{\mathrm{new}}[w] = \lambda\mathbf{E}[X_0[w-]]$$