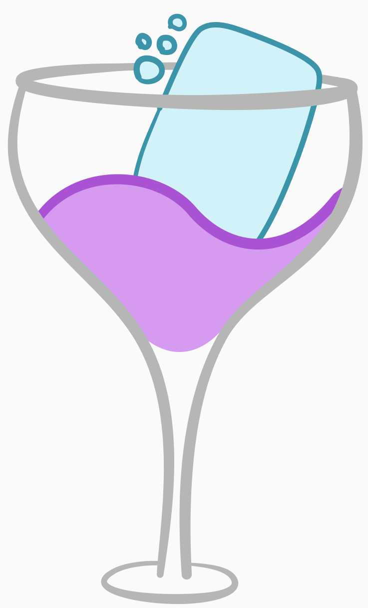
Uniform Bounds for **Scheduling** *with* **Job Size Estimates**

Ziv Scully
Cornell

Isaac Grosof
Carnegie Mellon → Northwestern

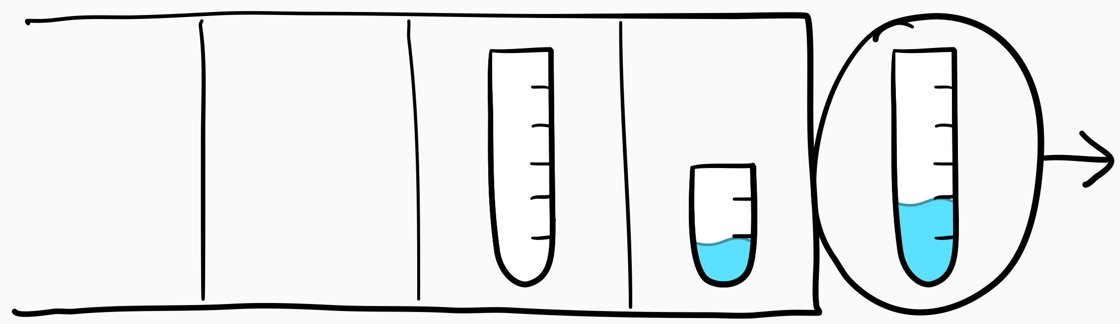
Michael Mitzenmacher
Harvard



algorithms

with

predictions



scheduling algorithms

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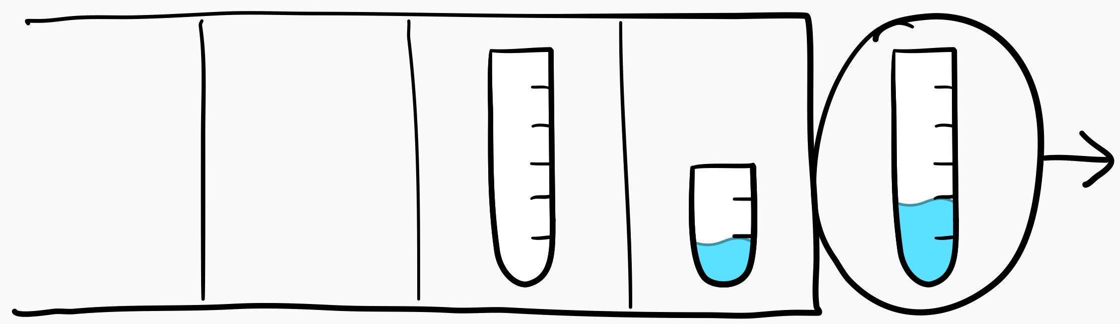

scheduling algorithms

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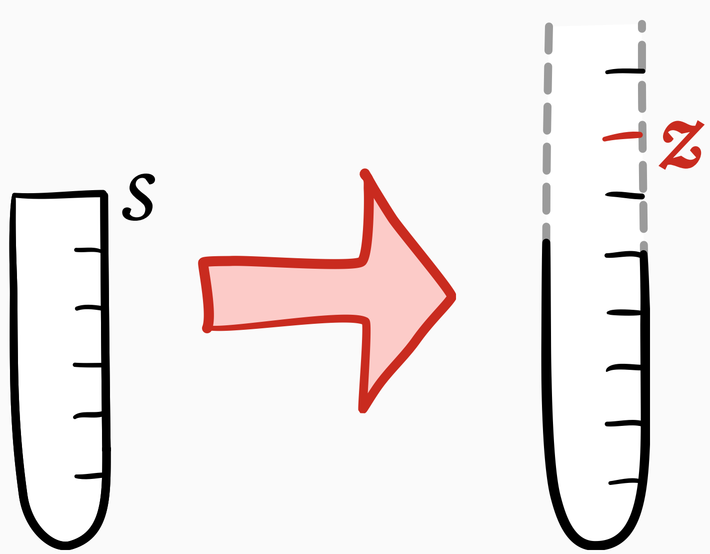

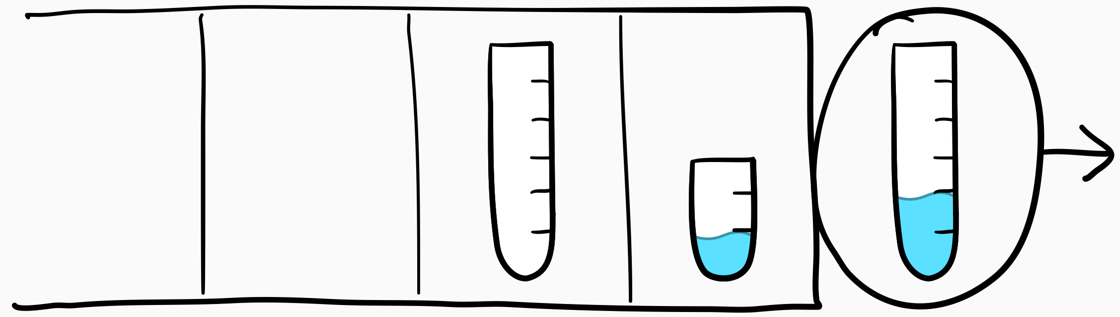

scheduling algorithms
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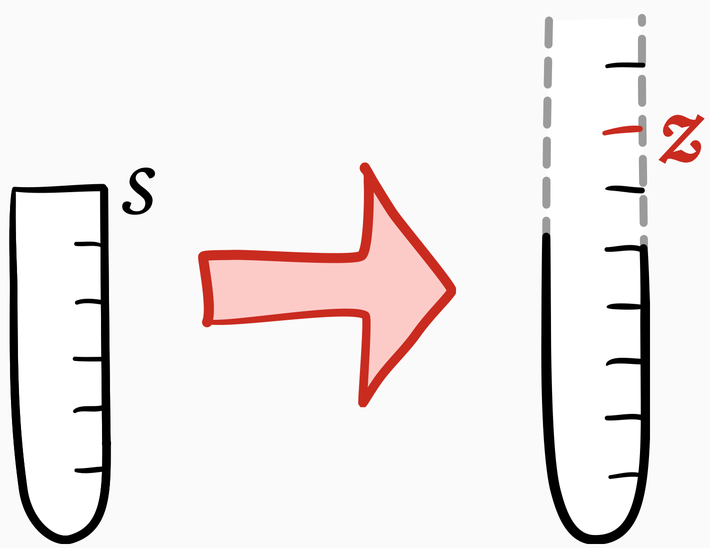


Twist: *stochastic* setting

How do we schedule
to minimize delay
with noisy size estimates?

How do we schedule to minimize delay with noisy size estimates?

What are scheduling and delay?

How do we schedule
to minimize delay
with noisy size estimates?

What are scheduling and delay?

What job size noise model?

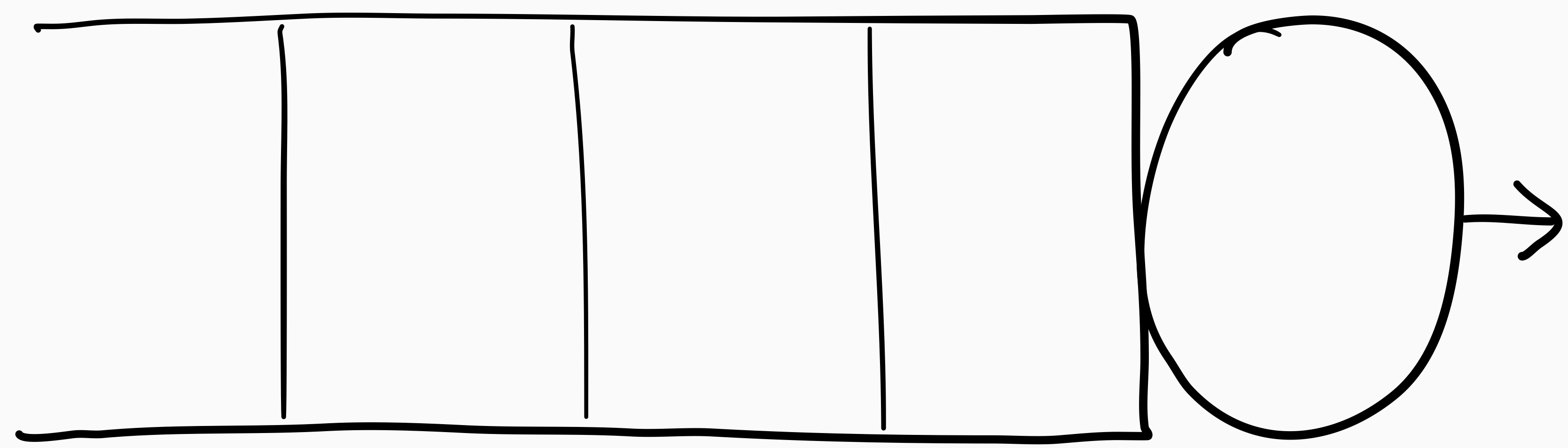
How do we schedule to minimize delay with noisy size estimates?

What are scheduling and delay?

What job size noise model?

What can we hope to achieve?

What are scheduling and delay?



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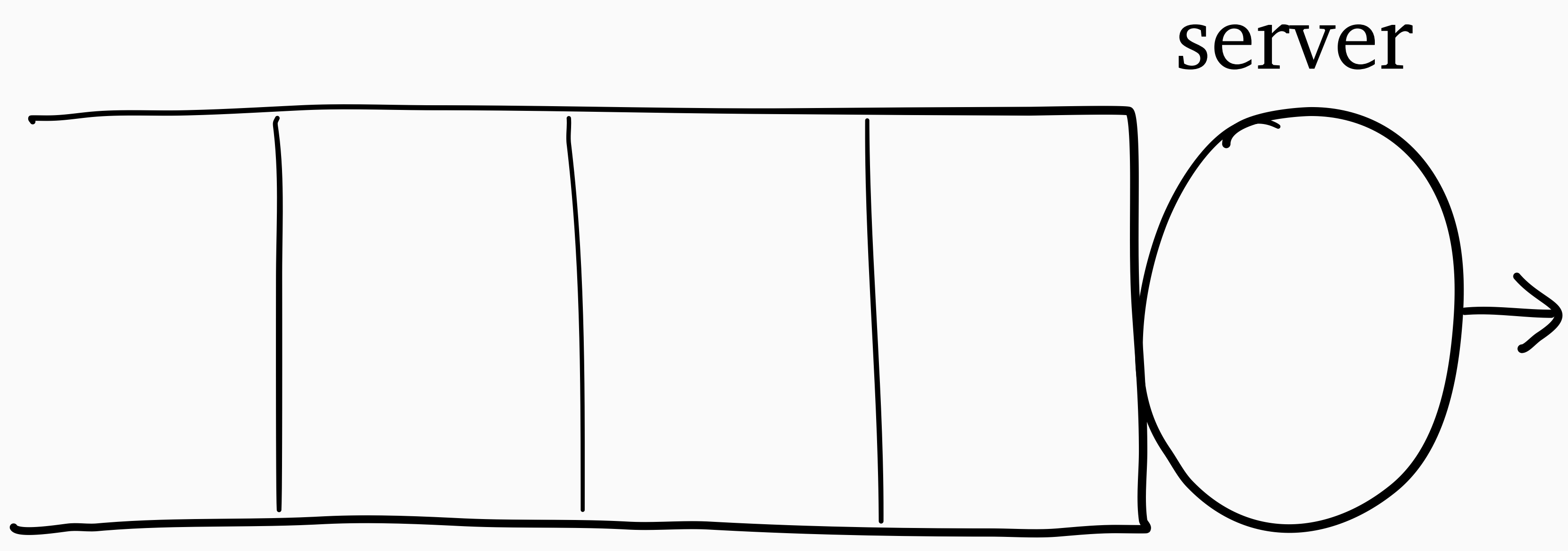

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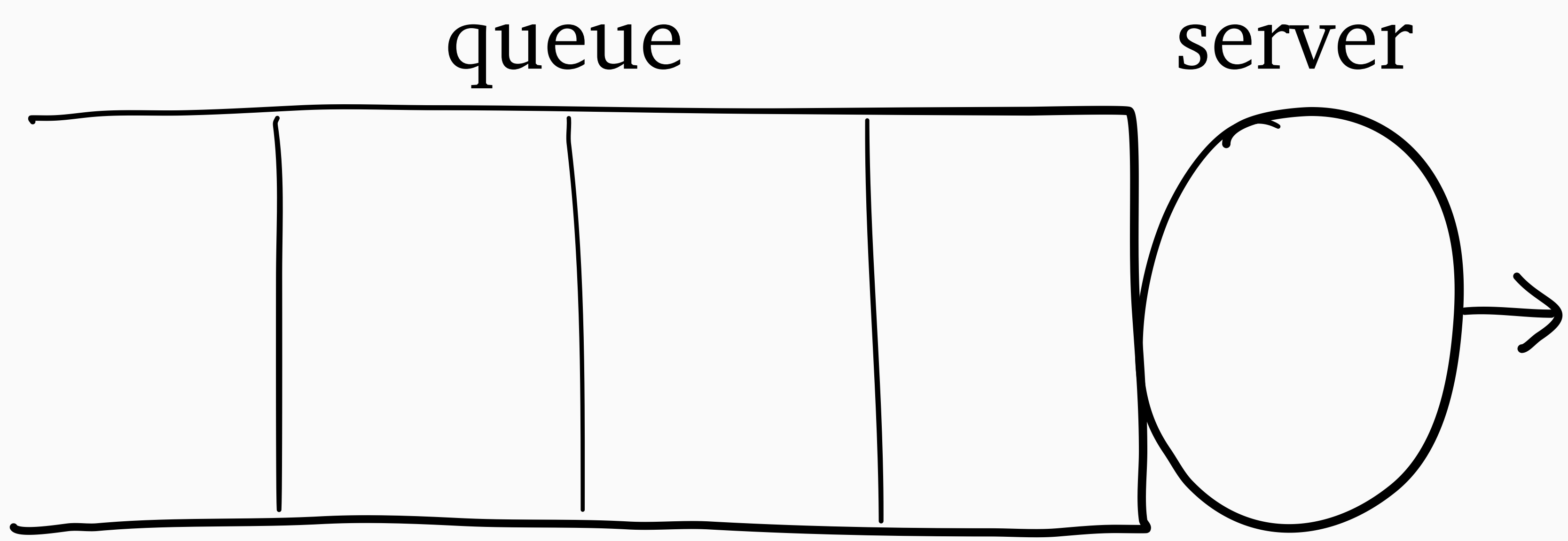

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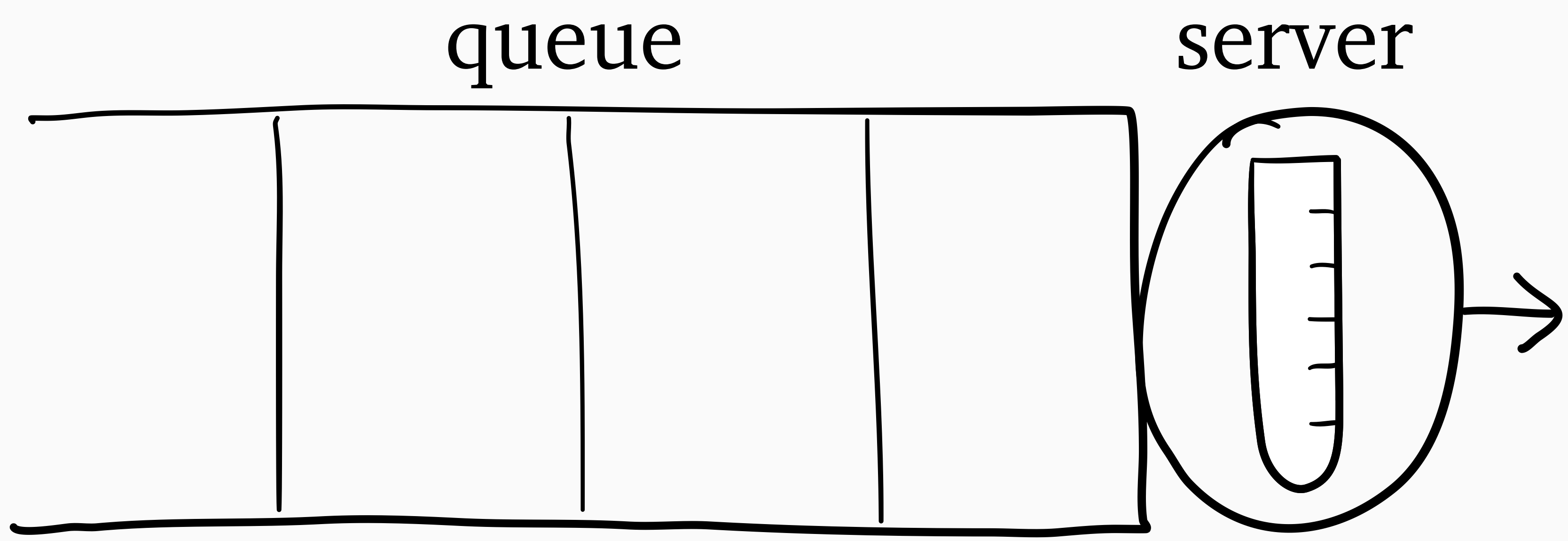

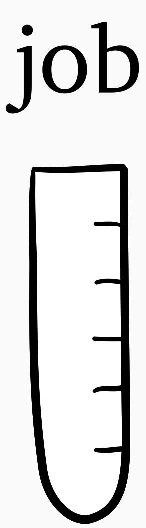

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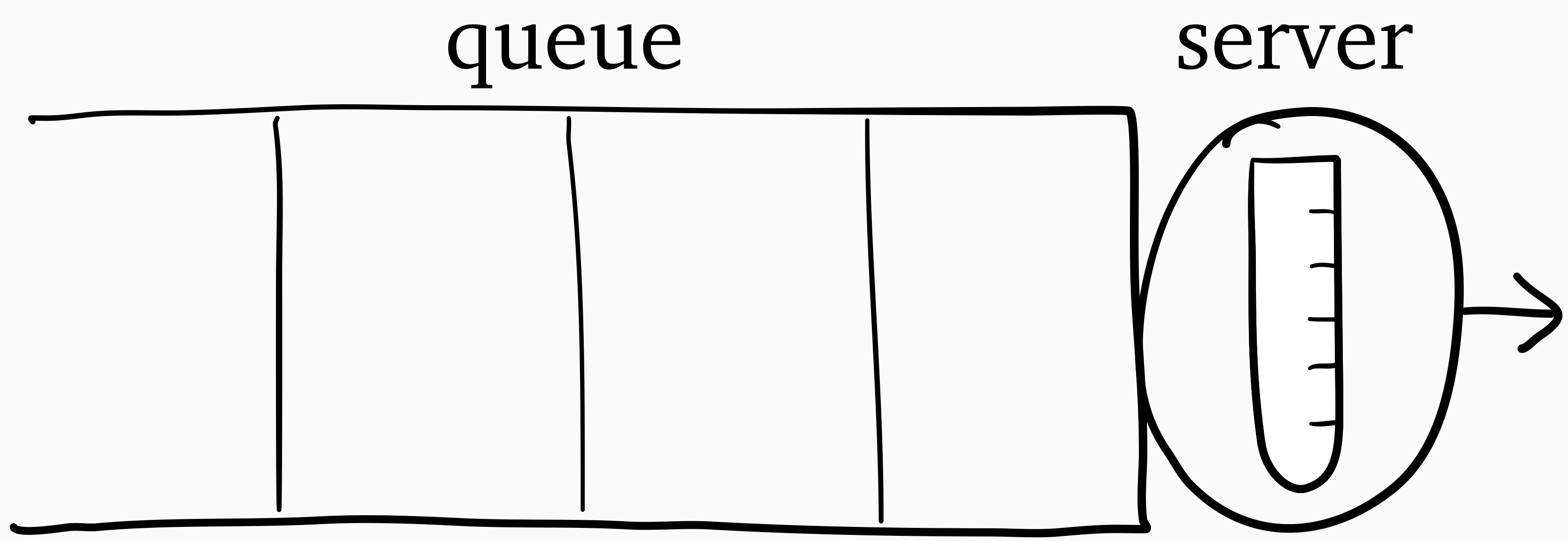

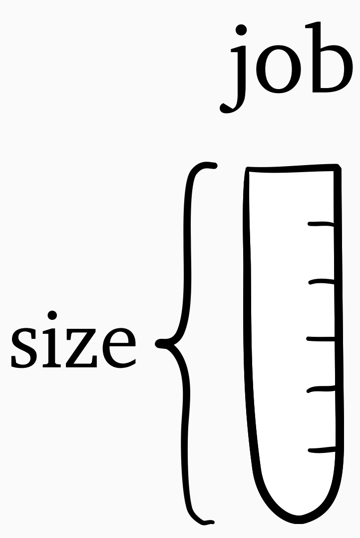

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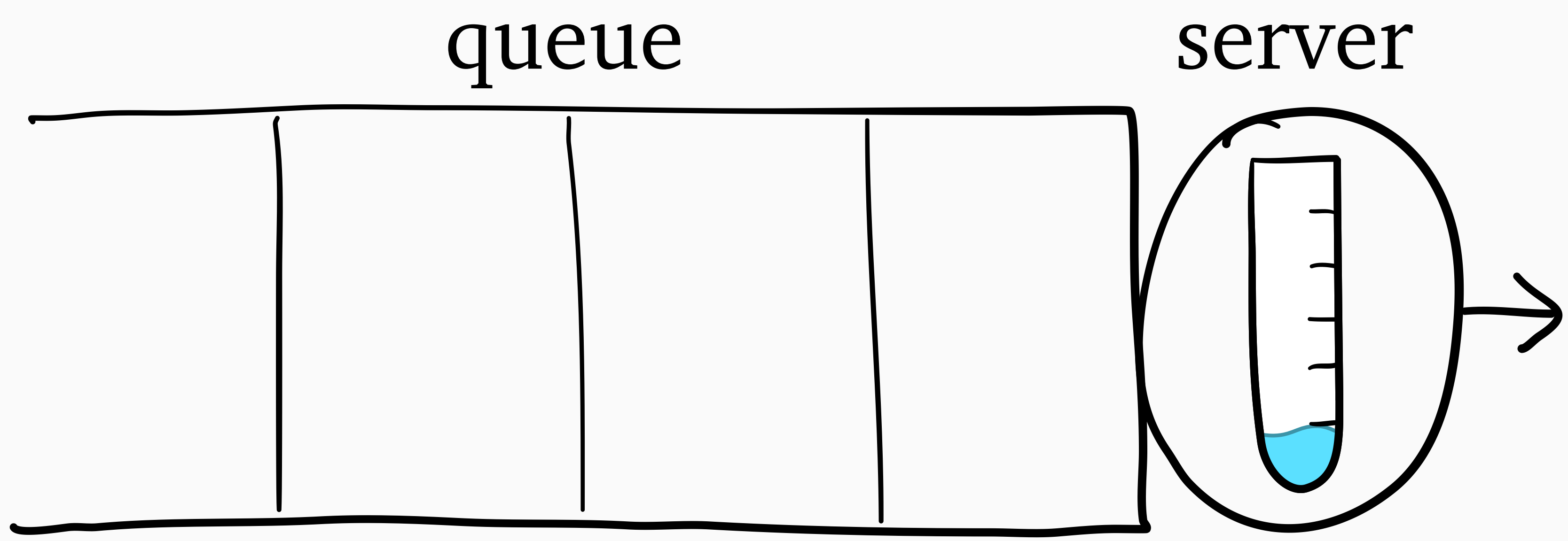

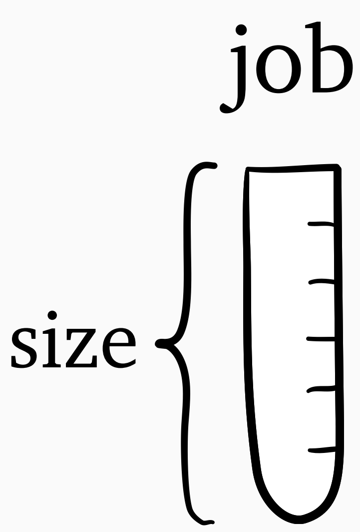

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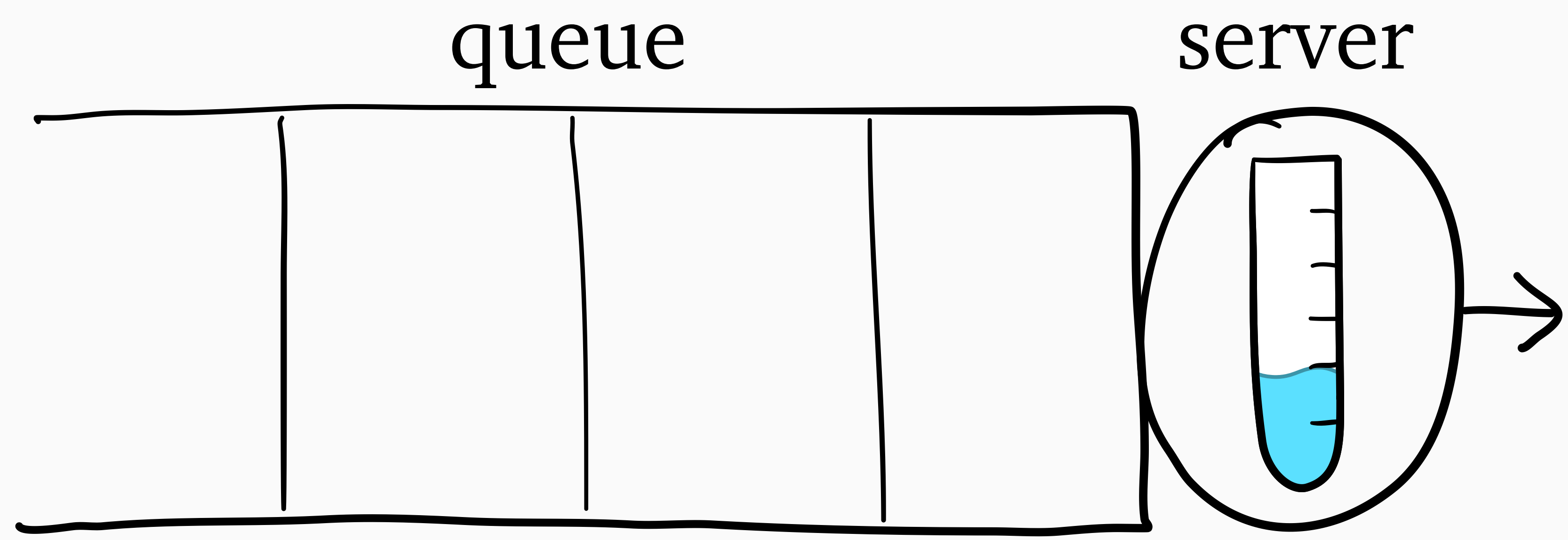

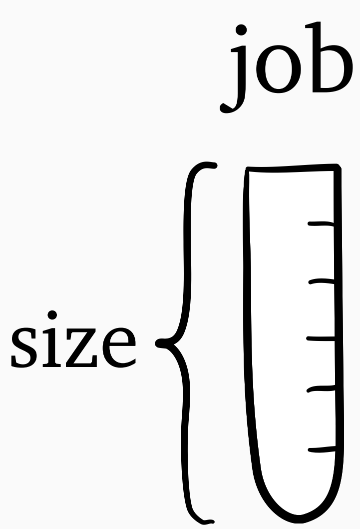

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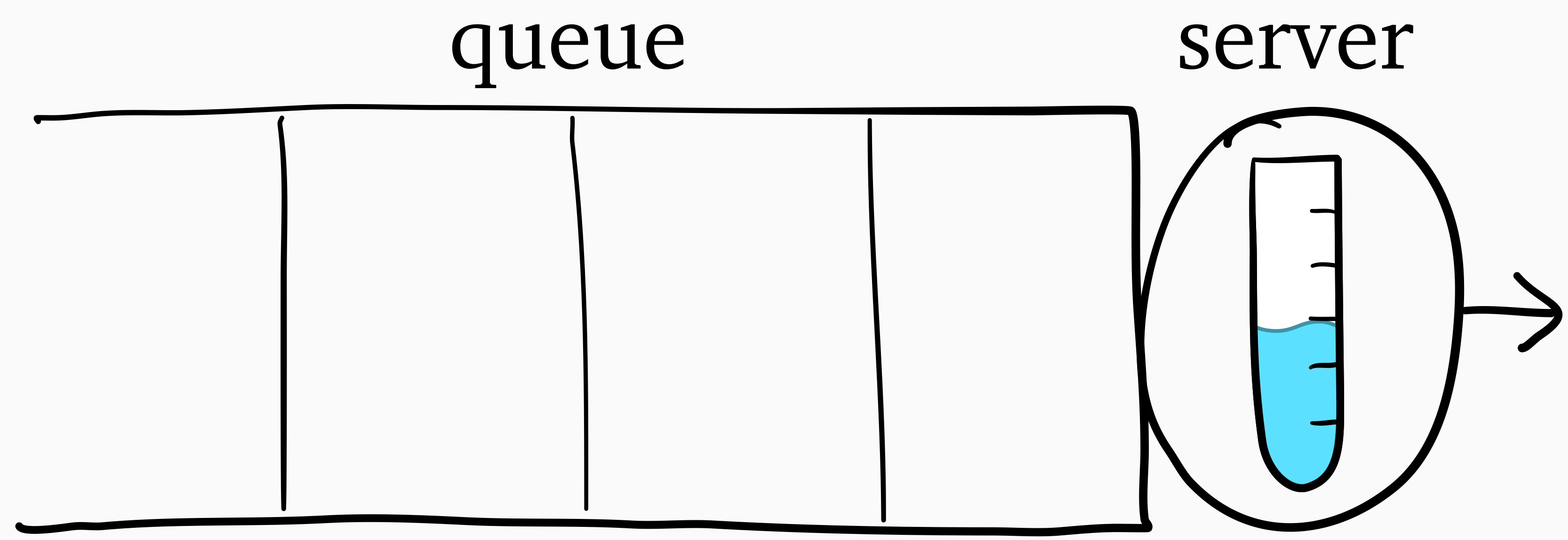

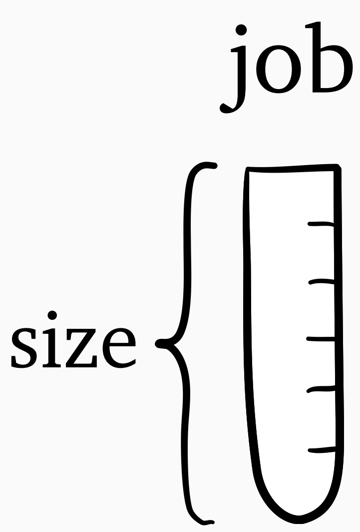

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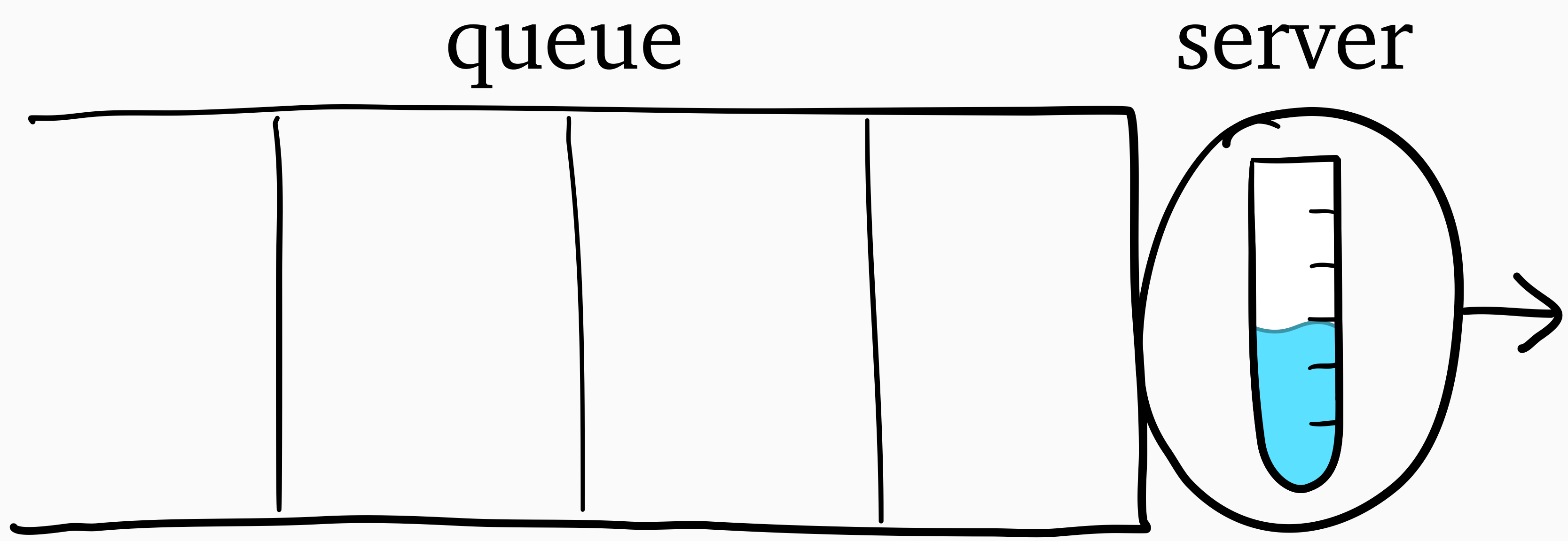

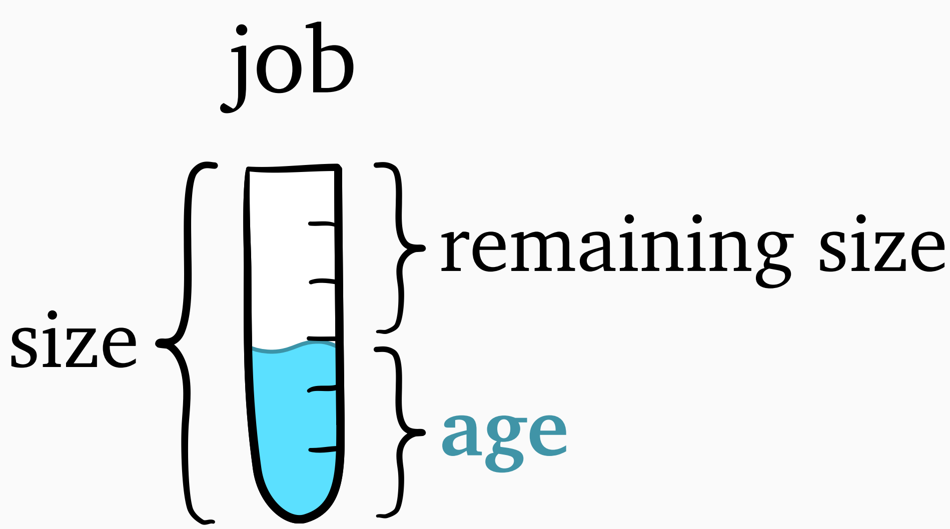

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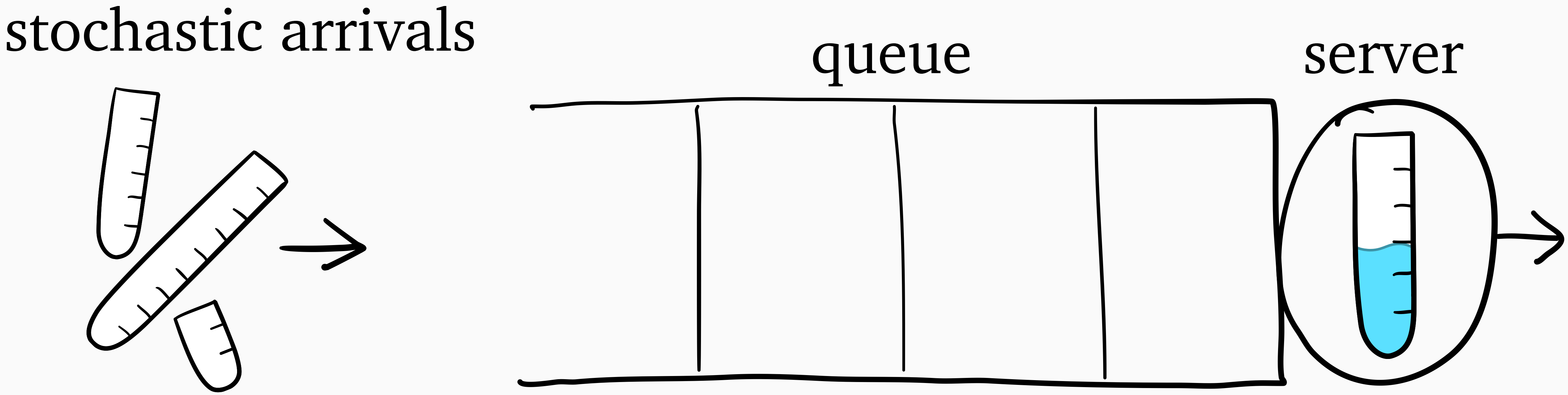

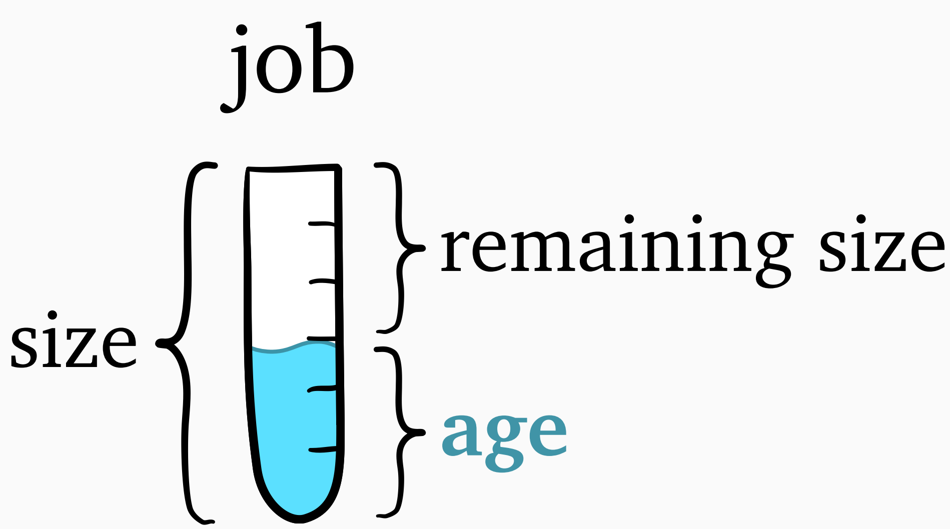

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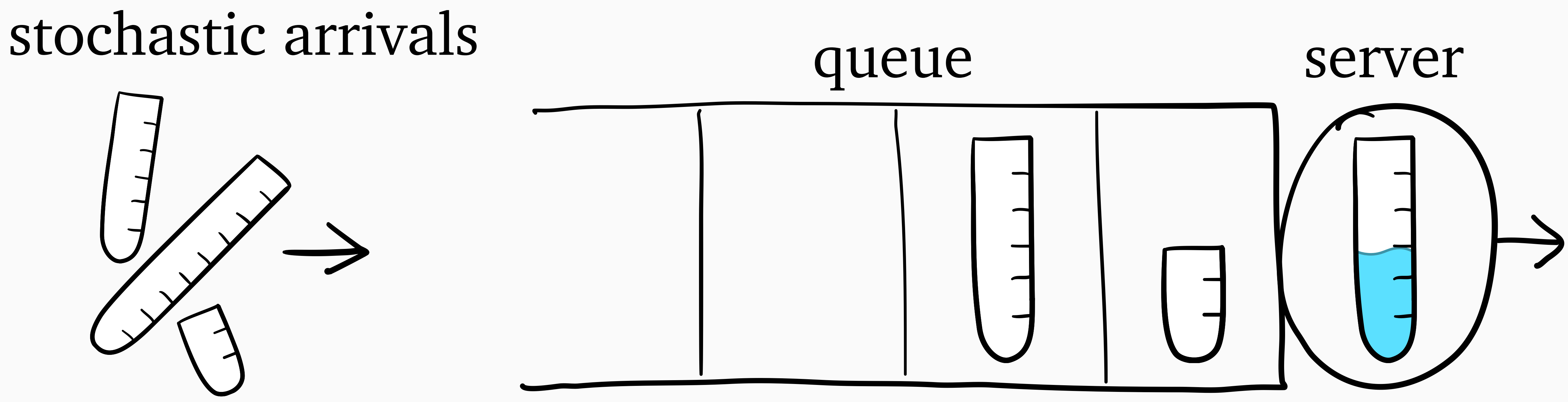

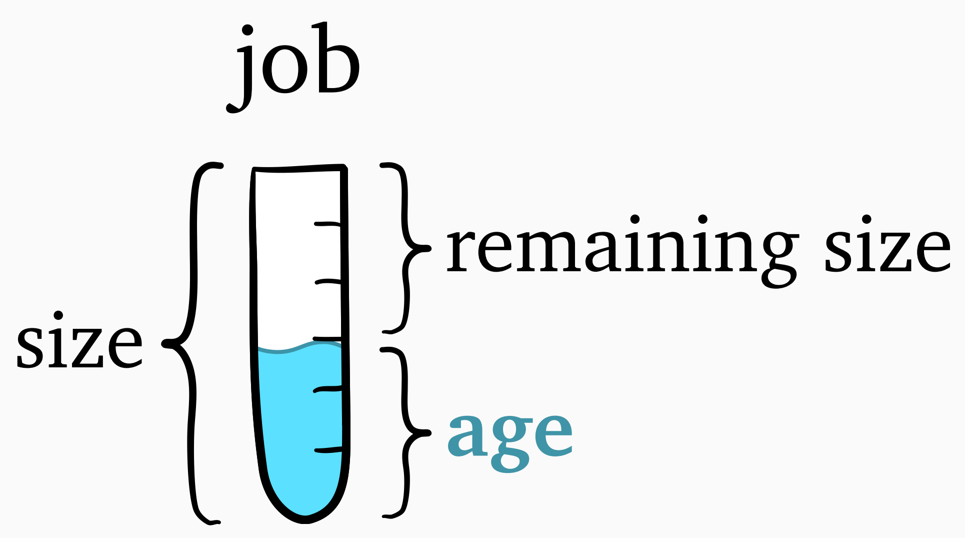

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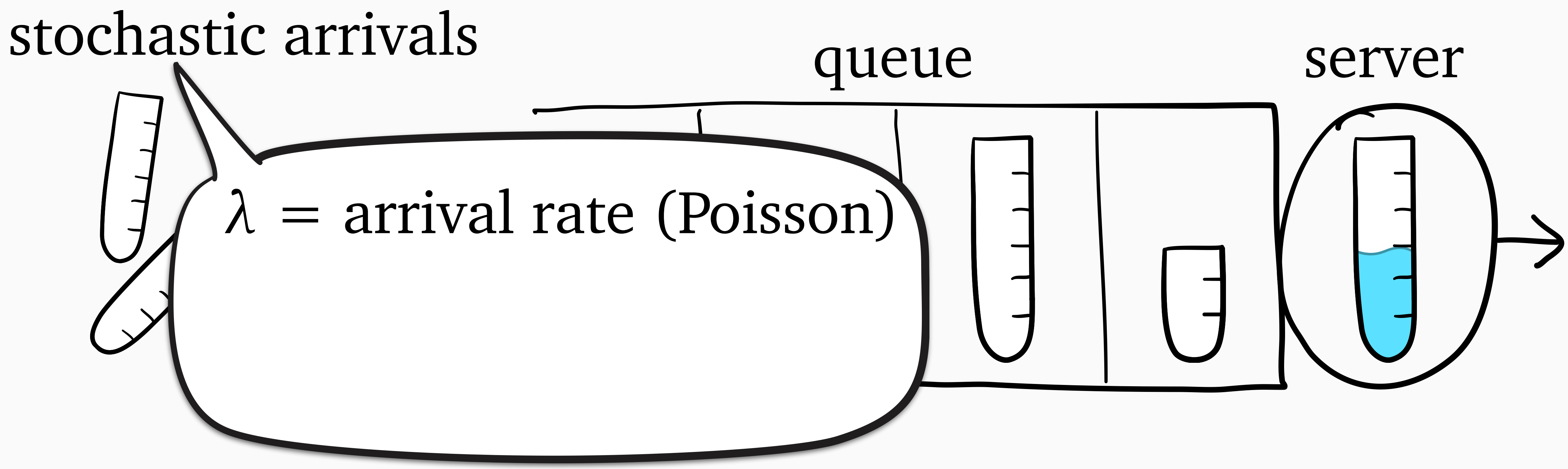

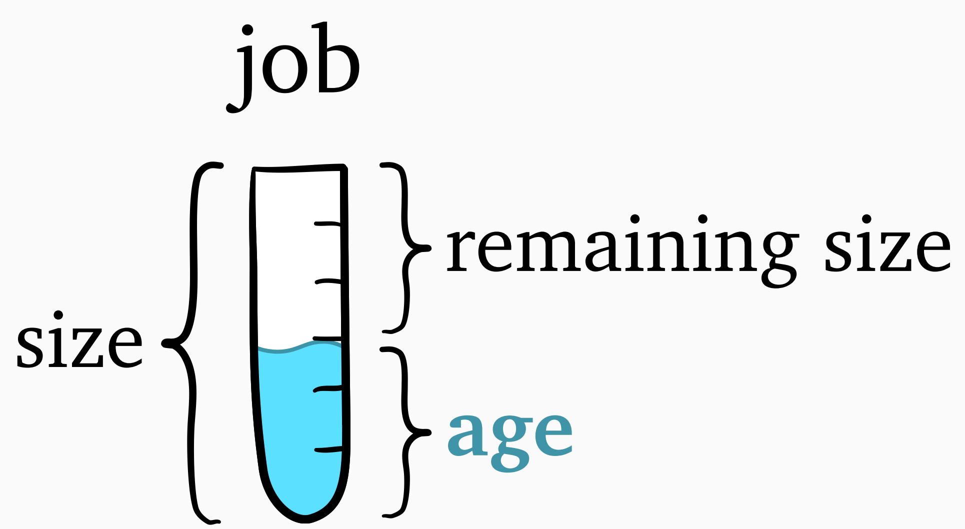

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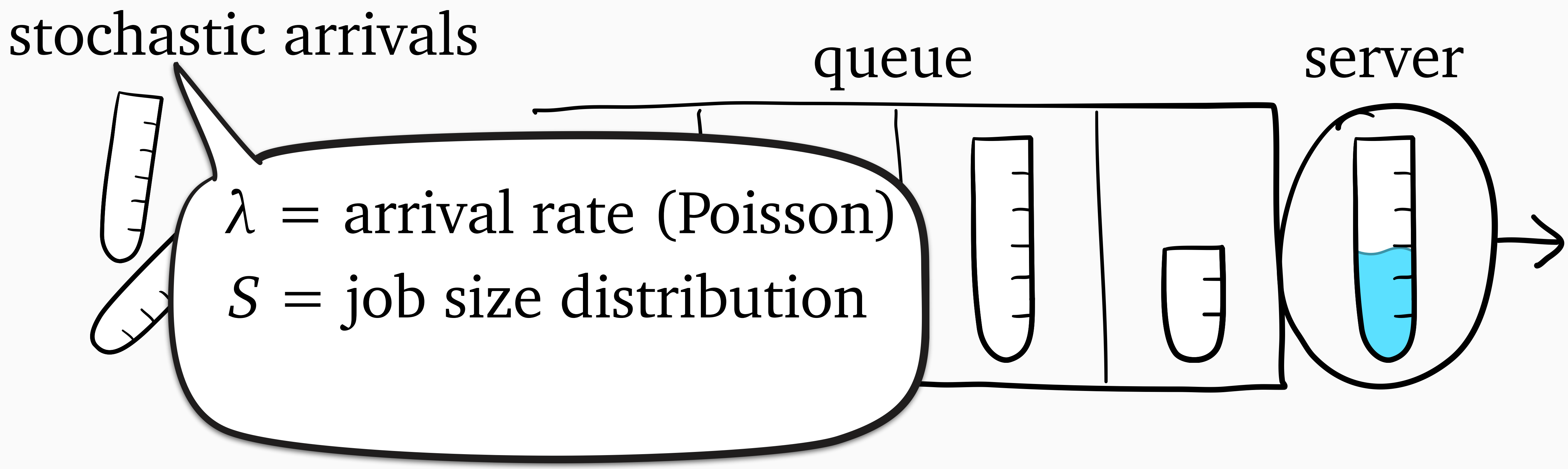

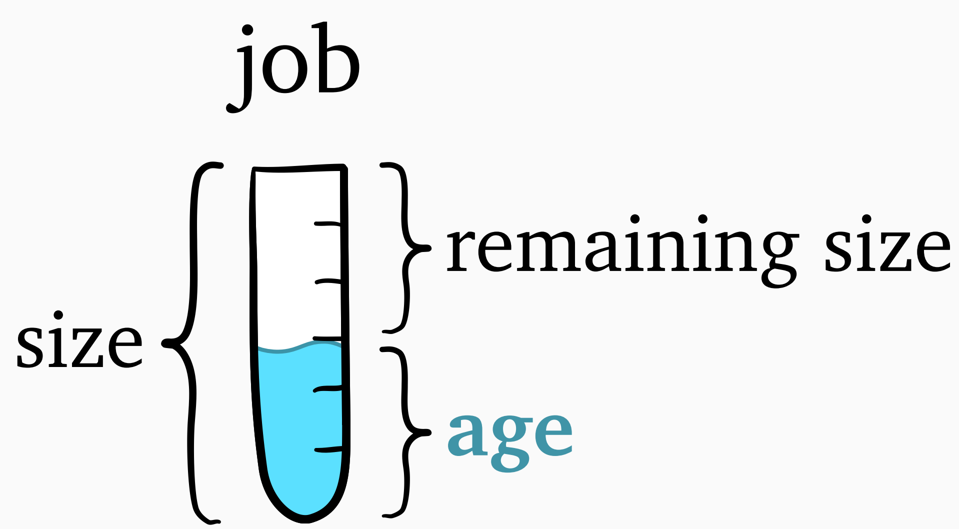

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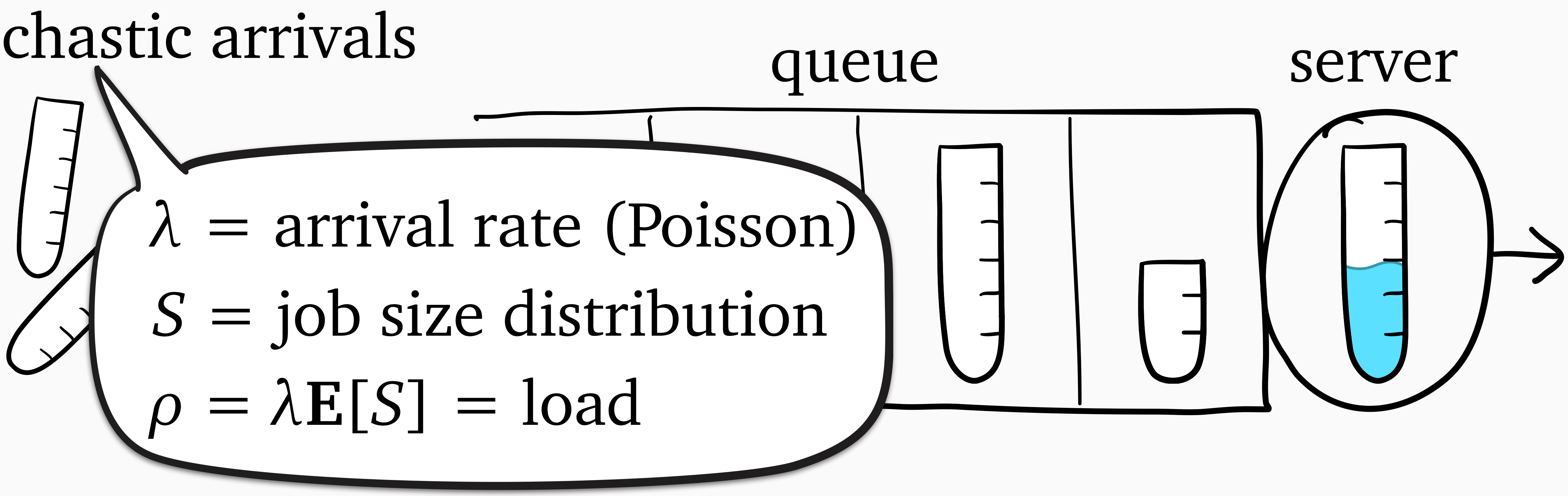

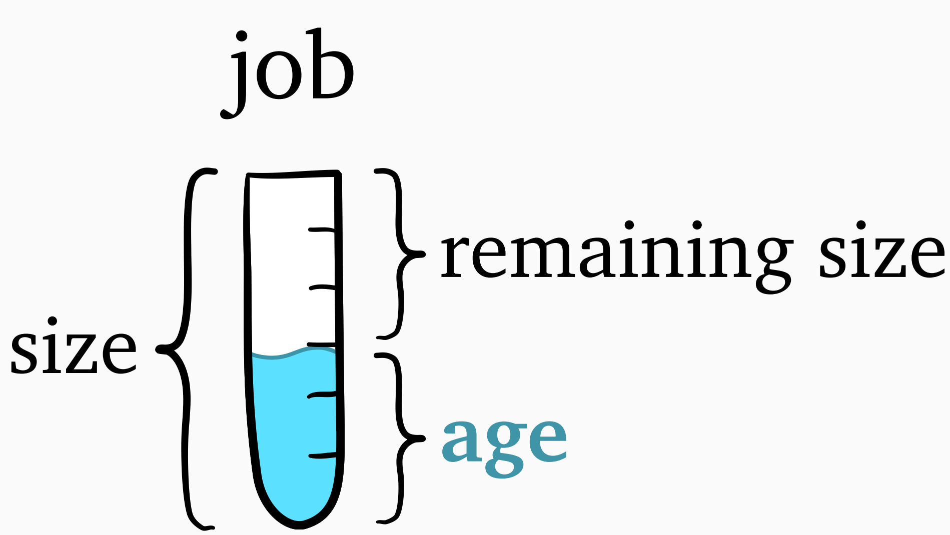

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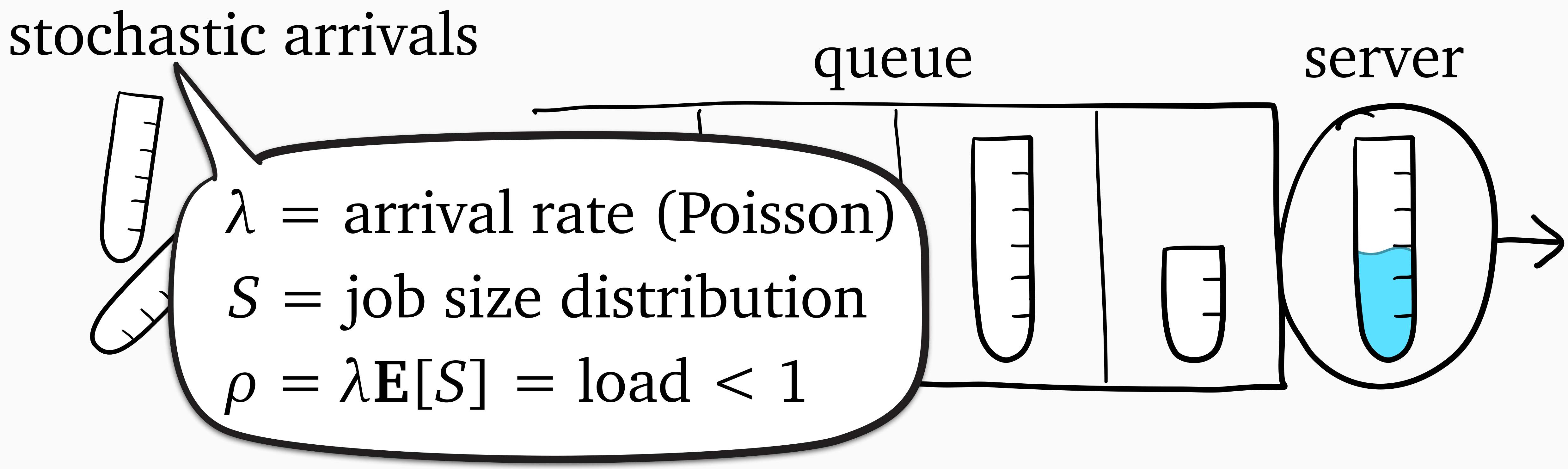

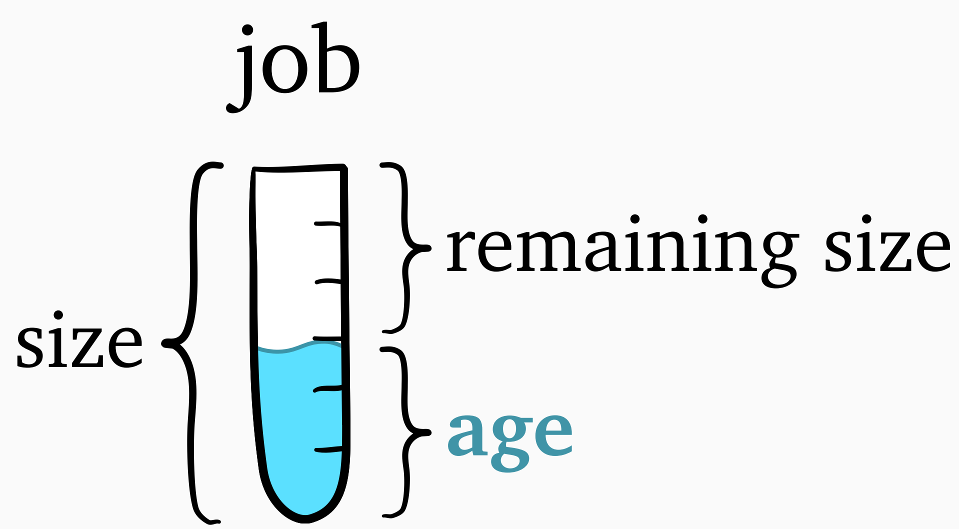

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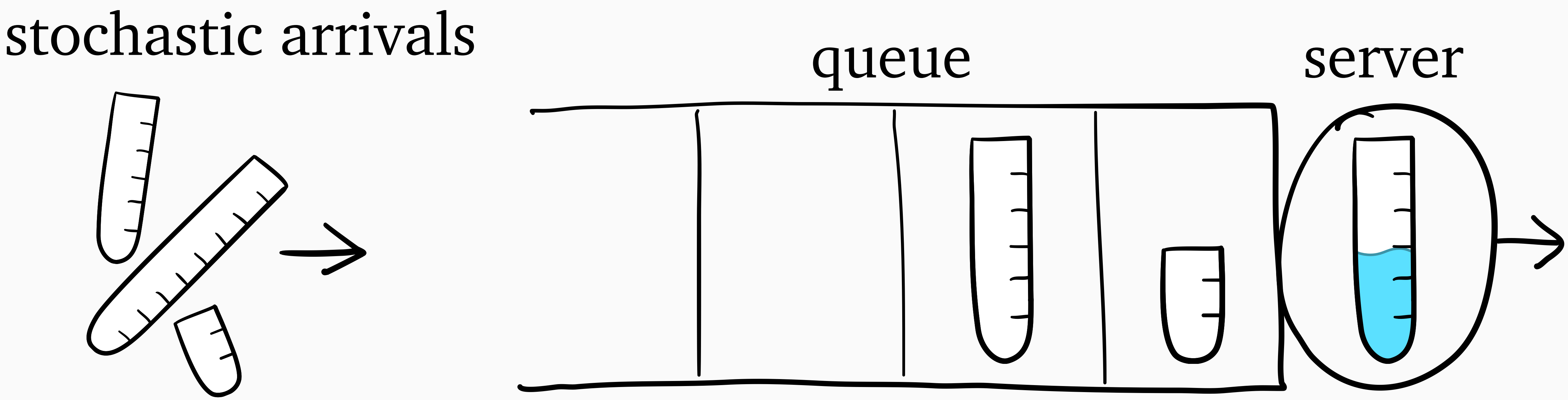

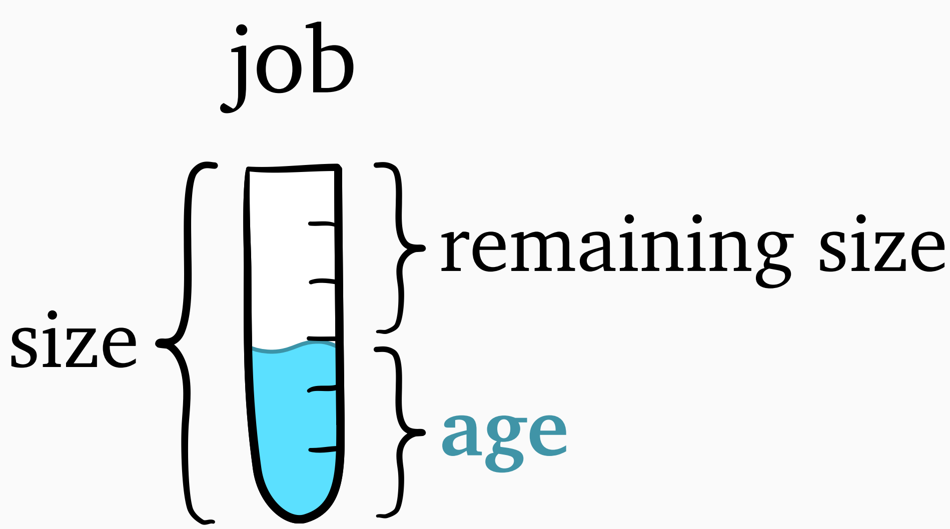

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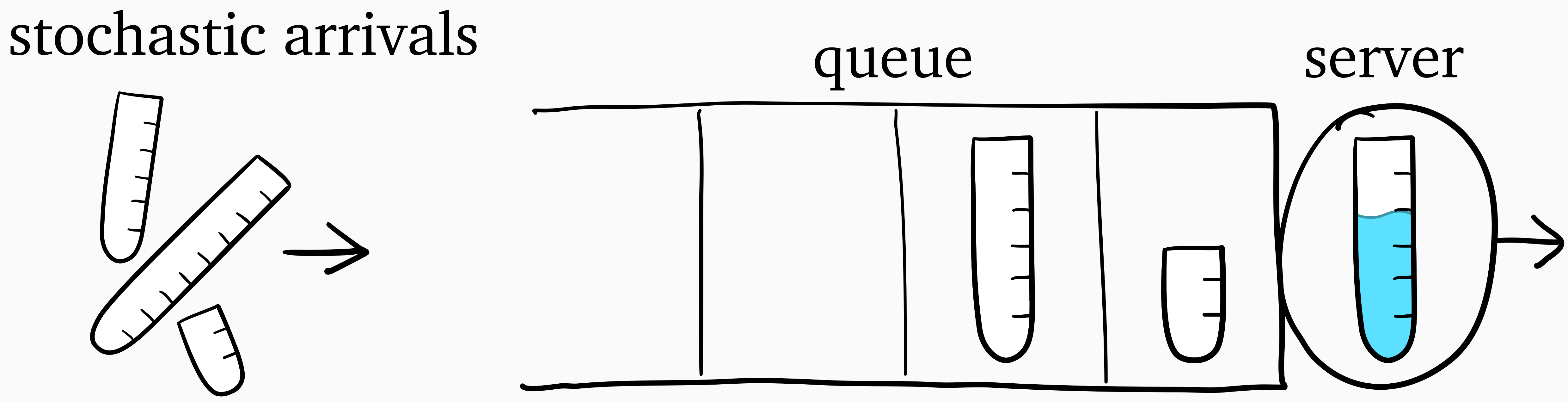

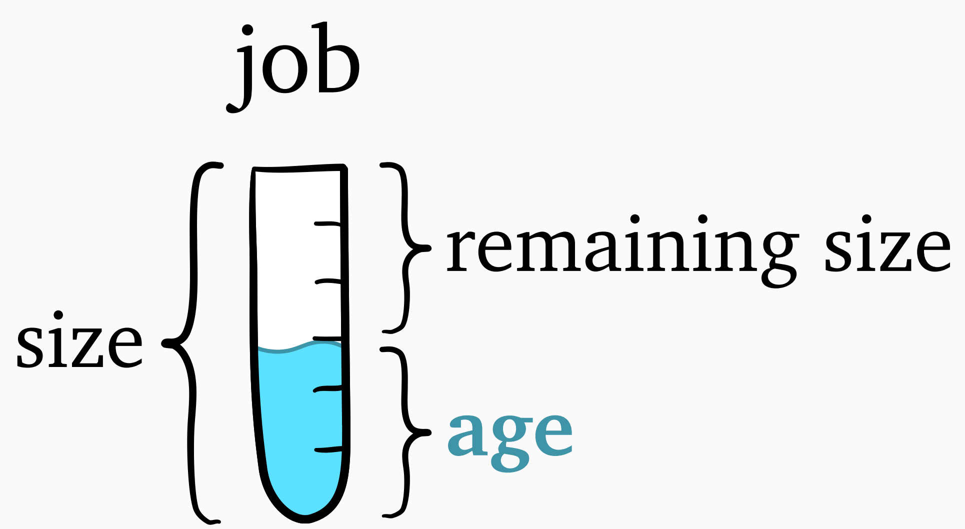

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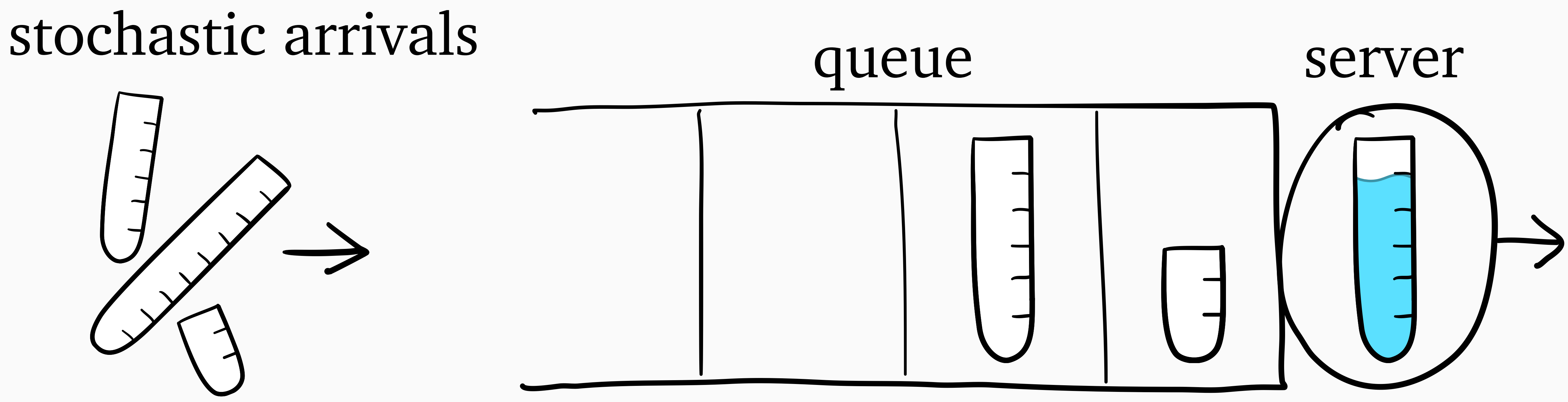

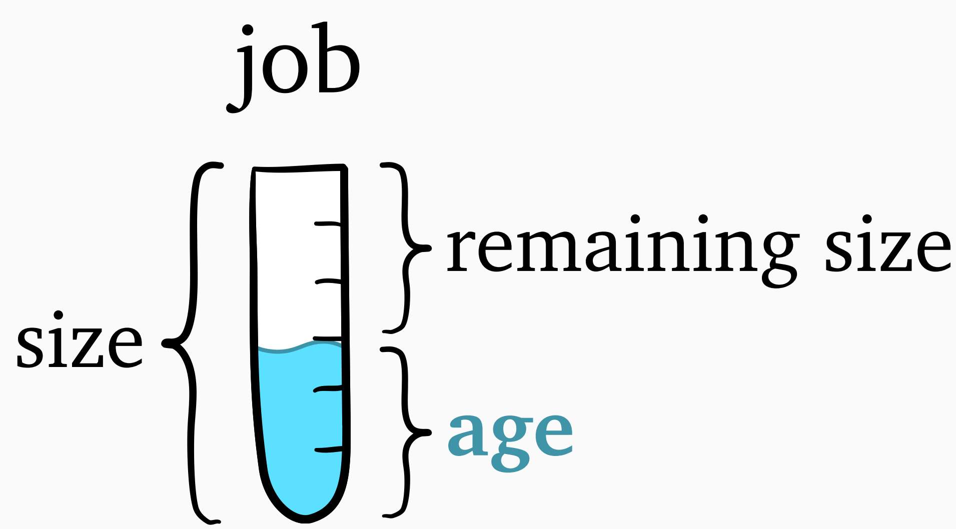

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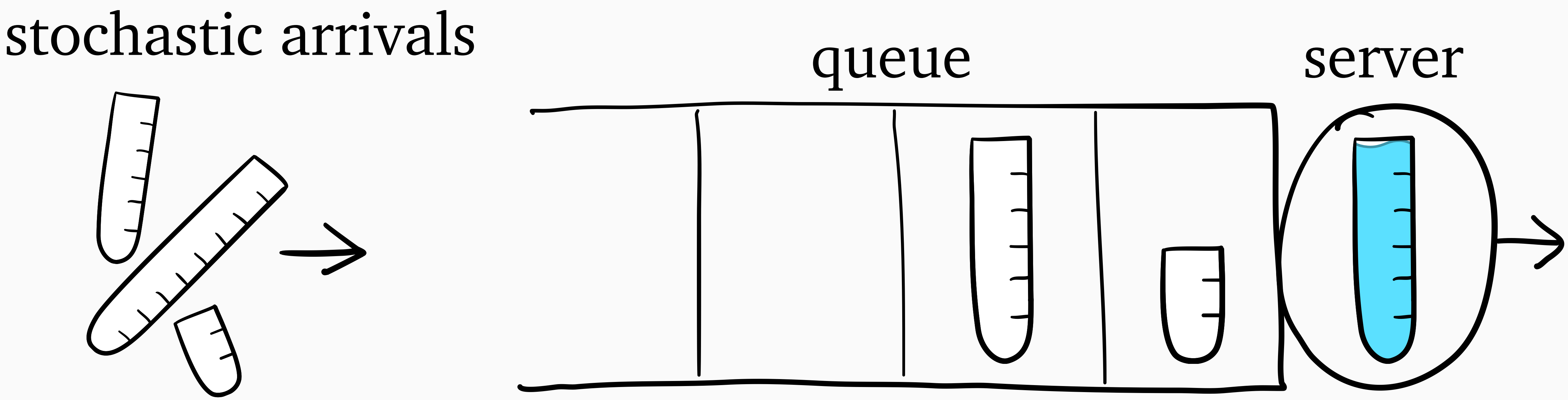

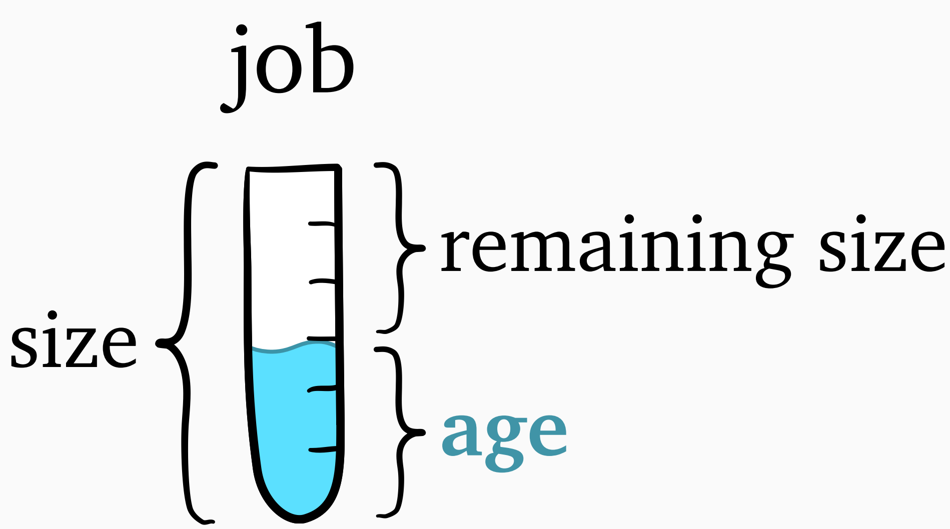

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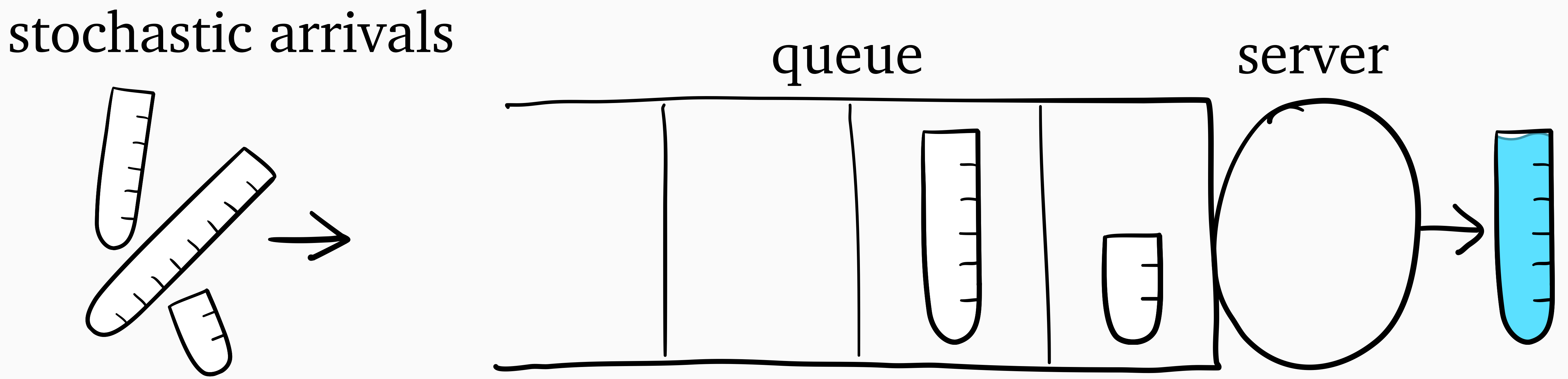

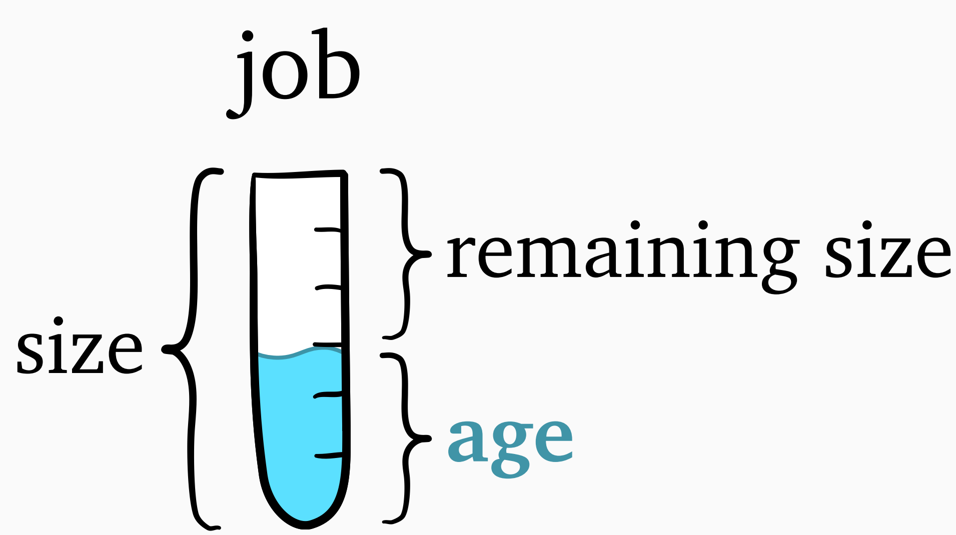

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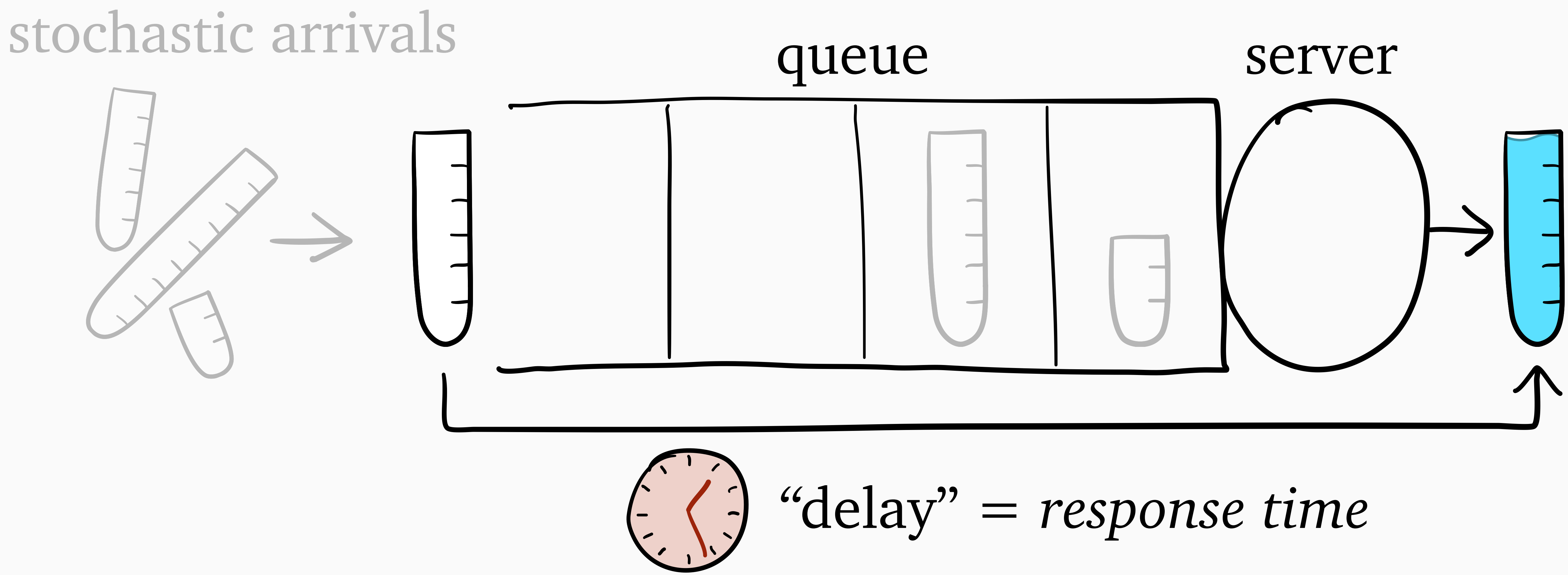

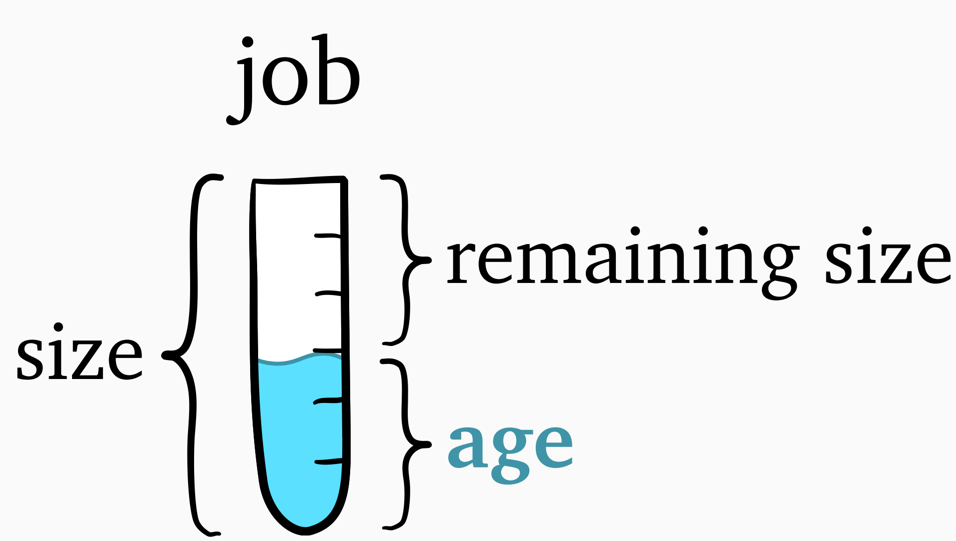

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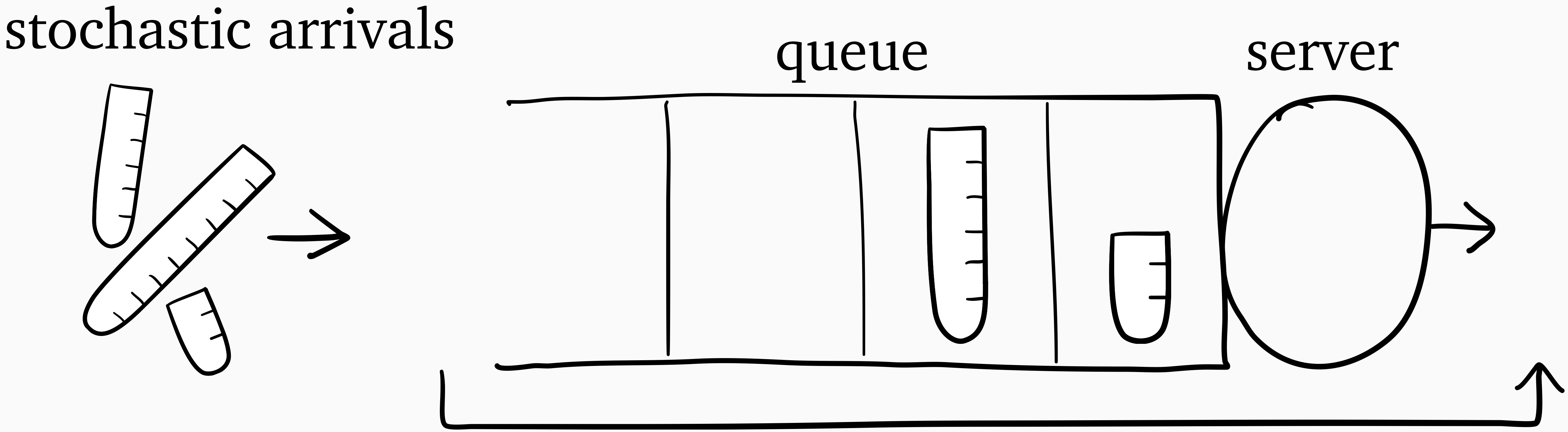
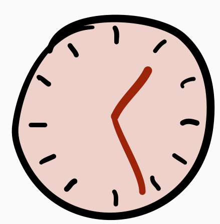
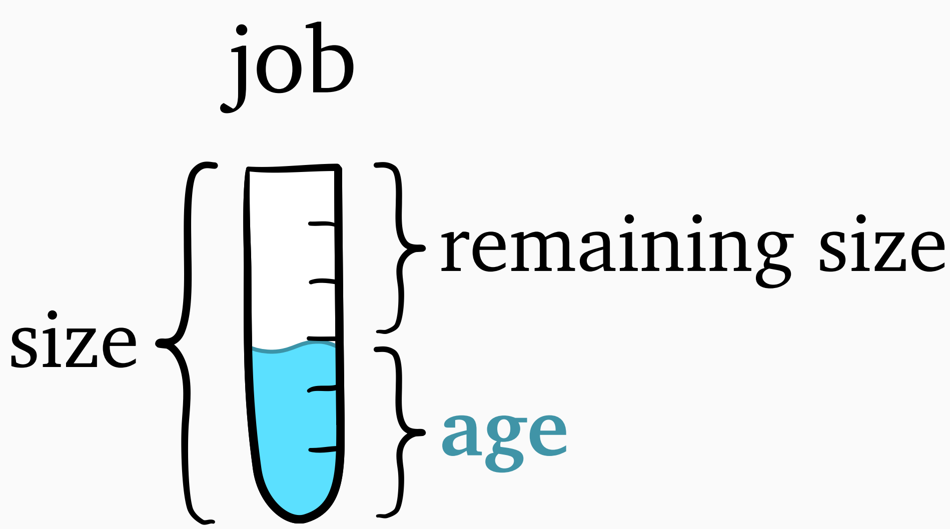




“delay” = *response time*

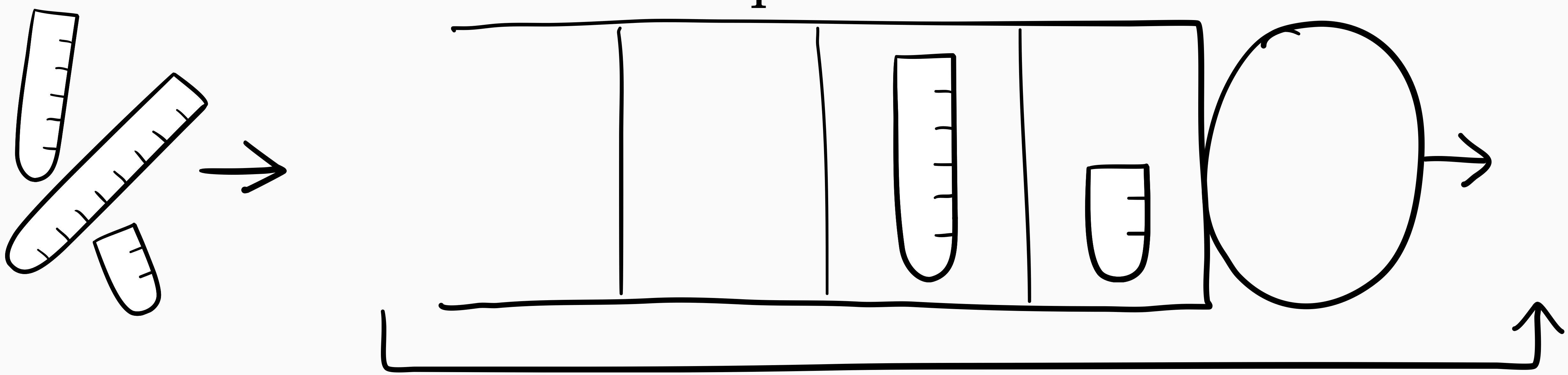


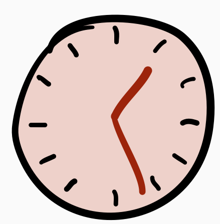
What are scheduling and delay?

stochastic arrivals

queue

server





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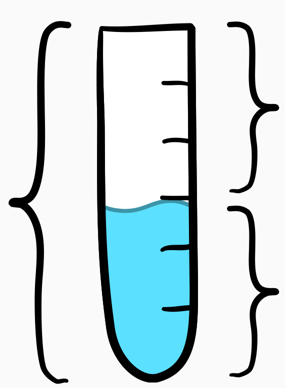
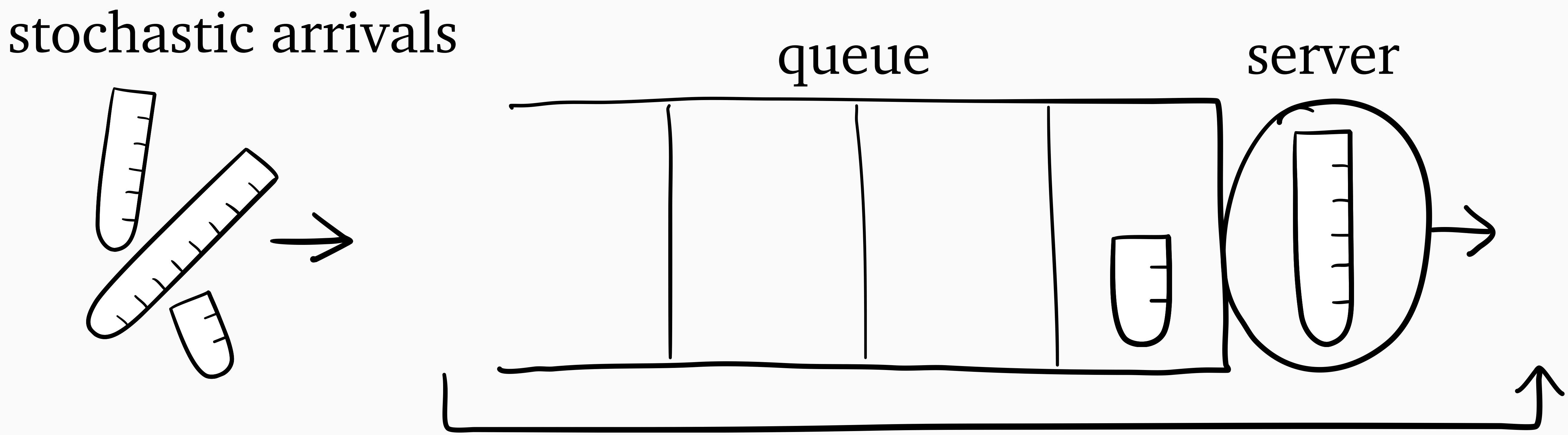
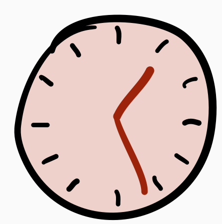
job



Scheduling policy:
decides which job to serve

What are scheduling and delay?





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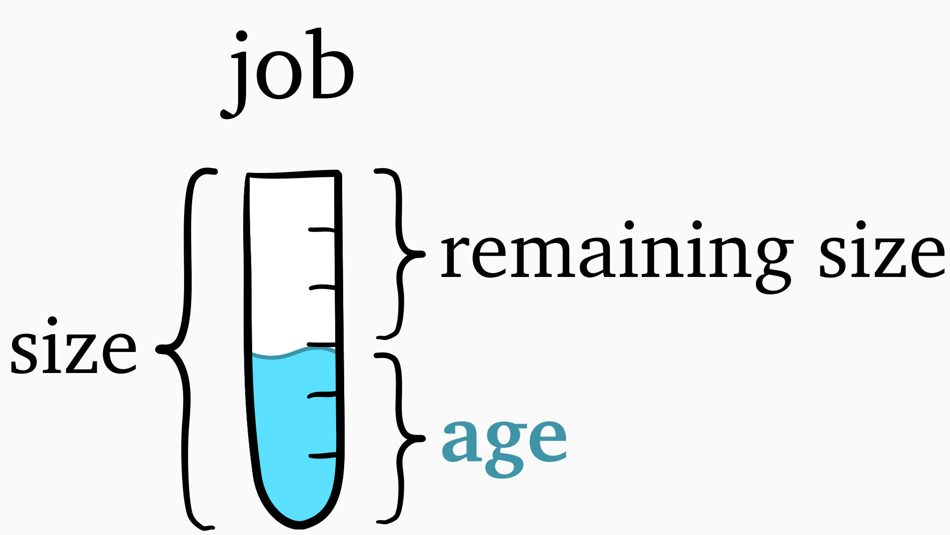

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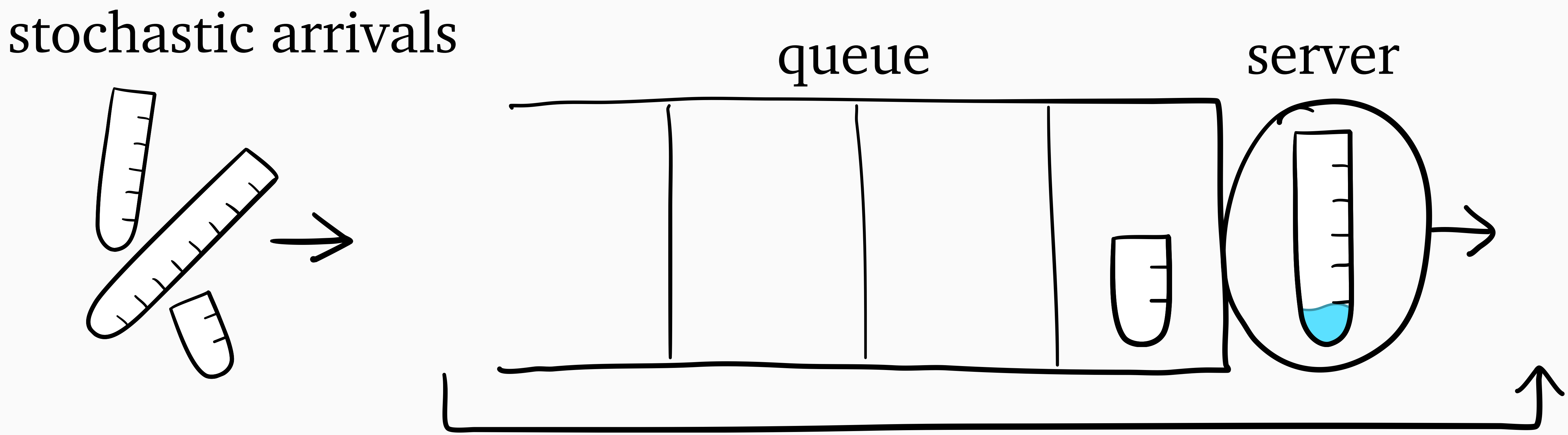

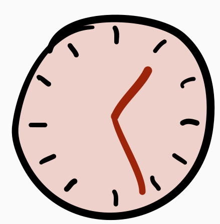

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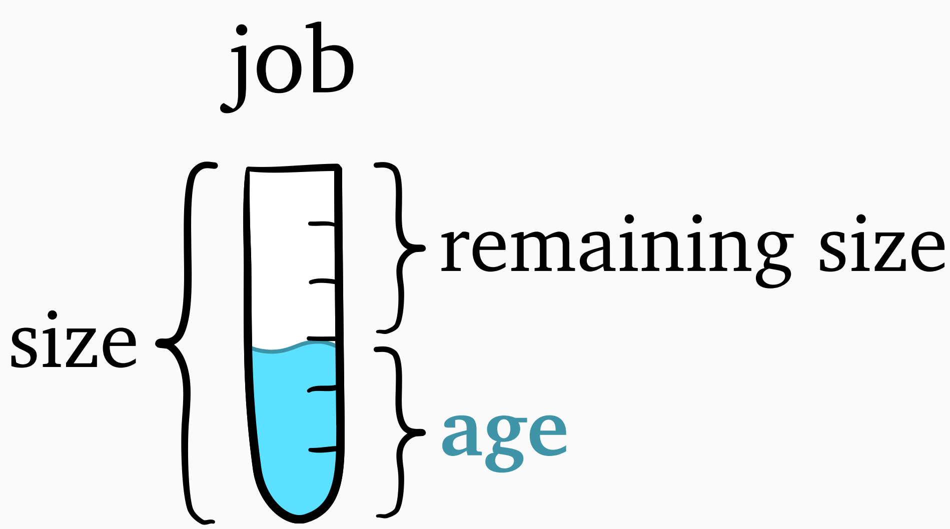

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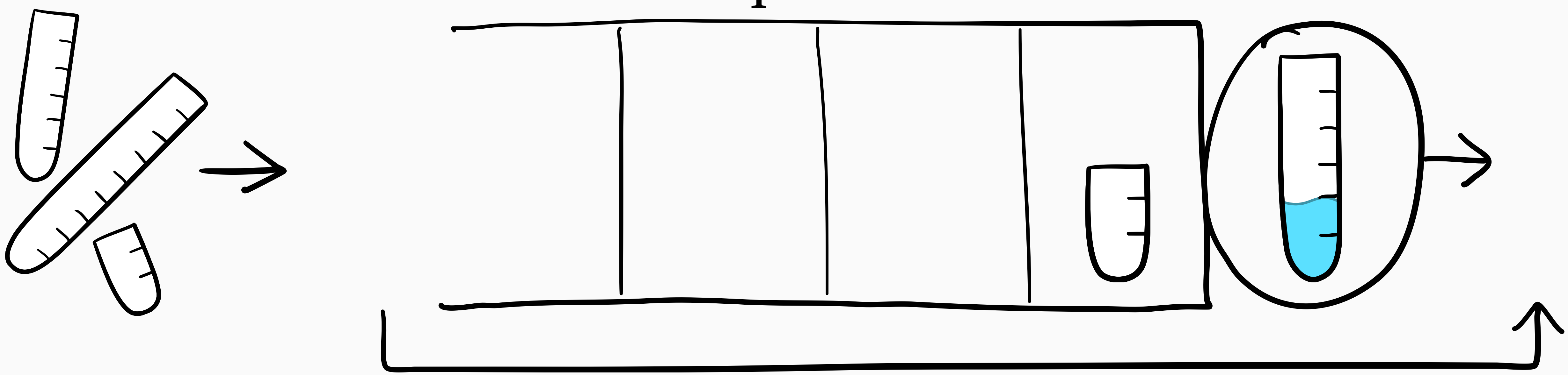

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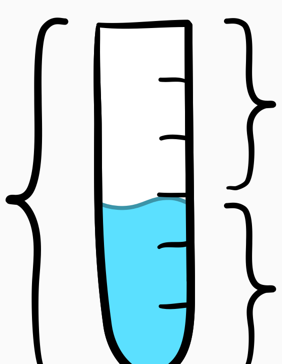

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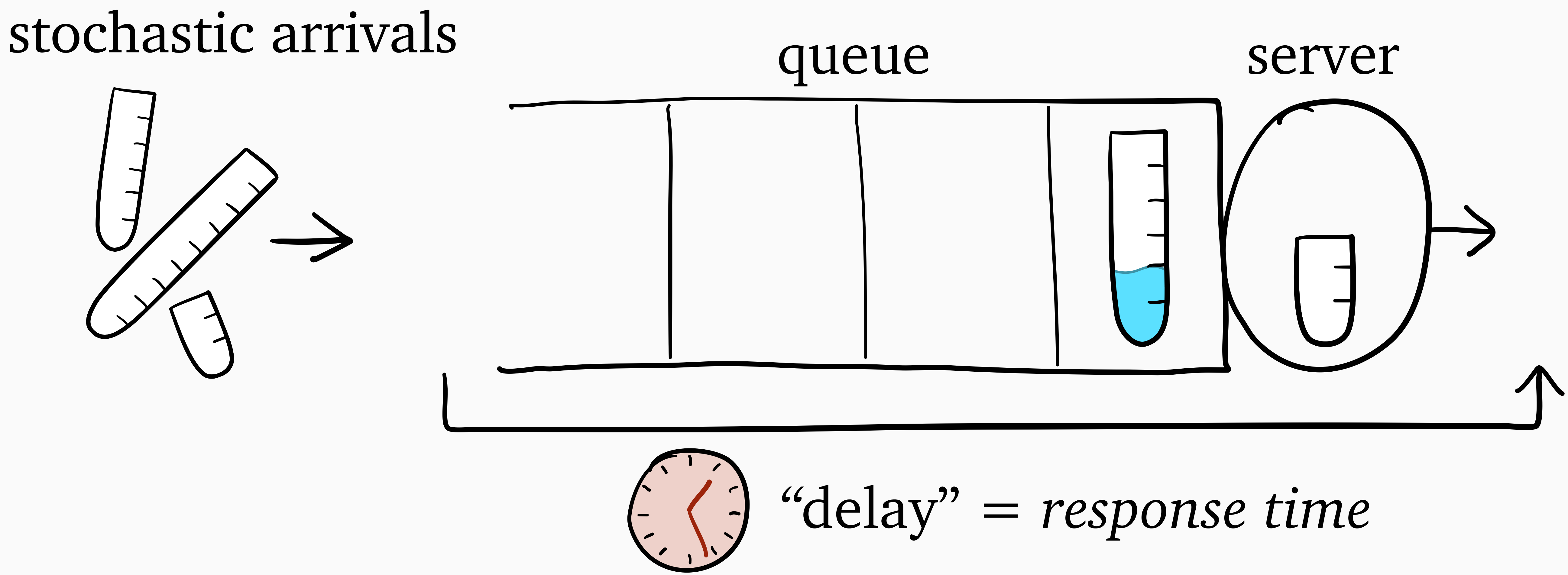

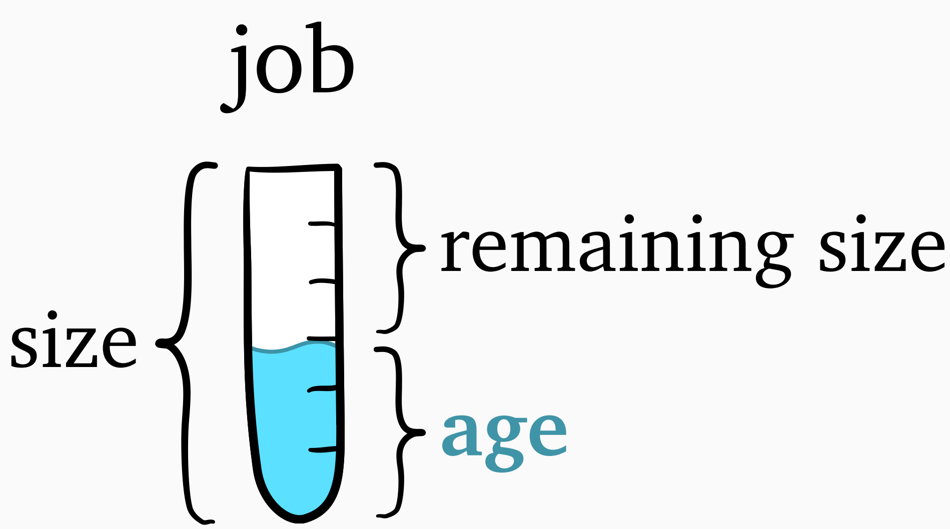

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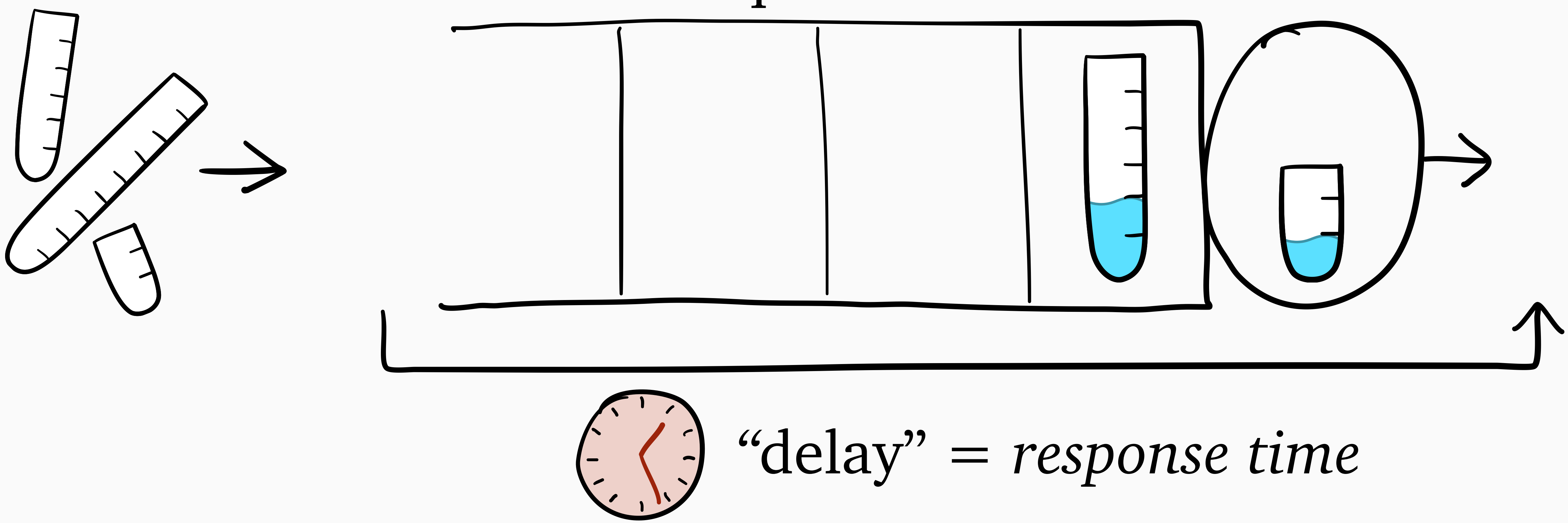

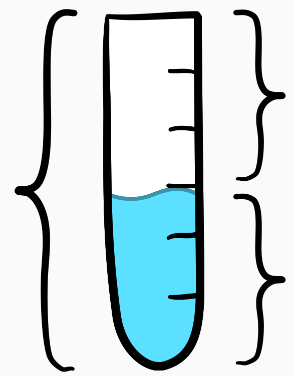

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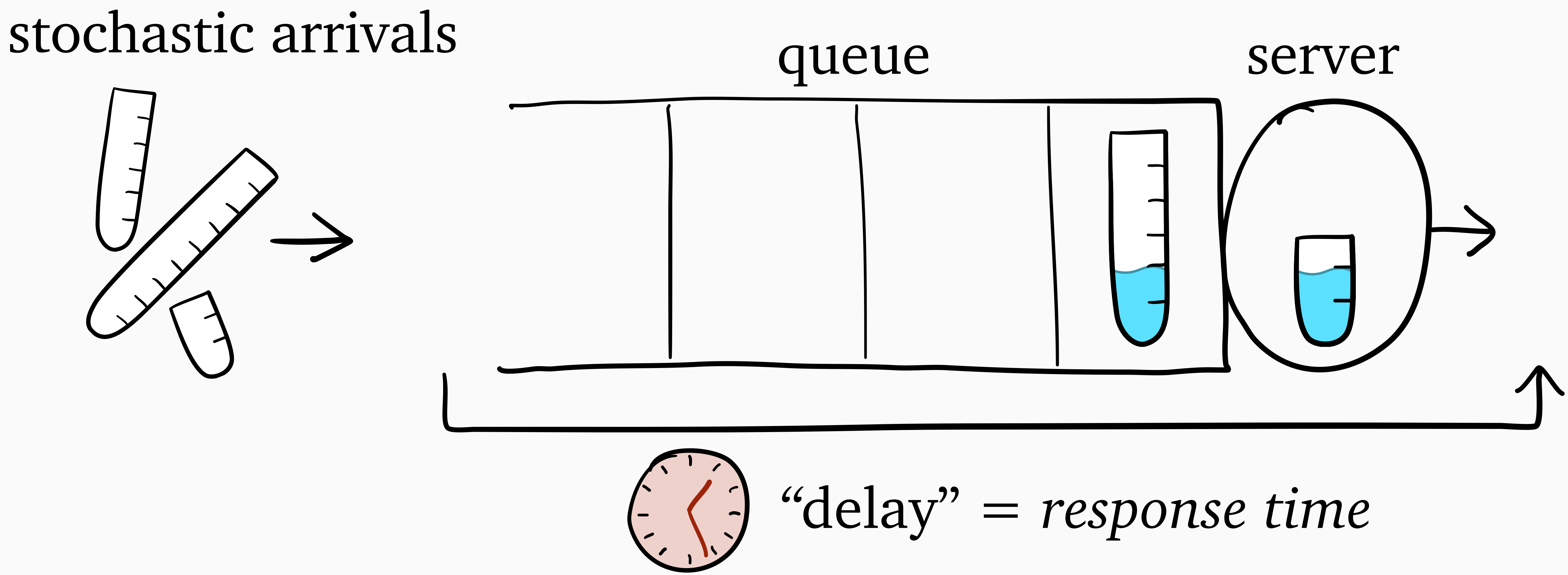

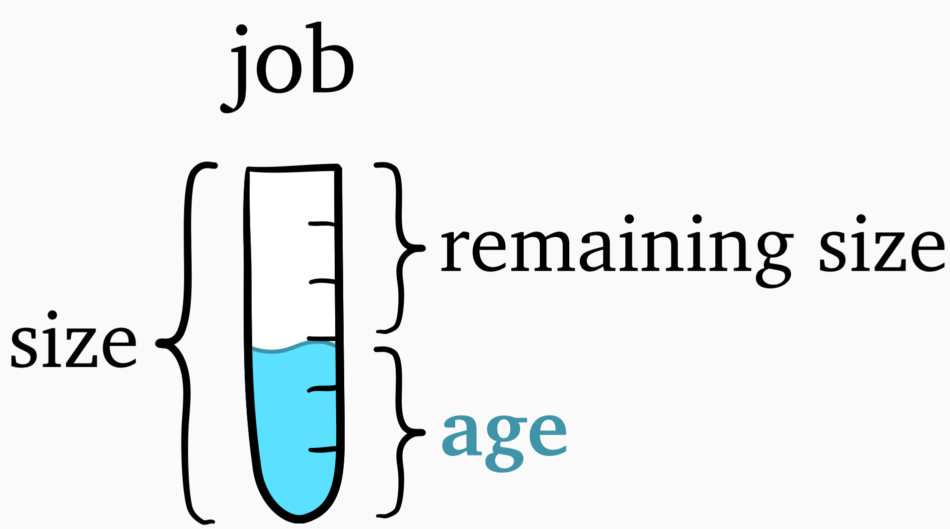

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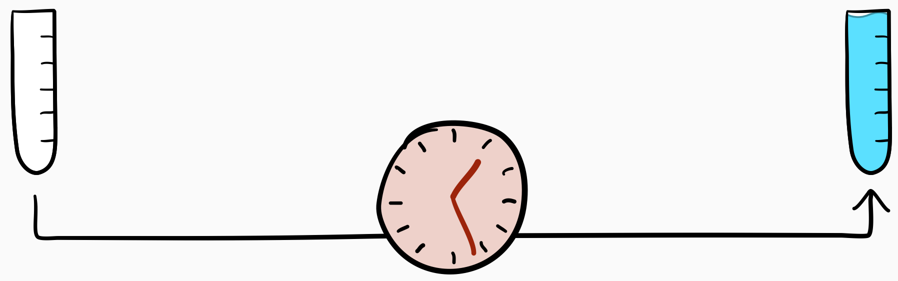
Optimizing delay

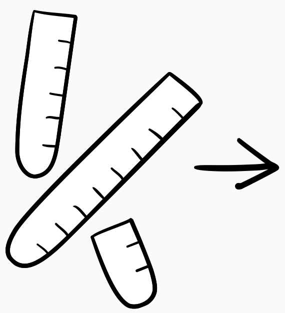
Optimizing delay

response time



Optimizing delay

stochastic arrival process λ, S



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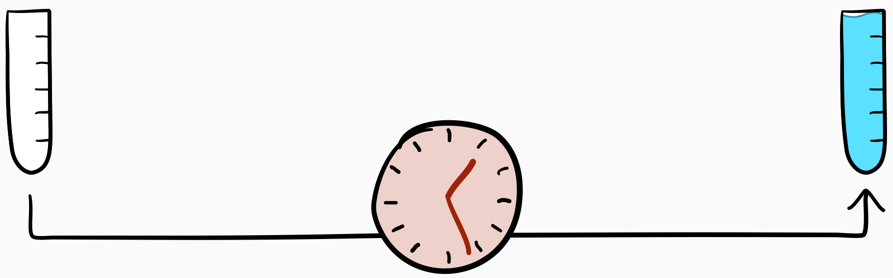

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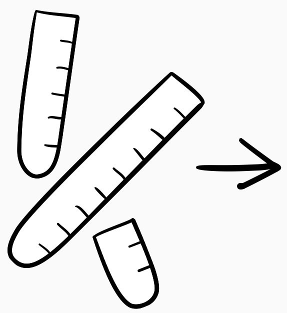
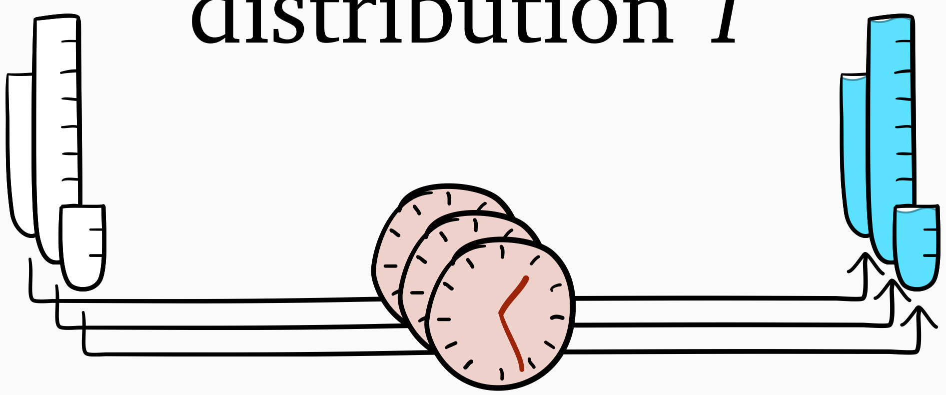


response time
distribution T



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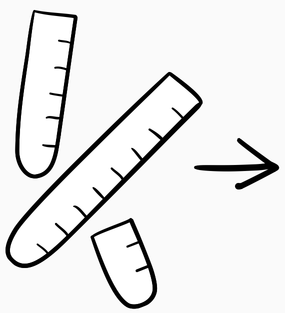
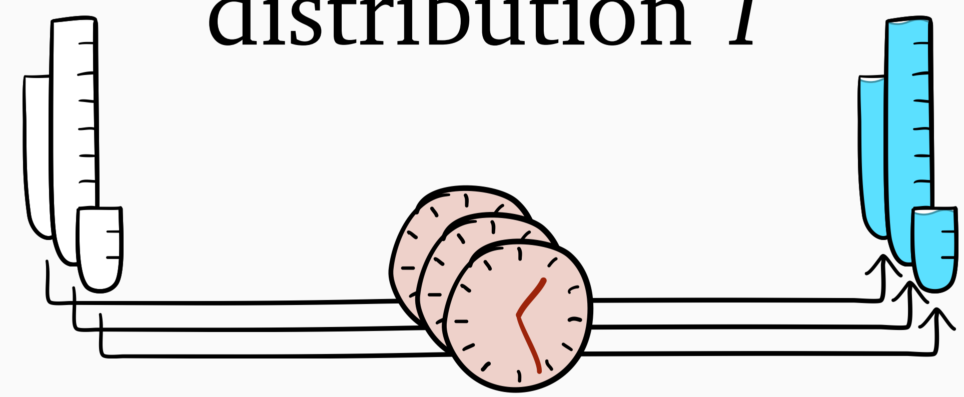
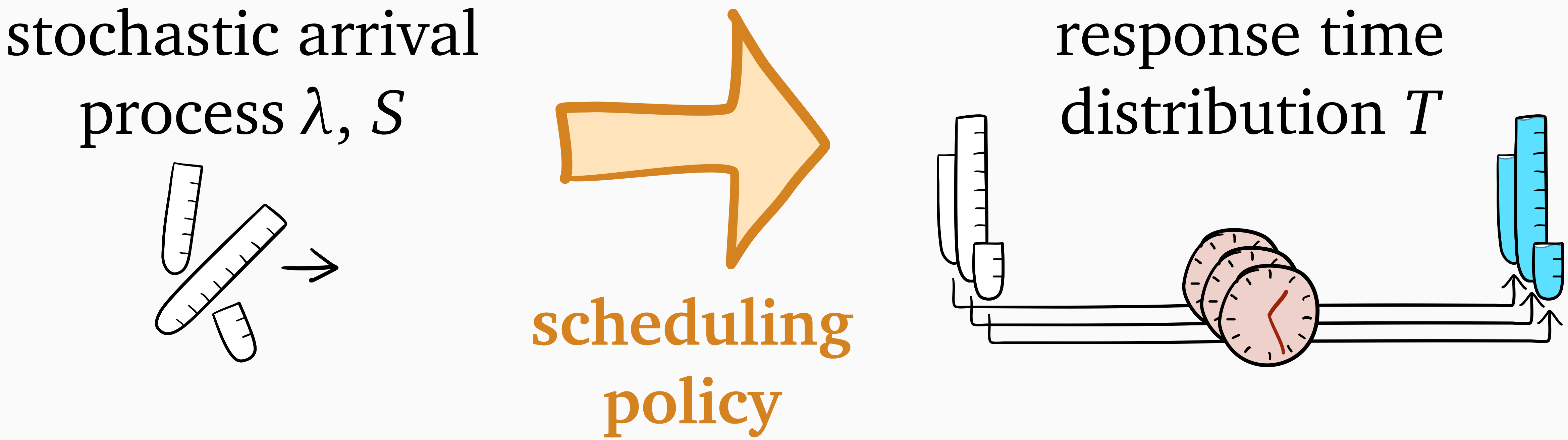


**scheduling
policy**

response time
distribution T

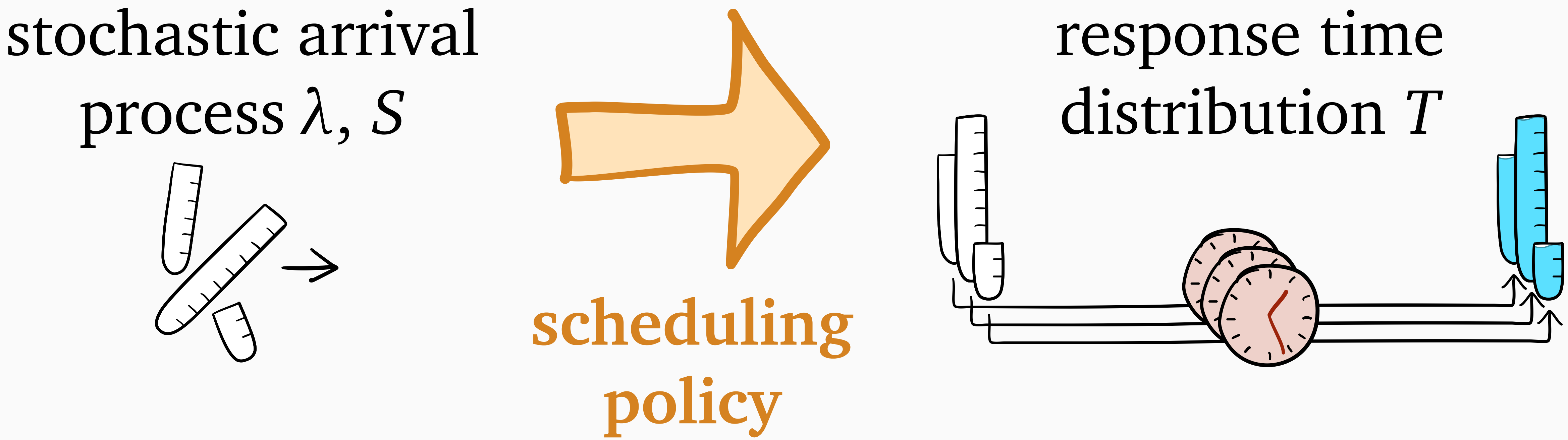


Optimizing delay



Goal: minimize *mean response time* $\mathbf{E}[T]$

Optimizing delay



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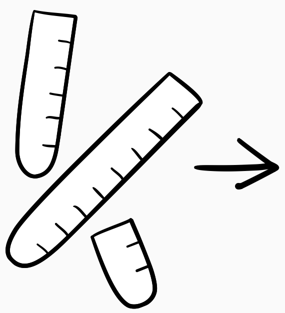
Warmup: if sizes known?

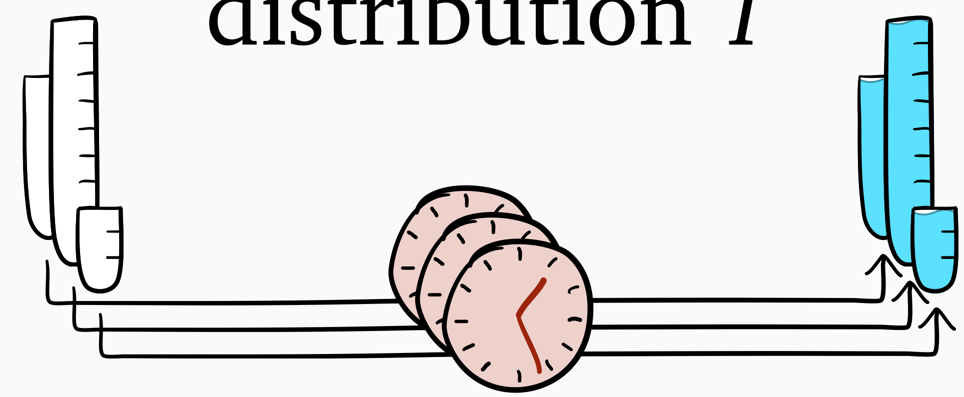
Optimizing delay

stochastic arrival process λ, S



scheduling policy

response time distribution T



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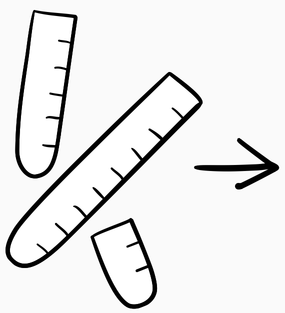

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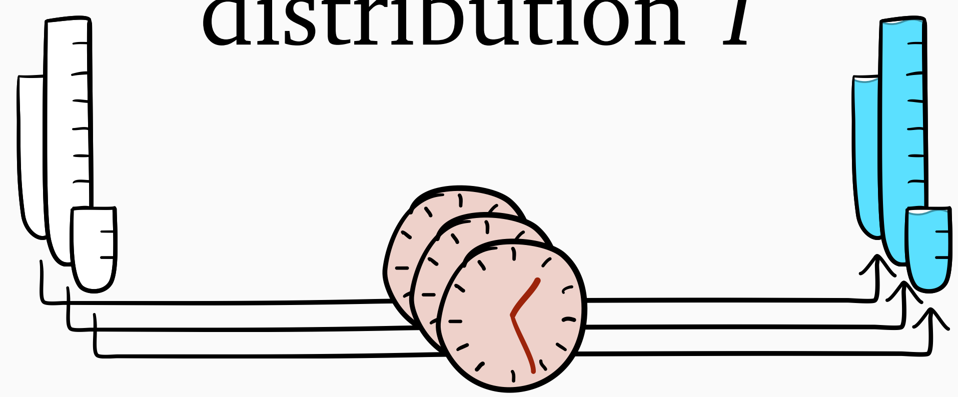

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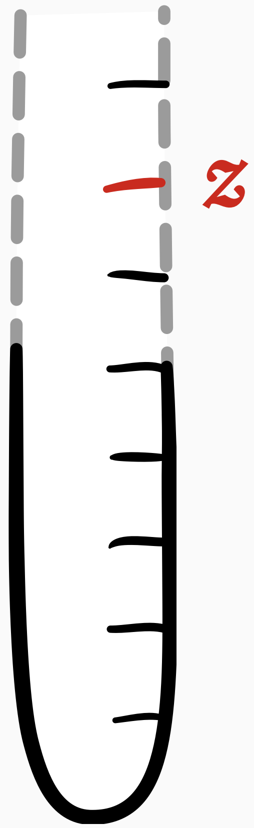
Warmup: if sizes known? **SRPT**

always serves job of *least remaining size*

What job size noise model?



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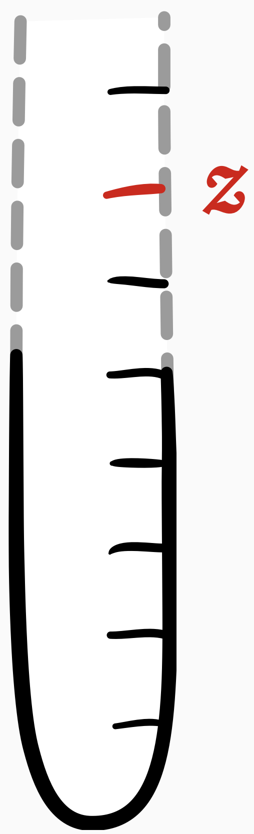

Model: (β, α)-bounded noise

true size s $\Rightarrow$ **estimated** size $z \in [\beta s, \alpha s]$

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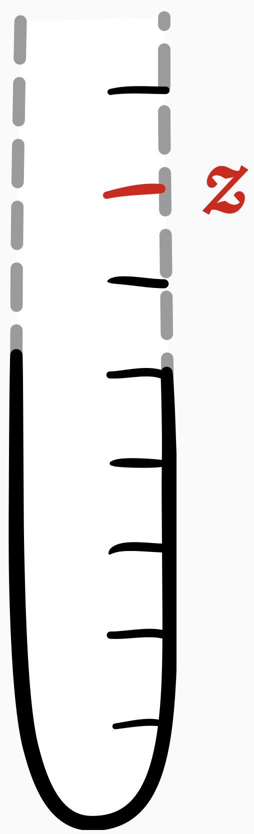

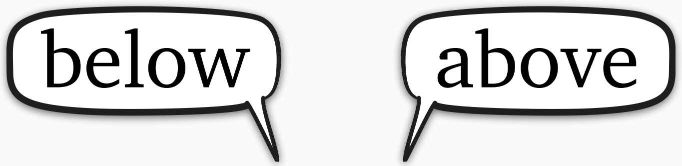

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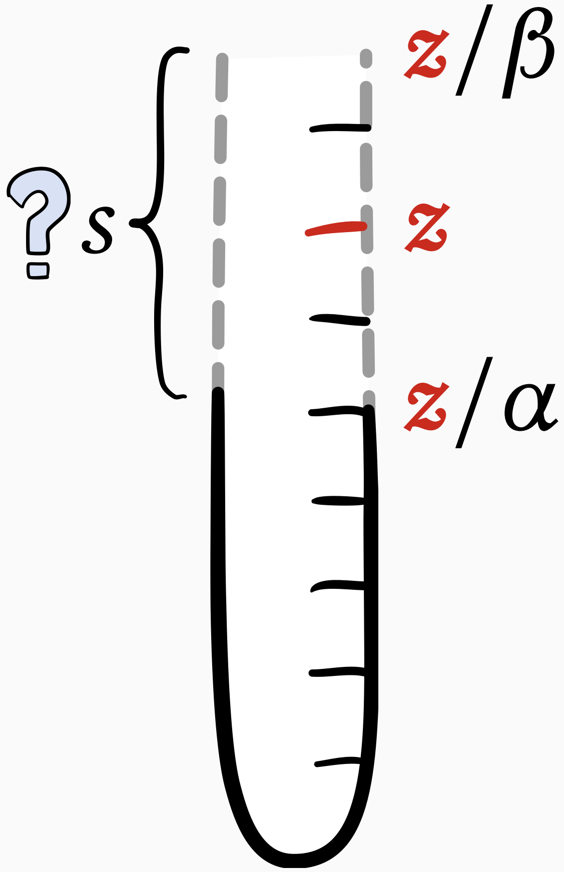

below above

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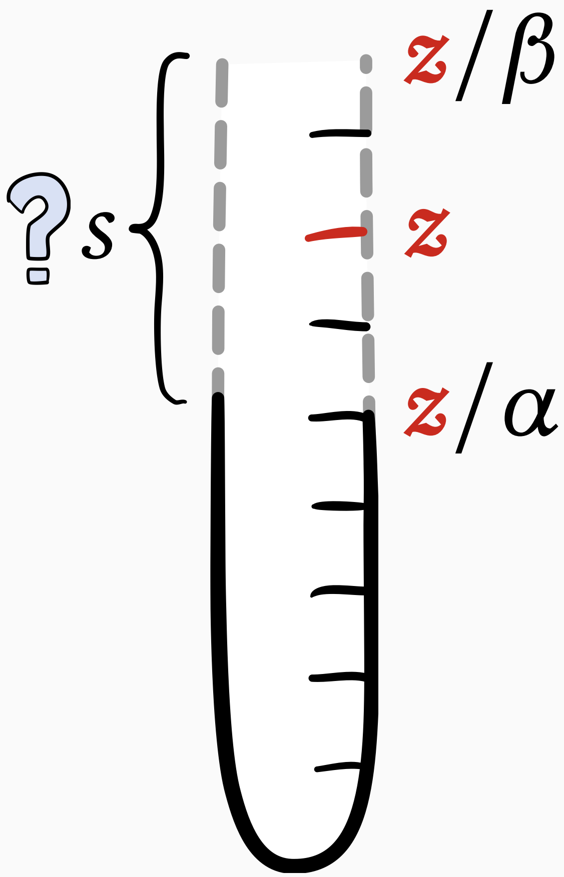

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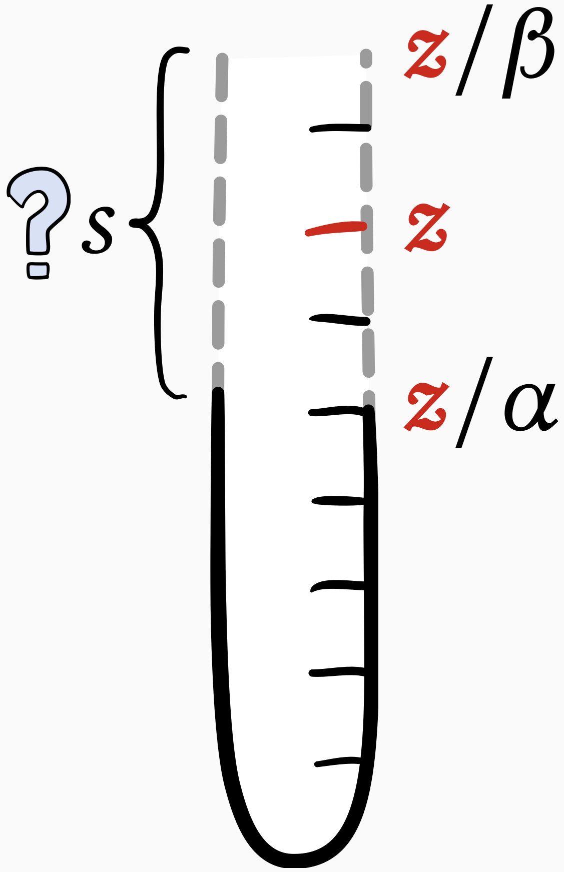

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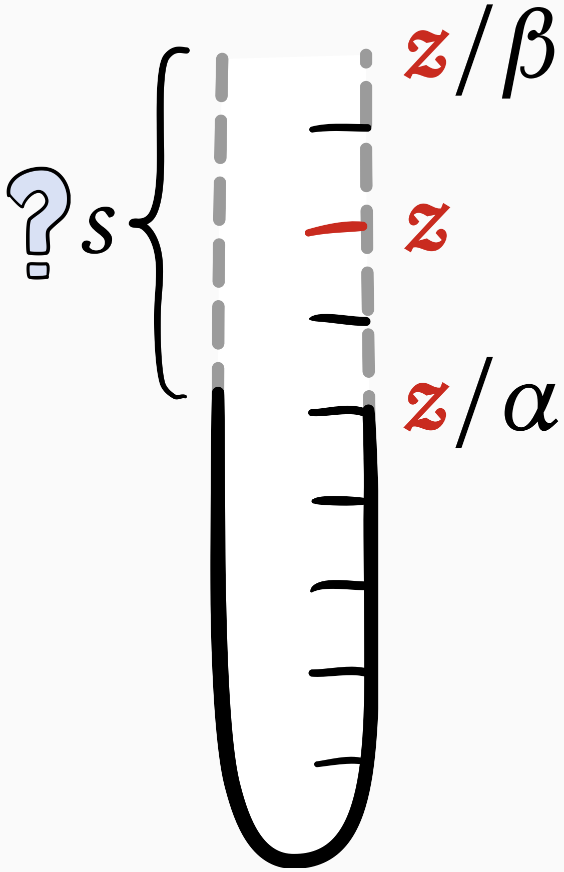

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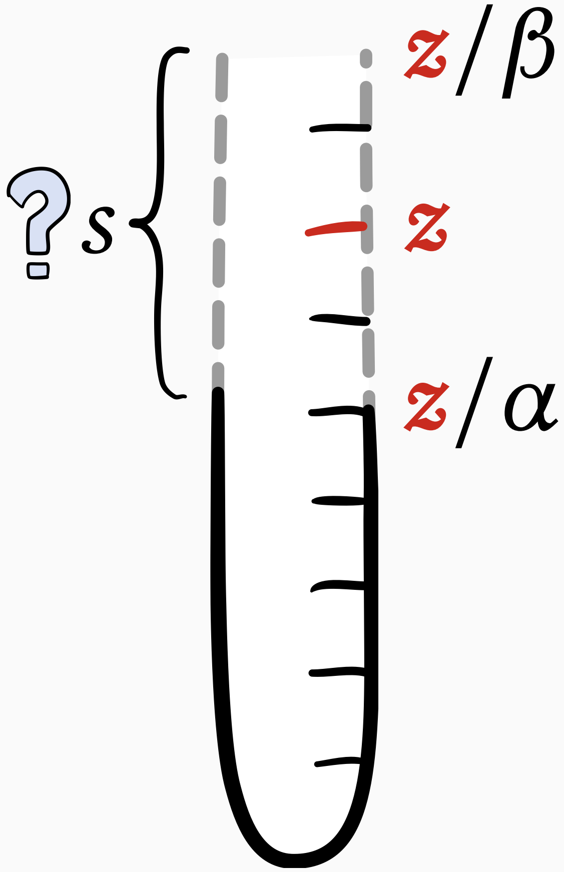
semi-adversarial

What job size noise model?



below above

Model: (β, α)-bounded n

Related: worst case
Azar, Leonardi, & Touitou
(STOC 2021, SODA 2022)

true size s $\Rightarrow$ **estimated** size $z \in [\beta s, \alpha s]$

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semi-adversarial

What can we hope to achieve?

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C-consistent: $$\frac{\mathbf{E}[T_{P}]}{\mathbf{E}[T_{\mathrm{SRPT}}]} \to C \qquad \text{as } \alpha, \beta \to 1$$

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C-consistent: $\dfrac{\mathbf{E}[T_P]}{\mathbf{E}[T_{\text{SRPT}}]} \to C$ as $\alpha, \beta \to 1$

G-graceful: $\dfrac{\mathbf{E}[T_P]}{\mathbf{E}[T_{\text{SRPT}}]} \leq G \cdot \dfrac{\alpha}{\beta}$ for all α, β

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Our contribution: *first policy* ***P*** that's consistent and graceful

- $G = 3.5$
- $C = 1$

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Our ... *first policy* P NEW!

that ... graceful

impossible in worst case
Azar, Leonardi, & Touitou
(STOC 2021)

- $G = $ ...
- NEW! $C = 1$

Our contribution: *first policy* $\boldsymbol{P}$ that's consistent and graceful

$C = 1$

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Our contribution: *first policy* $\boldsymbol{P}$ that's consistent and graceful

$C = 1$

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What is the new policy?

Our contribution: *first policy* ***P***
that's consistent and graceful

$C = 1$ $G = 3.5$

What is the new policy?

How do we bound its performance?

Our contribution: *first* policy P
that's consistent and graceful

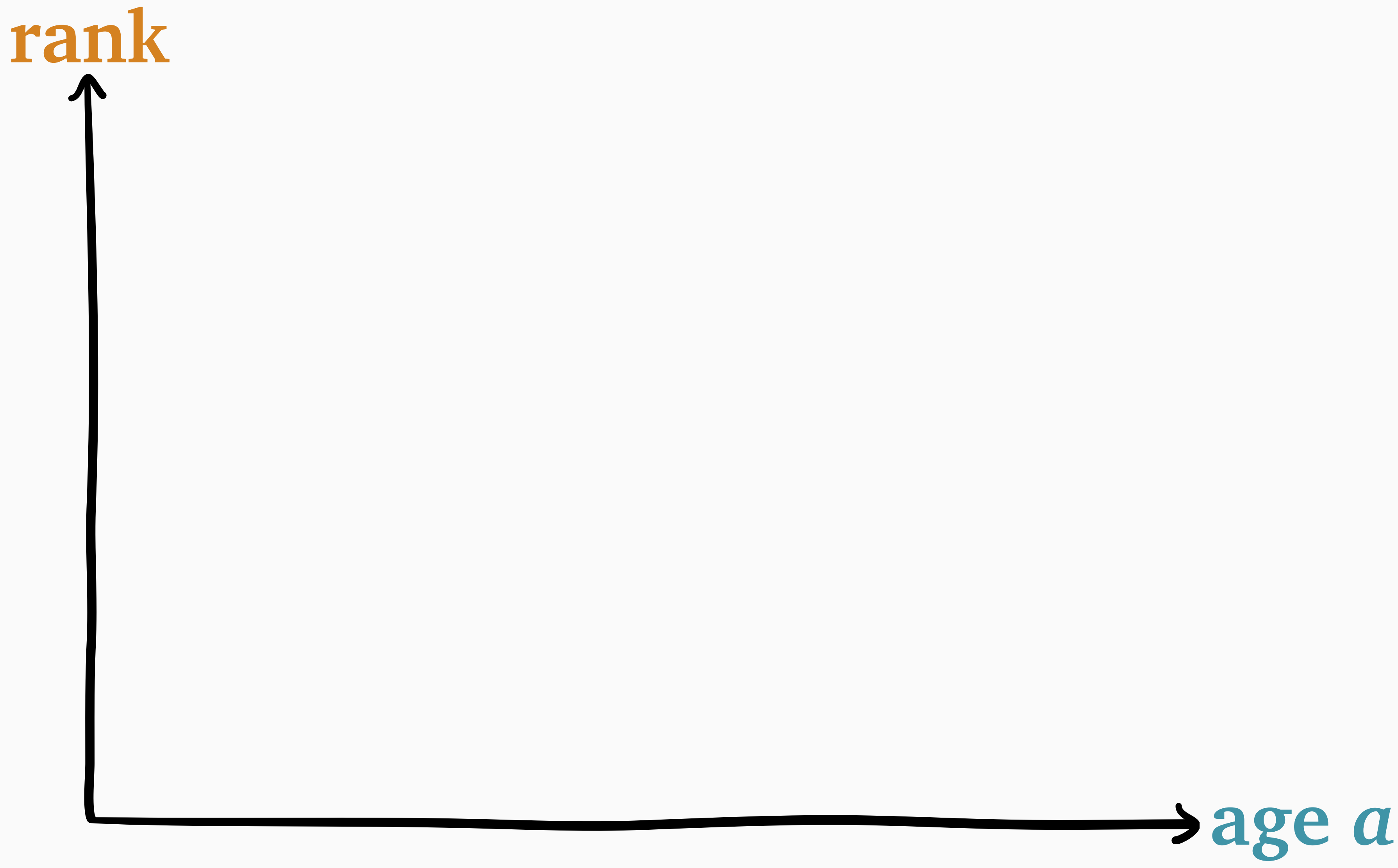
$C = 1$ $G = 3.5$

What does "policy" mean?

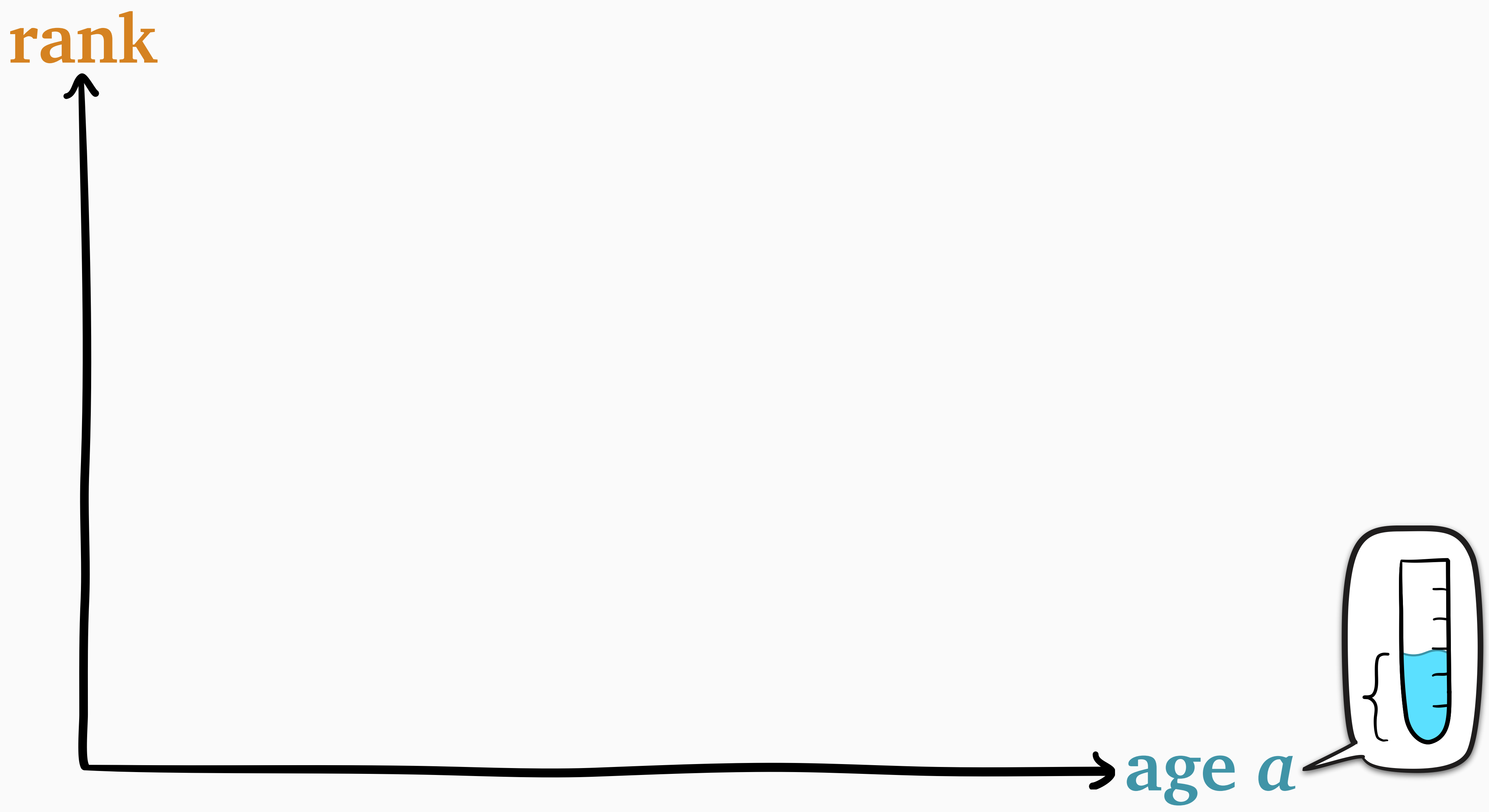
What is the new policy?

How do we bound its performance?

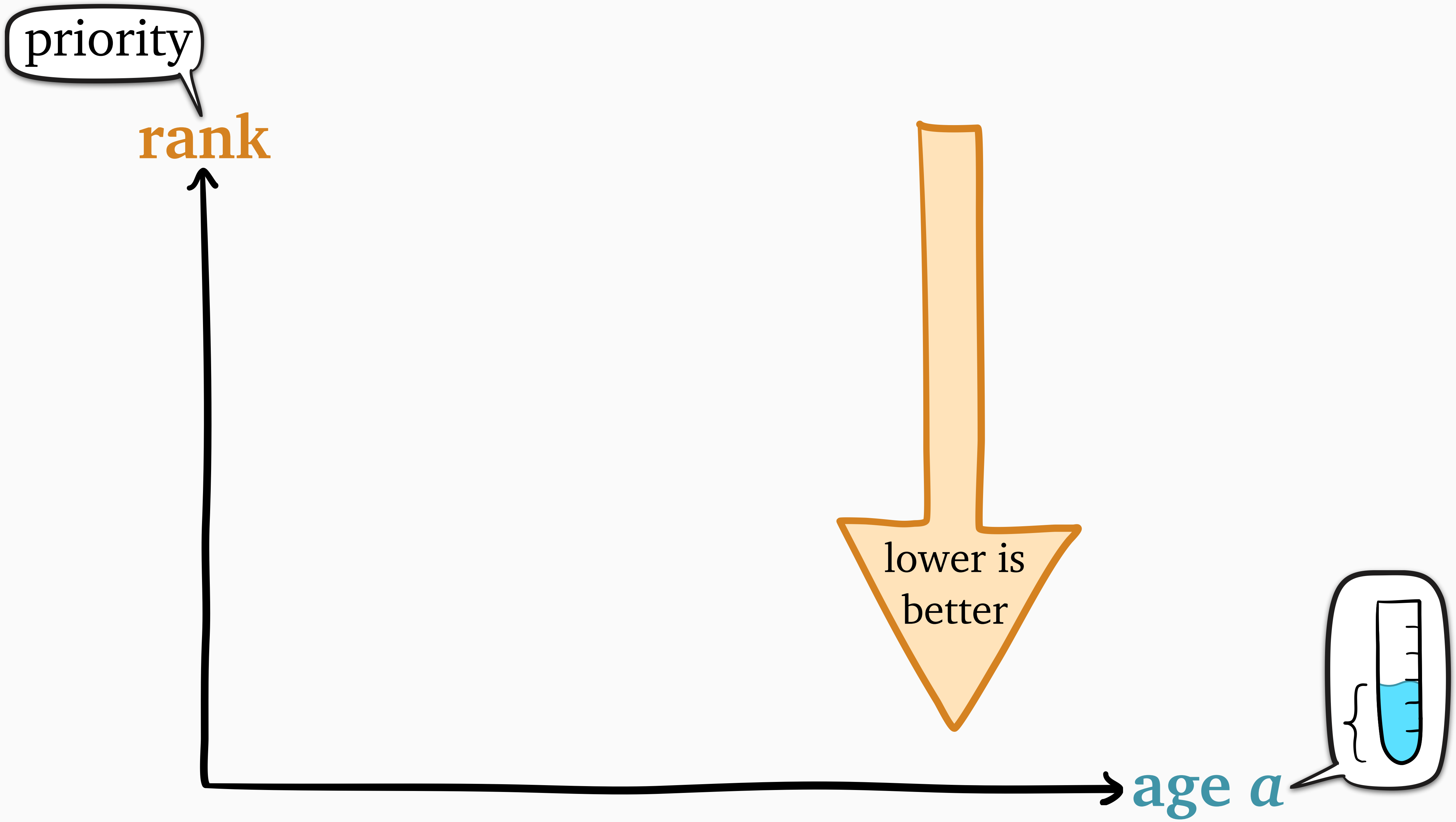
Scheduling with **rank** functions



Scheduling with **rank** functions



Scheduling with rank functions



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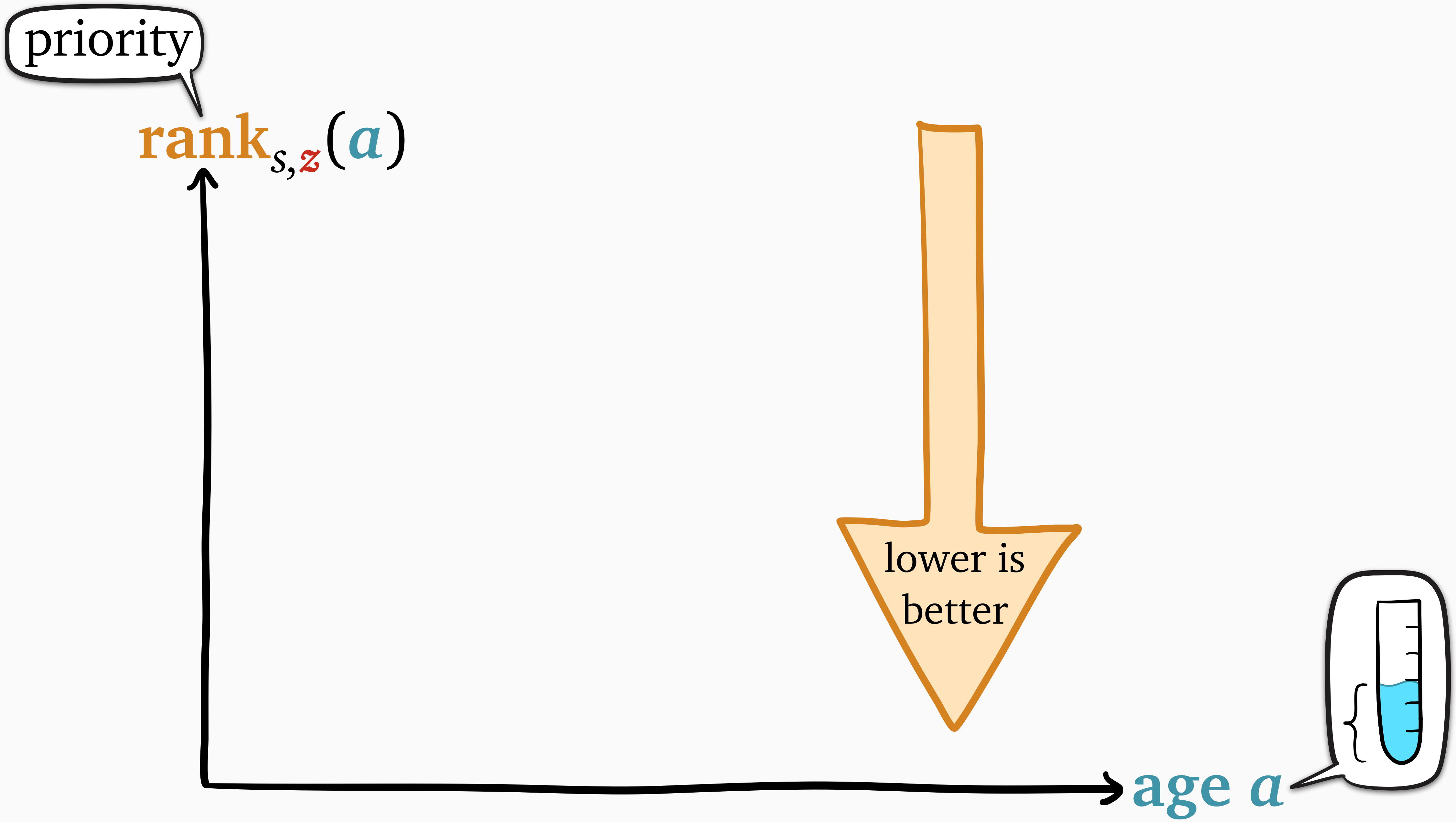

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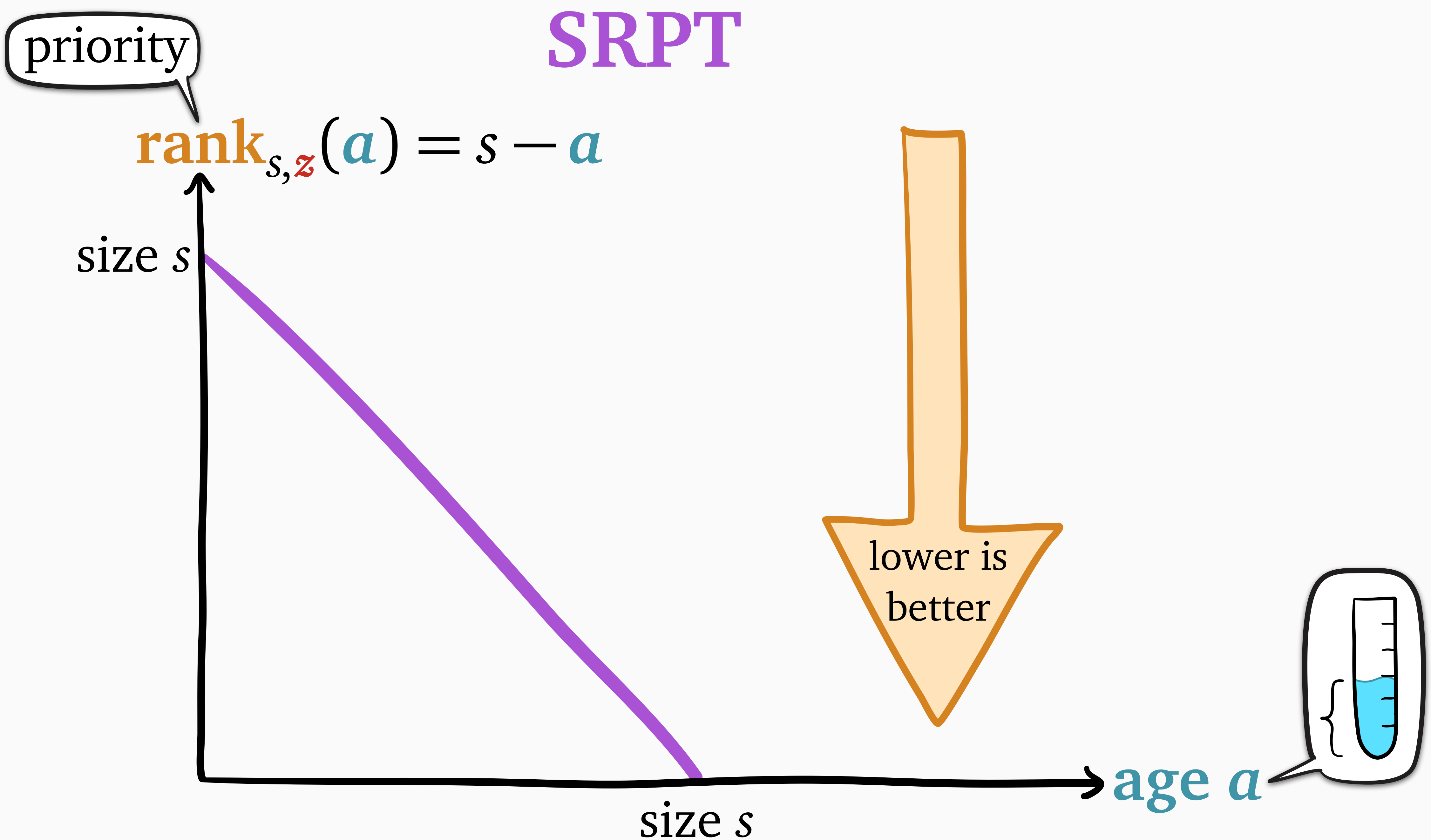
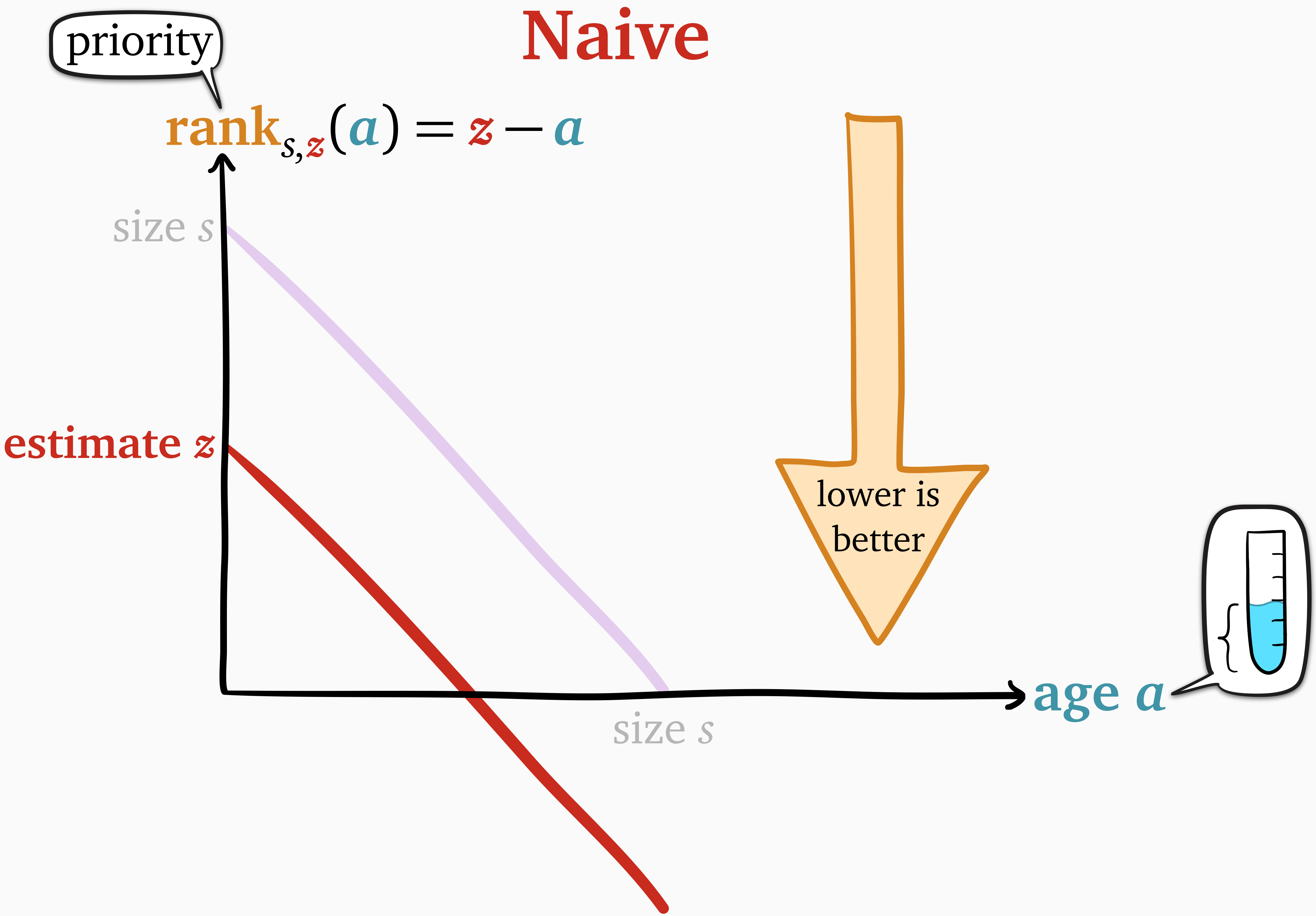
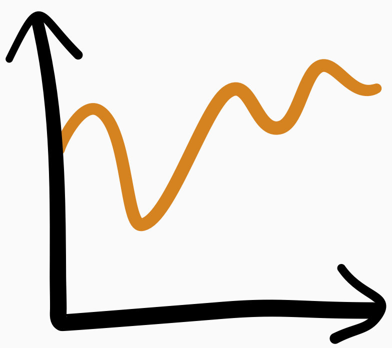


Scheduling with rank functions

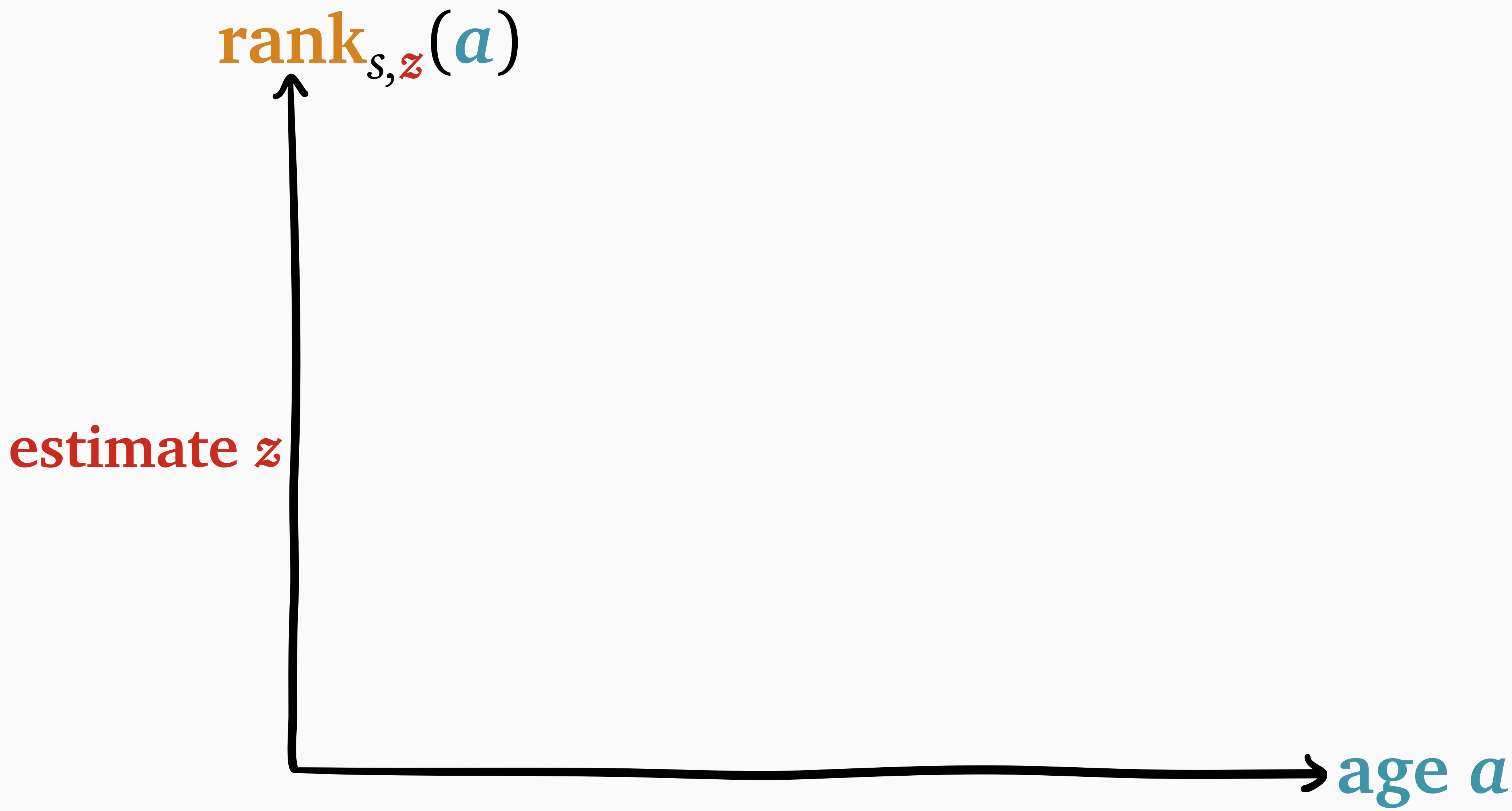




Policy design space:

rank functions

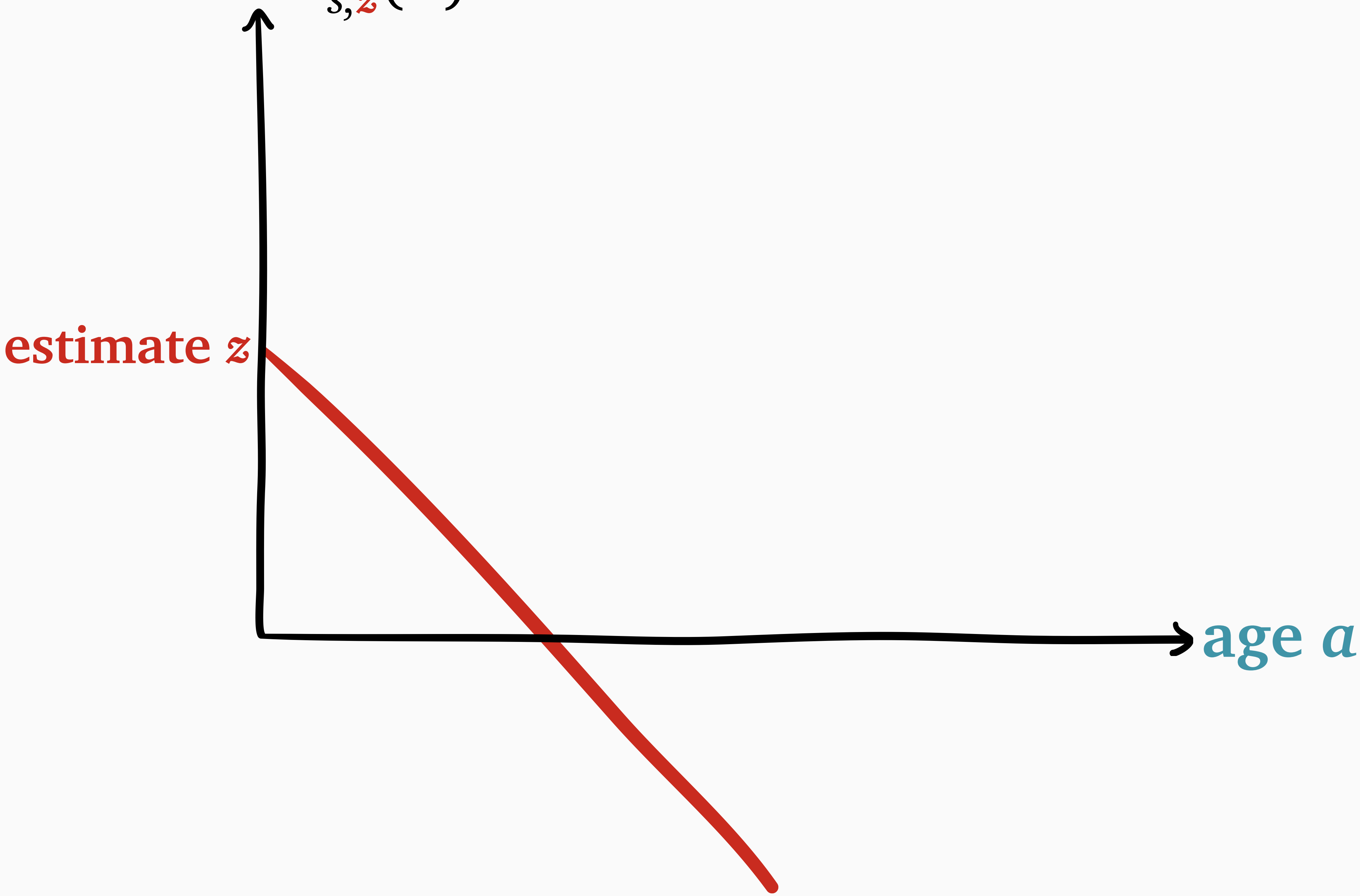
What's the right **rank** function?



What's the right **rank** function?

Naive

$\text{rank}_{s,z}(a) = z - a$



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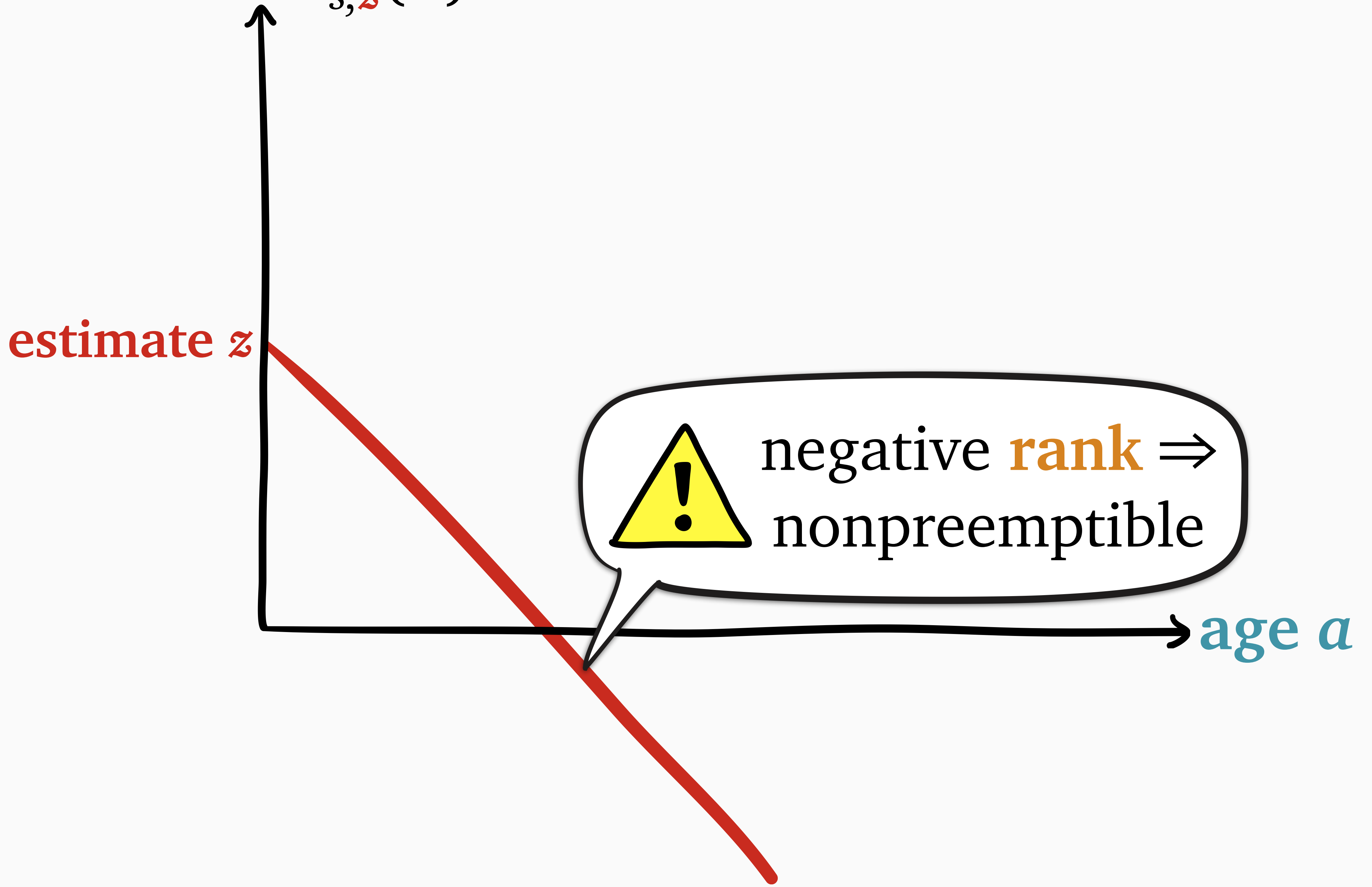

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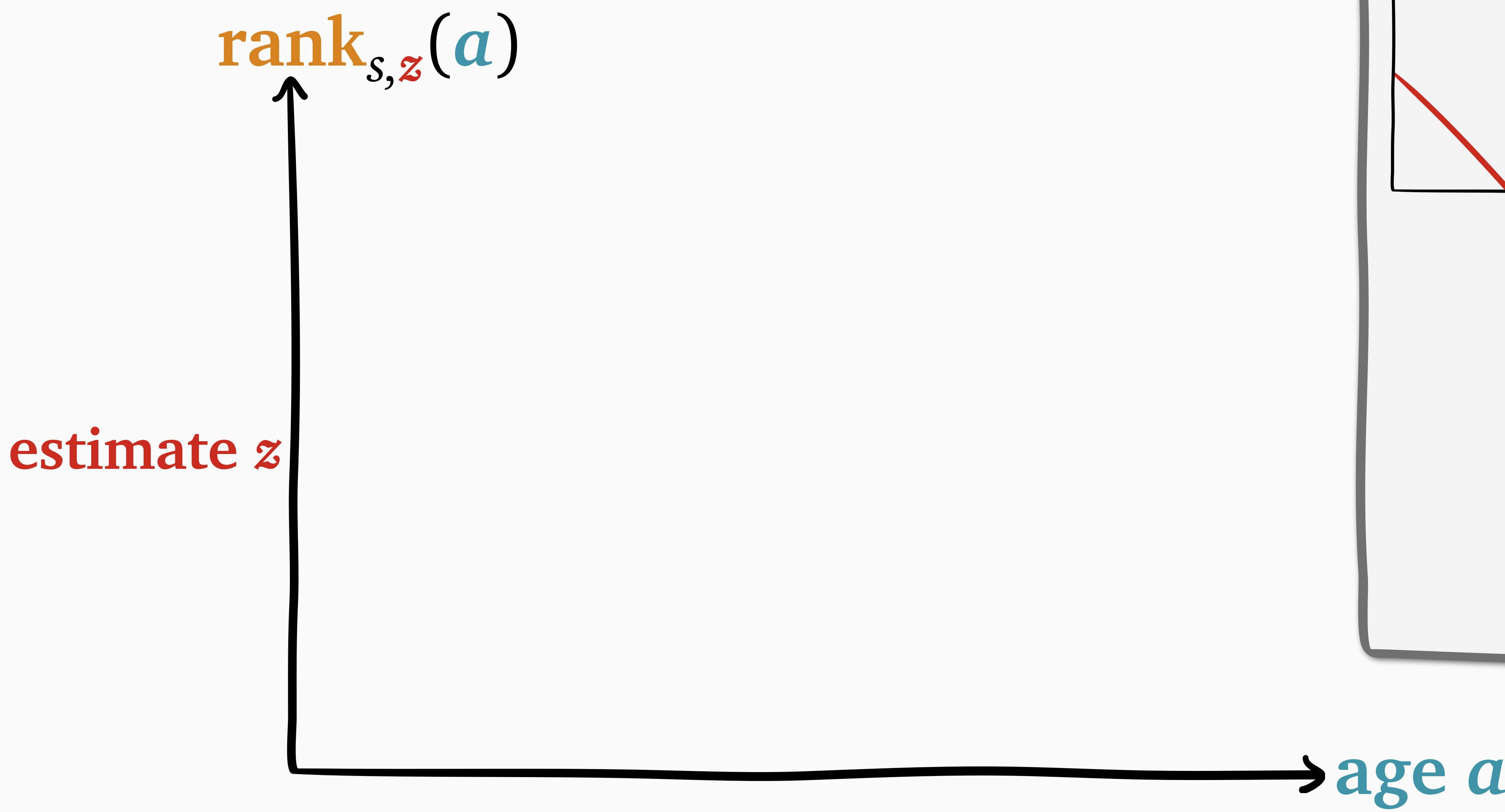
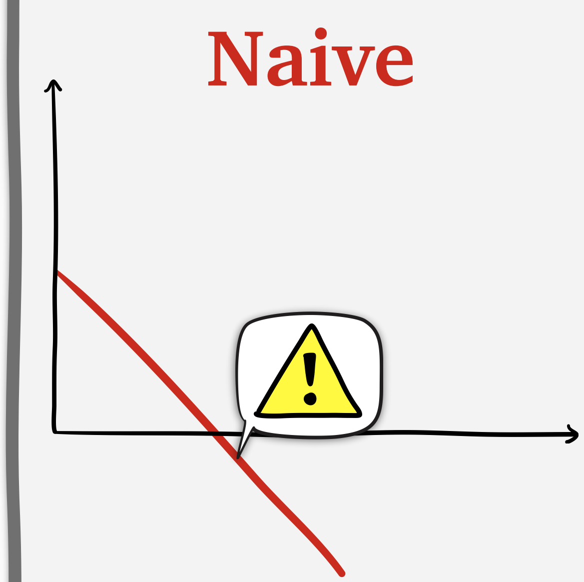


Not these…



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Checkmark

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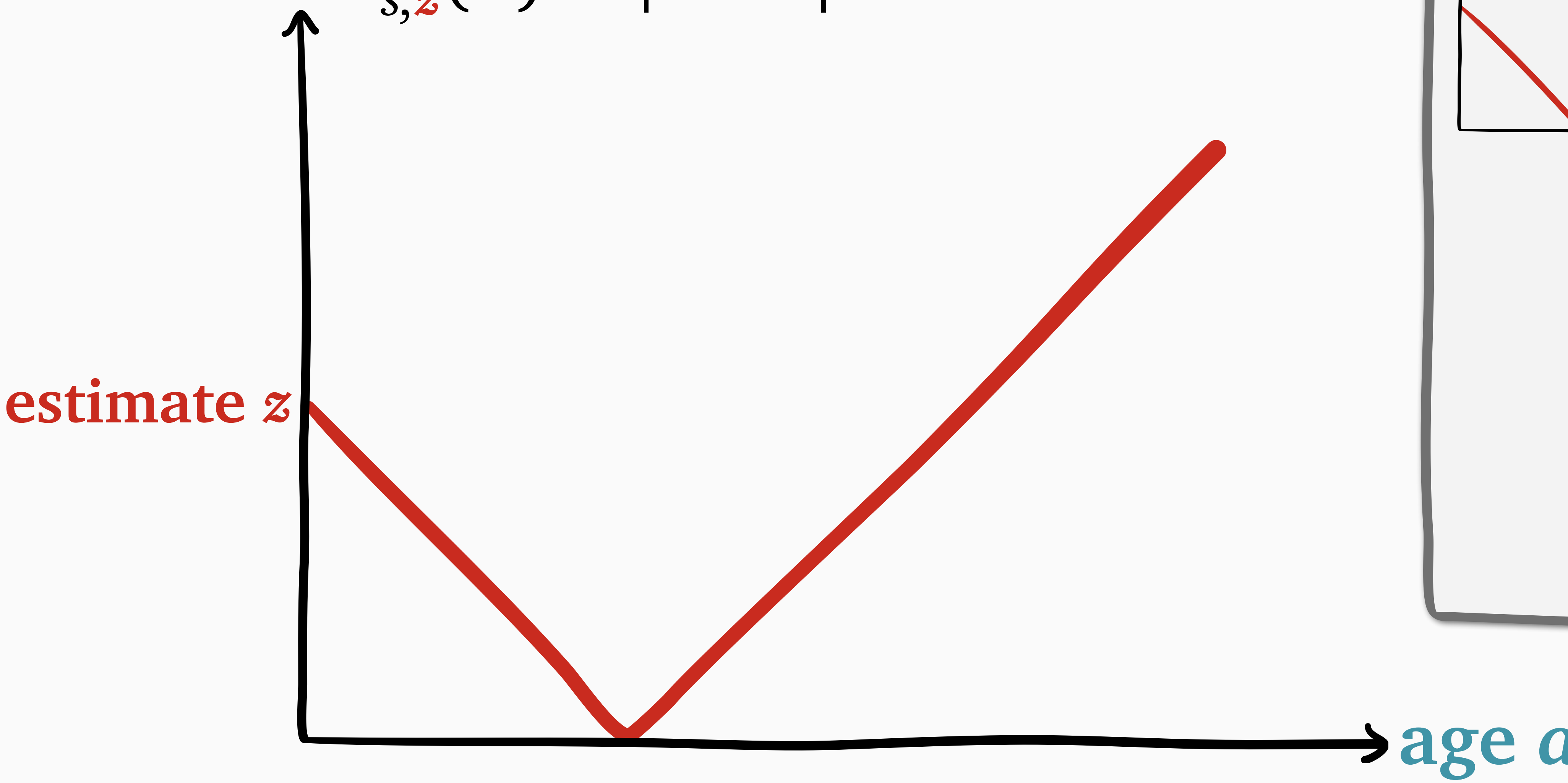

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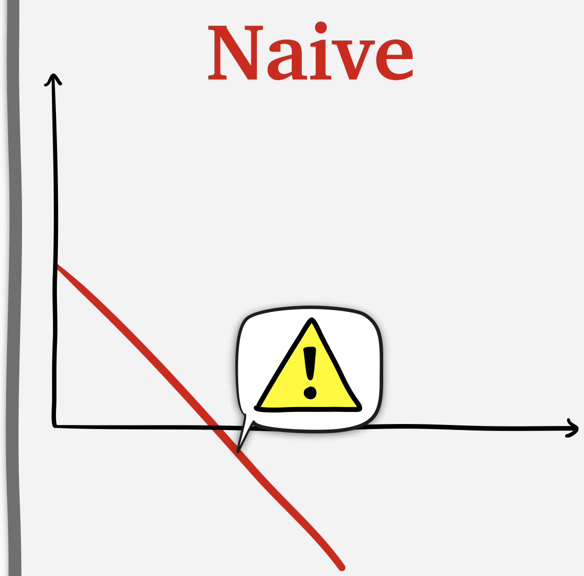

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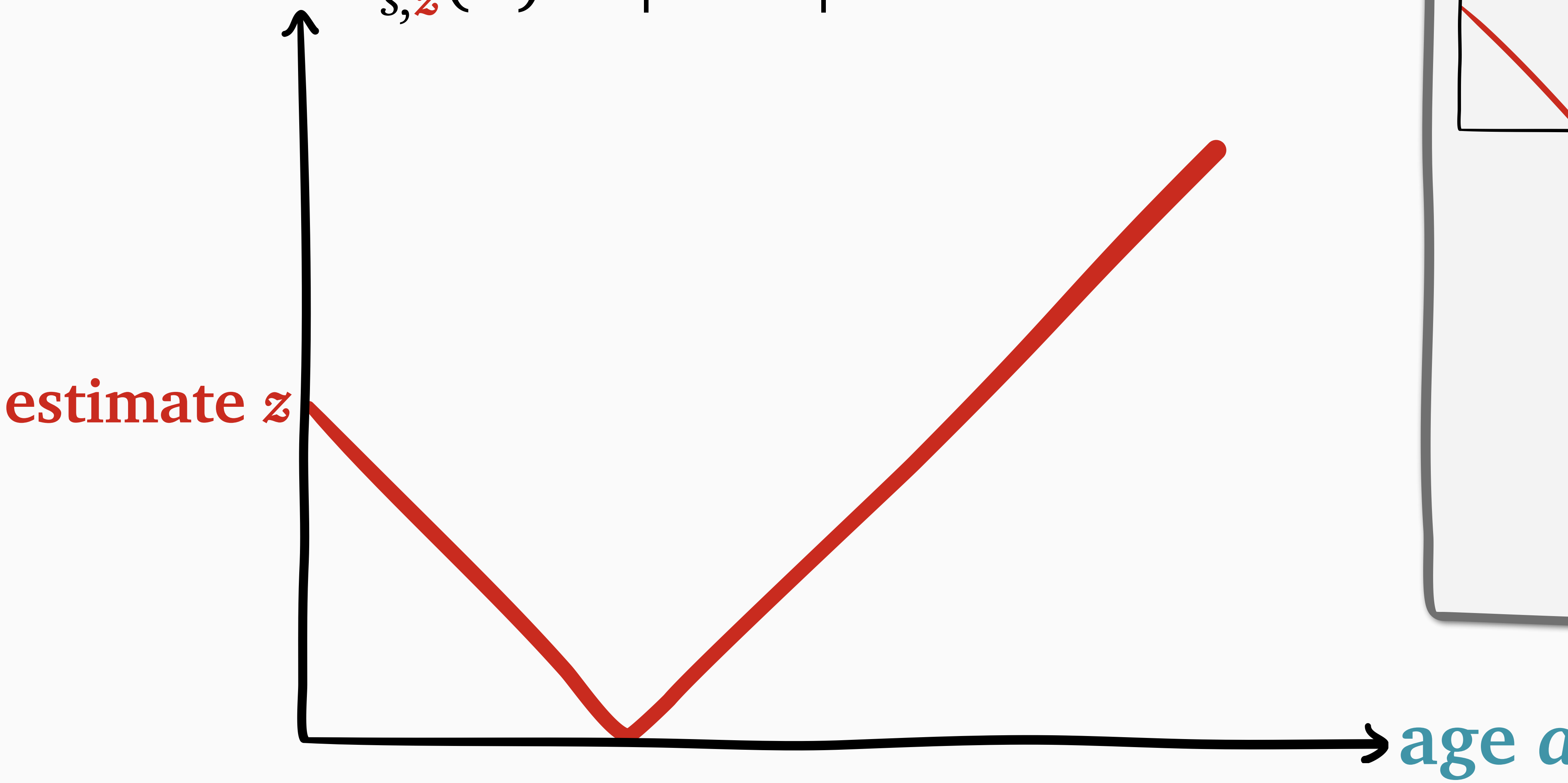

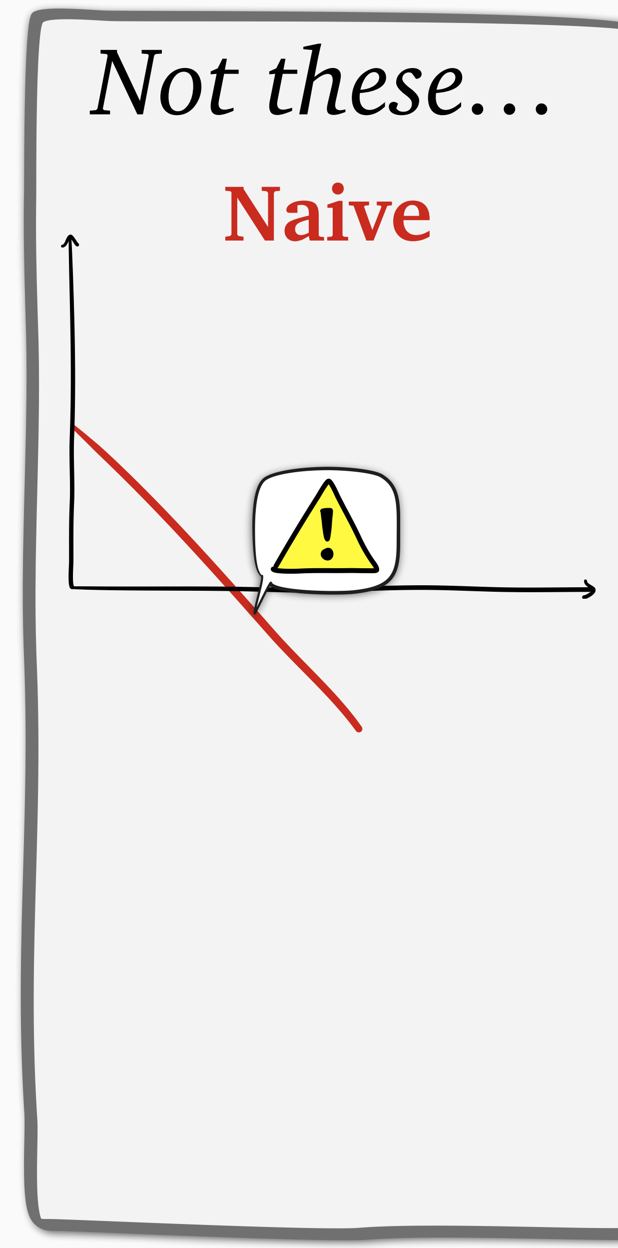

What if $\beta < \frac{1}{2}$?

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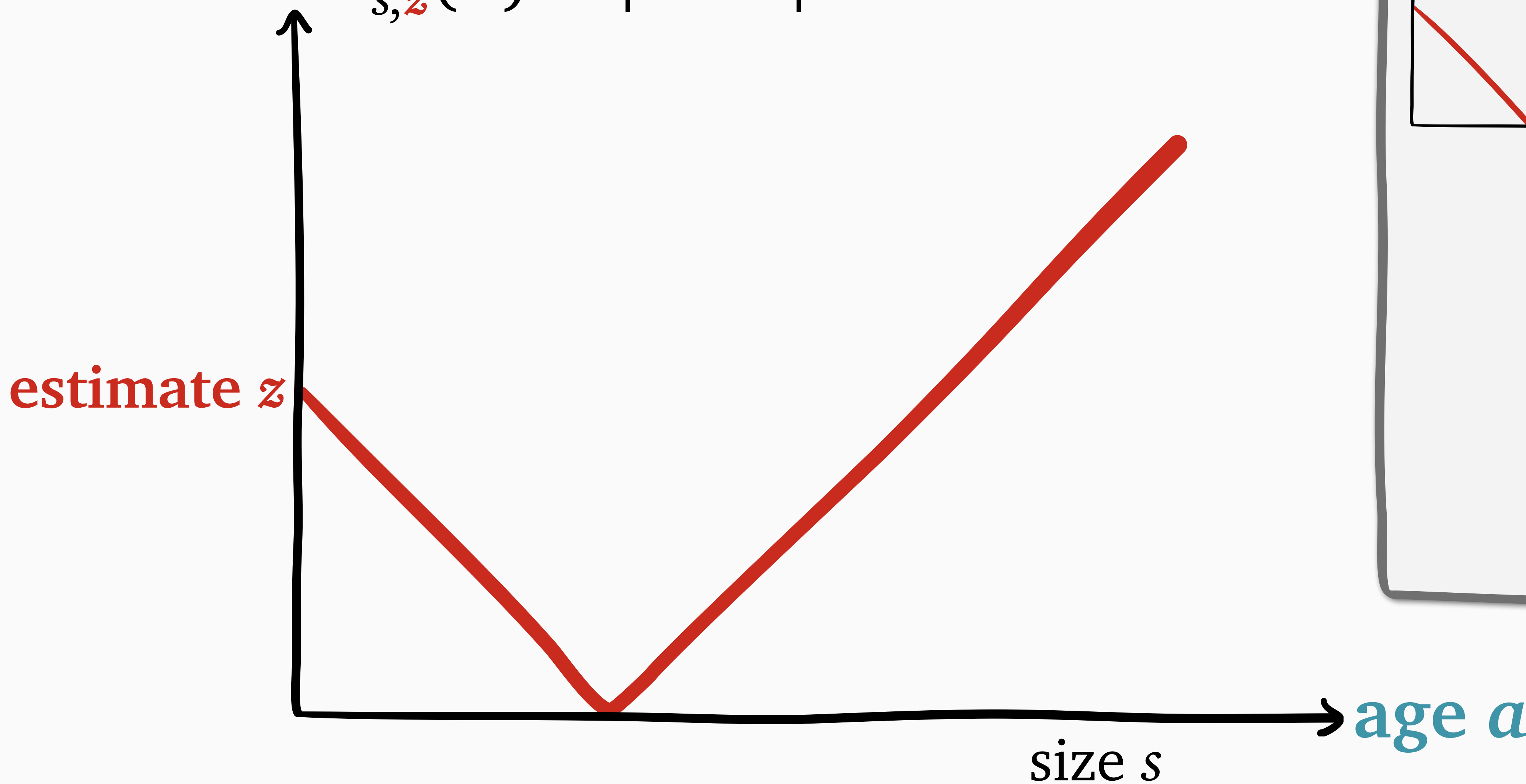

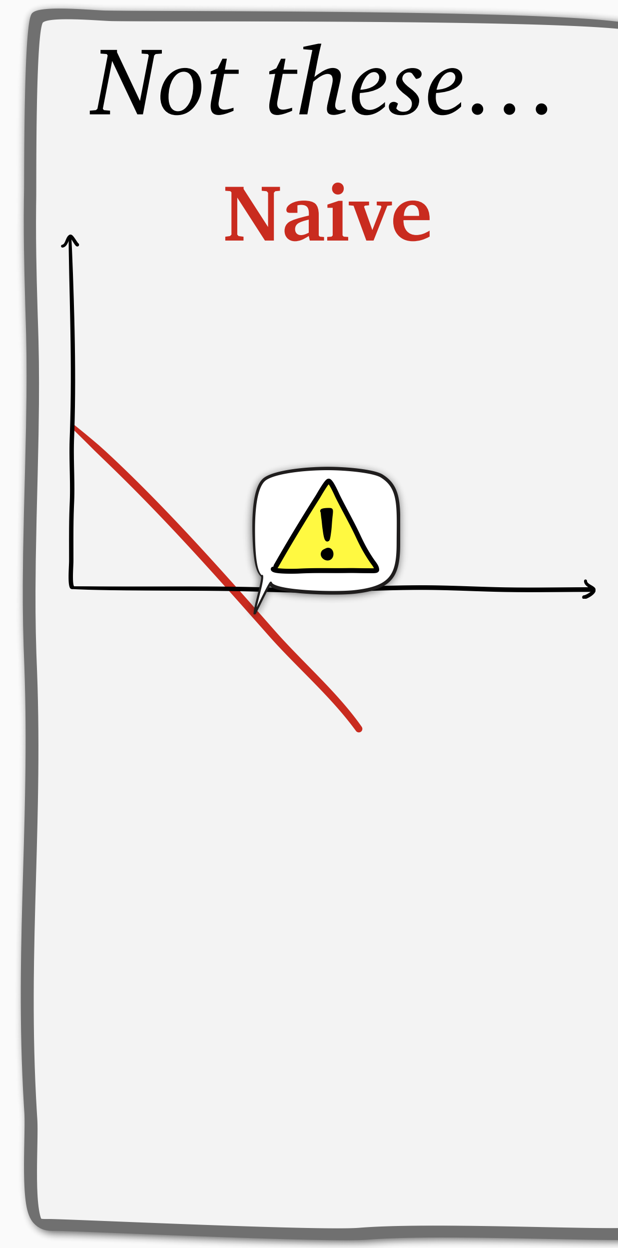

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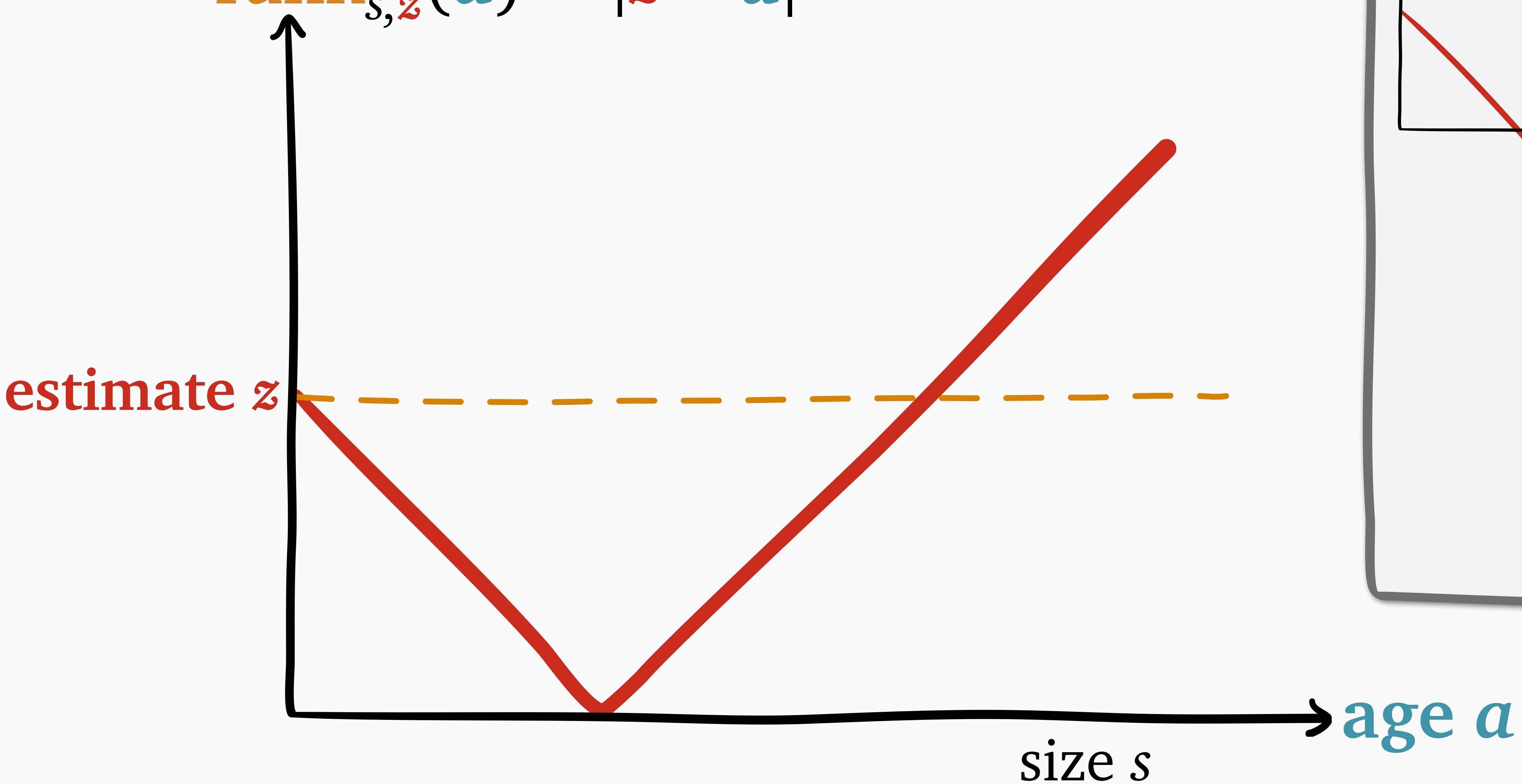

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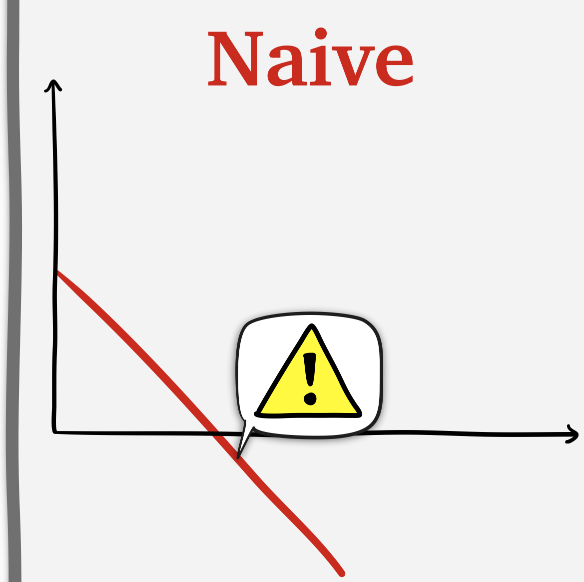

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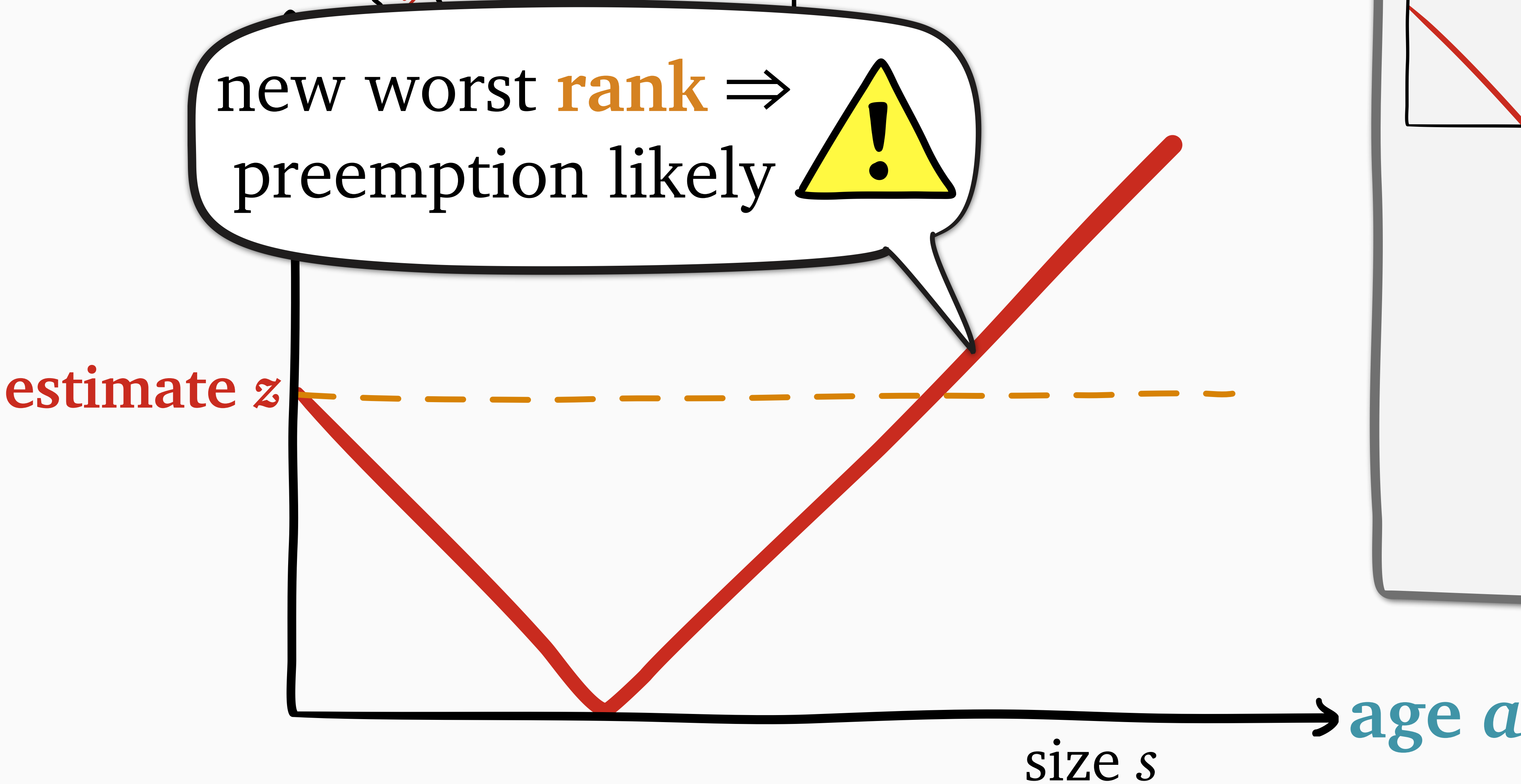

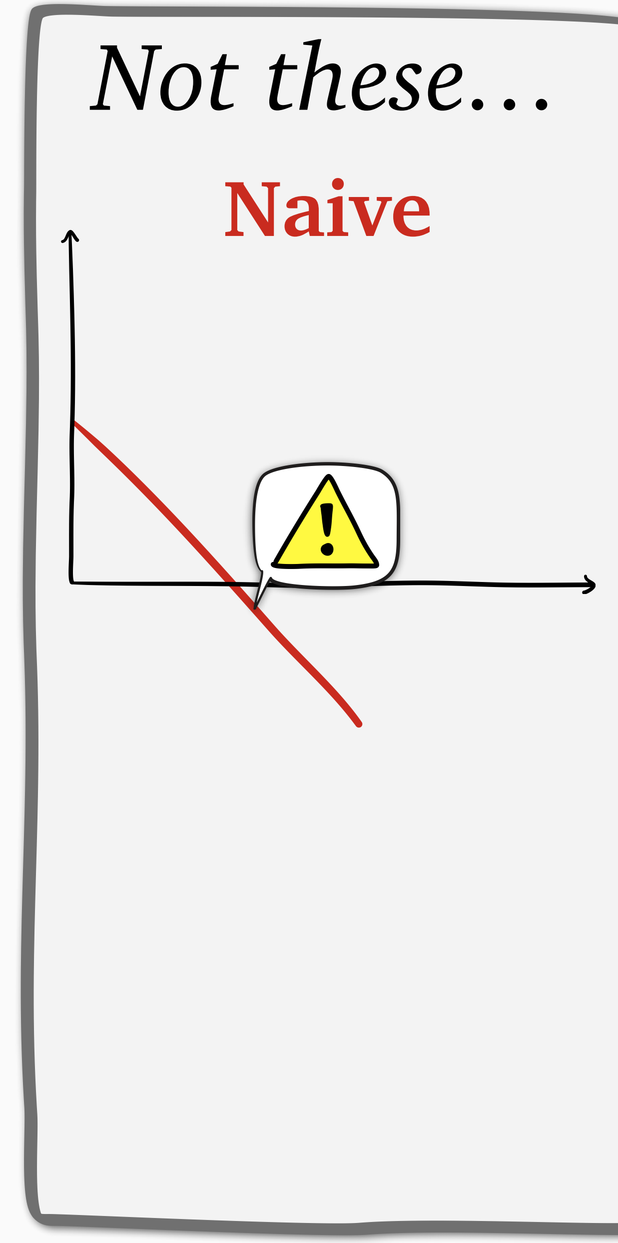

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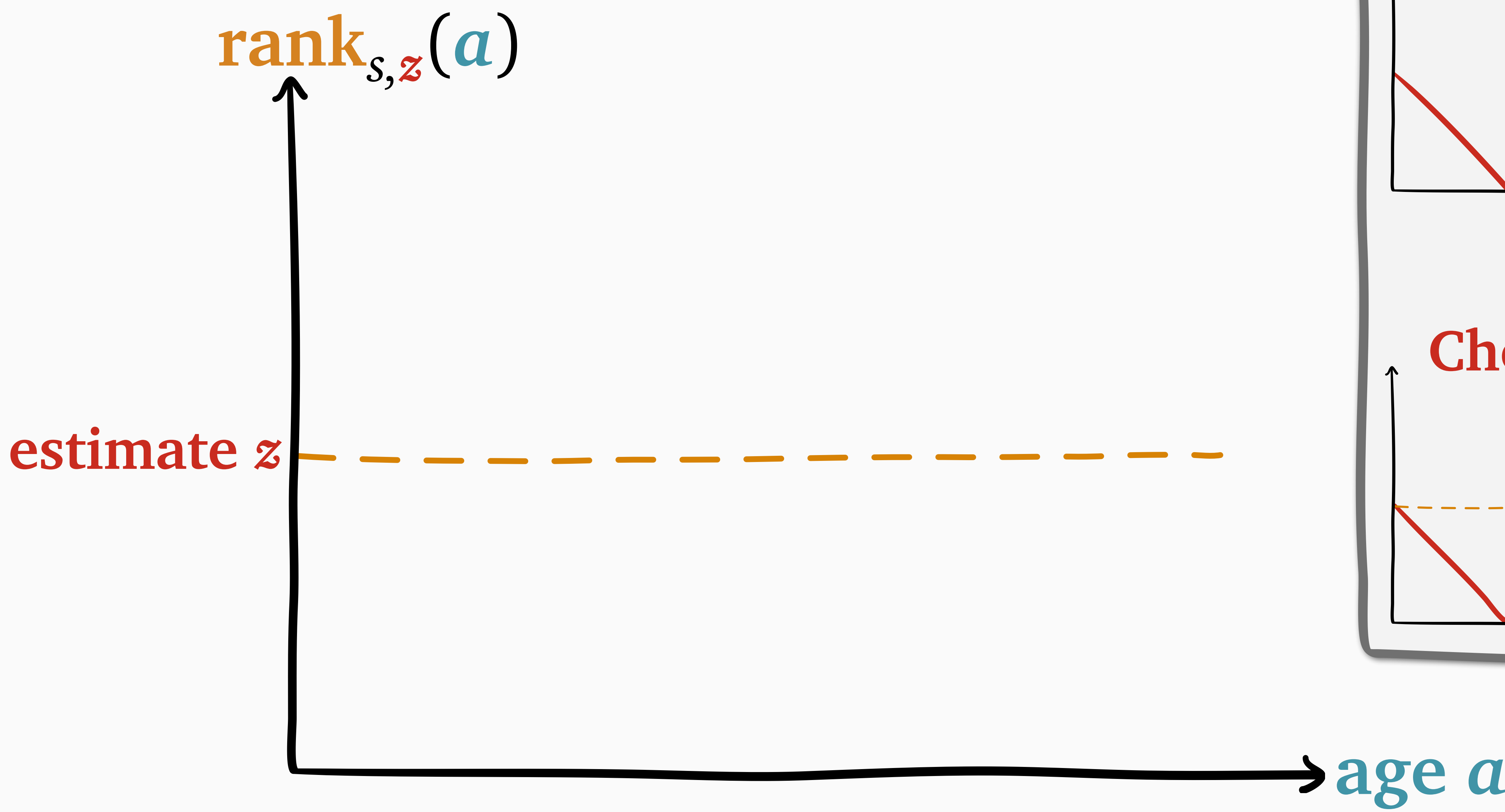
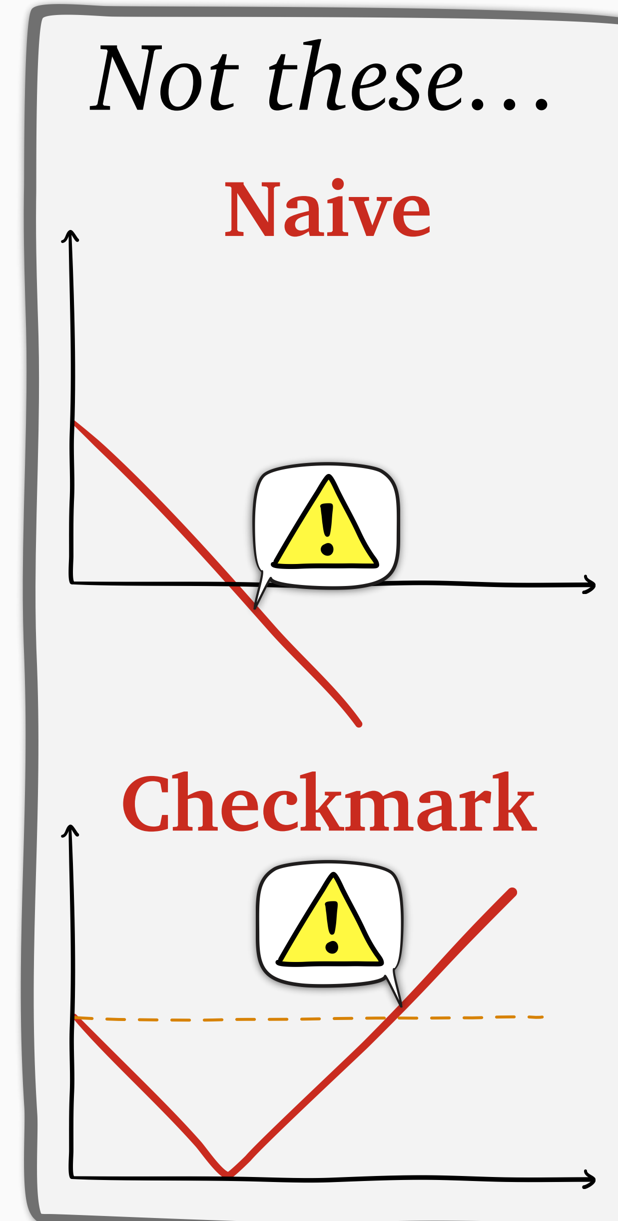
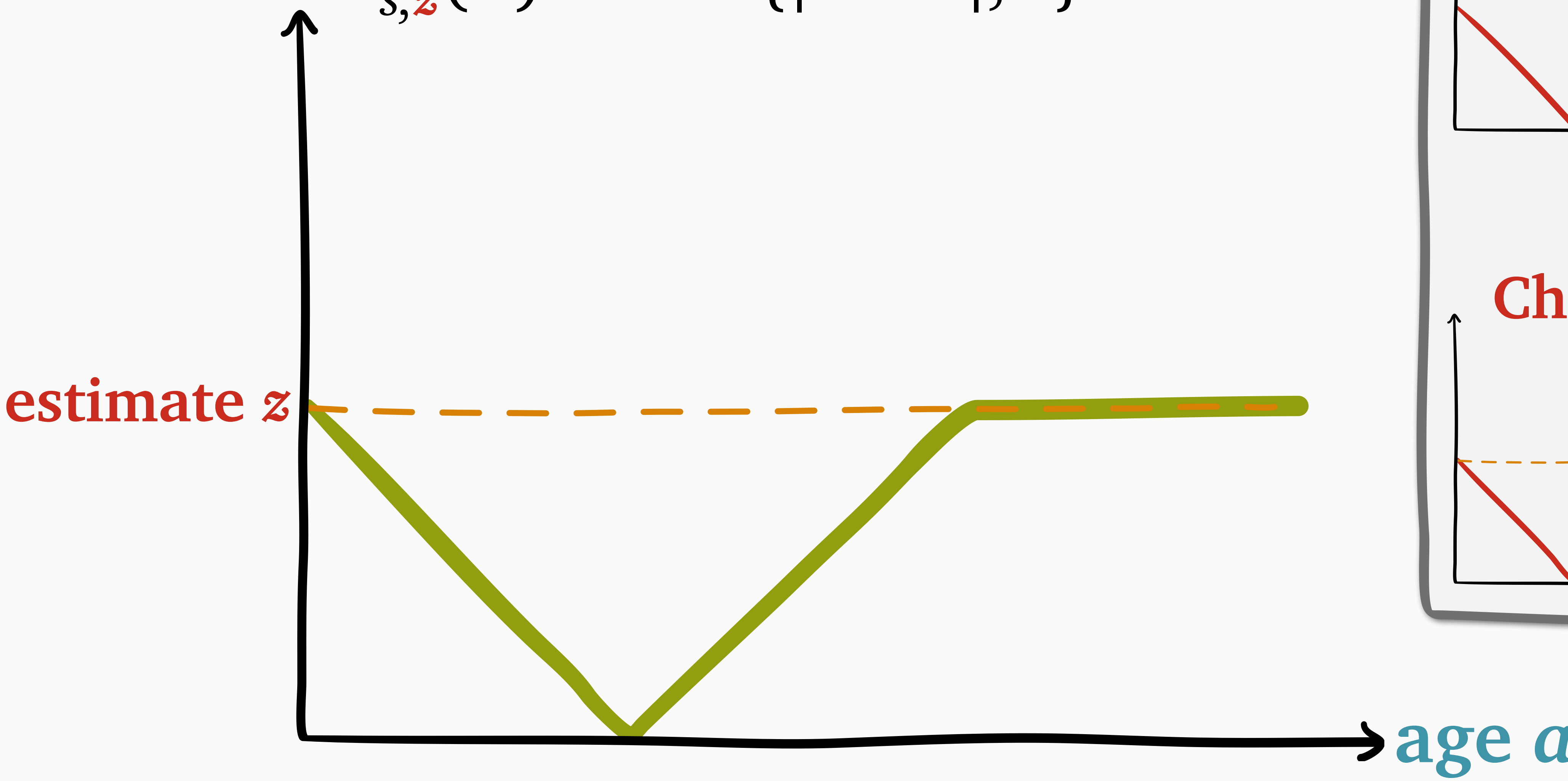


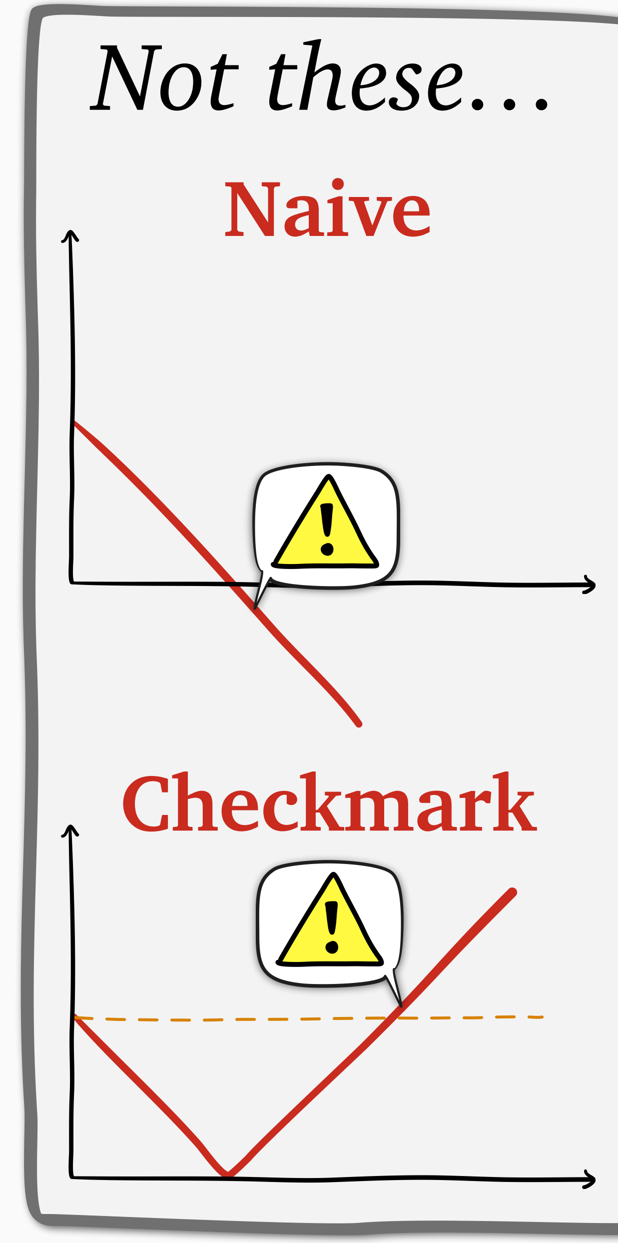


What’s the right **rank** function?

Radical

$$\text{rank}_{s,z}(a) = \min\{|z - a|, z\}$$





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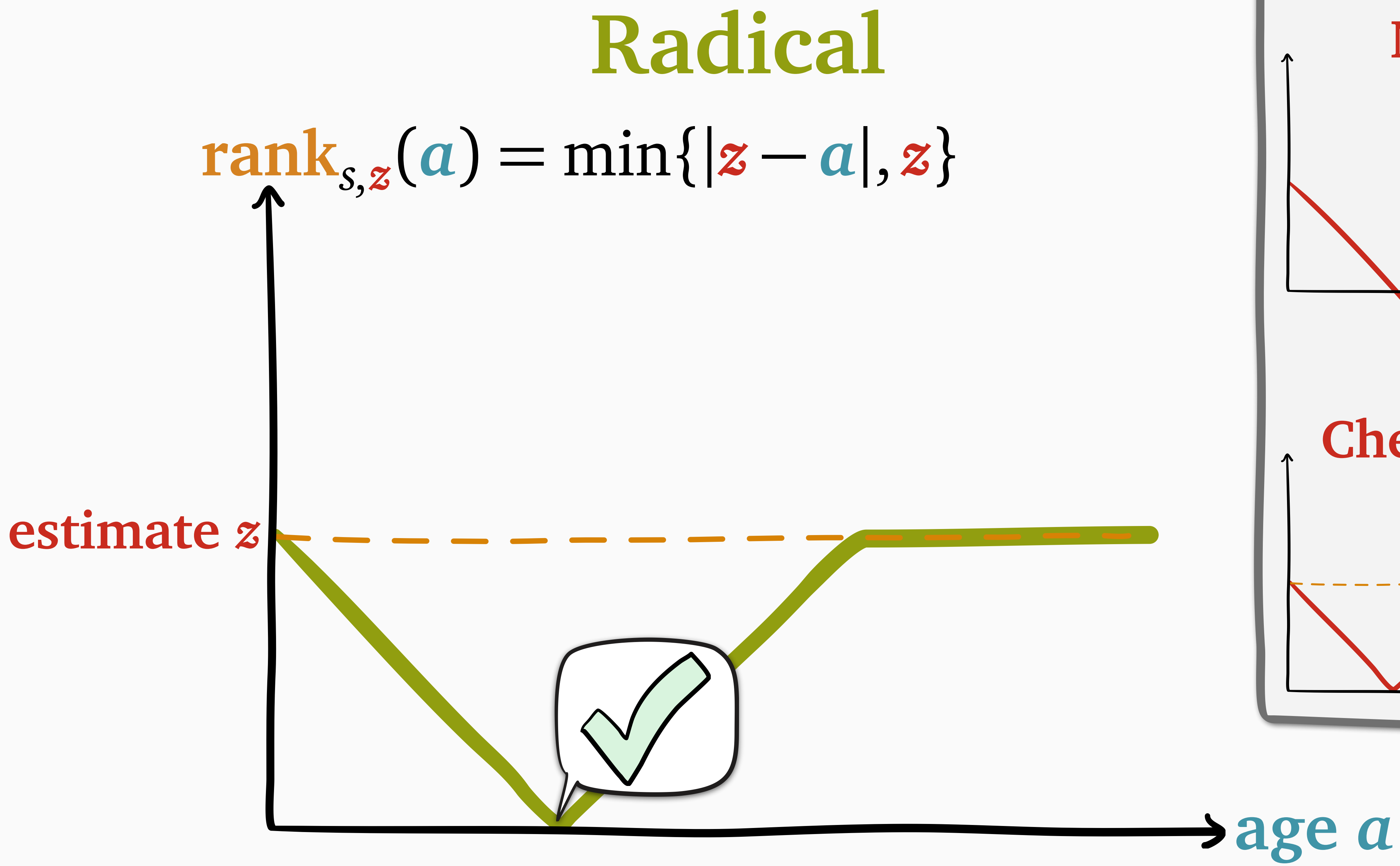

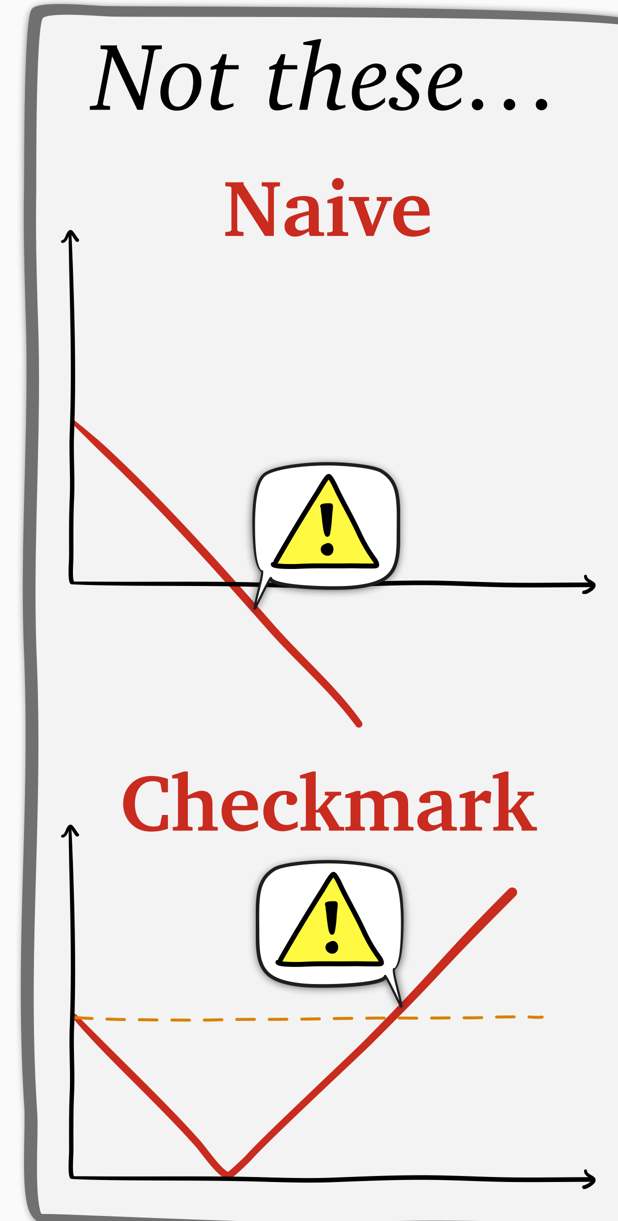

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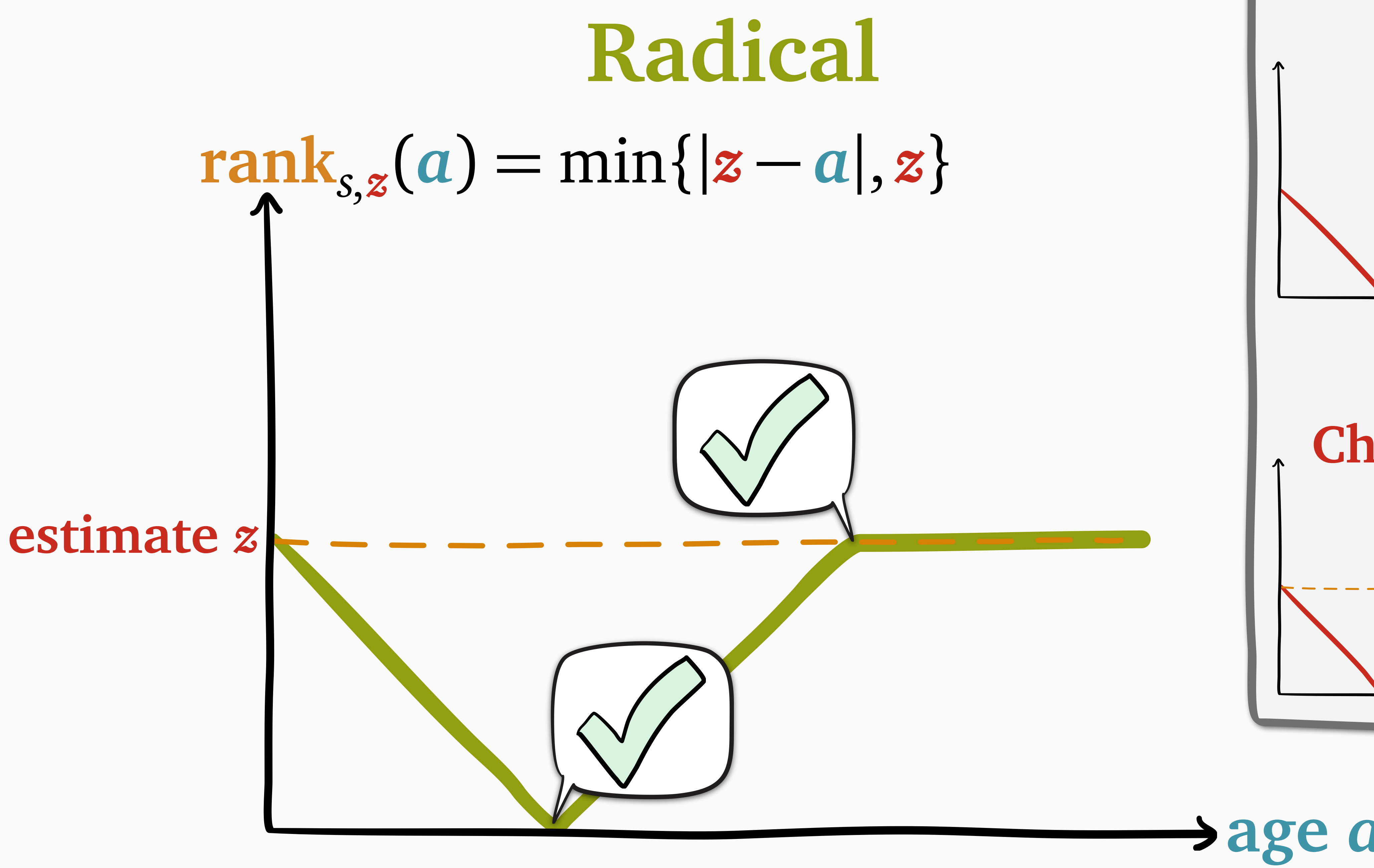

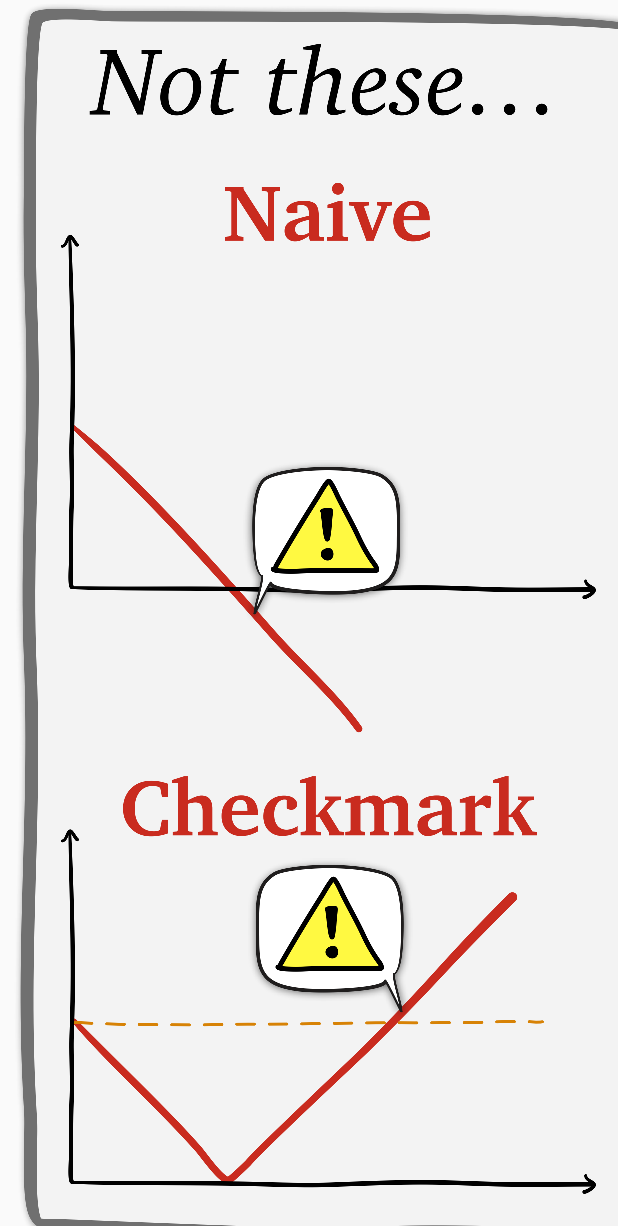

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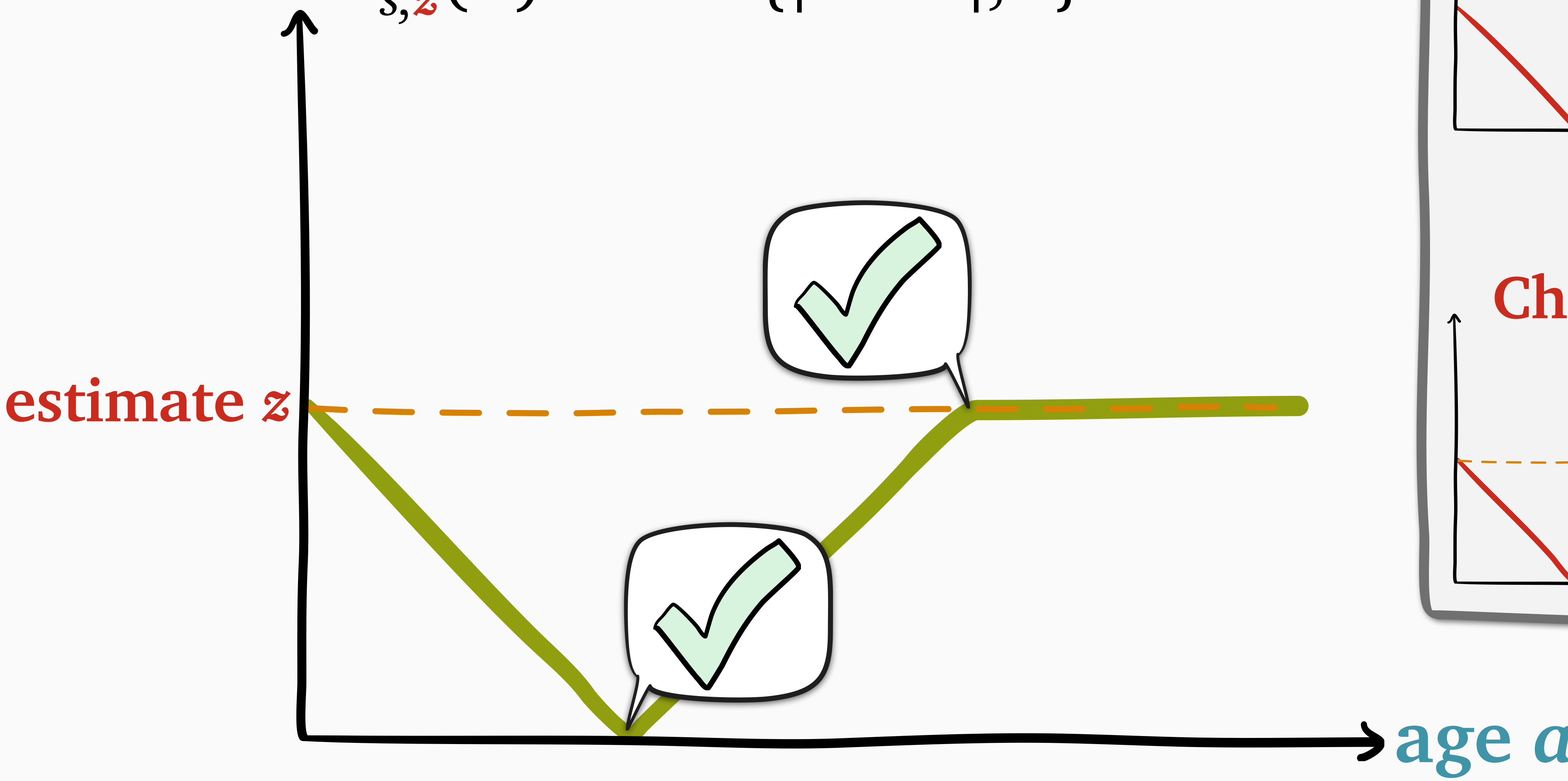
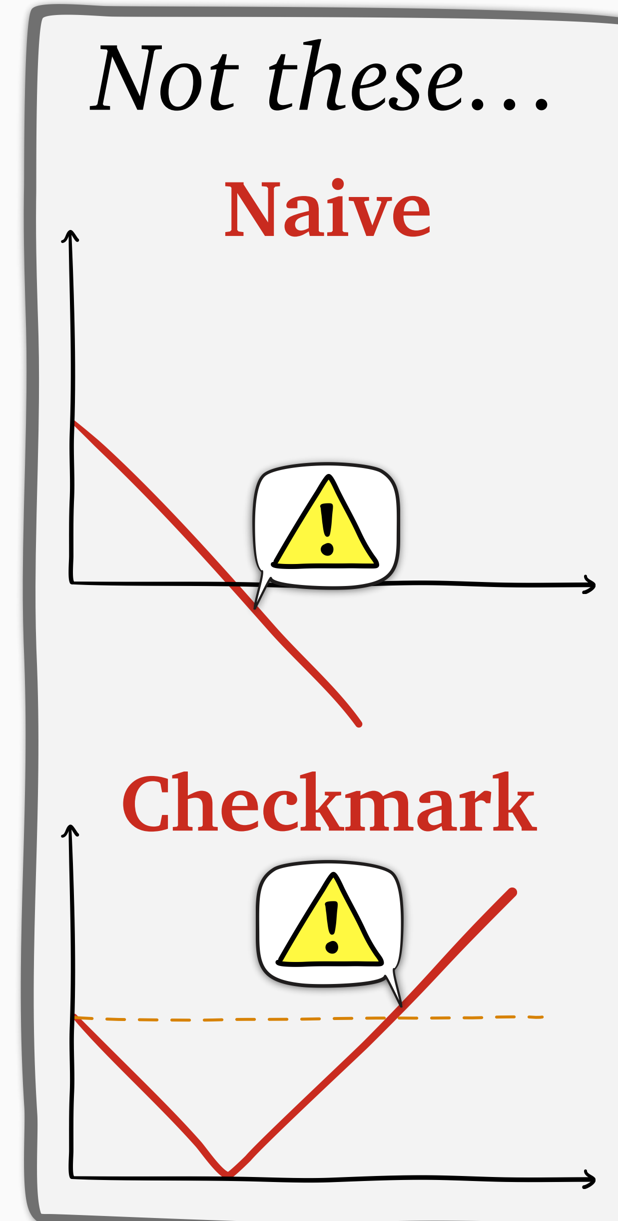
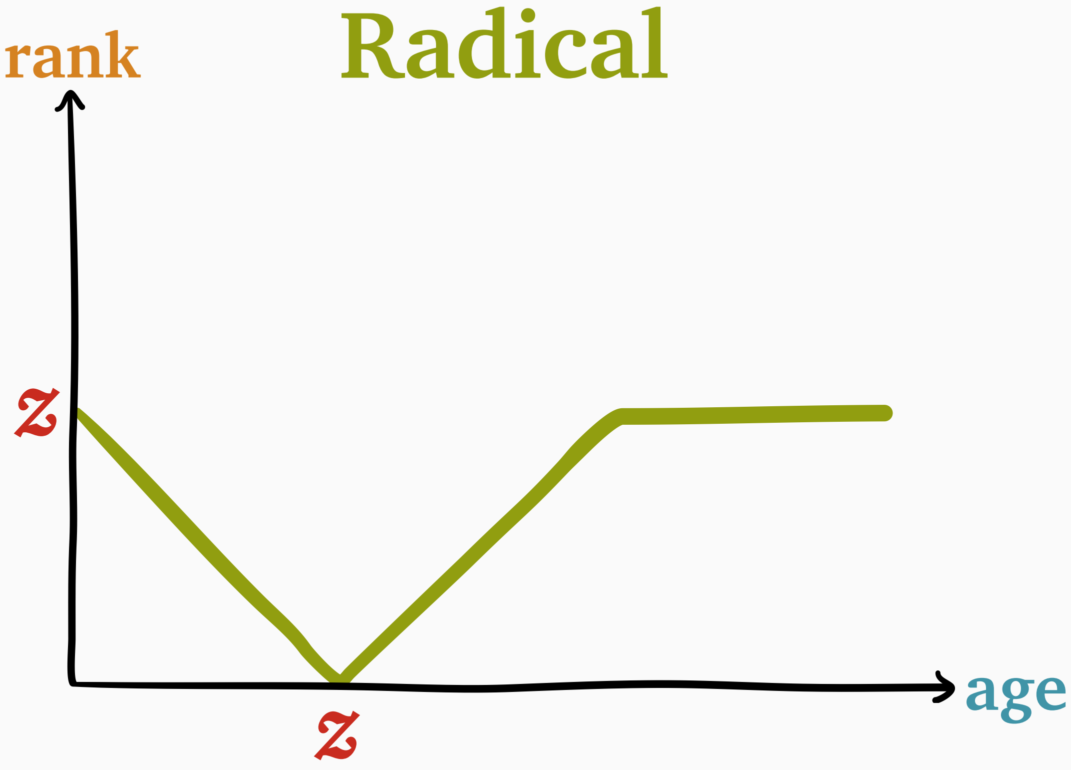
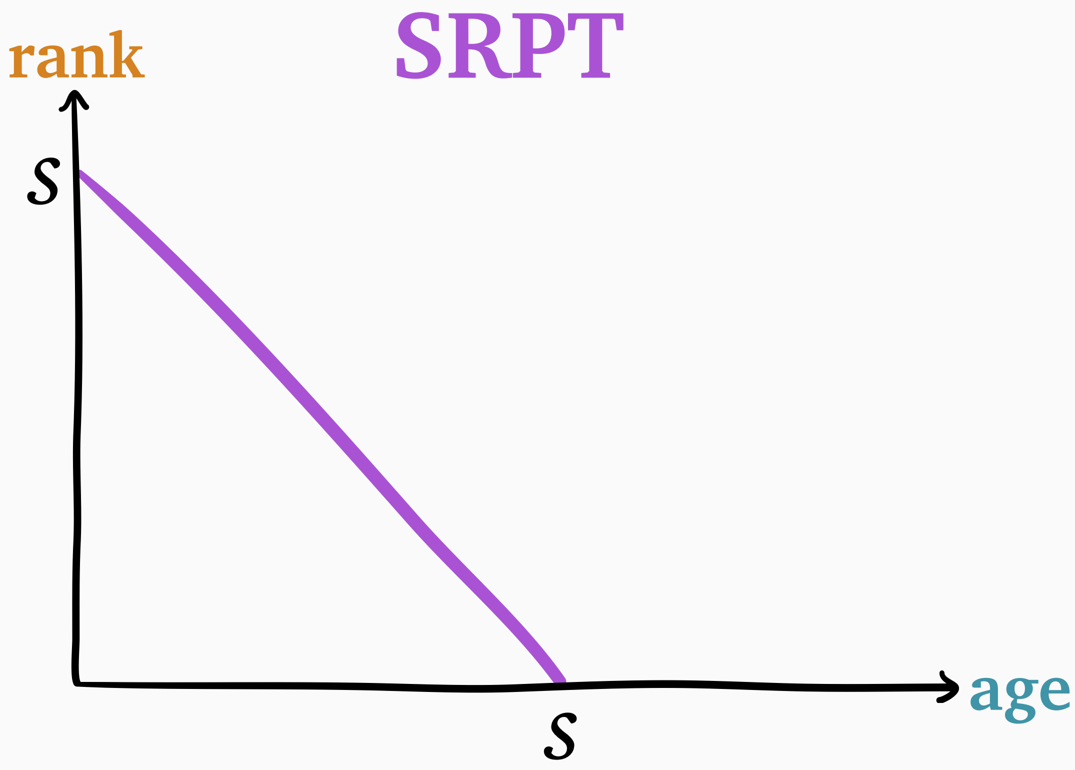




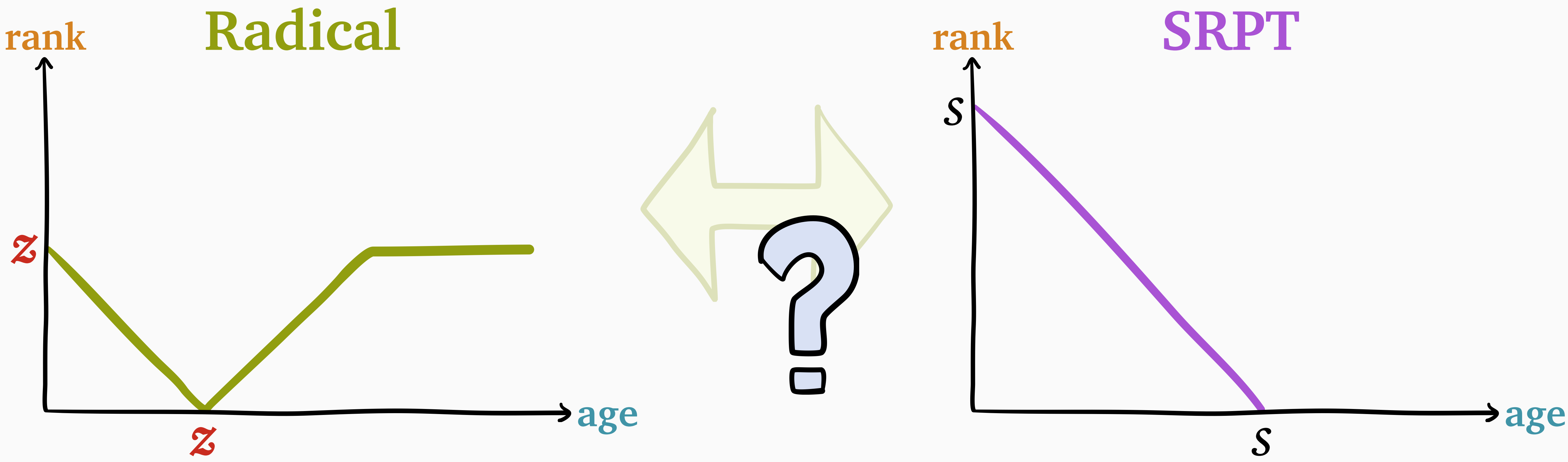
Theorem: Radical is 1-consistent, 3.5-graceful



Radical
rank
z
z
age



SRPT
rank
s
s
age


Radical
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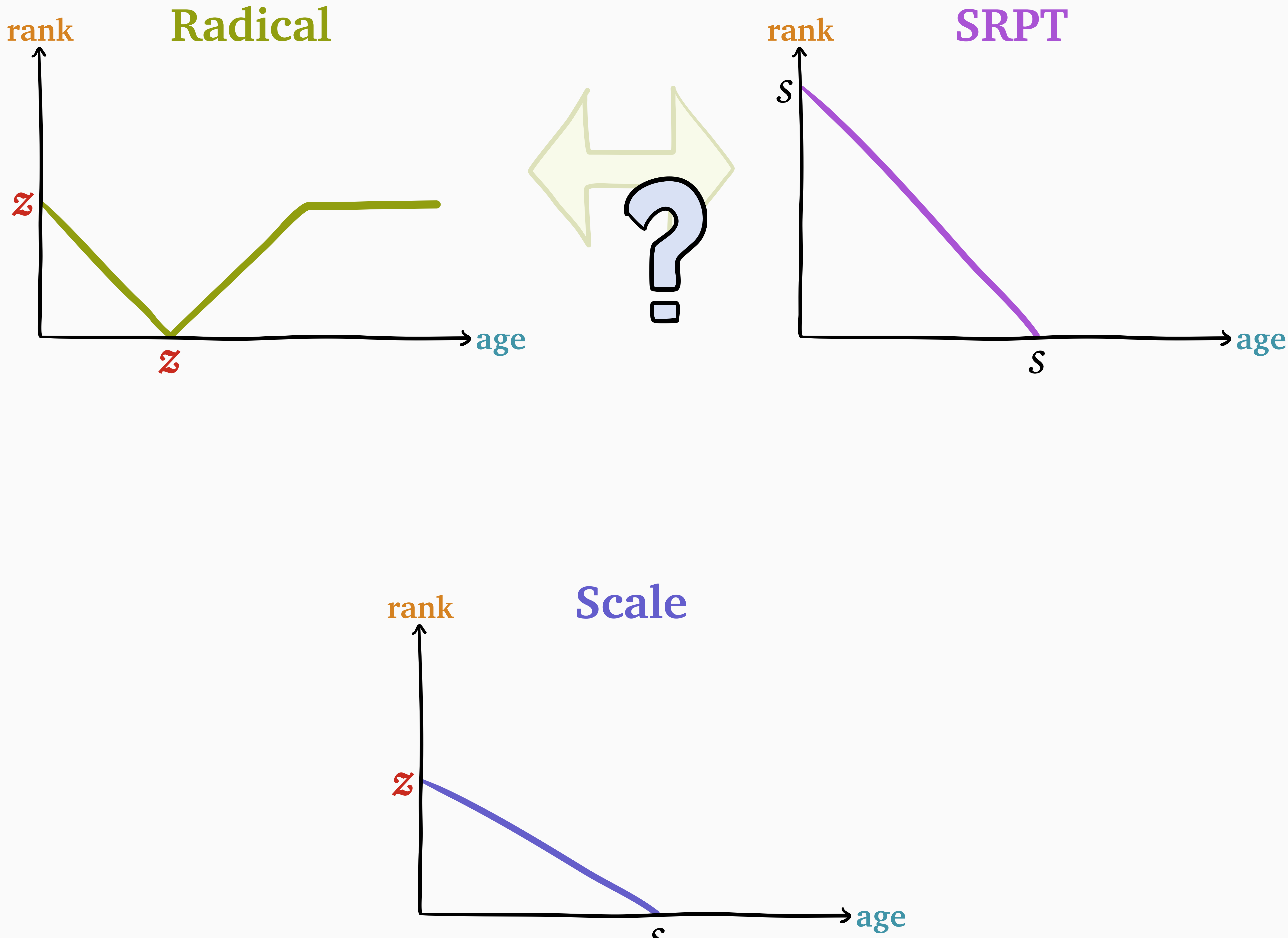
Radical
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Scale
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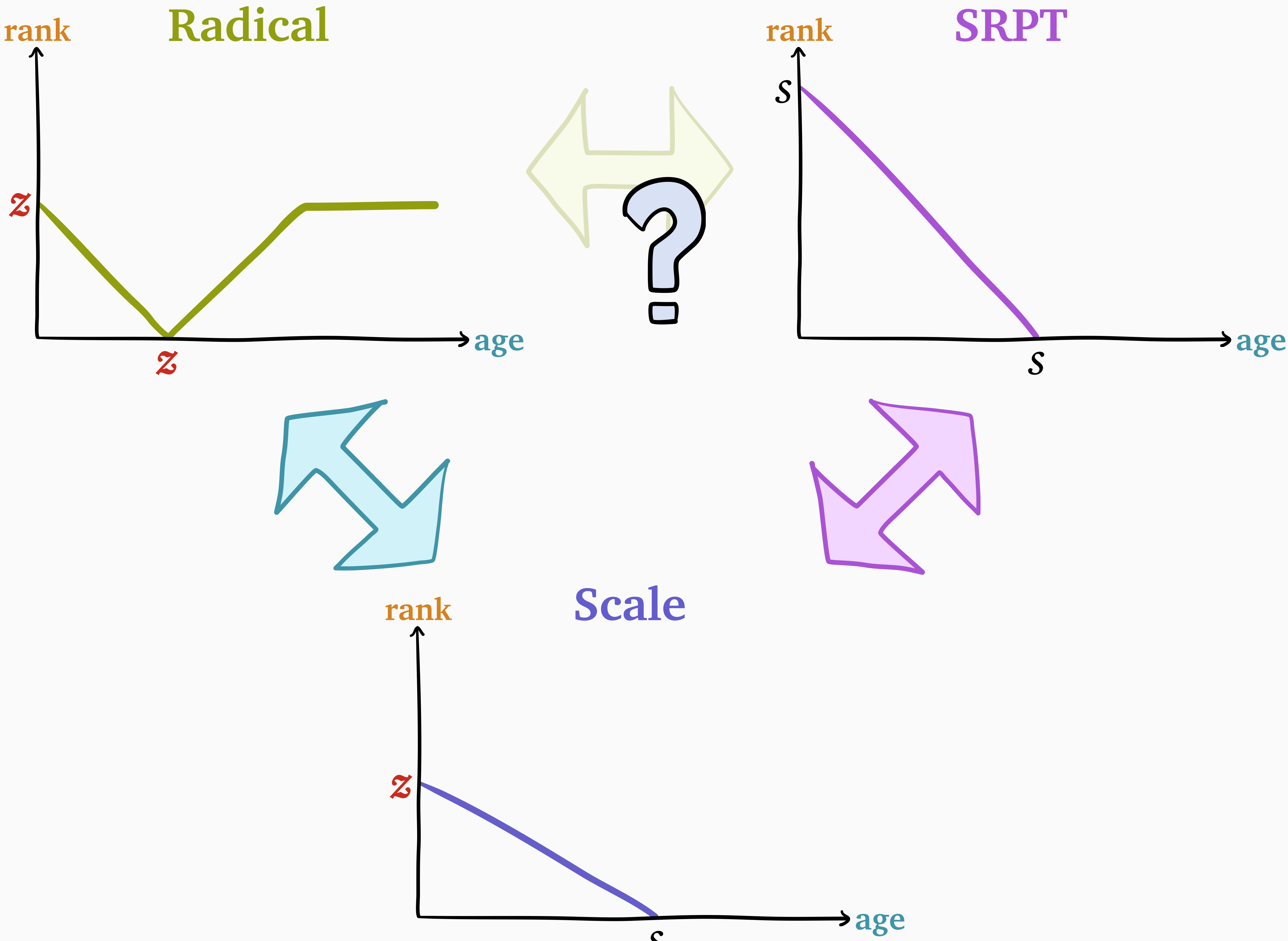
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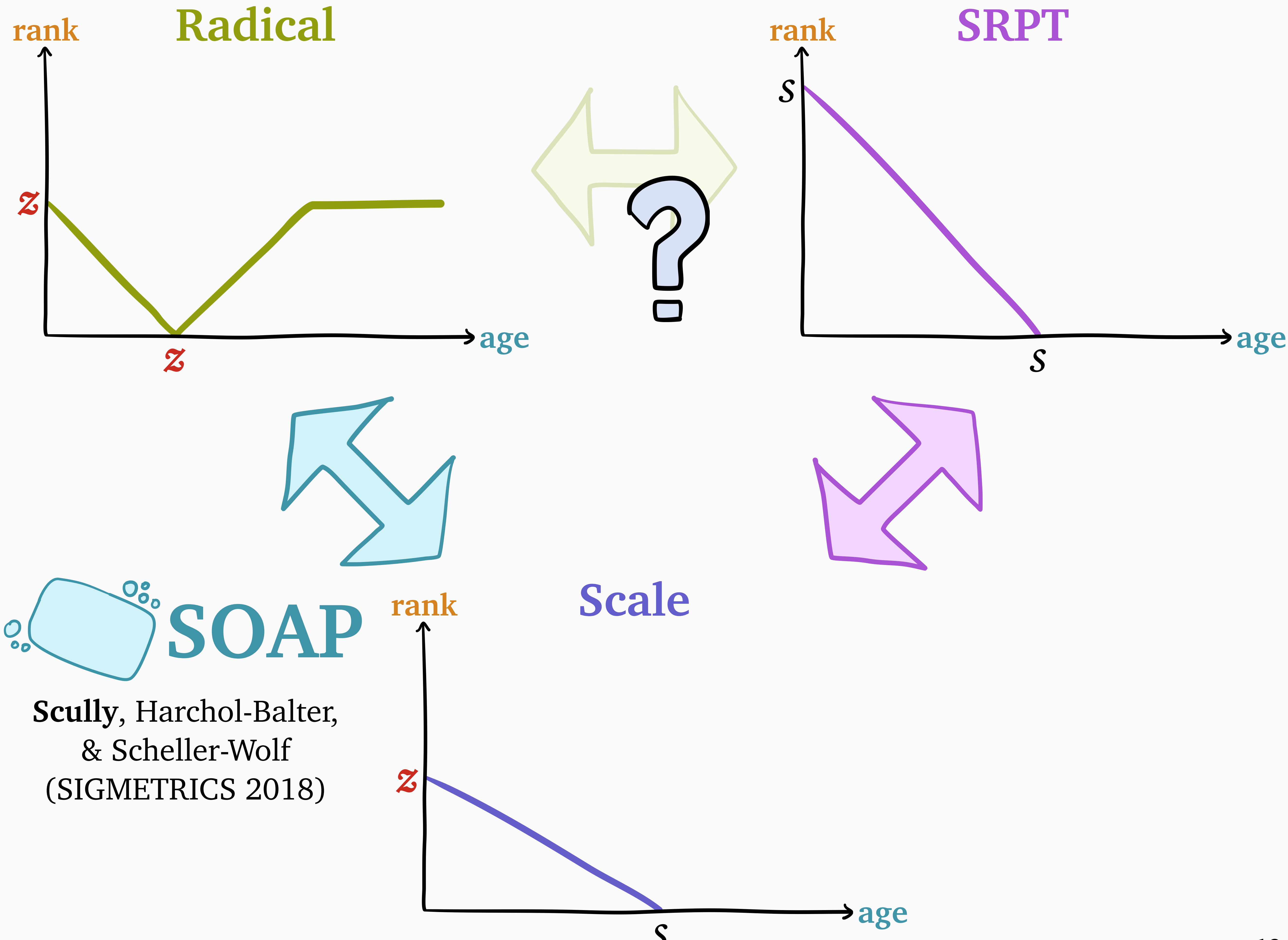
Radical
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SRPT
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SOAP
Scully, Harchol-Balter,
& Scheller-Wolf
(SIGMETRICS 2018)
Scale
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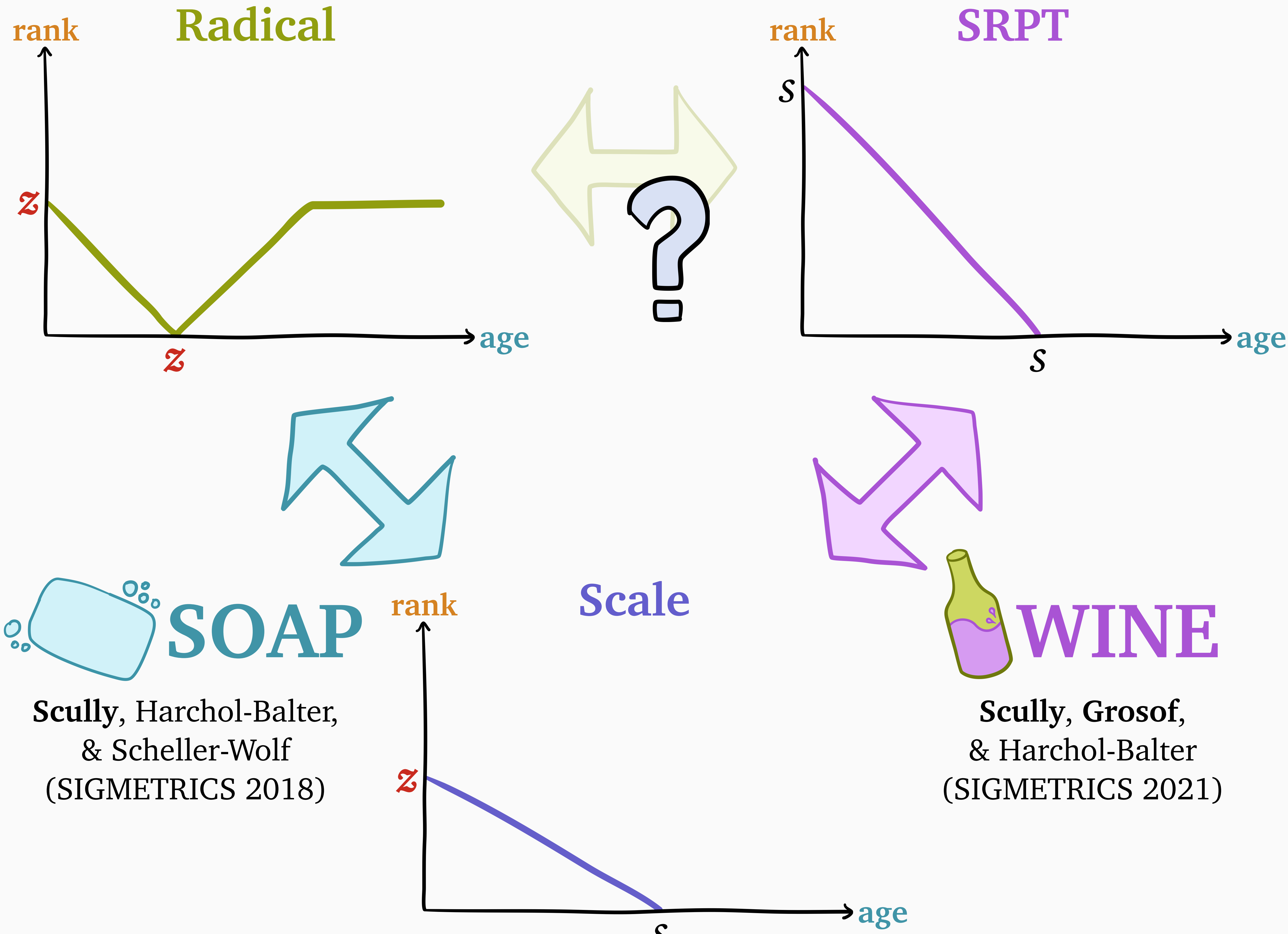
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Scully, Harchol-Balter,
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(SIGMETRICS 2018)
Scully, Grosof,
& Harchol-Balter
(SIGMETRICS 2021)

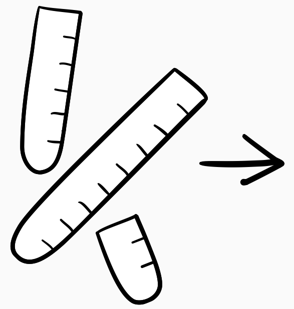
SOAP

Schedule Ordered by Age-based Priority

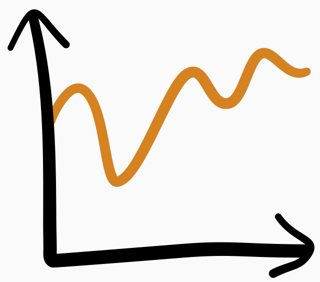
SOAP

Schedule **O**rdered by **A**ge-based **P**riority

stochastic arrival process λ, $(S, \boldsymbol{Z})$



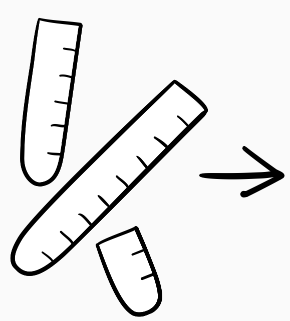
any **rank** function



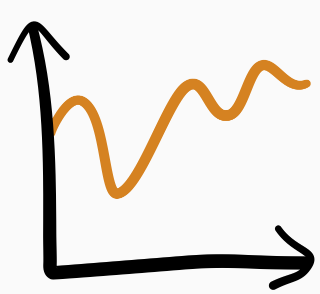
SOAP

Schedule **O**rdered by **A**ge-based **P**riority

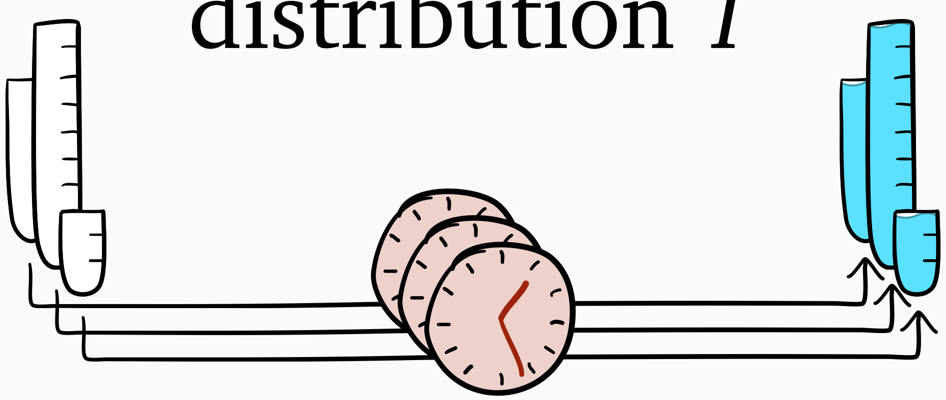
stochastic arrival process λ, $(S, \boldsymbol{Z})$

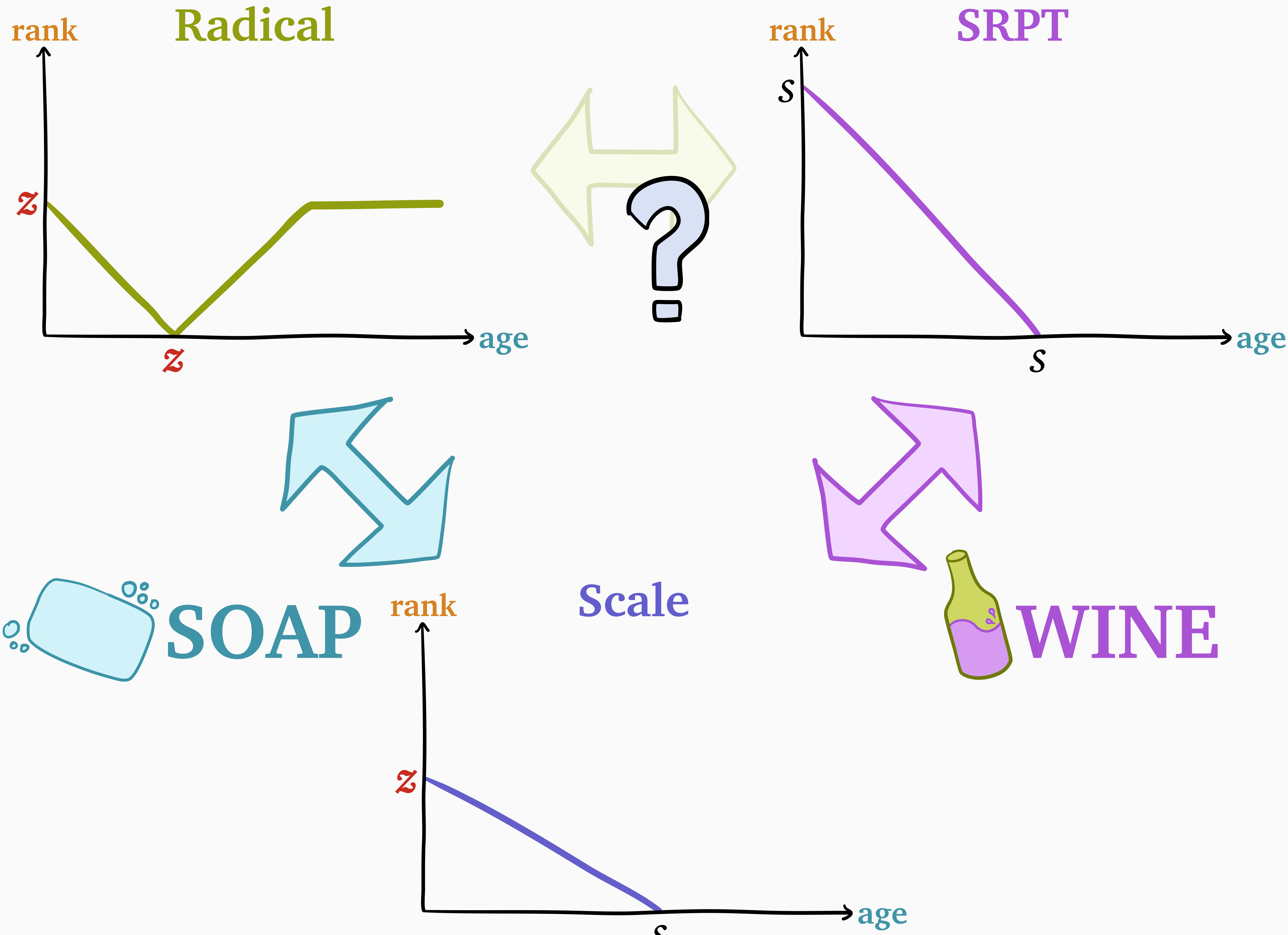


any **rank** function



response time distribution T

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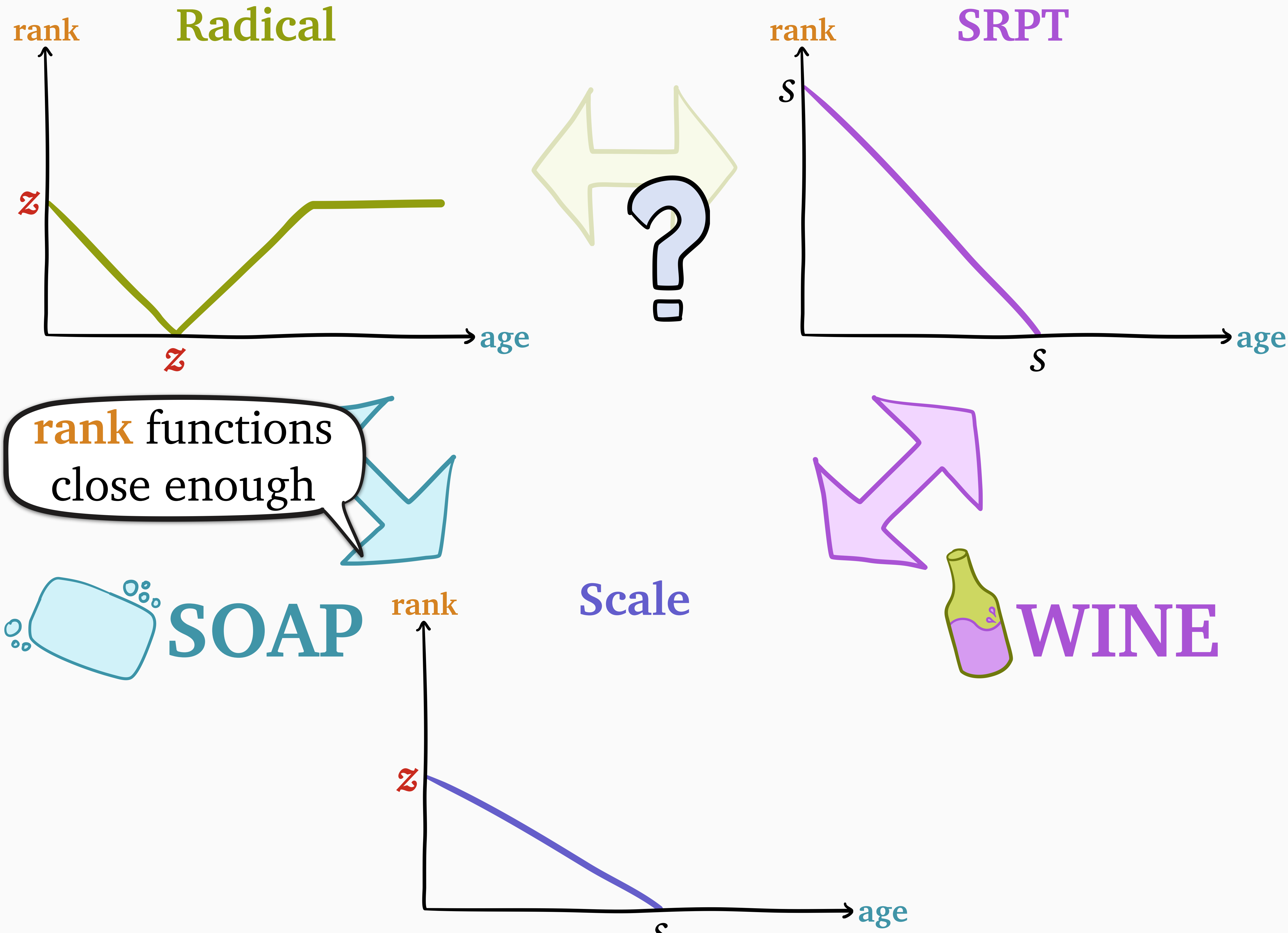

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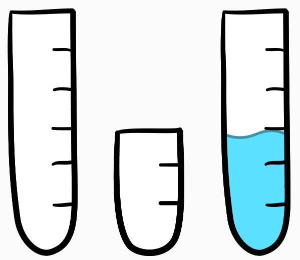
WINE

WINE

Work Integral Number Equality

WINE

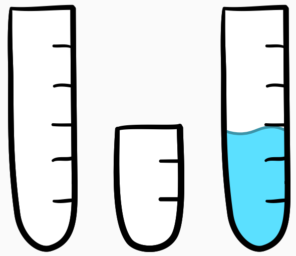
Work Integral Number Equality



number of jobs N

WINE

Work **I**ntegral **N**umber **E**quality

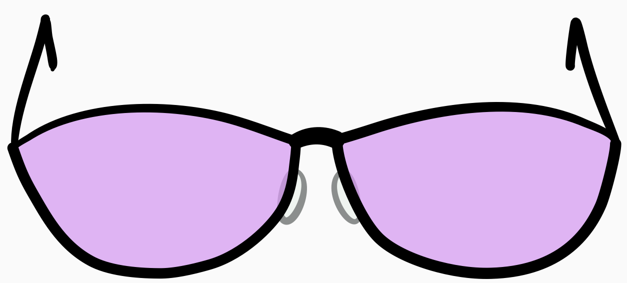


number of jobs N

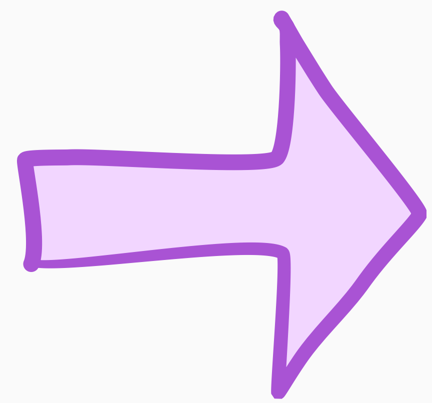
Little's Law:
$\mathbf{E}[T] = \mathbf{E}[N]/\lambda$

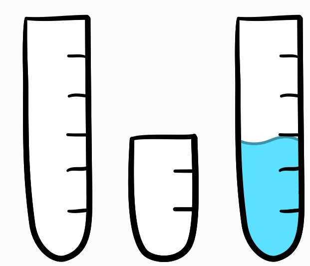
WINE

Work **I**ntegral **N**umber **E**quality



r-work $W(r)$





number of jobs N

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WINE

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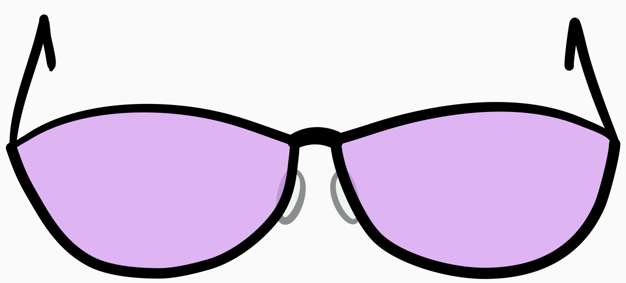

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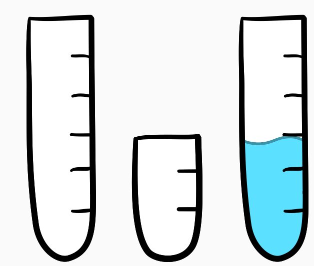


number of jobs N

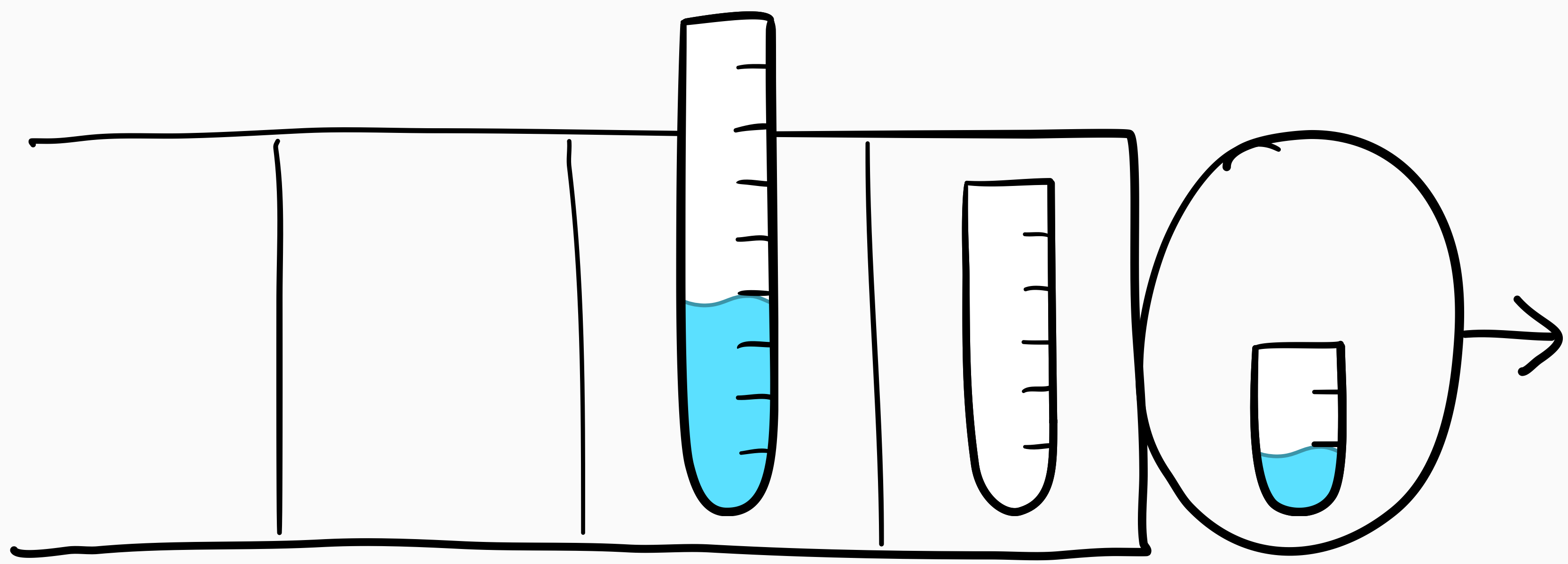
Little's Law:
$\mathbf{E}[T] = \mathbf{E}[N]/\lambda$

What is r-work?

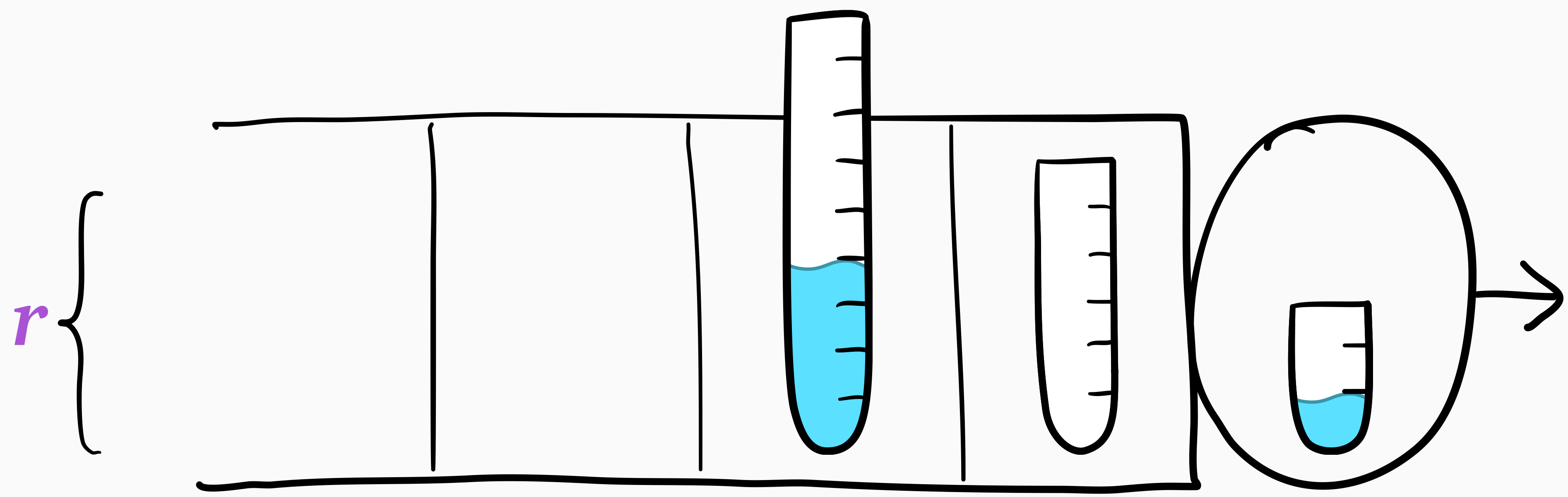
Get N from r-work?

Bound **Scale**'s $\mathbf{E}[T]$?

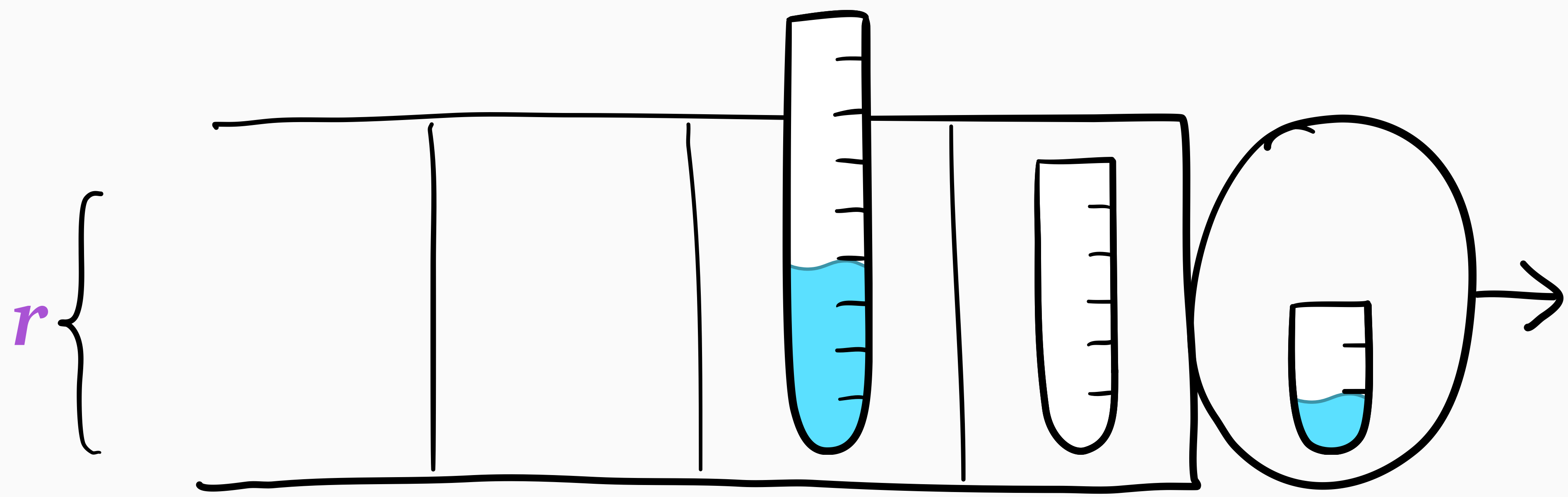
What is *r*-work?



What is *r*-work?



What is *r*-work?

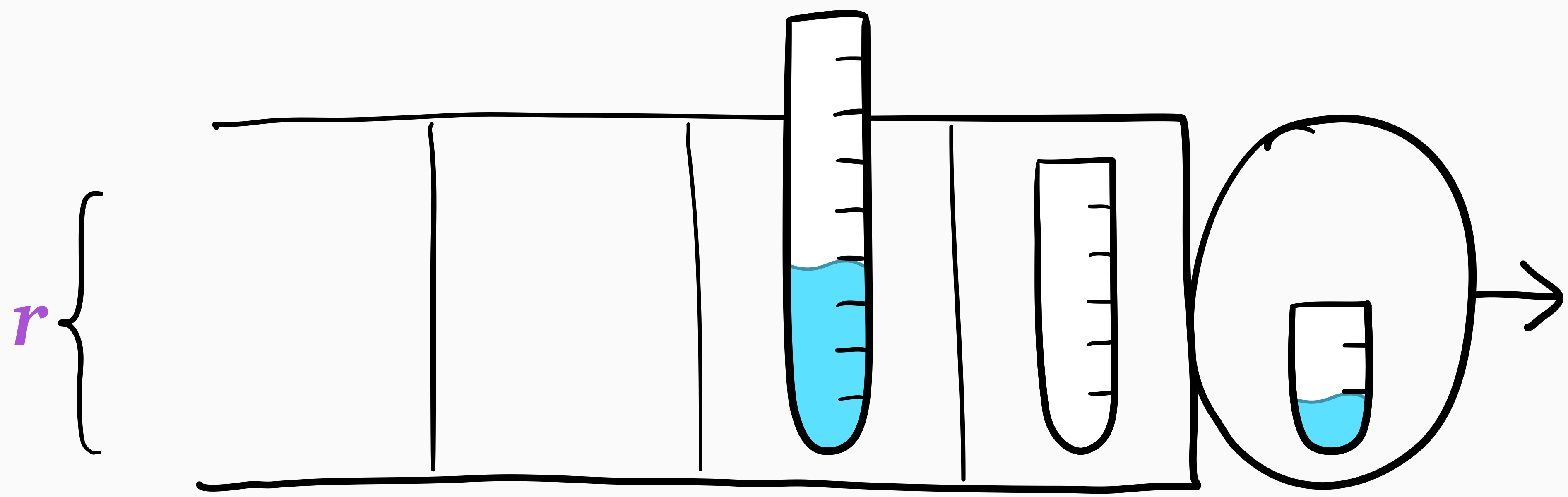


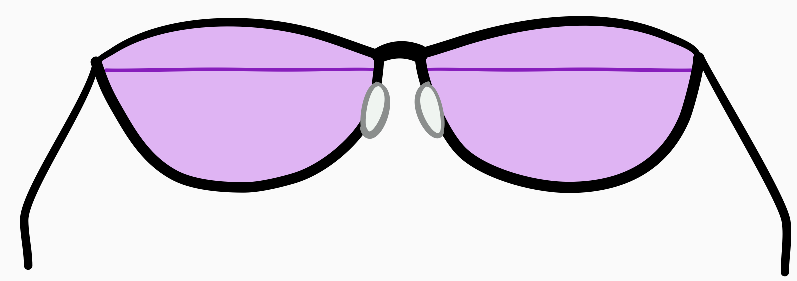
Definition:

$W(r)$ = total remaining size of jobs whose remaining size is $\leq r$

r-work

What is r-work?





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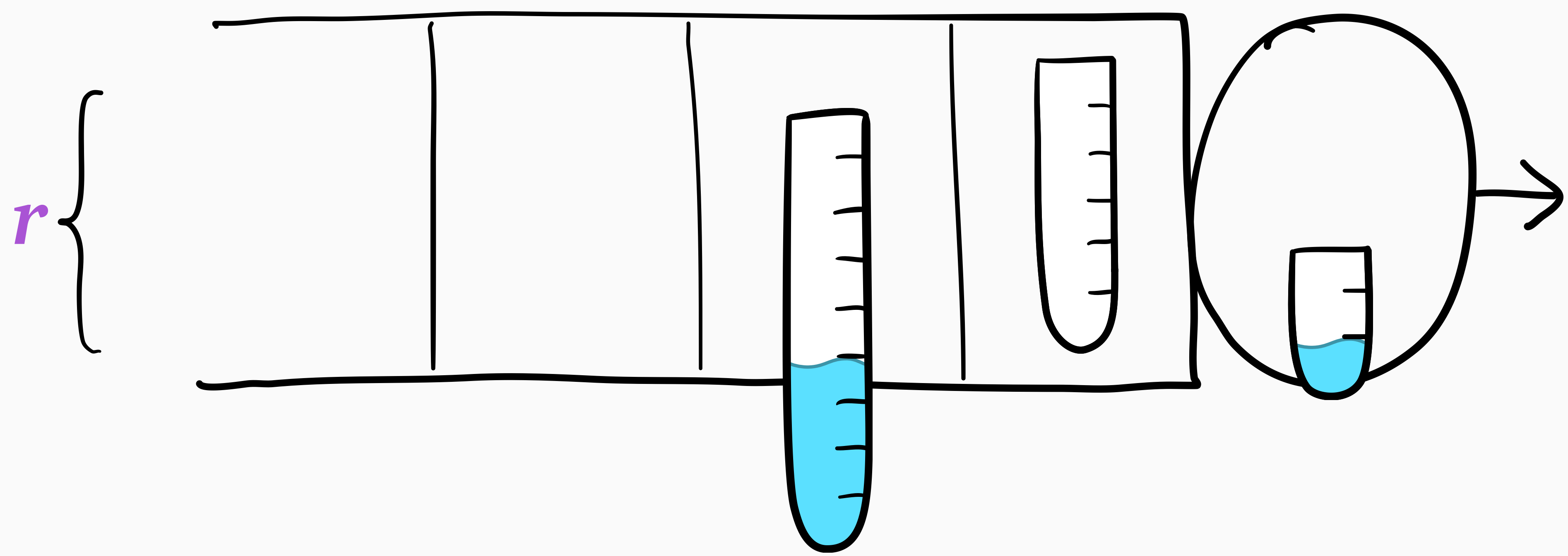

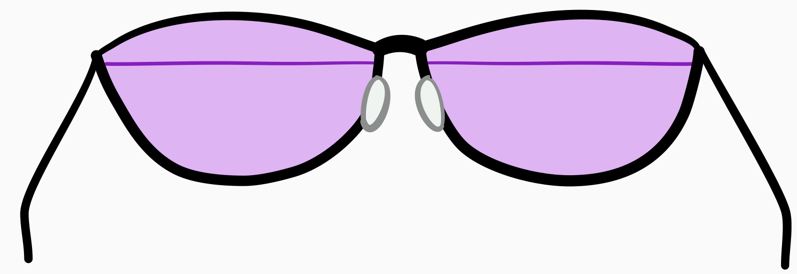

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r-work

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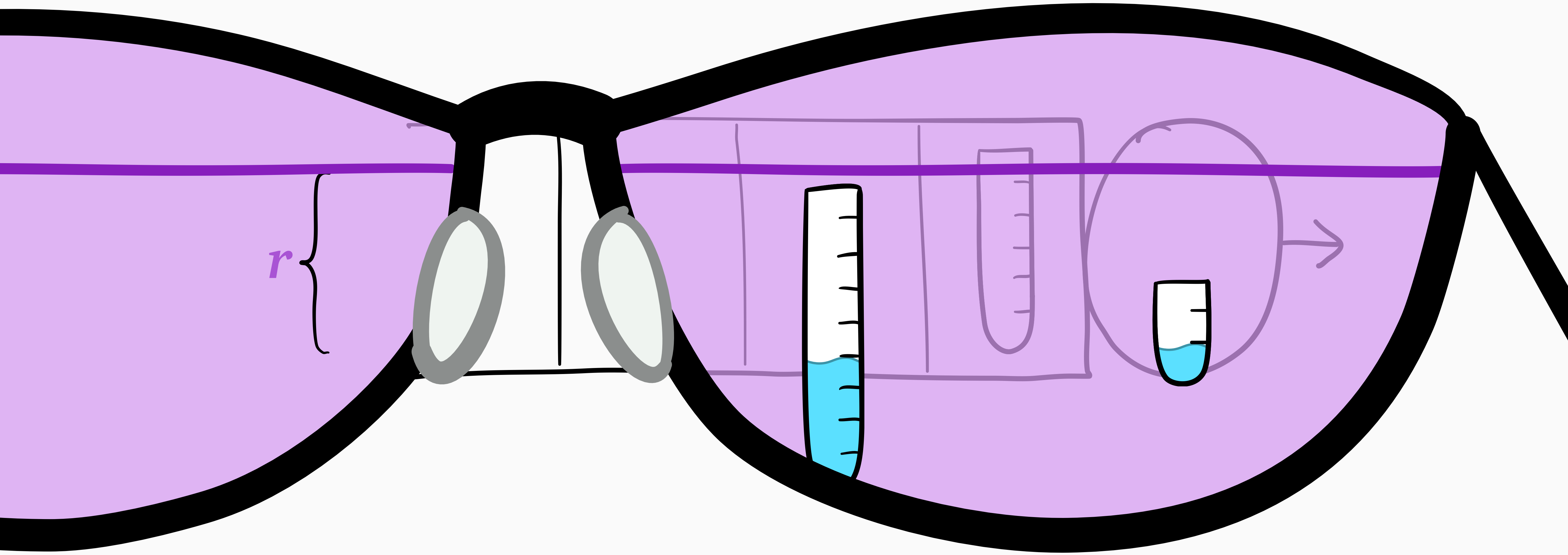

Definition:

$W(r)$ = total remaining size of jobs whose remaining size is $\leq r$

r-work

 **WINE**: under *any* scheduling policy,

$$N = \int_0^\infty \frac{W(r)}{r^2} \, \mathrm{d}r$$

WINE: under *any* scheduling policy,

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

NEW! **Lemma:**

$$\mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\frac{\alpha}{\beta} r\right)\right]$$

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$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

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$$\mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\tfrac{\alpha}{\beta} r\right)\right]$$

NEW! **Theorem:**

$$\mathbf{E}[T_{\mathrm{Scale}}] \le \tfrac{\alpha}{\beta}\mathbf{E}[T_{\mathrm{SRPT}}]$$

also holds in worst case

WINE: under *any* scheduling policy,

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

NEW!

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WINE: under *any* scheduling policy,

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

NEW!

Lemma:

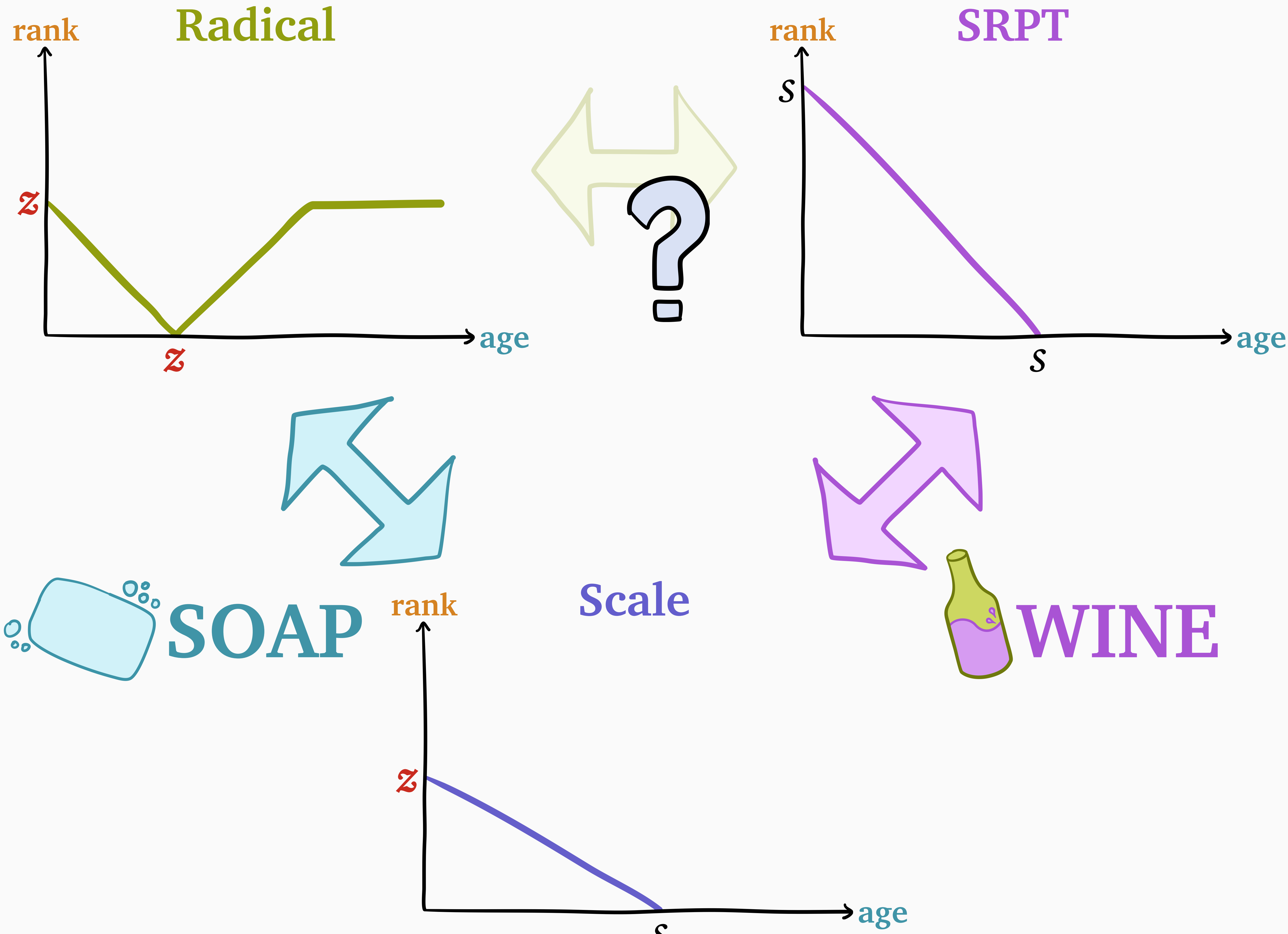
$$\mathbf{E}[W_{\text{Scale}}(r)] \le \mathbf{E}\Big[W_{\text{SRPT}}\Big(\tfrac{\alpha}{\beta} r\Big)\Big]$$

also holds with “noisy scaling”

NEW!

Theorem:

$$\mathbf{E}[T_{\text{Scale}}] \le \tfrac{\alpha}{\beta}\mathbf{E}[T_{\text{SRPT}}]$$


Radical
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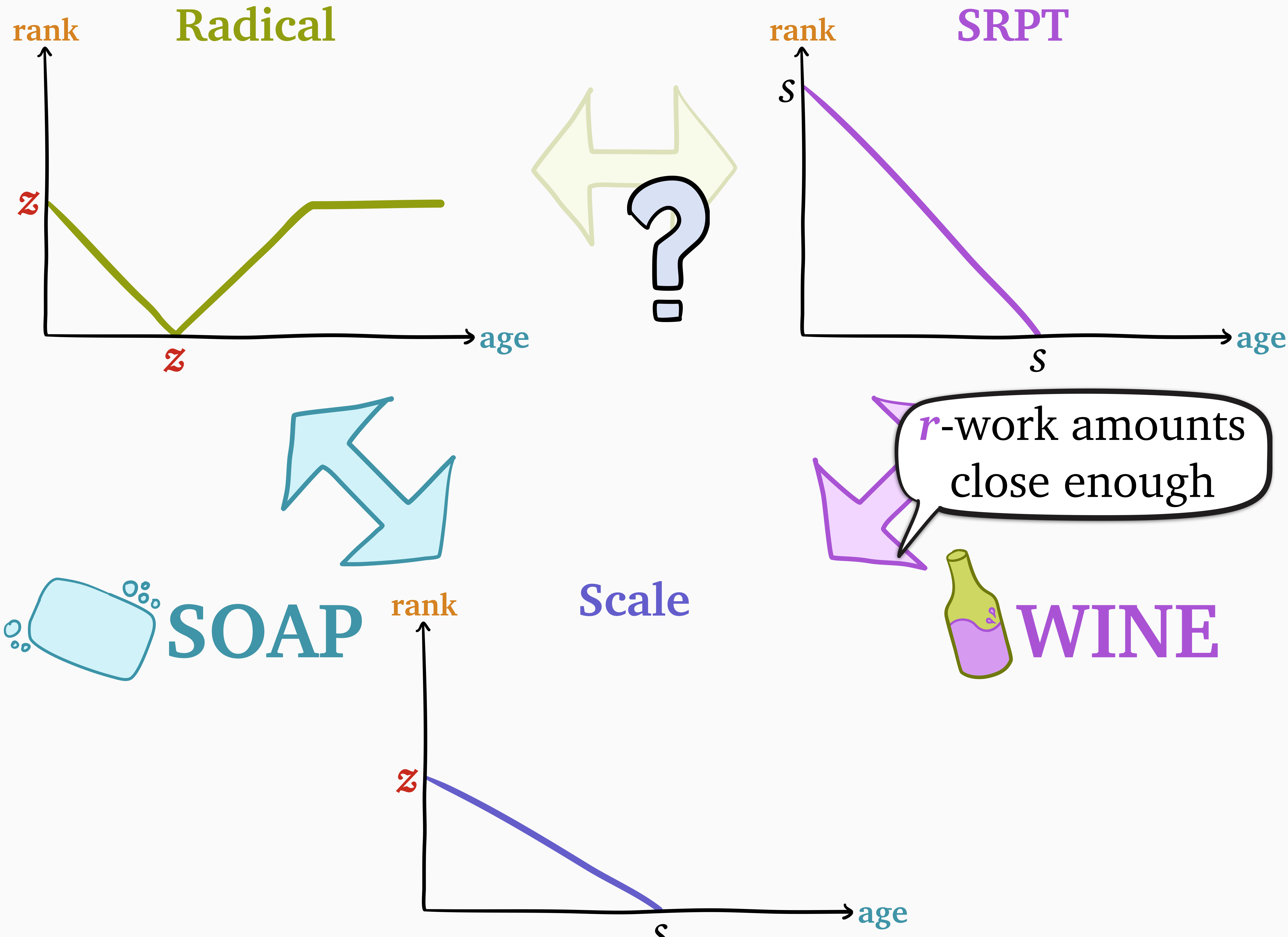
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r-work amounts close enough
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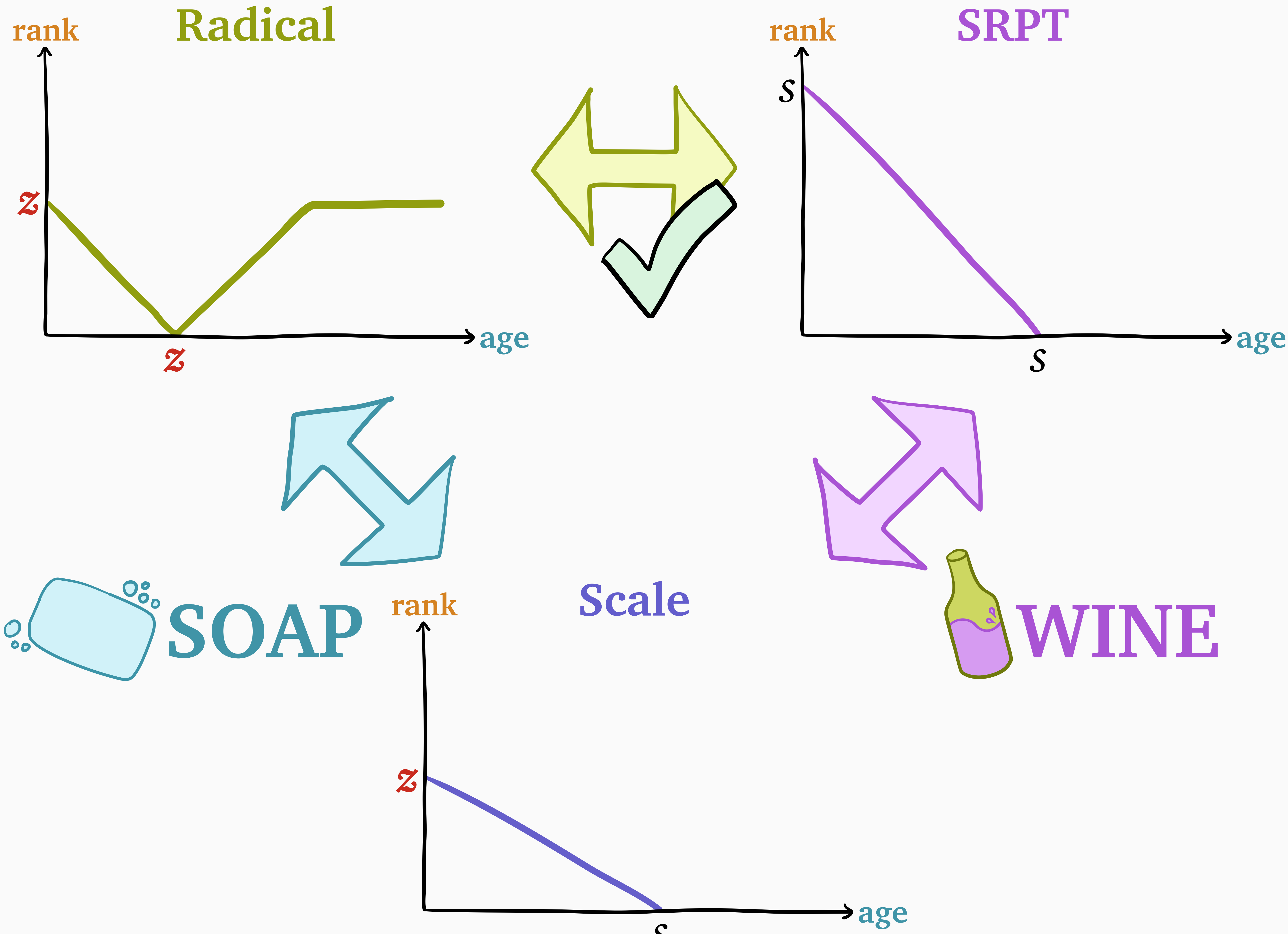

Radical
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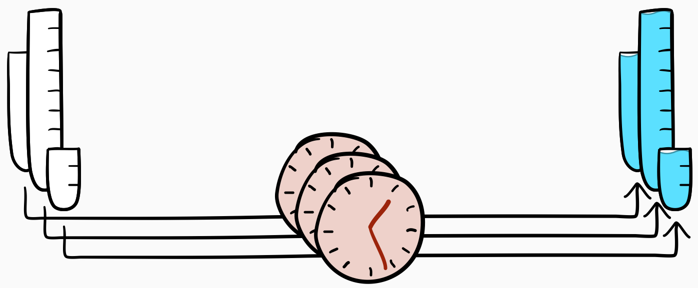
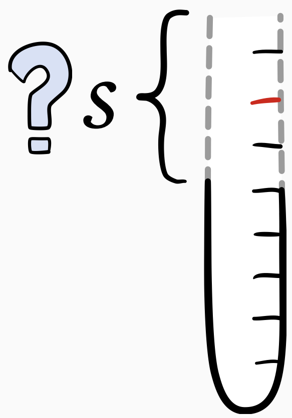
WINE

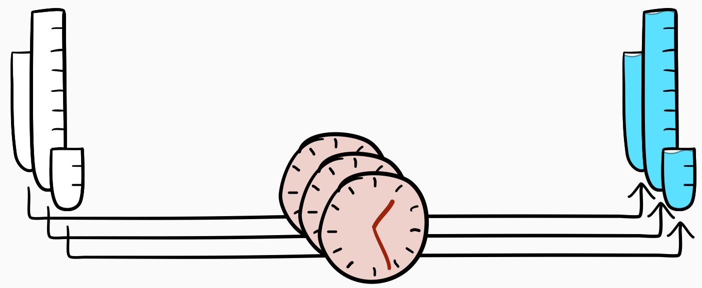
Problem: minimize $\mathbf{E}[T]$
with noisy information

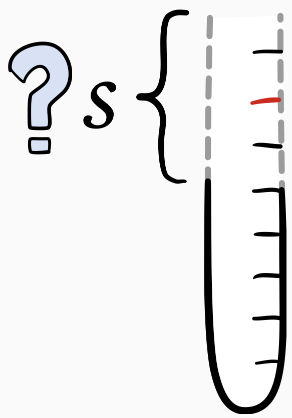




$s \quad z \in [\beta s, \alpha s]$

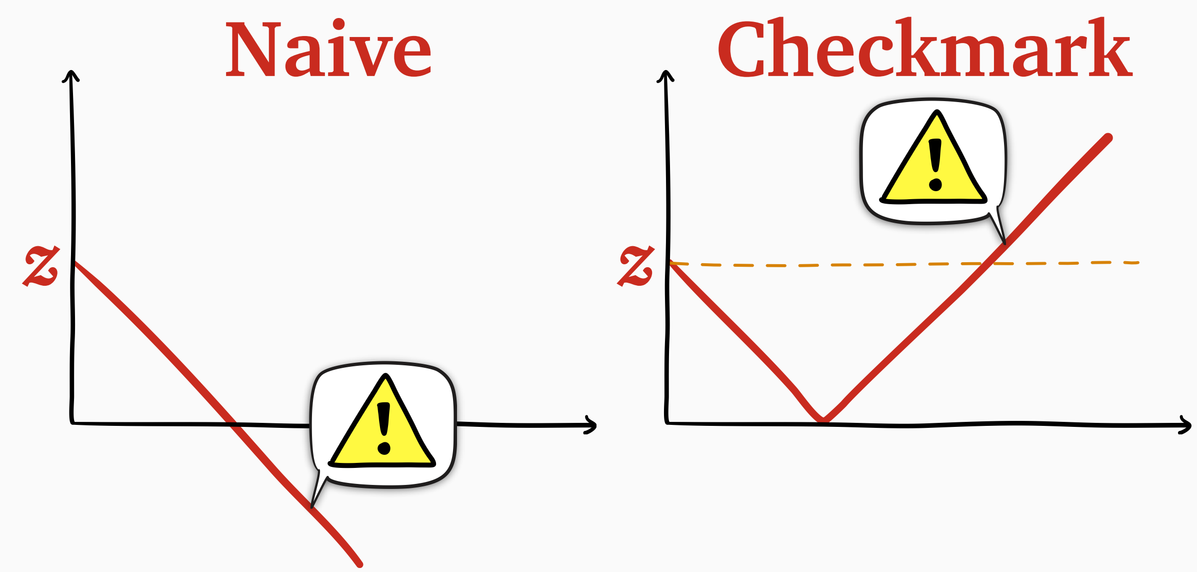
Problem: minimize $\mathbf{E}[T]$ with noisy information





$z \in [\beta s, \alpha s]$

Obstacle: natural **rank** functions perform badly



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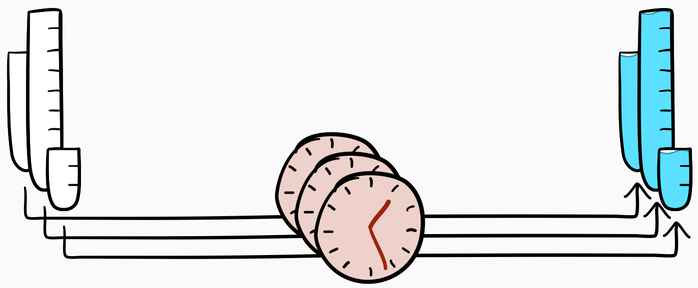

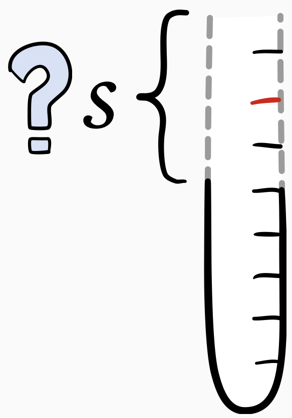

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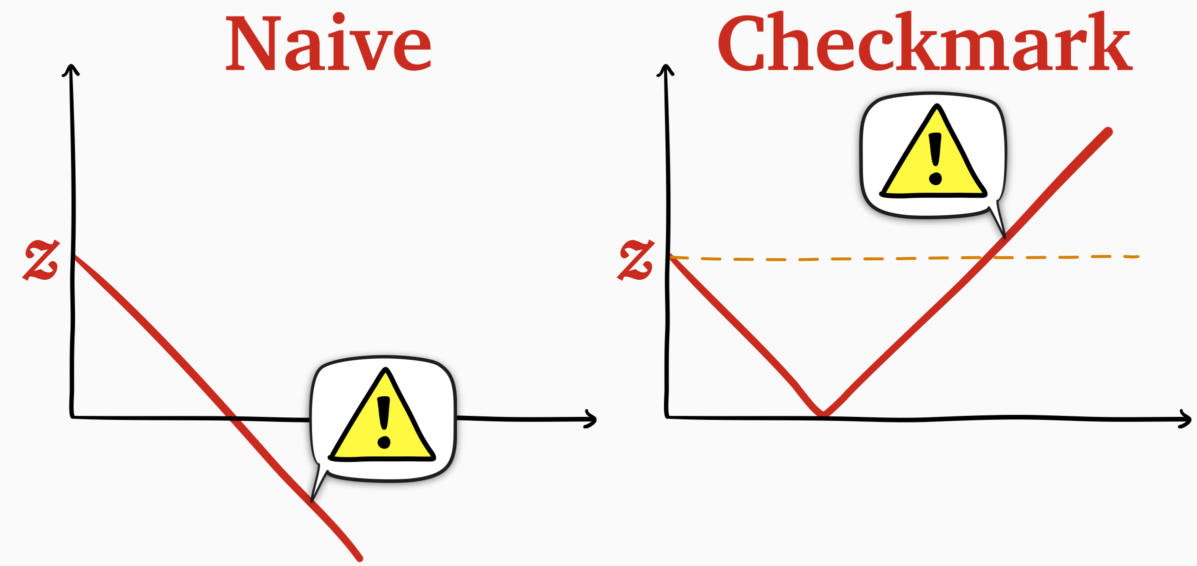
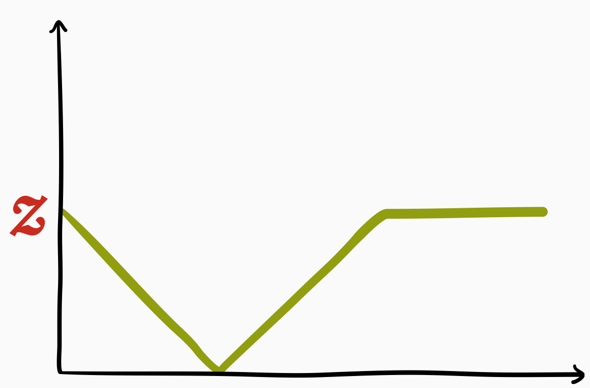


Solution: new policy, Radical, with provably bounded $\mathbf{E}[T]$



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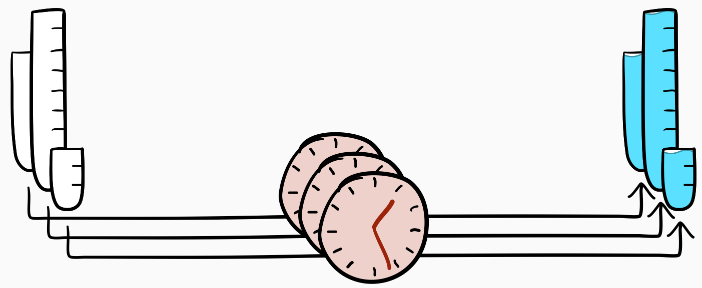

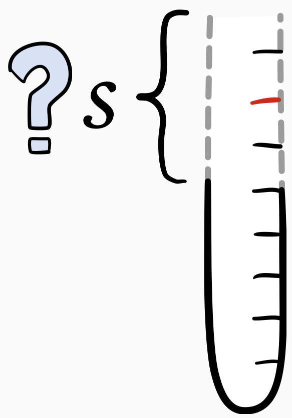

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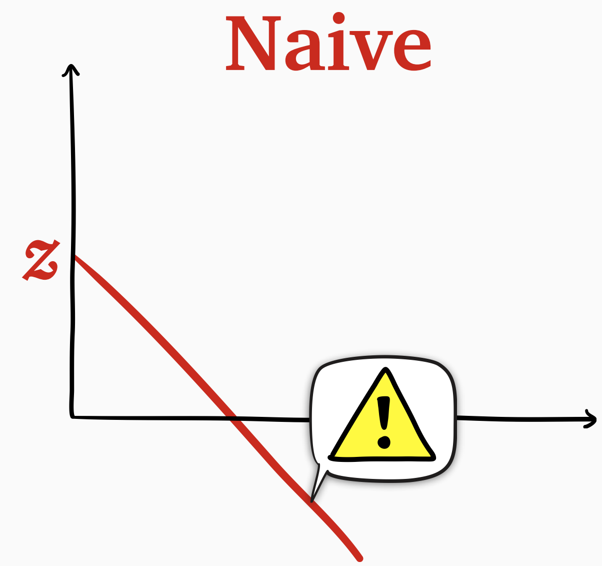
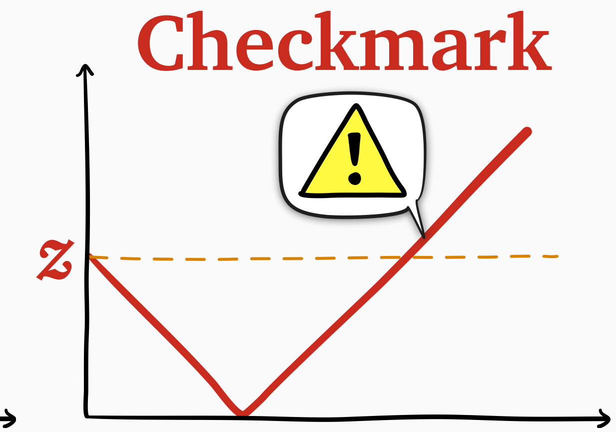
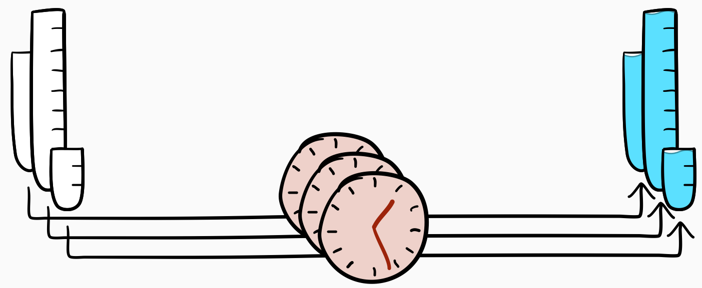


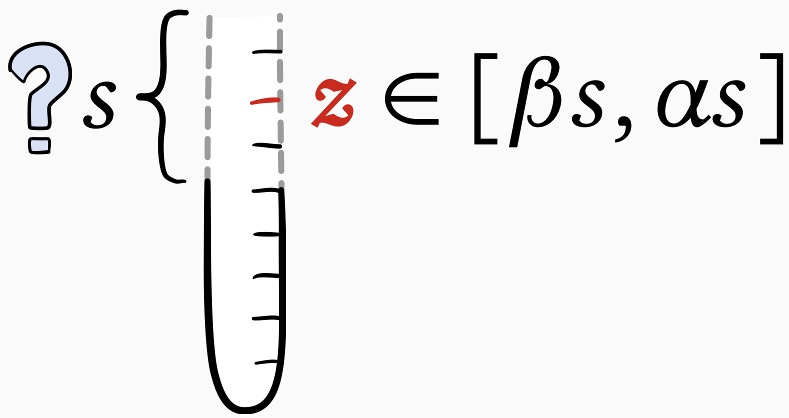


Solution: new policy, **Radical**, with provably bounded $\mathbf{E}[T]$

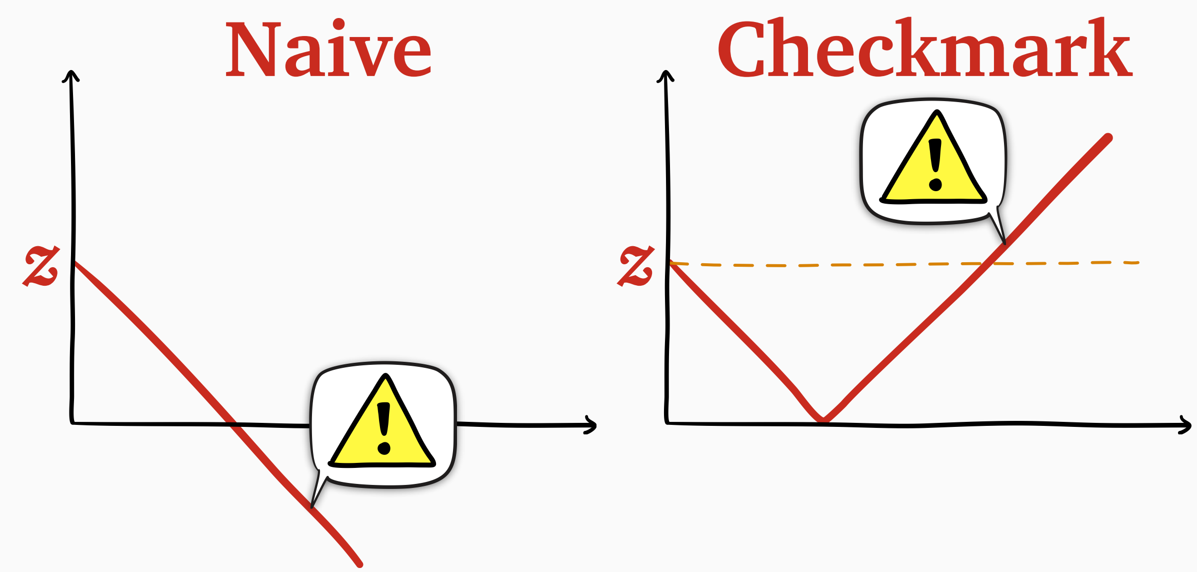
1-consistent,
3.5-graceful

Problem: minimize $\mathbf{E}[T]$ with noisy information





Obstacle: natural rank functions perform badly



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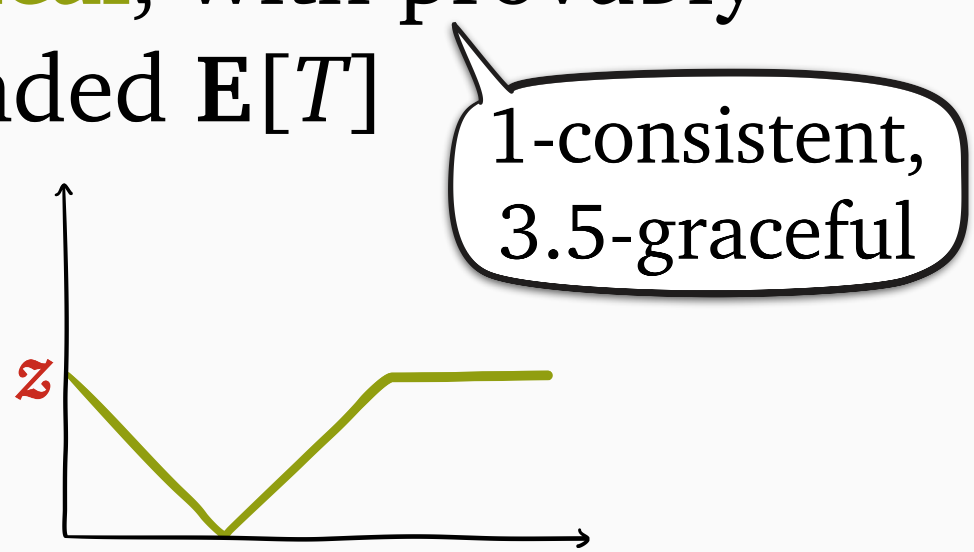

Method: two new tools from queueing theory

Consistency-robustness tradeoff?

✓ C-consistent: $\dfrac{\mathbf{E}[T_{\mathbf{P}}]}{\mathbf{E}[T_{\mathrm{SRPT}}]} \to C$ as $\alpha, \beta \to 1$

✓ G-graceful: $\dfrac{\mathbf{E}[T_{\mathbf{P}}]}{\mathbf{E}[T_{\mathrm{SRPT}}]} \le G \cdot \dfrac{\alpha}{\beta}$ for all α, β

✗ R-robust: $\dfrac{\mathbf{E}[T_{\mathbf{P}}]}{\mathbf{E}[T_{\mathrm{SRPT}}]} \le R$ for all α, β

Our contribution: *first policy* ***P*** that's consistent and graceful

- $G = 3.5$
- $C = 1$

Consistency-robustness tradeoff?

✓ C-consistent: $$\frac{\mathbf{E}[T_P]}{\mathbf{E}[T_{\text{SRPT}}]} \to C \quad \text{as } \alpha, \beta \to 1$$

✓ G-graceful: $$\frac{\mathbf{E}[T_P]}{\mathbf{E}[T_{\text{SRPT}}]} \le G \cdot \frac{\alpha}{\beta} \quad \text{for all } \alpha, \beta$$

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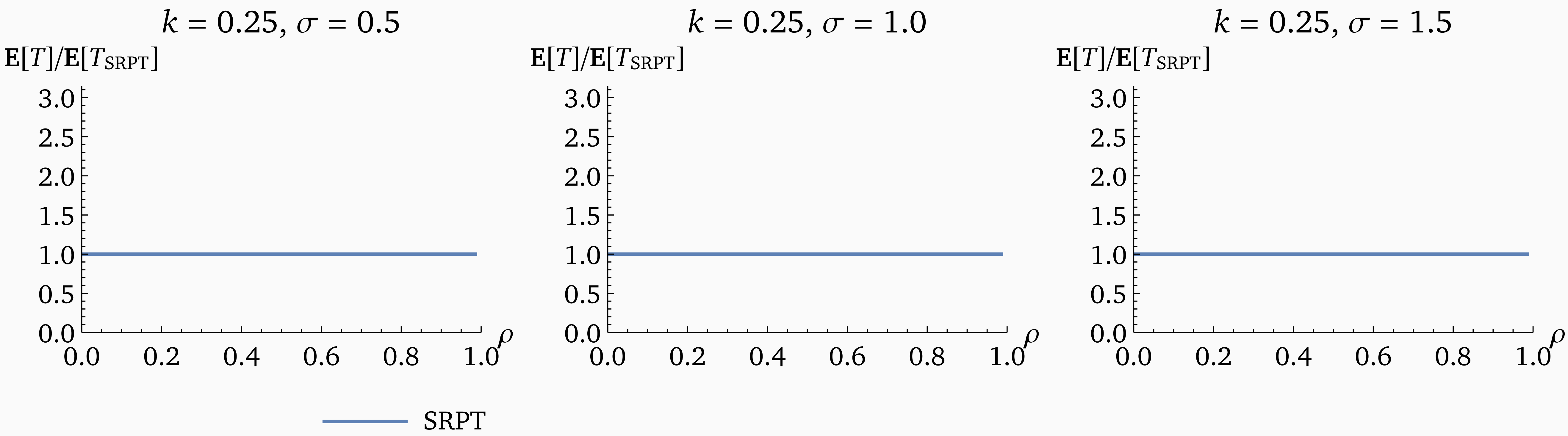
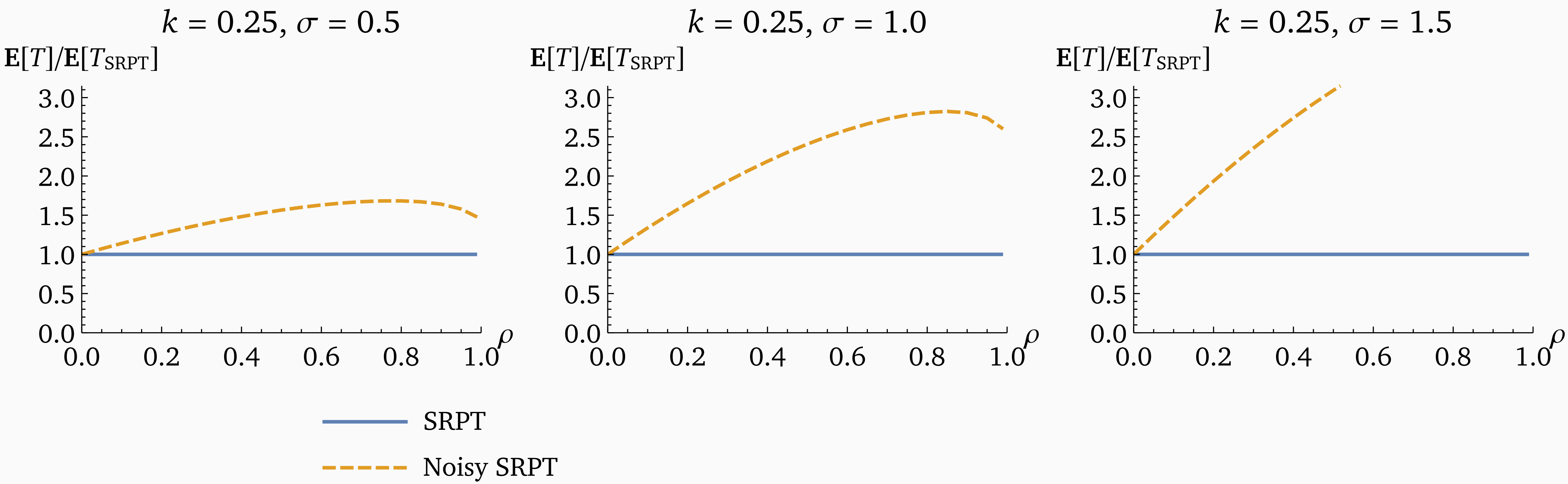
Our contribut ... P

that's consisten ...

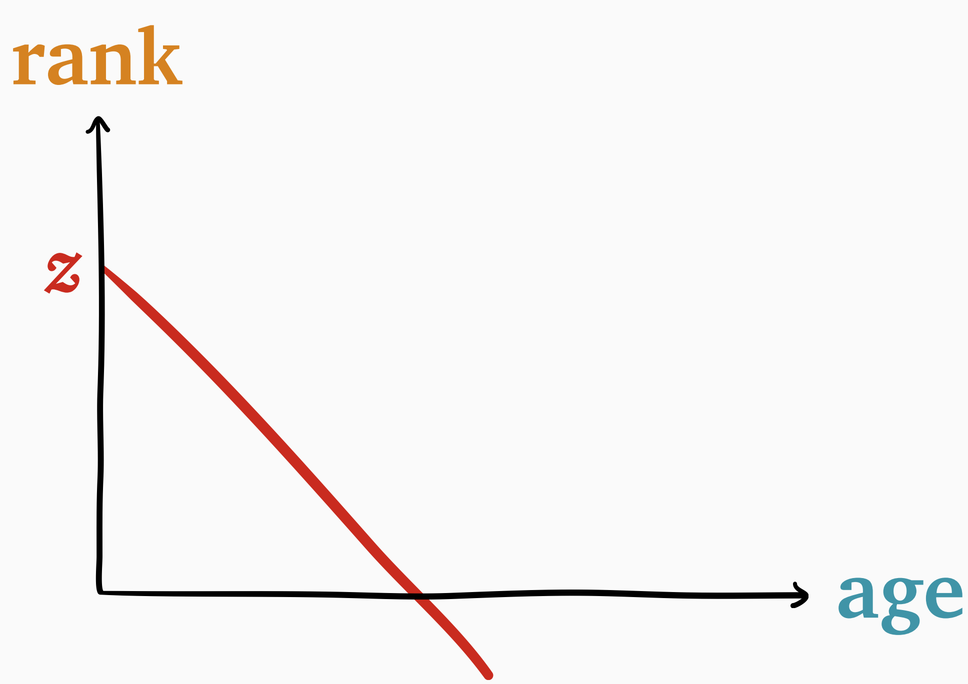
- $G = 3.5$
- $C = 1$

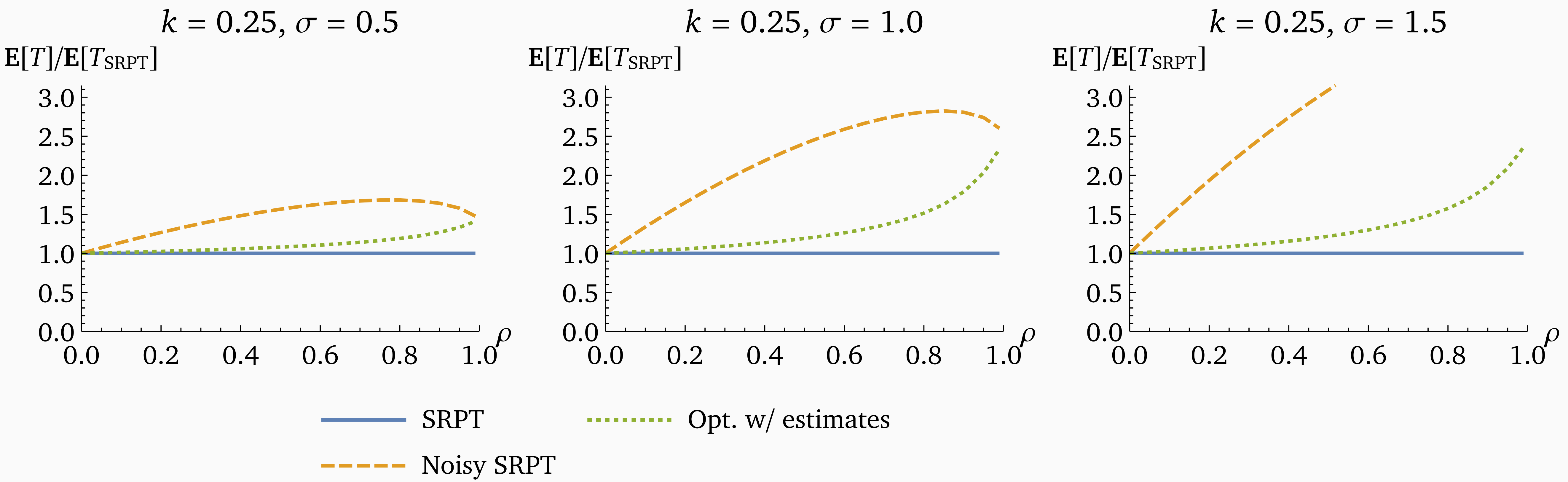


k = 0.25, σ = 0.5
k = 0.25, σ = 1.0
k = 0.25, σ = 1.5
E[T]/E[T_SRPT]
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
ρ
SRPT

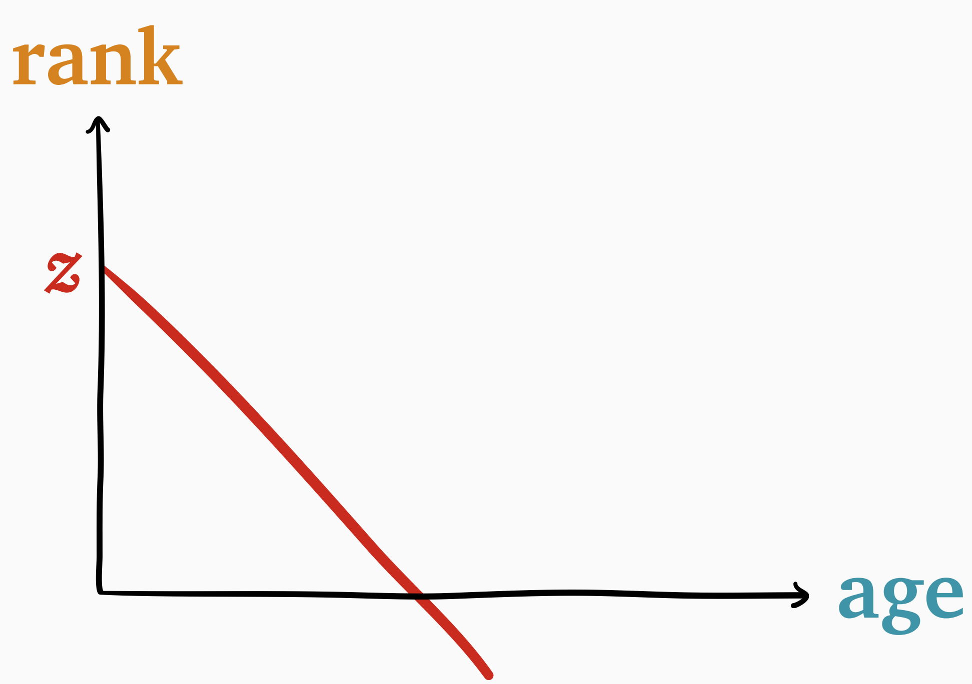


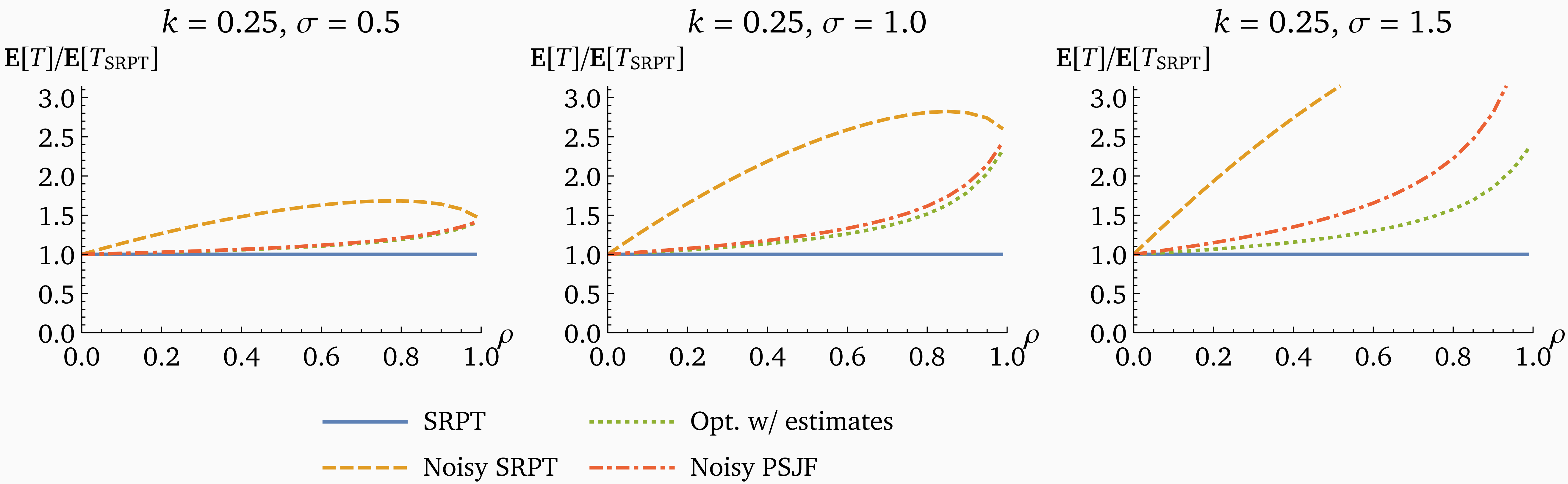
Noisy SRPT (Naive)





Noisy SRPT (Naive)





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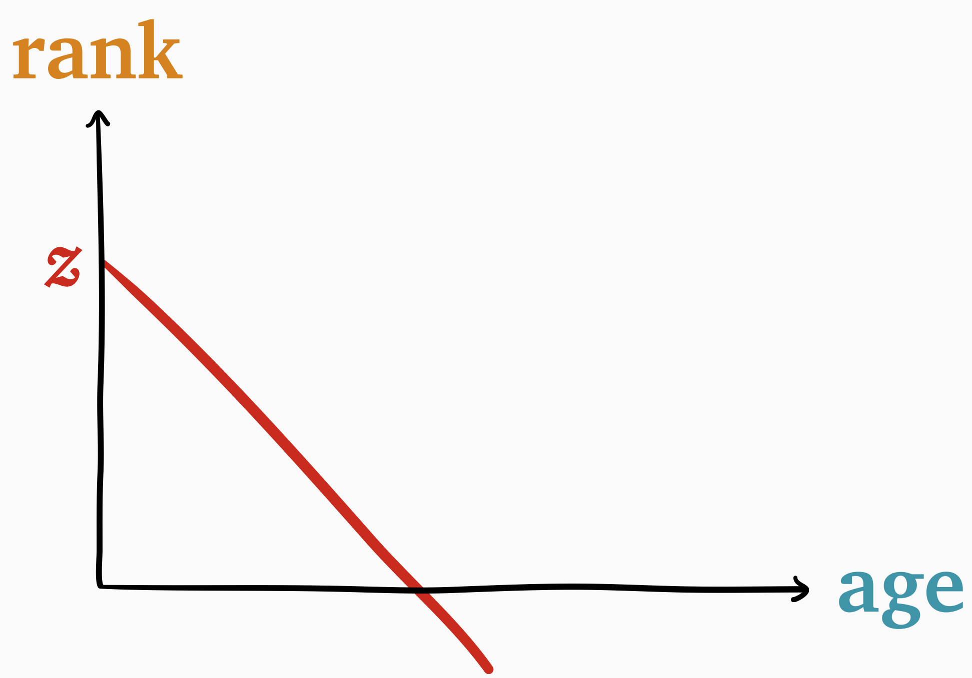
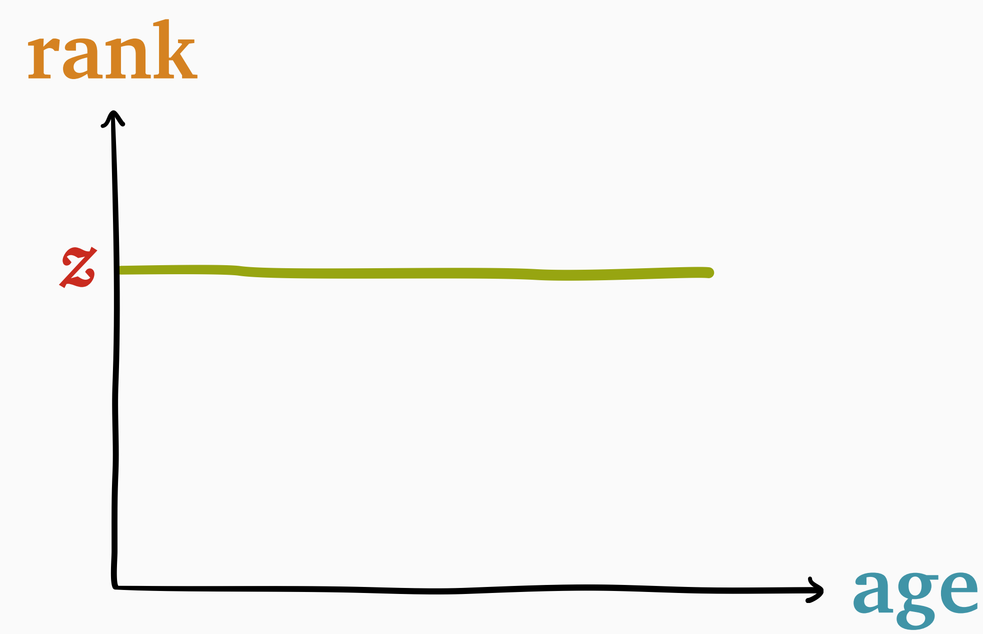
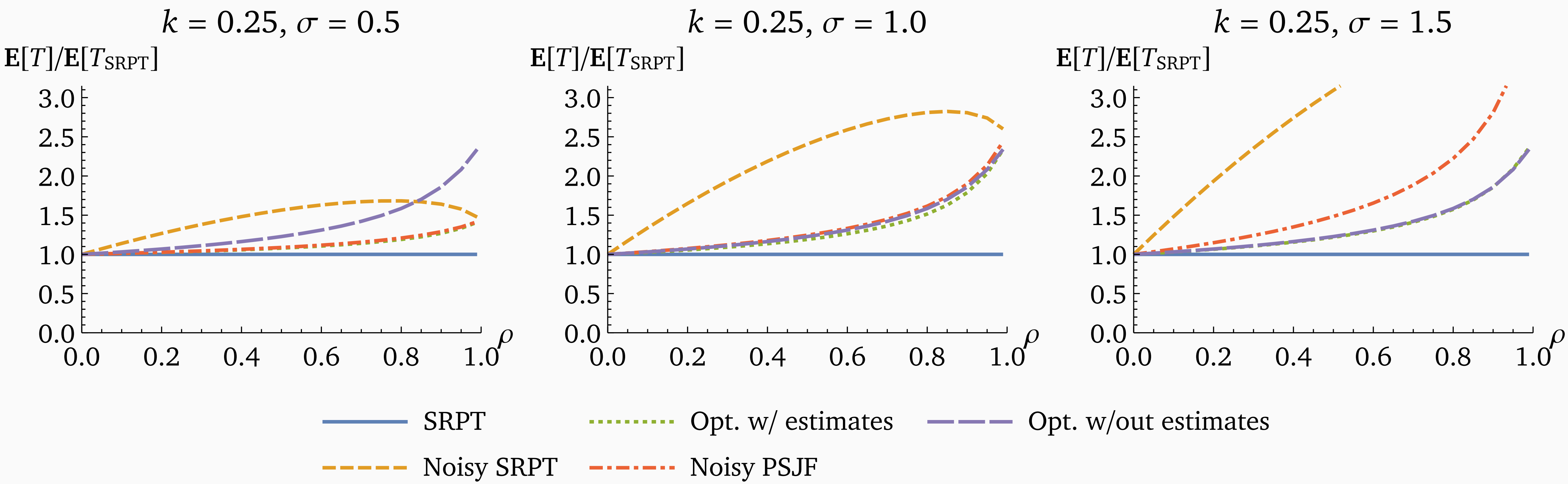
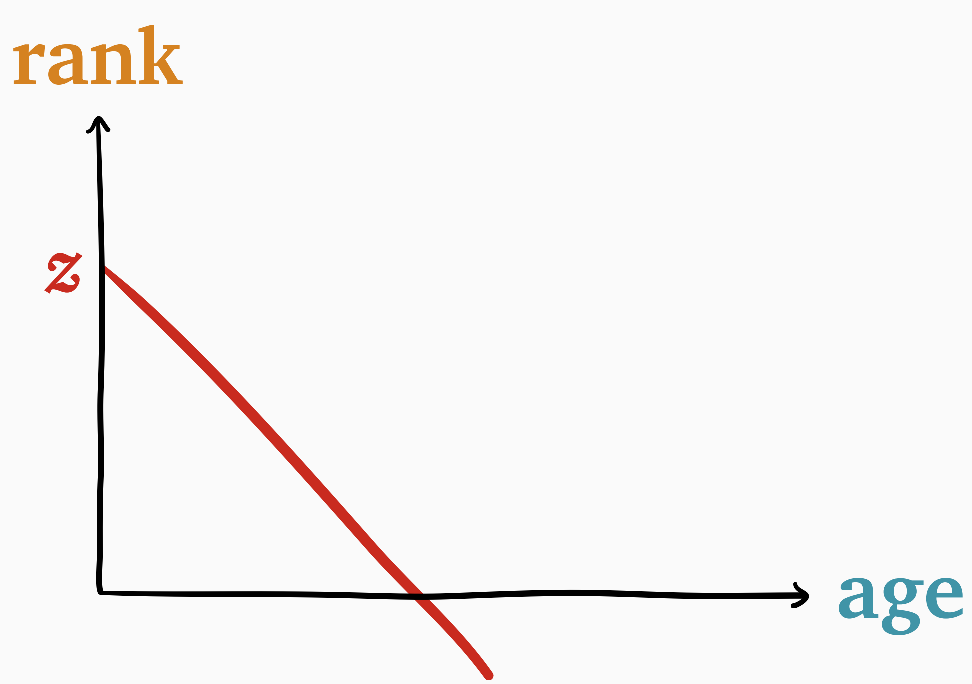


Noisy PSJF (1.5-graceful)

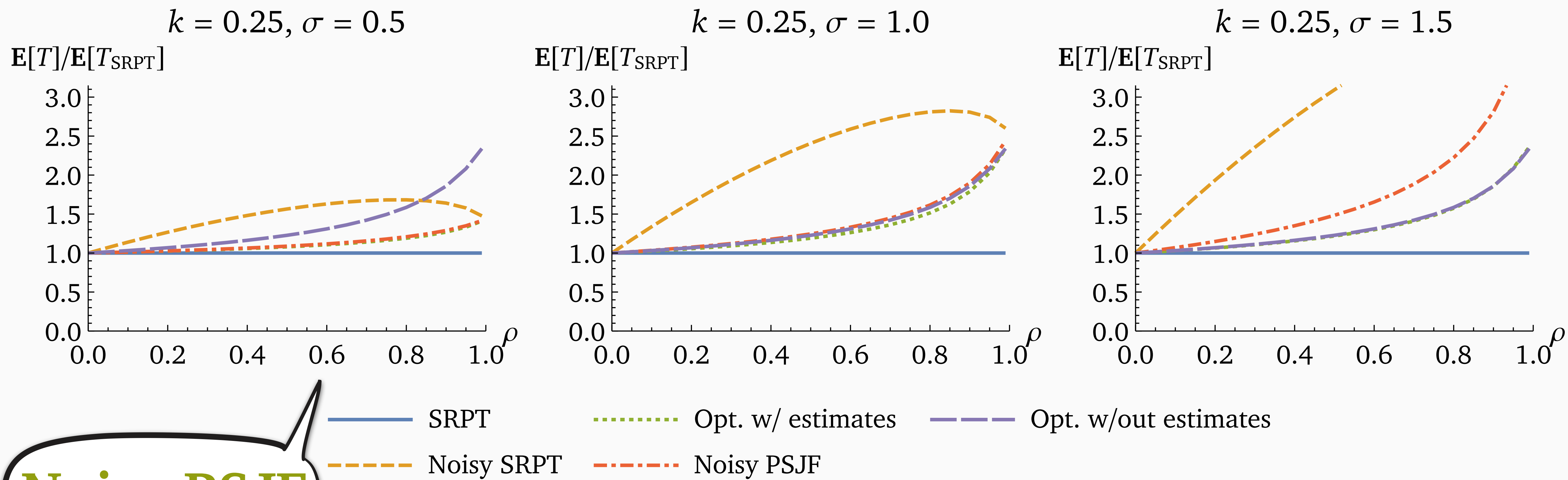




Noisy SRPT (Naive)

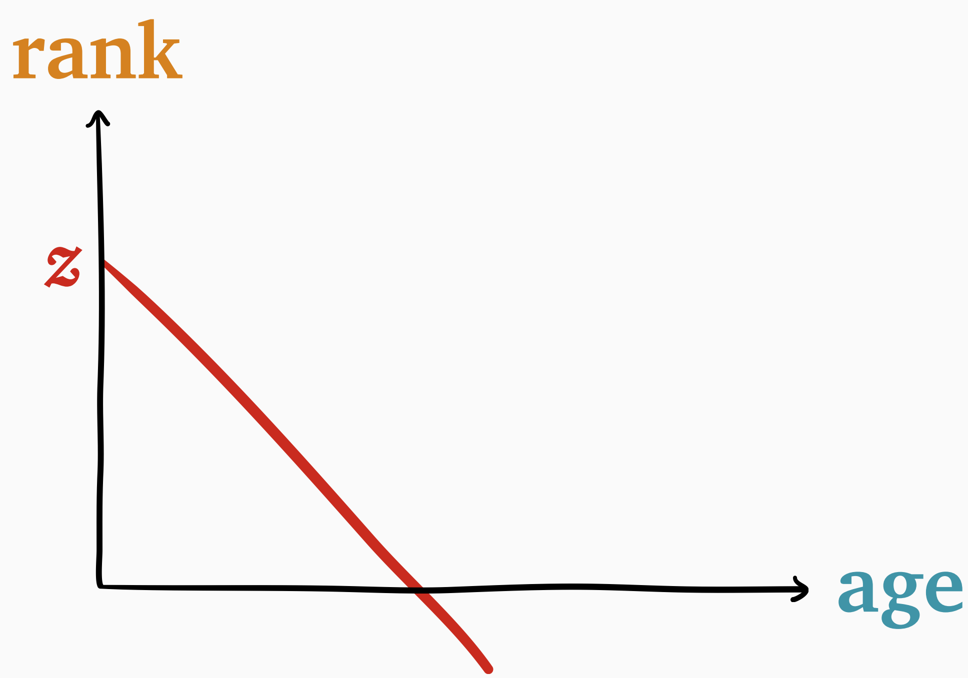


Noisy PSJF (1.5-graceful)

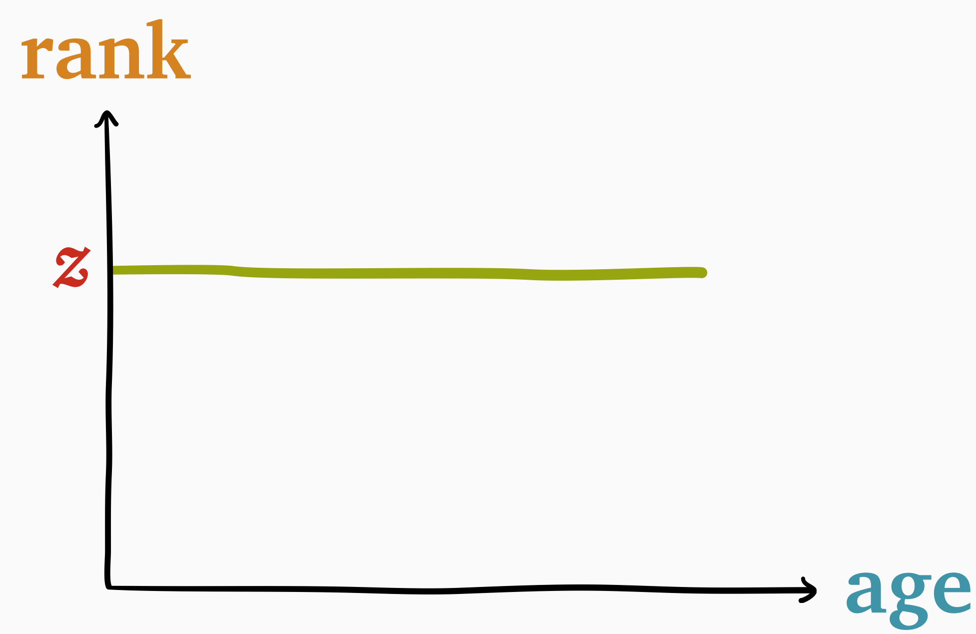


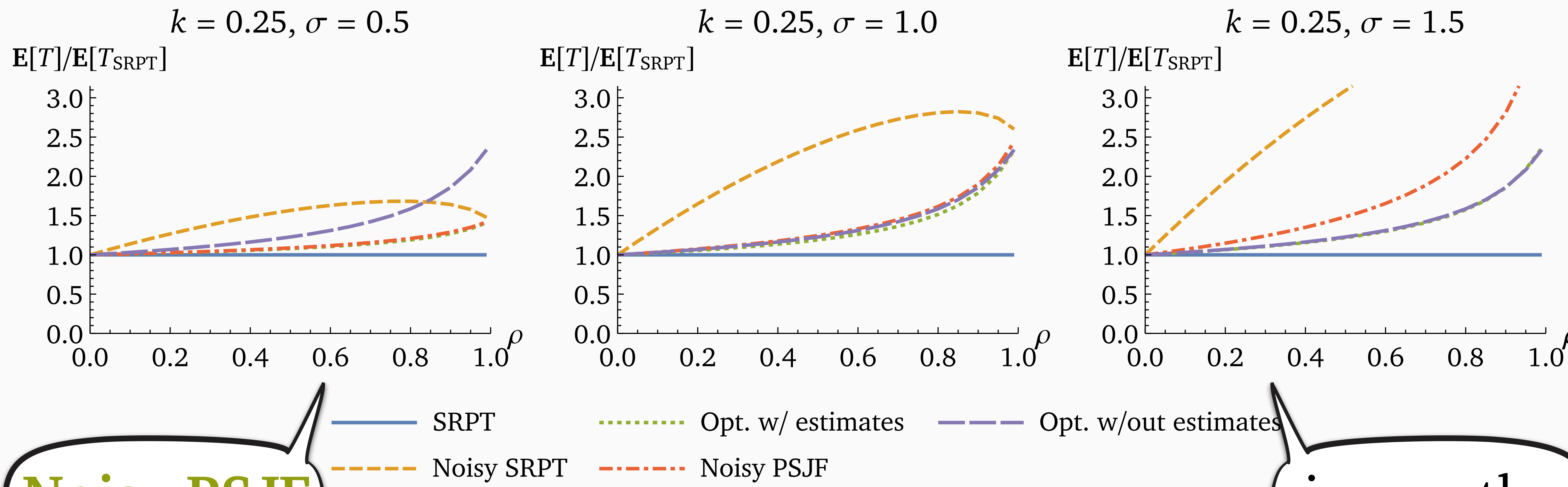
Noisy PSJF suffices

Noisy SRPT (Naive)



Noisy PSJF (1.5-graceful)

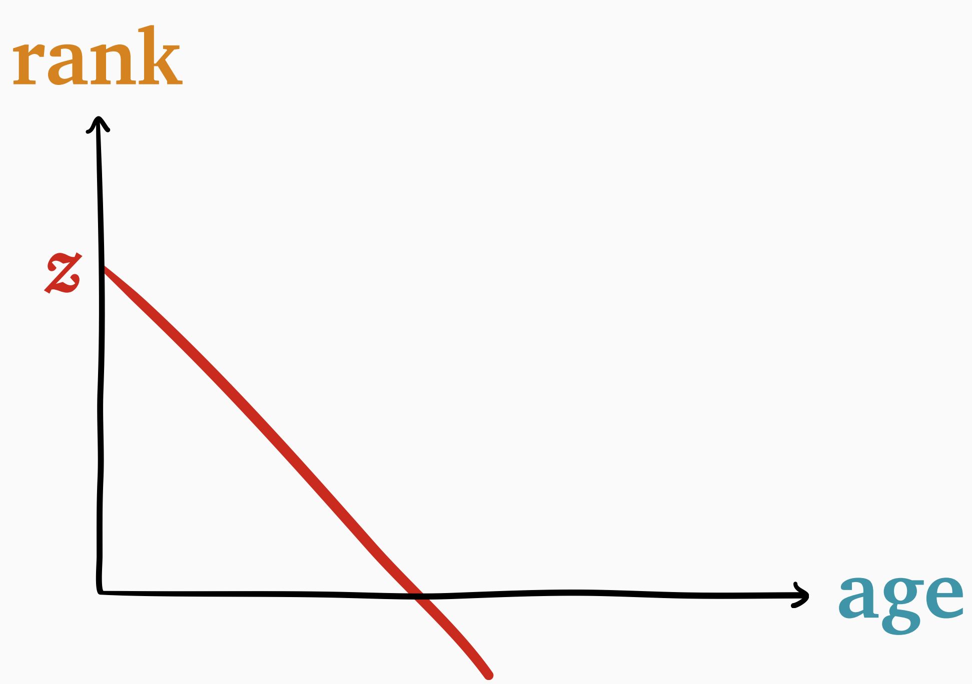




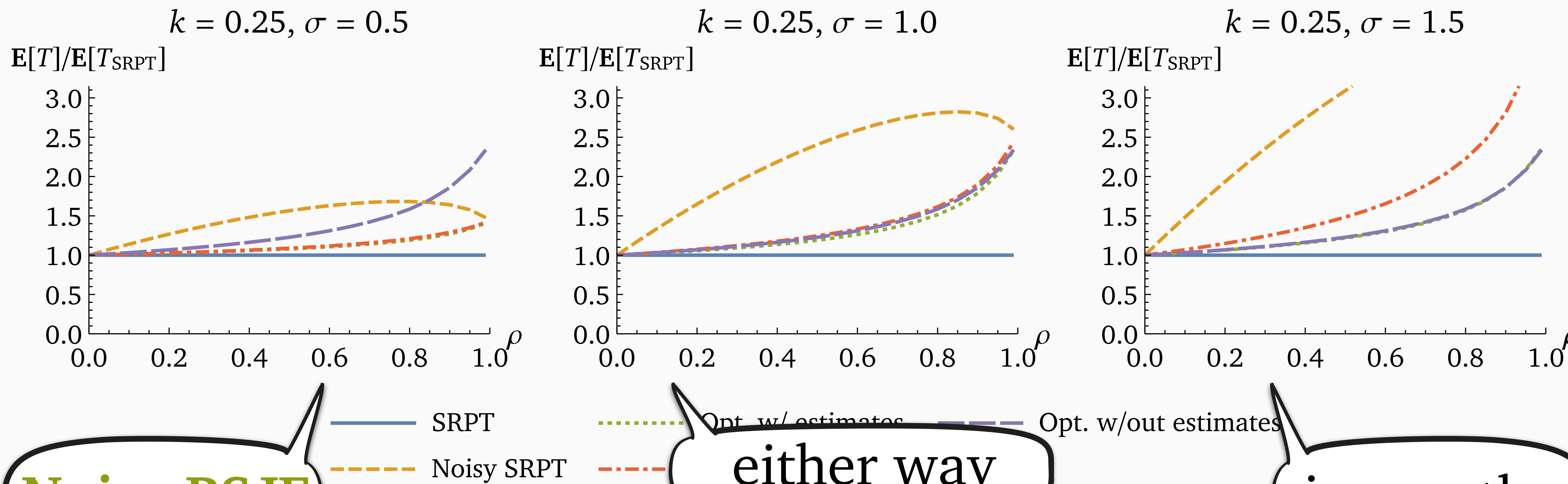
Noisy PSJF suffices

ignore the estimates

Noisy SRPT (Naive)



Noisy PSJF (1.5-graceful)

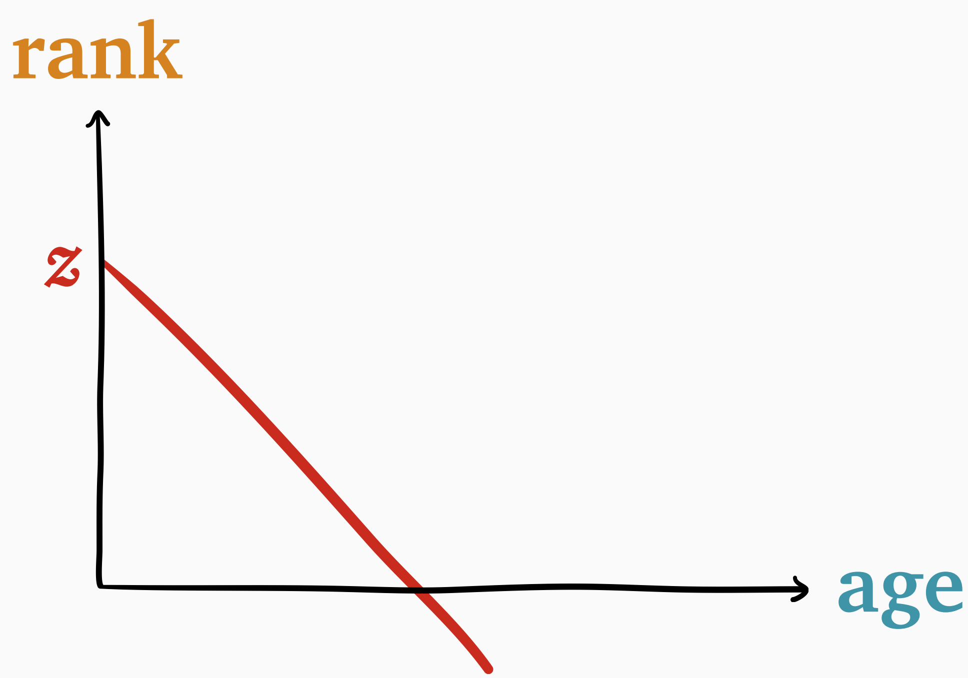


Noisy PSJF suffices

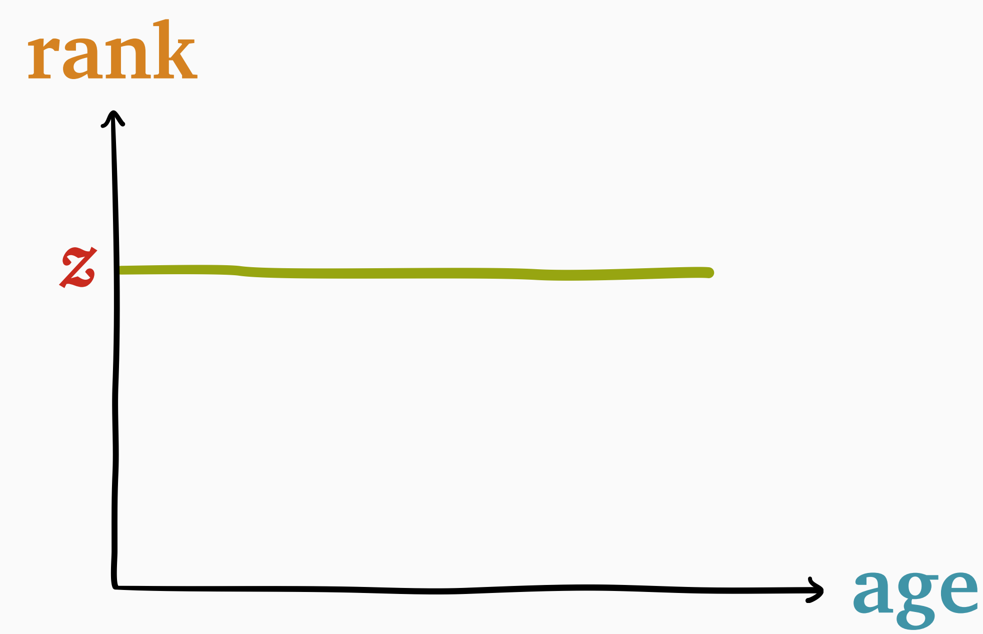
either way

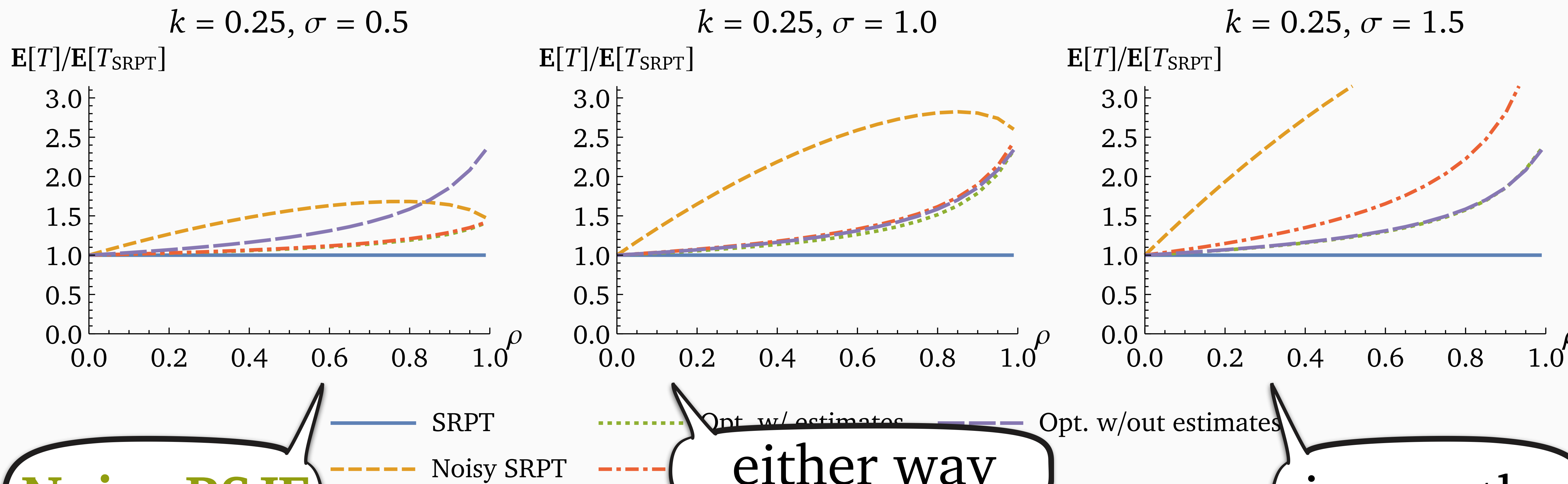
ignore the estimates

Noisy SRPT (Naive)



Noisy PSJF (1.5-graceful)





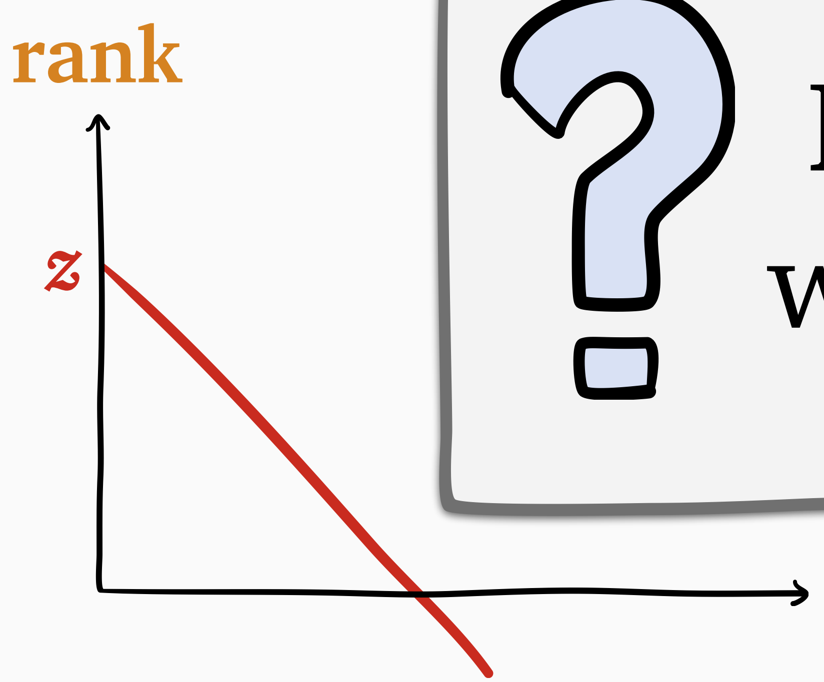
Noisy PSJF suffices

either way

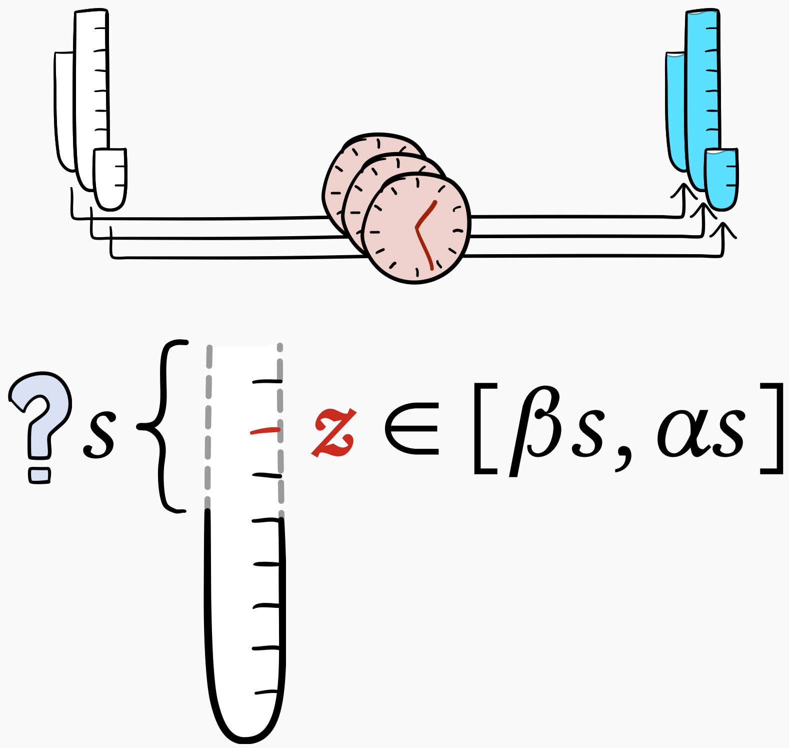
ignore the estimates

Noisy SR … .5-graceful)

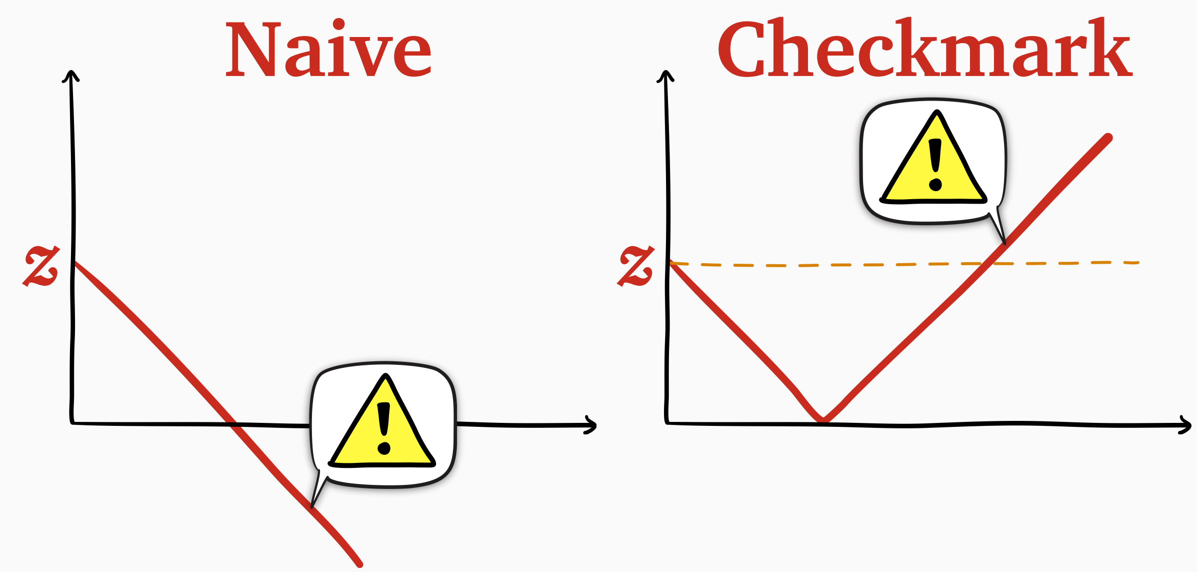
Mix Noisy PSJF with Opt. w/out?



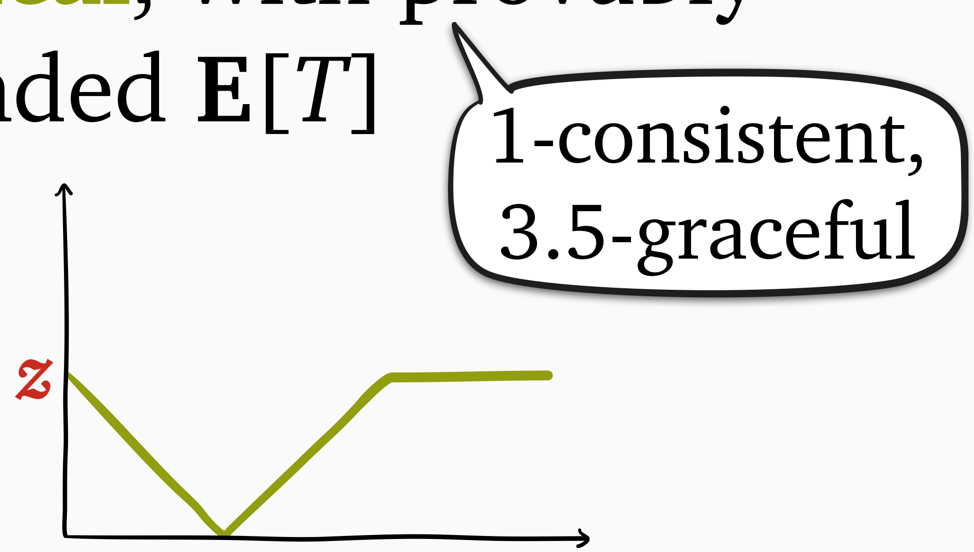
Problem: minimize $\mathbf{E}[T]$ with noisy information



Obstacle: natural rank functions perform badly



Solution: new policy, Radical, with provably bounded $\mathbf{E}[T]$



Method: two new tools from queueing theory

SOAP
& WINE

Proof sketch

Lemma:

$$\mathbf{E}[W_{\mathbf{Scale}}(\boldsymbol{r})] \le \mathbf{E}\left[W_{\mathbf{SRPT}}\left(\frac{\alpha}{\beta}\boldsymbol{r}\right)\right]$$

Proof sketch

Lemma:

$$\mathbf{E}[W_{\mathrm{SRPT}}(r)] \le \mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\frac{\alpha}{\beta}r\right)\right]$$

Proof sketch

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Key steps:

Proof sketch

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Key steps:

1. **SRPT** minimizes mean $\boldsymbol{r}$-work

Proof sketch

Lemma:

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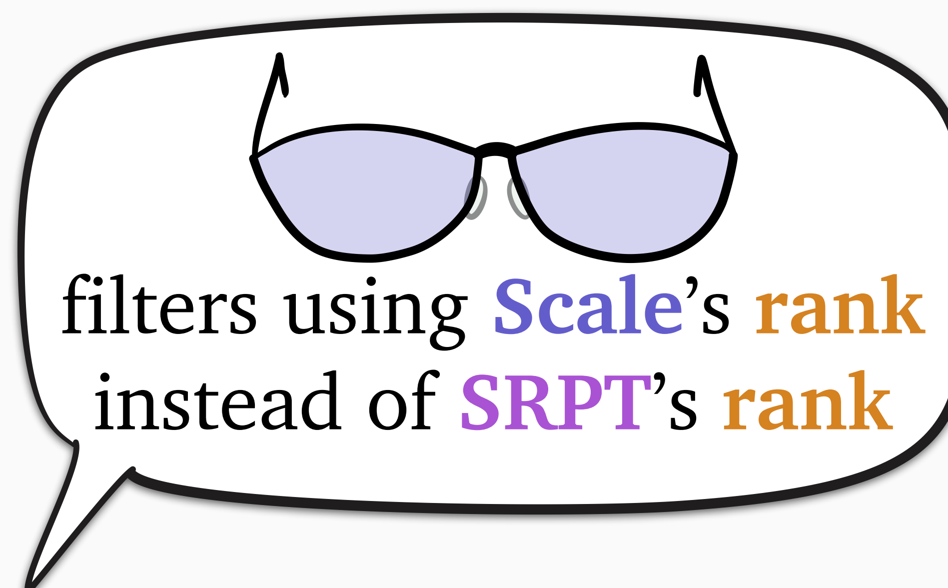
Key steps:

1. **SRPT** minimizes mean r-work
2. **Scale** minimizes mean **noise-scaled**-r-work

Proof sketch

Lemma:

$$\mathbf{E}[W_{\mathrm{SRPT}}(r)] \le \mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\frac{\alpha}{\beta} r\right)\right]$$



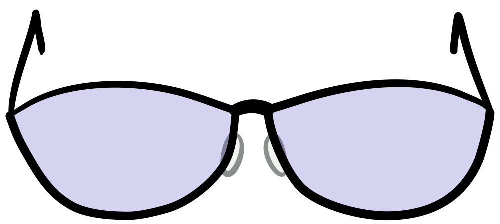
Key steps:

1. **SRPT** minimizes mean r-work
2. **Scale** minimizes mean **noise-scaled**-r-work

Proof sketch

Lemma:

$$\mathbf{E}[W_{\mathrm{SRPT}}(r)] \le \mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\tfrac{\alpha}{\beta} r\right)\right]$$



filters using **Scale**'s **rank** instead of **SRPT**'s **rank**

Key steps:

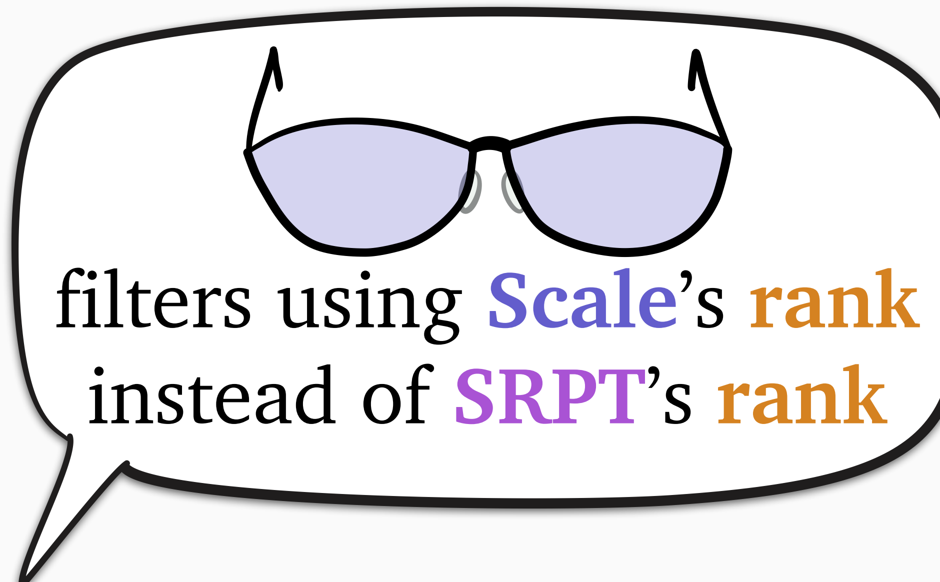
1. **SRPT** minimizes mean r-work
2. **Scale** minimizes mean **noise-scaled**-r-work
3. Under any policy,

Proof sketch

Lemma:

$$\mathbf{E}[W_{\mathrm{SRPT}}(r)] \le \mathbf{E}[W_{\mathrm{Scale}}(r)] \le \mathbf{E}\left[W_{\mathrm{SRPT}}\left(\frac{\alpha}{\beta} r\right)\right]$$

Key steps:



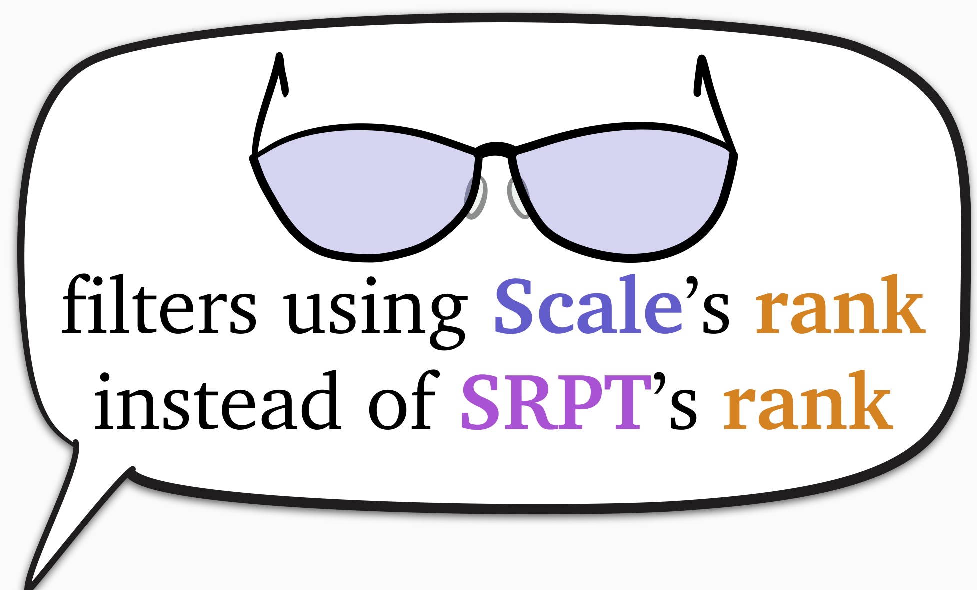
1. SRPT minimizes mean r-work
2. Scale minimizes mean **noise-scaled**-r-work
3. Under any policy,

   r-work ≤ **noise-scaled**-αr-work ≤ $\frac{\alpha}{\beta} r$-work

Proof sketch

Lemma:


$$\mathbf{E}[W_{\text{SRPT}}(r)] \le \mathbf{E}[W_{\text{Scale}}(r)] \le \mathbf{E}\left[W_{\text{SRPT}}\left(\tfrac{\alpha}{\beta} r\right)\right]$$



Key steps:

1. SRPT minimizes mean r-work
2. Scale minimizes mean noise-scaled-r-work
3. Under any policy,

   r-work $\le$ noise-scaled-αr-work $\le \frac{\alpha}{\beta} r$-work