A New Toolbox *for* Scheduling Theory

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University of Washington

Kunhe Yang
UC Berkeley

Performance

Performance

efficiency *throughput* *delay* *reliability* *safety* *value*

Performance

efficiency *throughput* *delay* *reliability* *safety* *value*

Contention

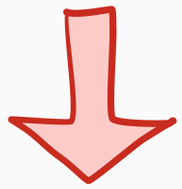
Queueing

Delay

healthcare

Contention

Queueing



Delay

supply chains

healthcare

Contention

Queueing

Delay

your local supermarket

supply chains

healthcare

Contention

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supply chains **Contention** call centers

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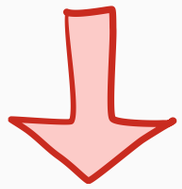


Delay

your local supermarket

supply chains **Contention** call centers

healthcare **Queueing** transportation

Delay

your local supermarket

supply chains

Contention

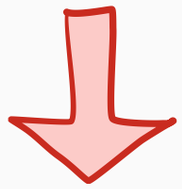
call centers

healthcare

transportation

Queueing

databases



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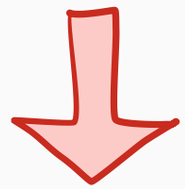
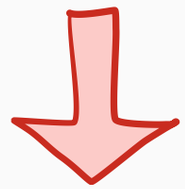


transportation

Queueing



databases

Delay

networks

your local supermarket

supply chains **Contention** call centers

healthcare transportation

Queueing

databases

Delay networks

operating systems

your local supermarket

supply chains **Contention** call centers

healthcare **Queueing** transportation

databases

computer architecture **Delay** networks

operating systems

your local supermarket

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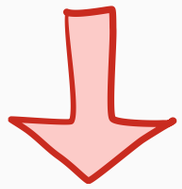

supercomputing databases

computer architecture **Delay** networks

operating systems

your local supermarket

supply chains **Contention** call centers

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supercomputing databases

computer architecture **Delay** networks

operating systems

How to reduce delays?

your local supermarket

supply chains **Contention** call centers

healthcare **Queueing** transportation

supercomputing databases

computer architecture **Delay** networks

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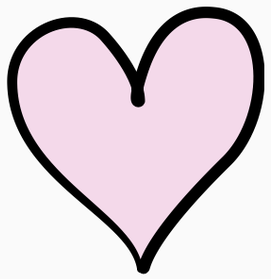
How to reduce delays?

Scheduling

Good news:

scheduling can reduce delay



Good news:

scheduling can reduce delay

Bad news:

limited understanding of scheduling

Good news:

scheduling can reduce delay

Bad news:

limited understanding of scheduling

evaluation

Good news:

scheduling can reduce delay

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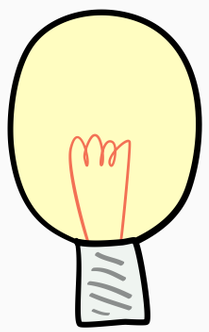


We need:

rigorous theory of scheduling

Good news:

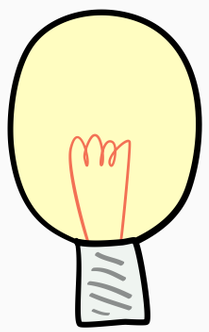
scheduling can reduce delay

Bad news:

limited understanding of scheduling

design

evaluation



We need:

rigorous theory of scheduling

CS, EE, OR, OM

Good news:

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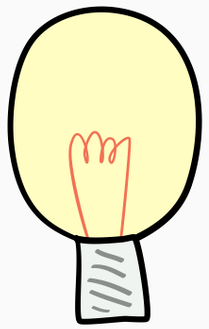

We need:

rigorous theory of scheduling

applied math, probability, control, learning theory

CS, EE, OR, OM

Theory lags behind practice

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Uncertainty:
unknown job sizes, noisy estimates

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Multiple servers:
even FCFS hard to understand

Theory lags behind practice

Uncertainty:
unknown job sizes, noisy estimates

Multiple servers:
even FCFS hard to understand

Preemption practicalities:
limitations, overhead

Theory lags behind practice

This talk:
new theoretical tools

Uncertainty:
unknown job sizes, noisy estimates

Multiple servers:
even FCFS hard to understand

Preemption practicalities:
limitations, overhead

Outline: two new tools

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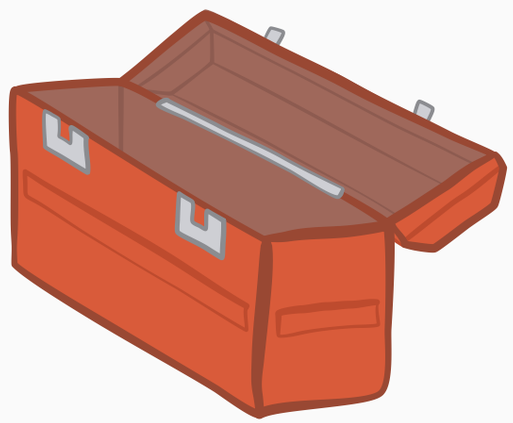

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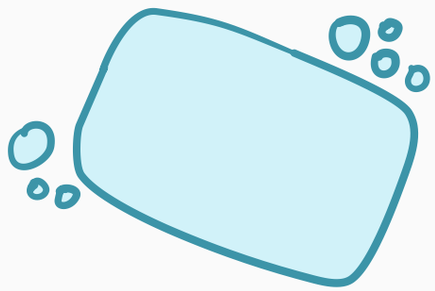
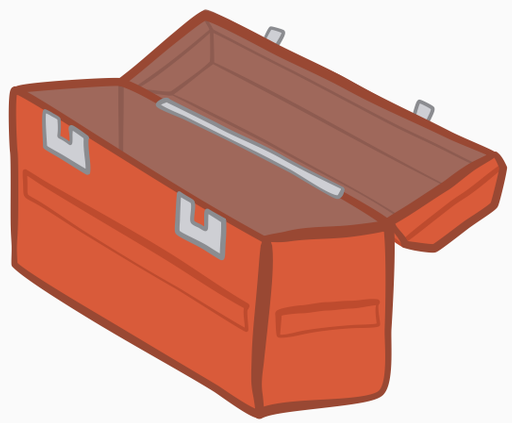


SOAP

new unifying theory of
single-server scheduling



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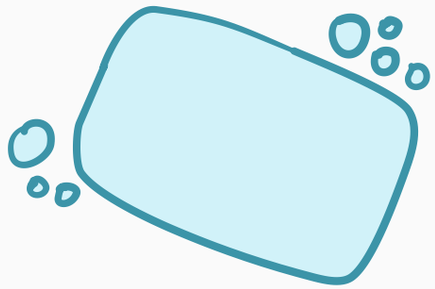
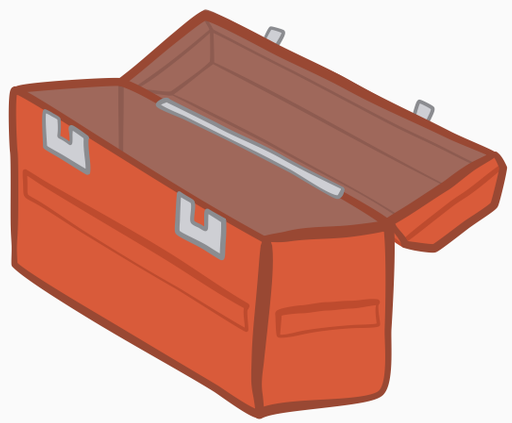


SOAP

greatly increases **number of policies** we can analyze

new unifying theory of single-server scheduling



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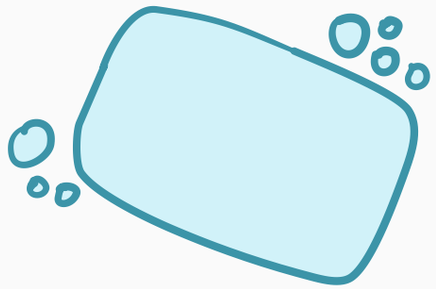
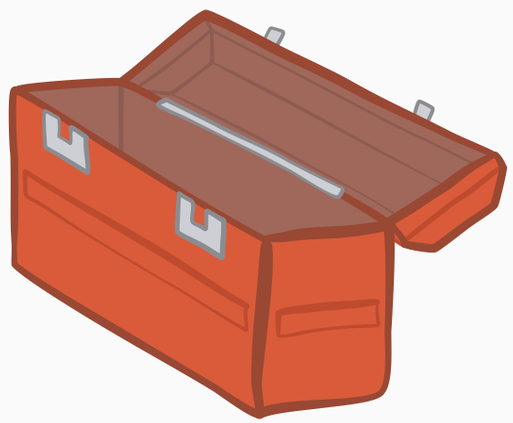


SOAP

new unifying theory of single-server scheduling

greatly increases **number of policies** we can analyze

- unknown sizes
- preemption limitations



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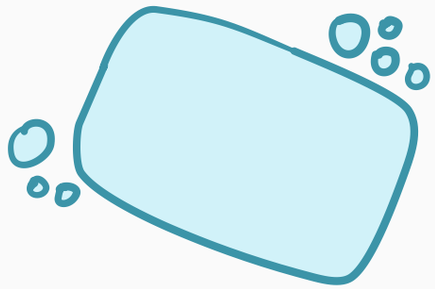
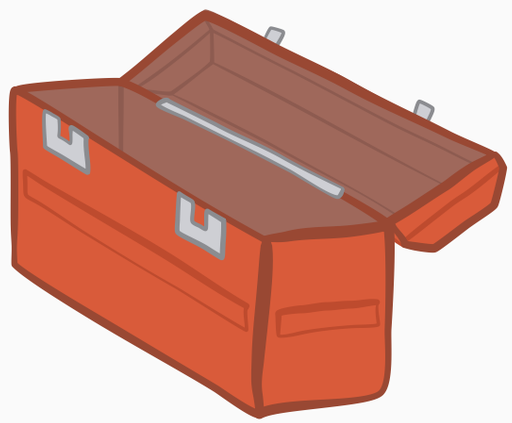


SOAP

new unifying theory of single-server scheduling

unknown sizes

preemption limitations



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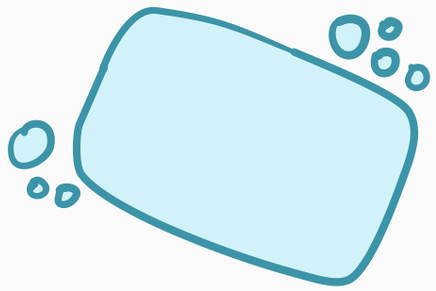


SOAP

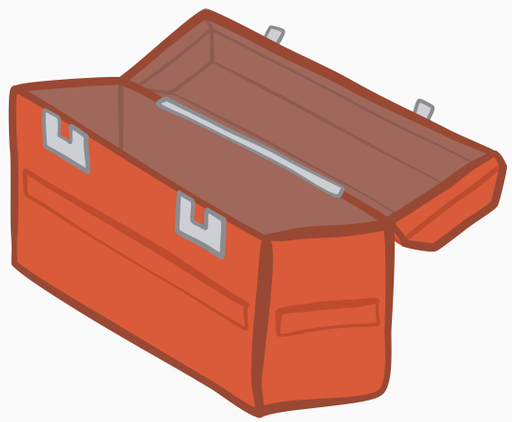
new unifying theory of single-server scheduling

→ unknown sizes

→ preemption limitations

WINE

new queueing identity to complement Little's Law



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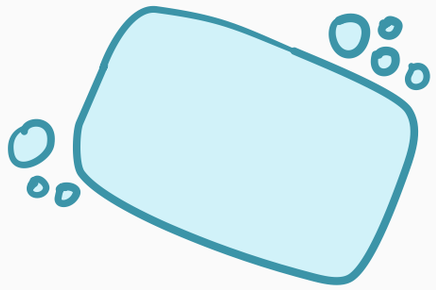

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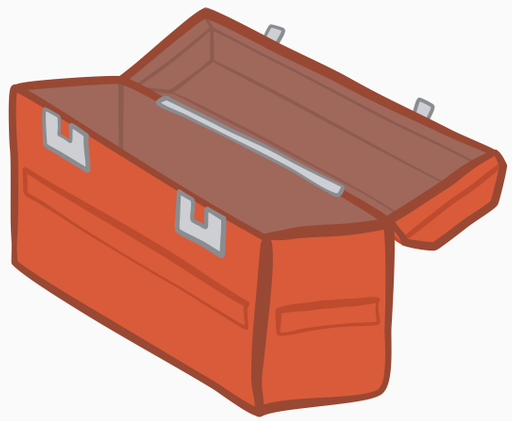
reduces **complex systems** to related **simple systems**



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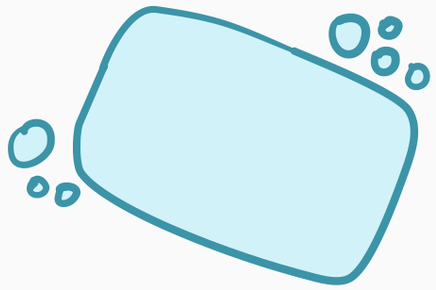

SOAP

new unifying theory of single-server scheduling

⇒ unknown sizes

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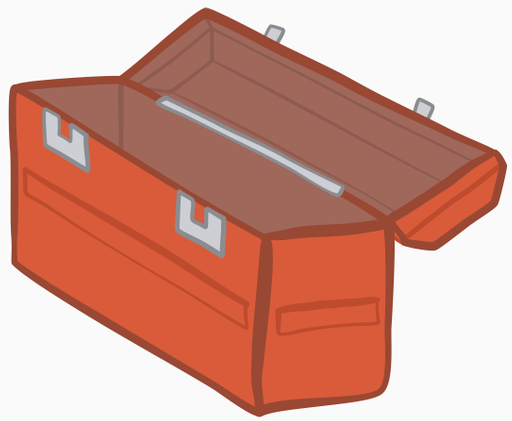
reduces **complex systems** to related **simple systems**

⇒ multiserver systems

⇒ noisy size estimates



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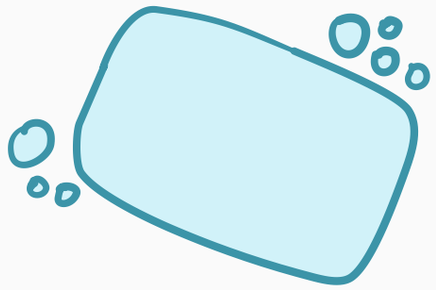

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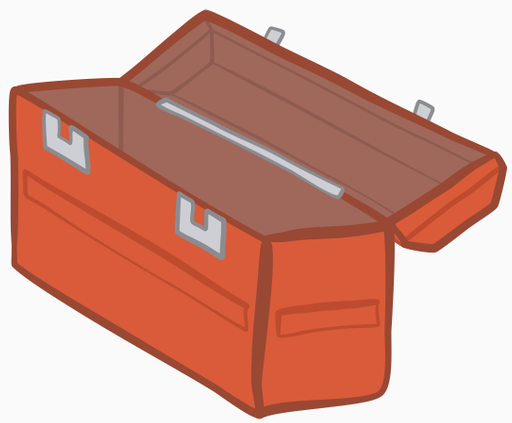
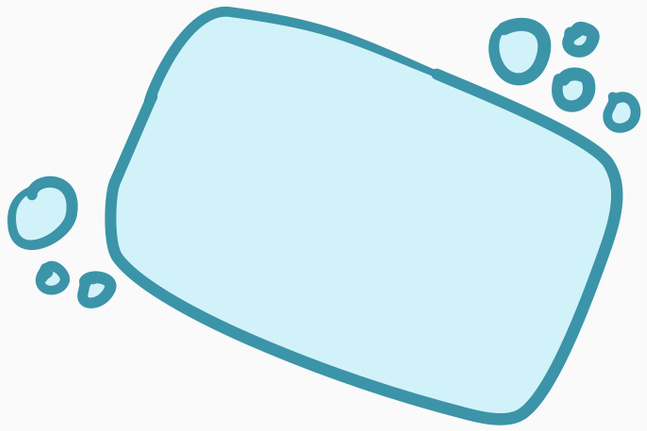
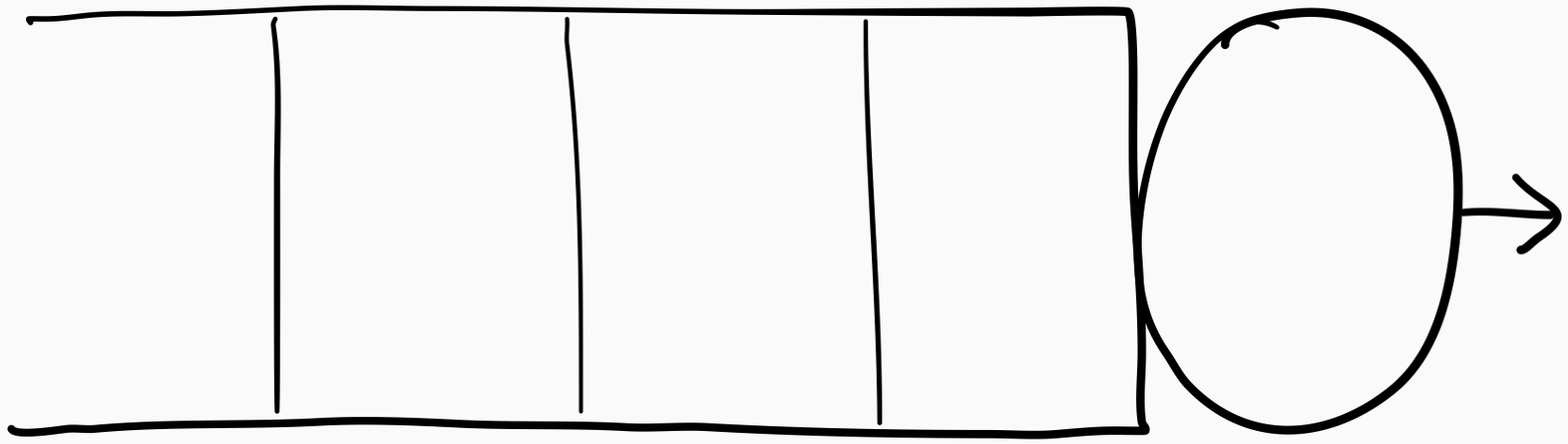




Part 1

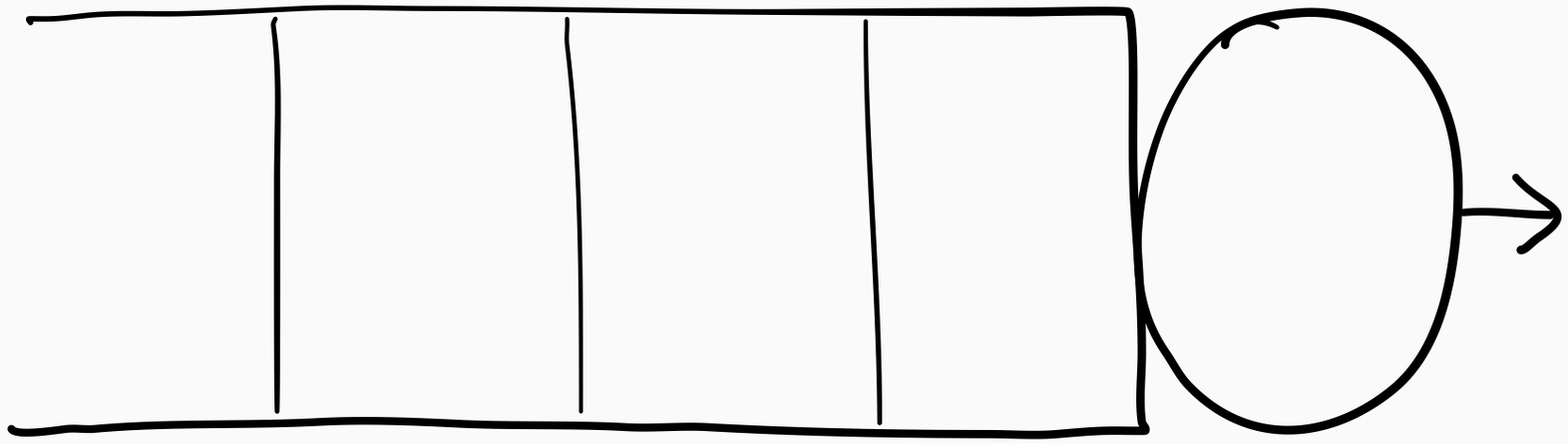
SOAP

Single-server queueing system



Single-server queueing system

M/G/1



Single-server queueing system

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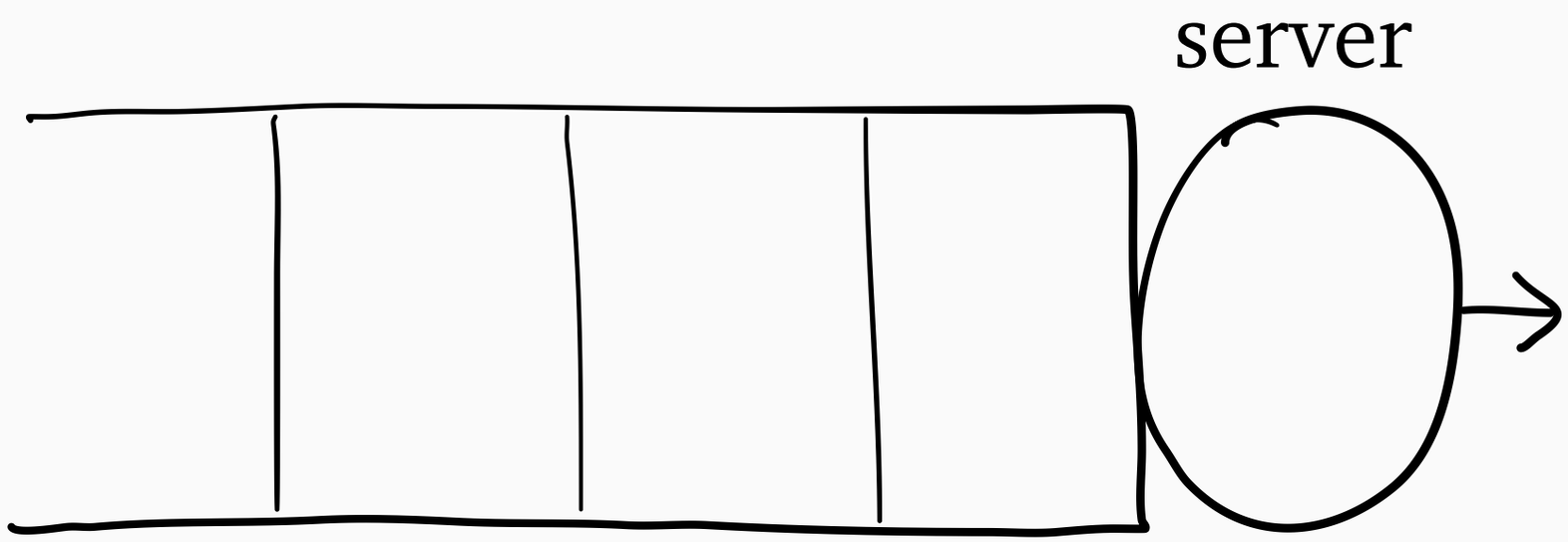

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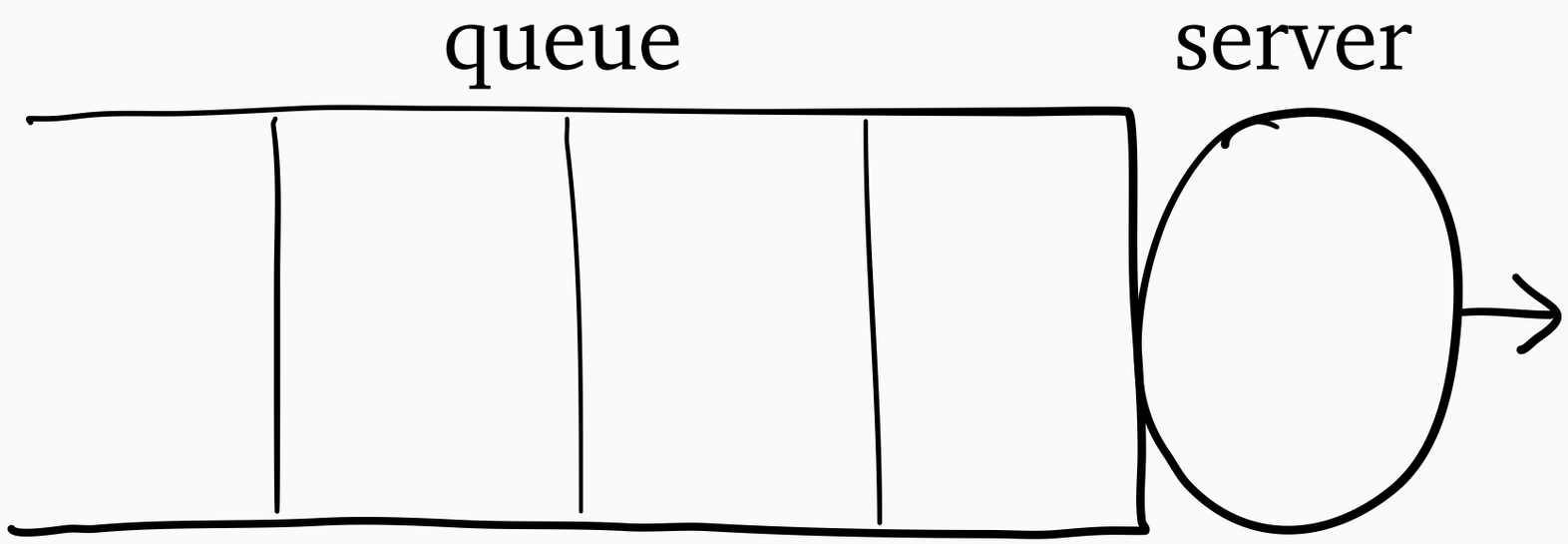

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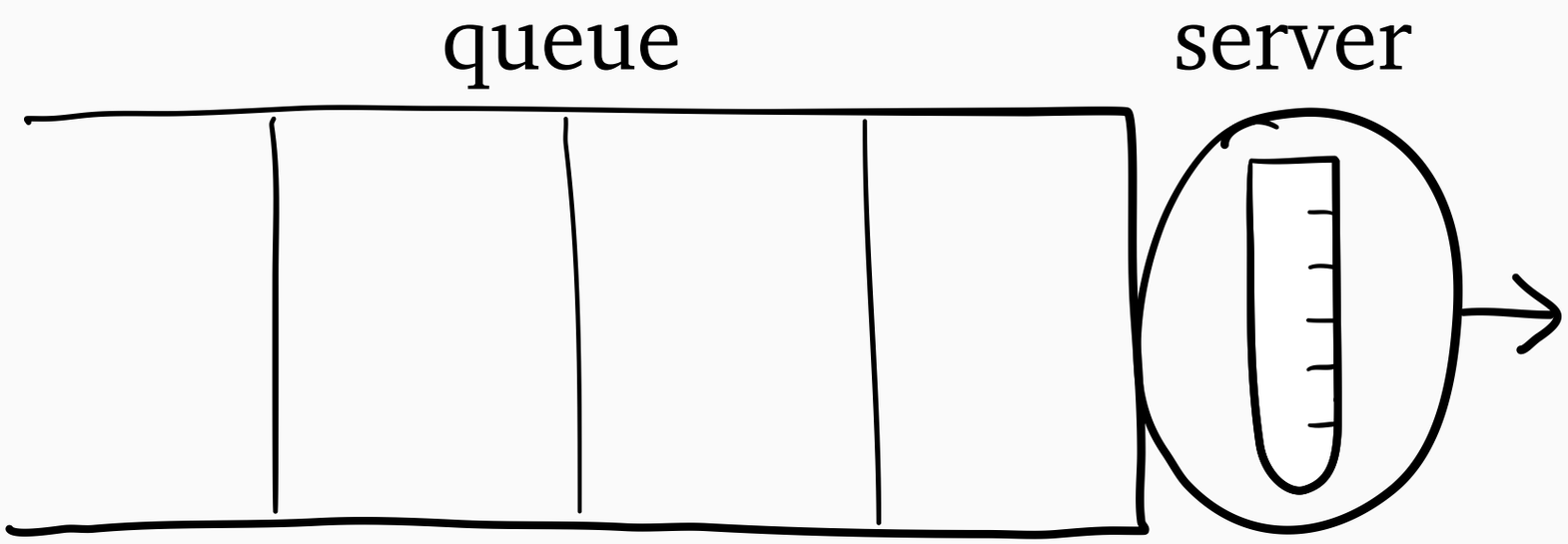
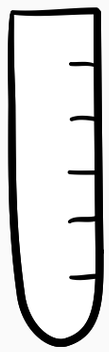


job



Single-server queueing system

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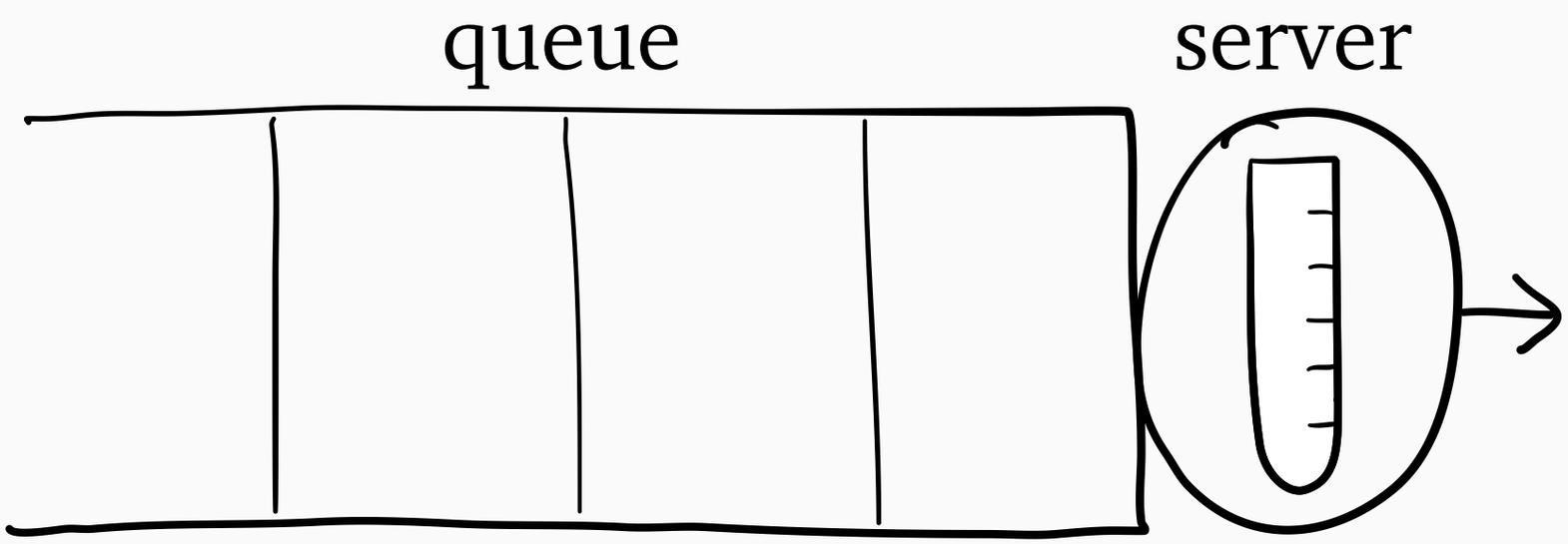

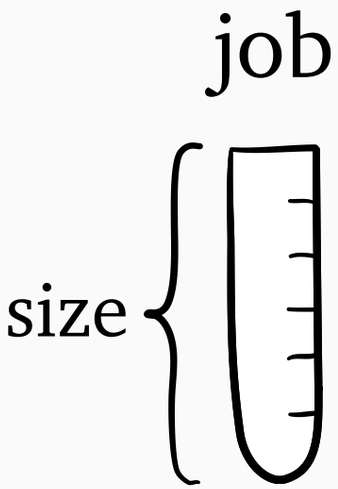

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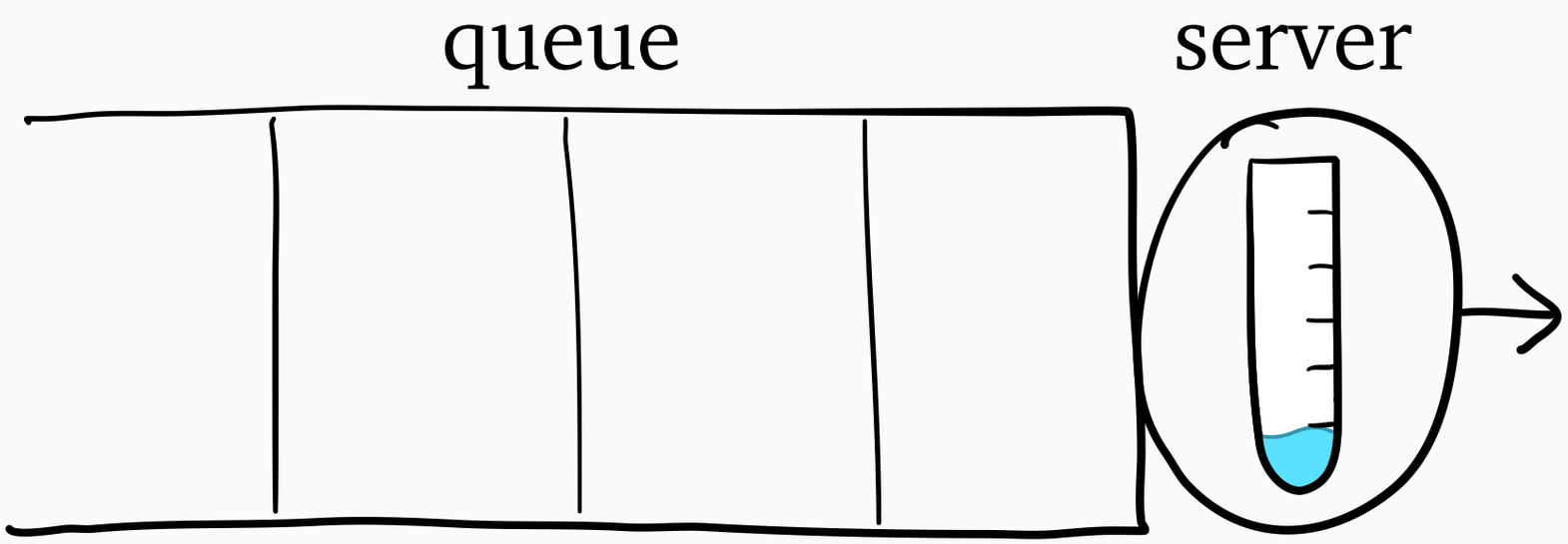

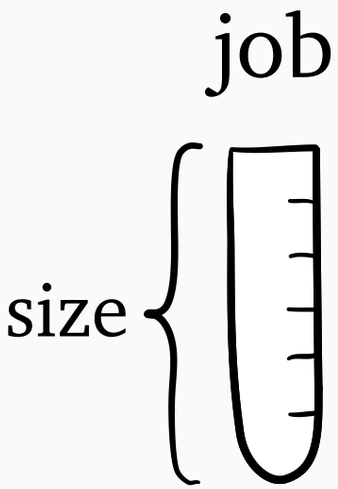

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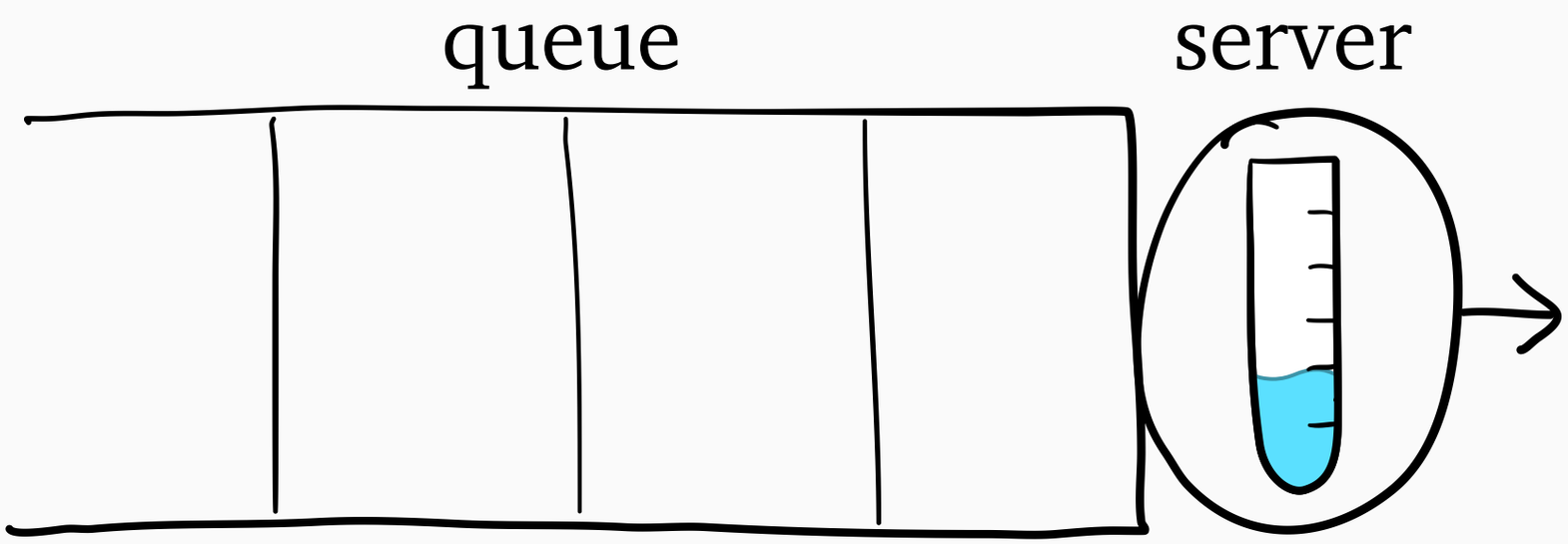

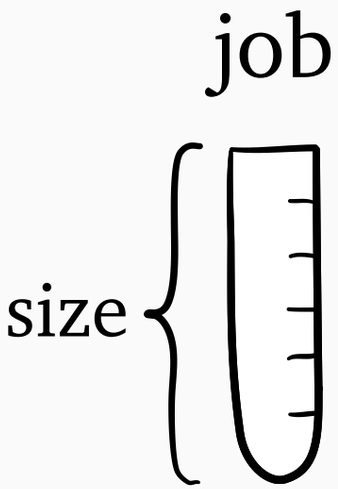

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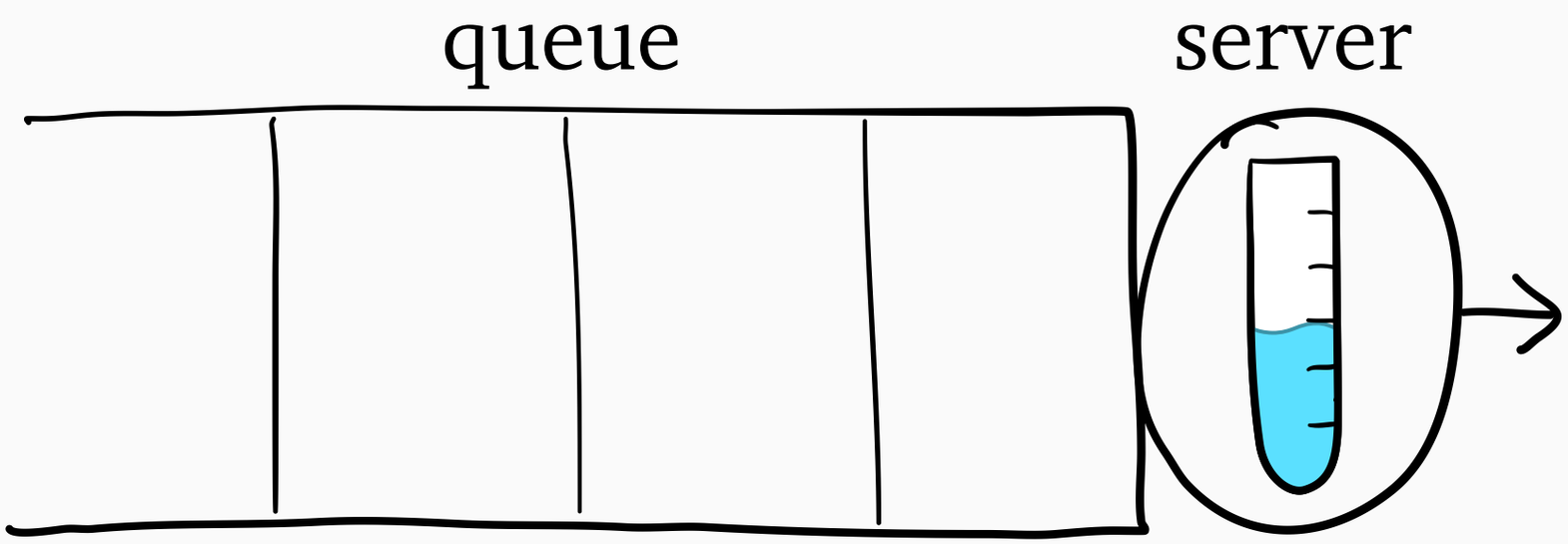
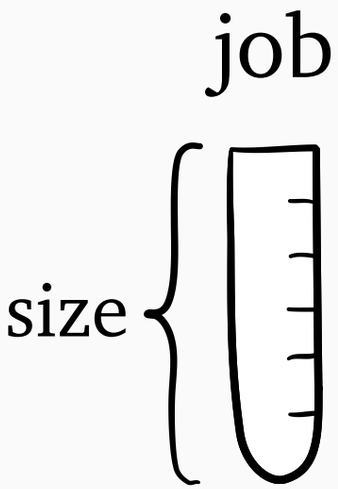
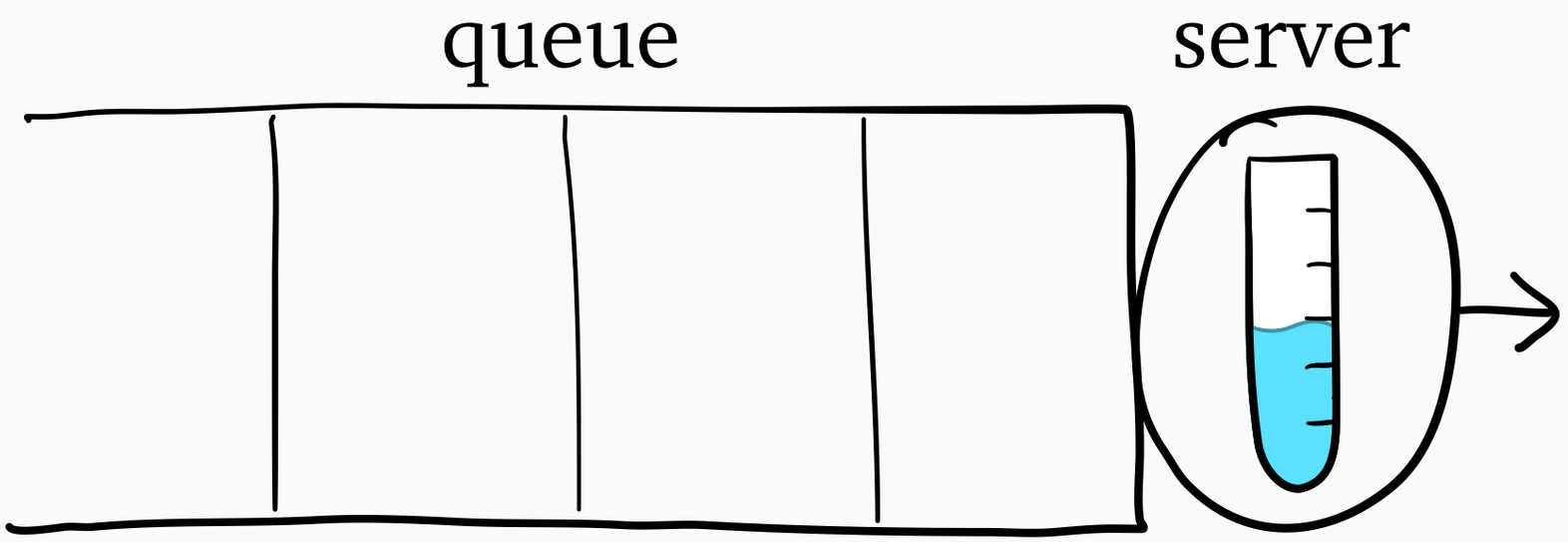
M/G/1

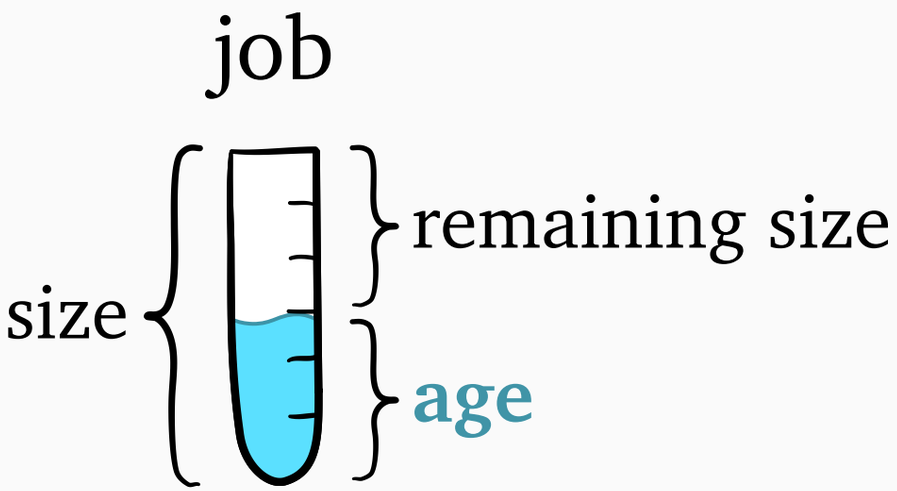




Single-server queueing system

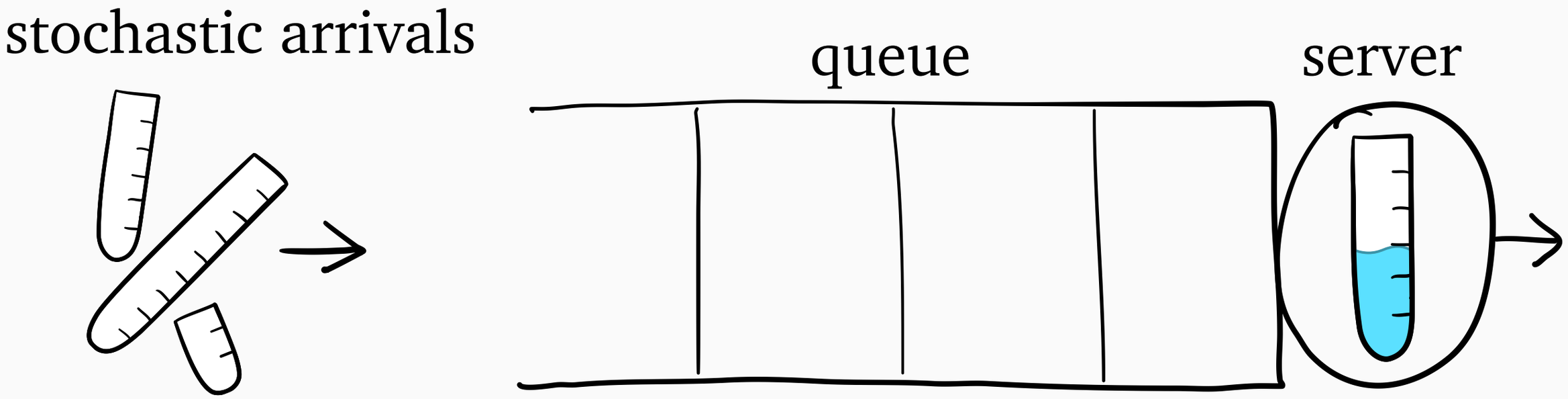
M/G/1

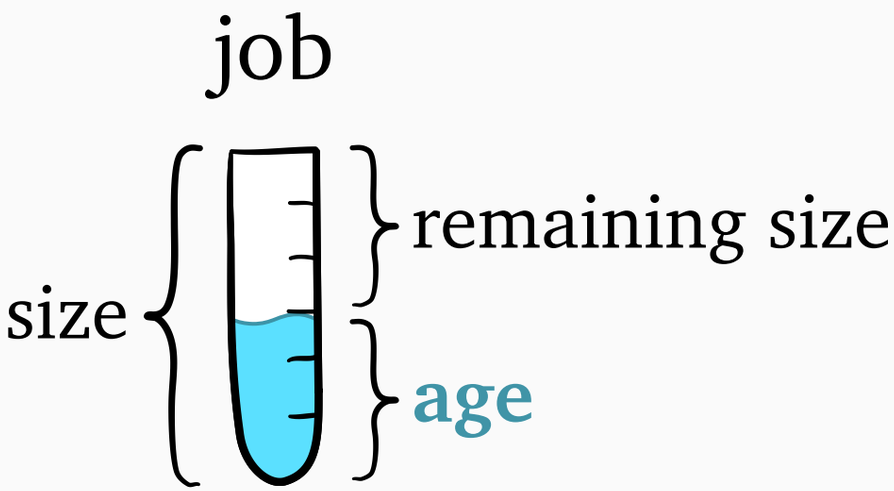




Single-server queueing system

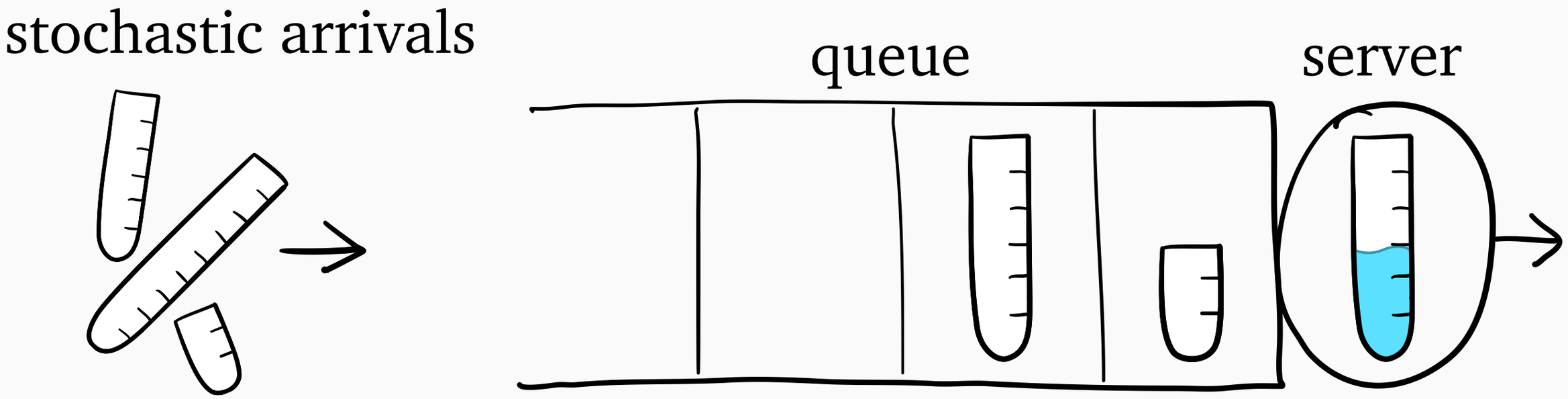
M/G/1

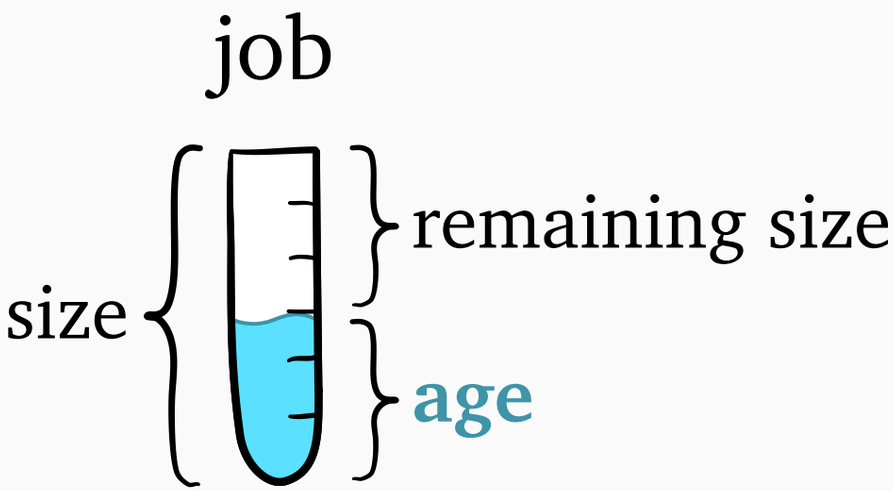




Single-server queueing system

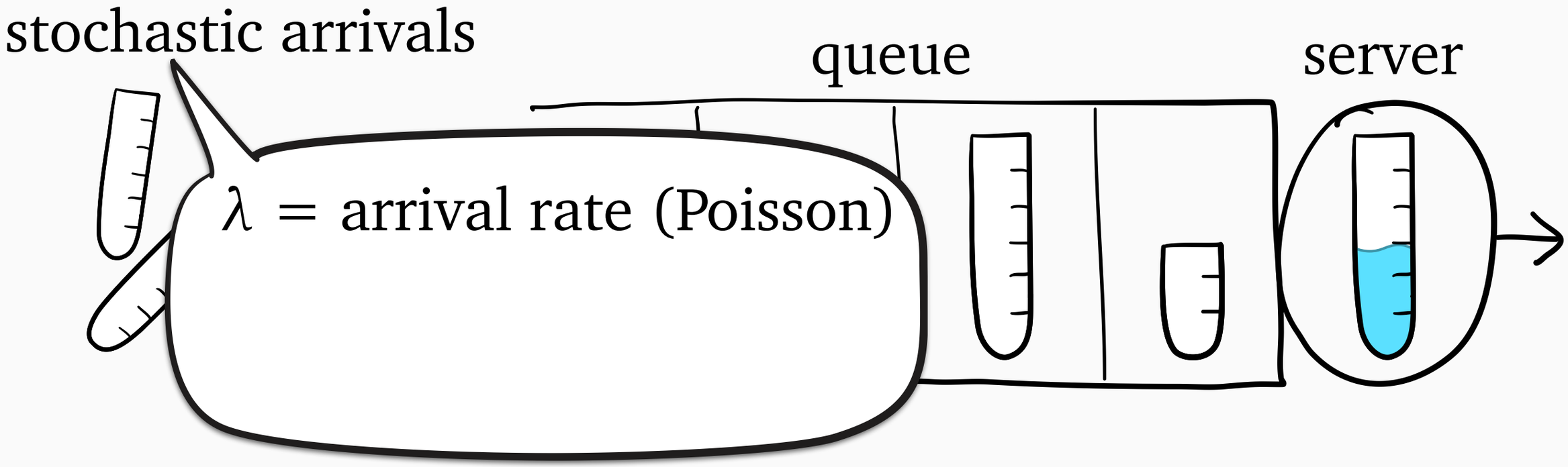
M/G/1

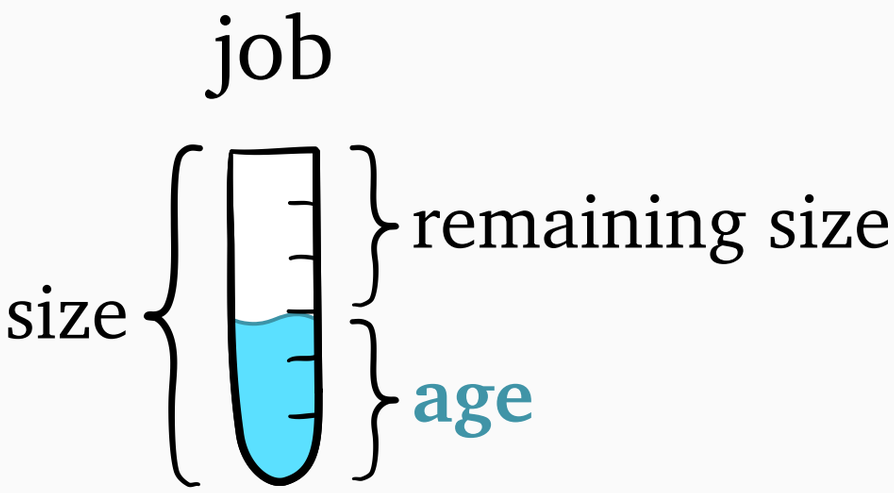




Single-server queueing system

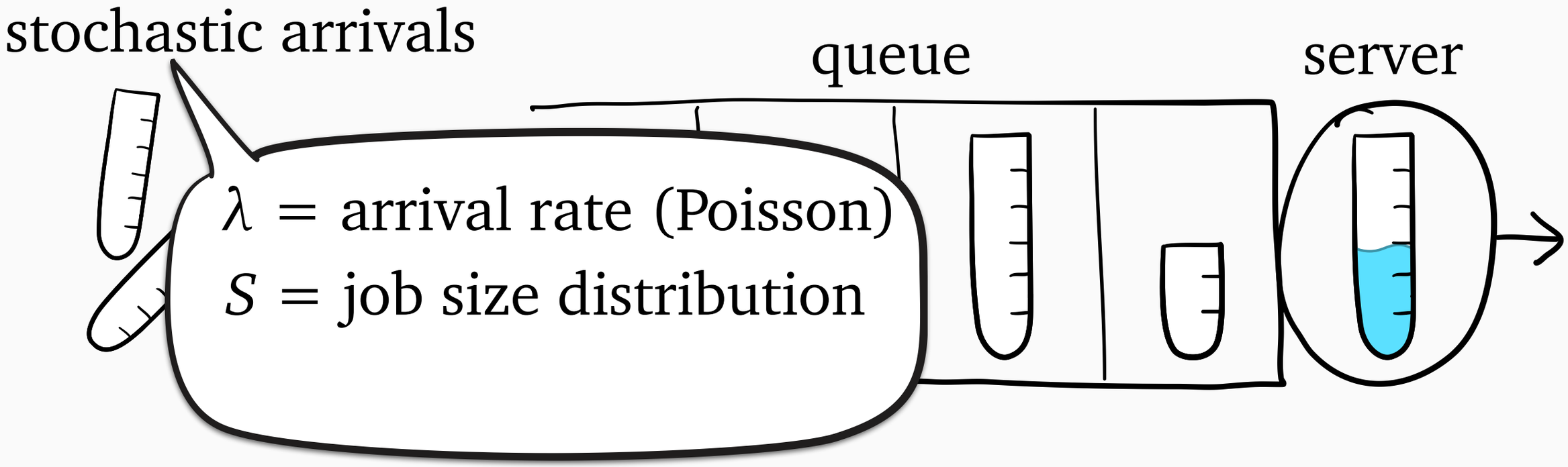
M/G/1

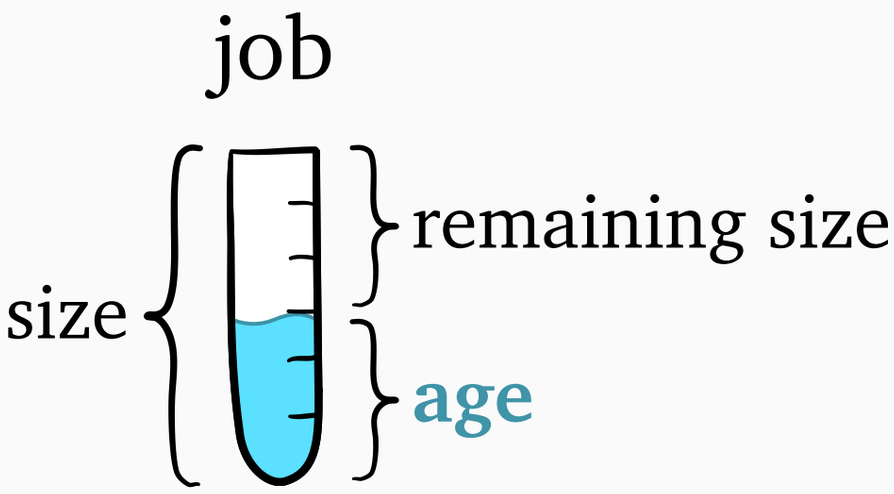




Single-server queueing system

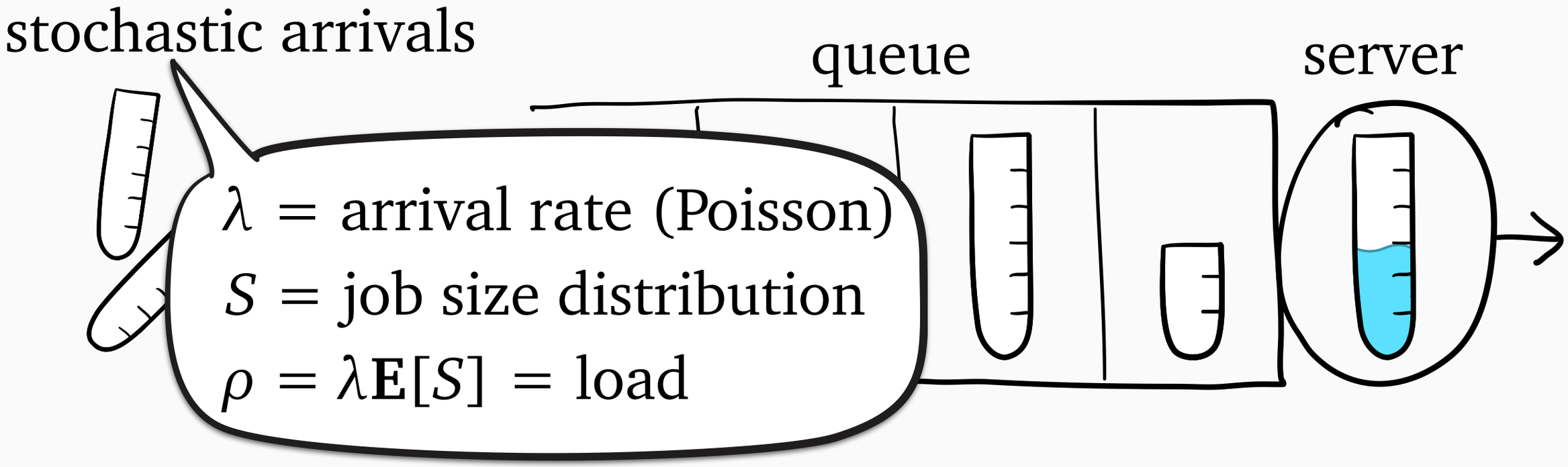
M/G/1

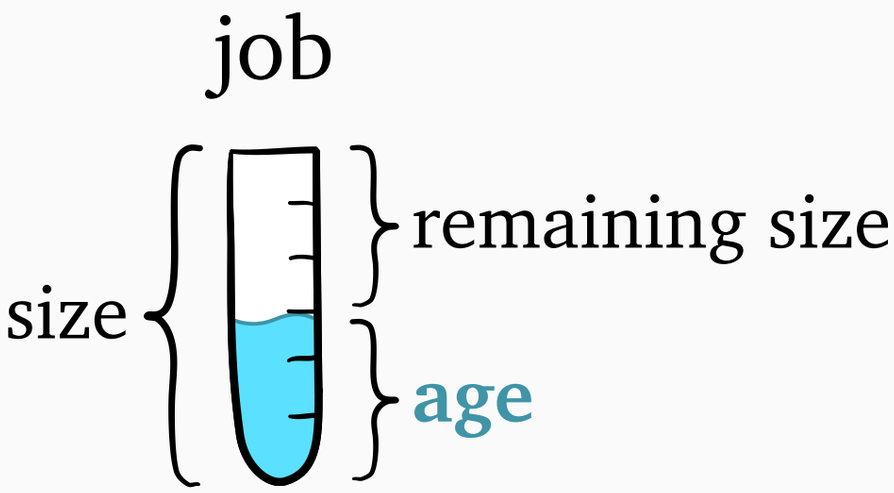




Single-server queueing system

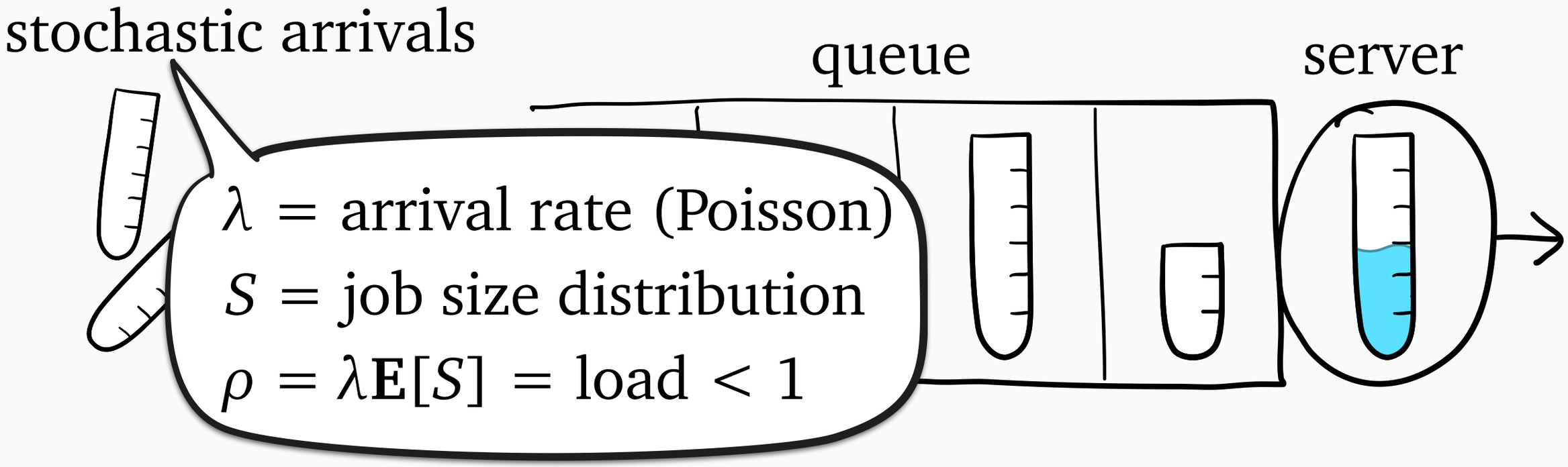
M/G/1

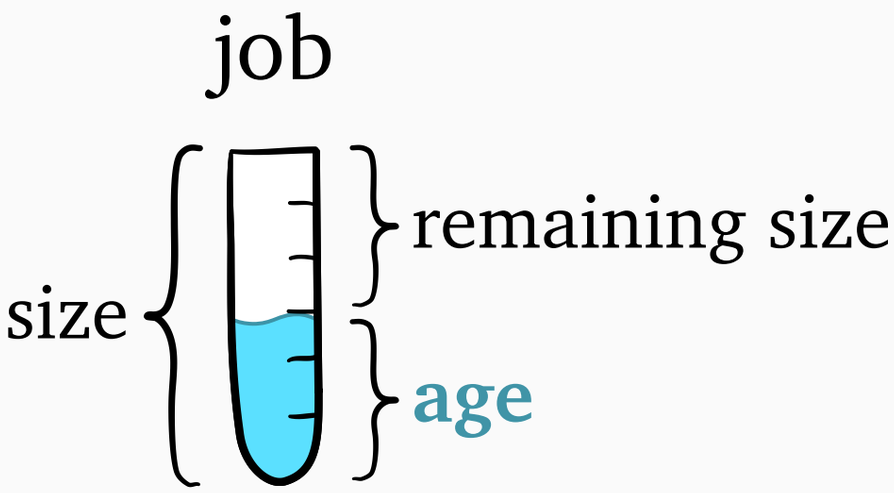




Single-server queueing system

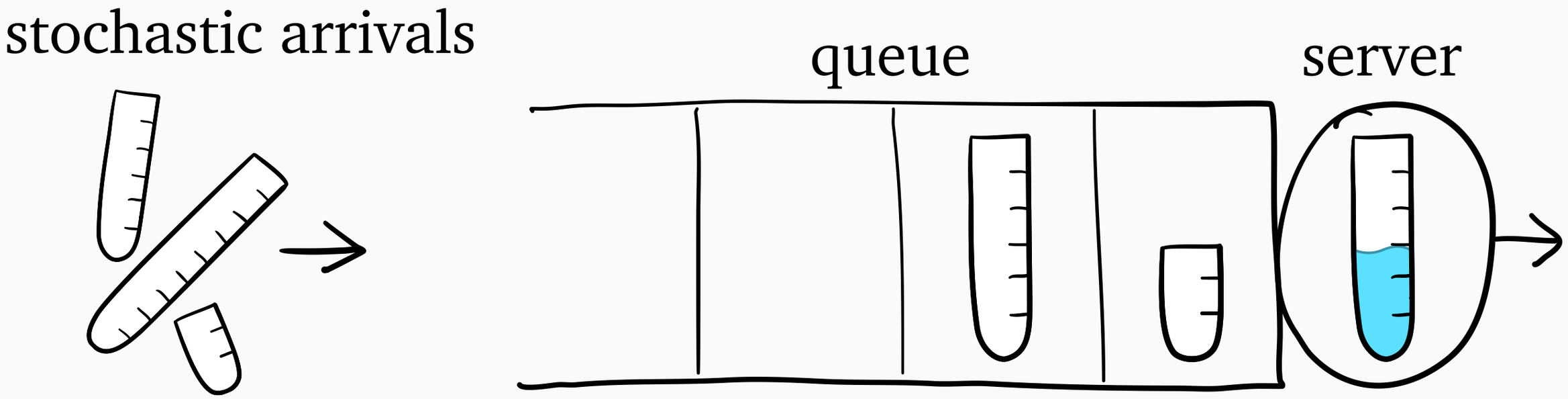
M/G/1

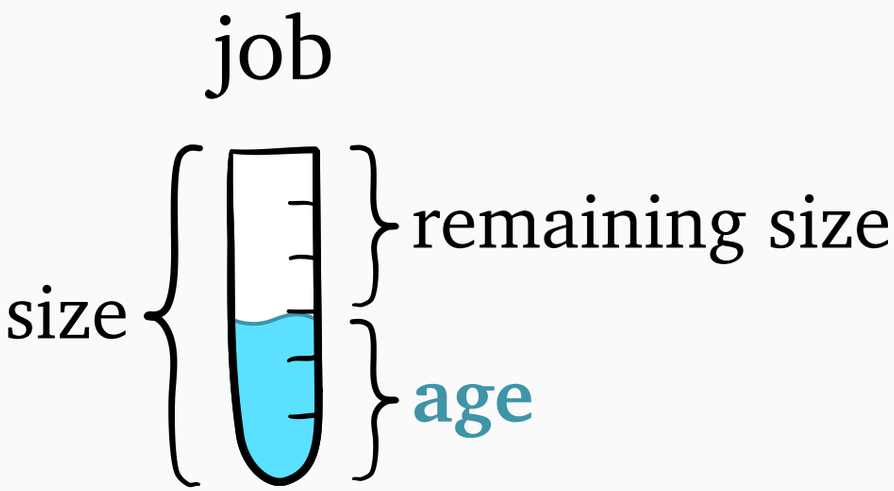




Single-server queueing system

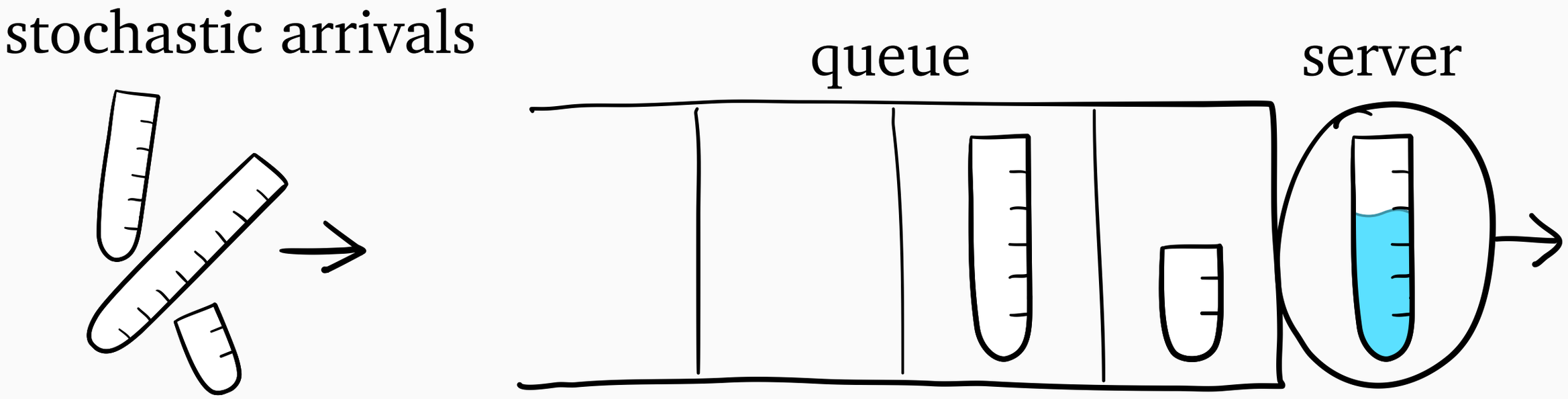
M/G/1

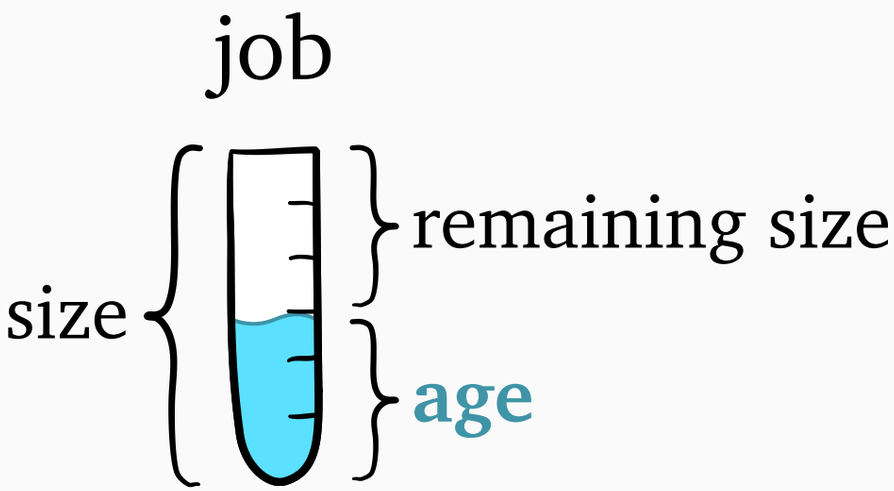




Single-server queueing system

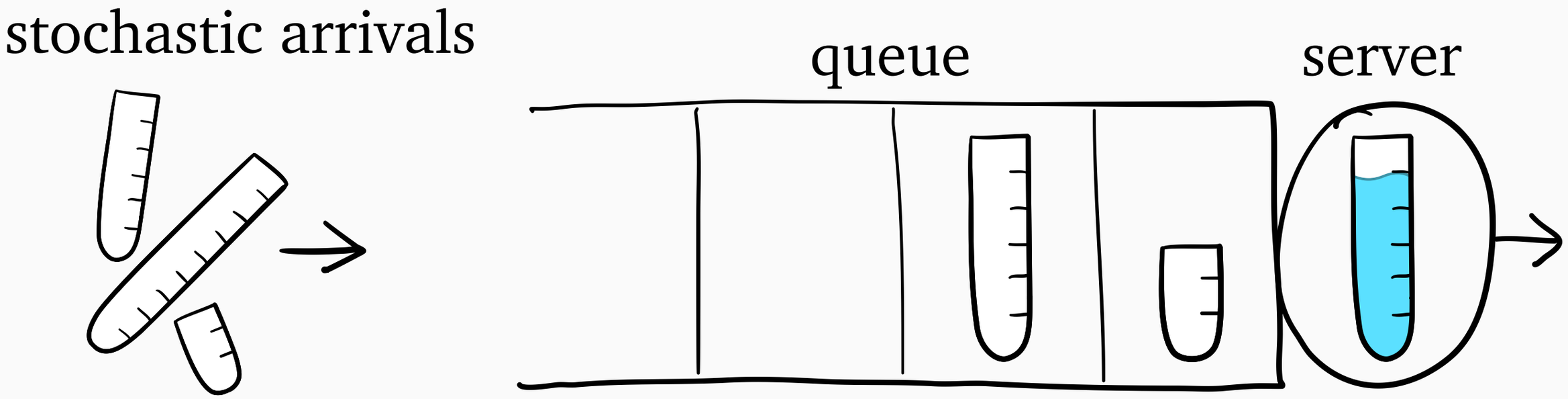
M/G/1

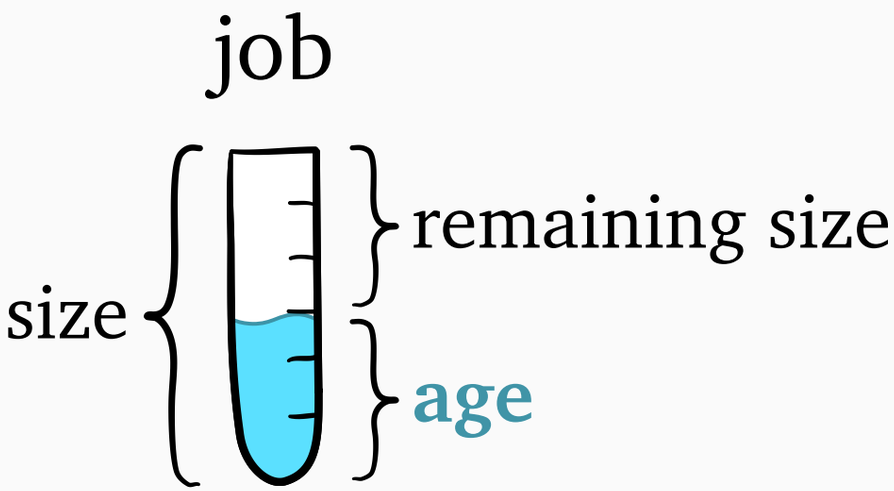




Single-server queueing system

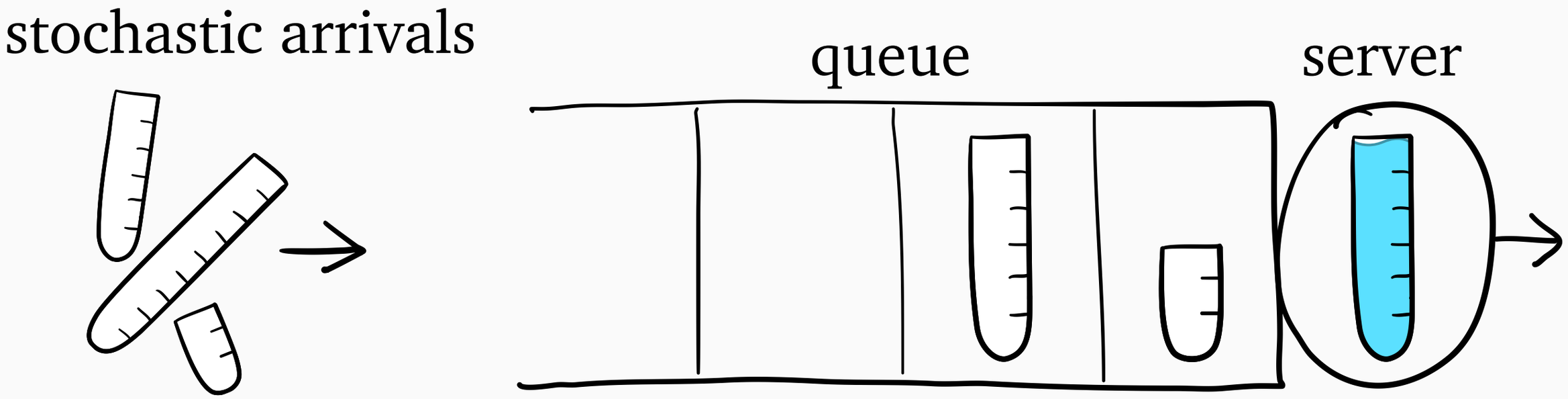
M/G/1

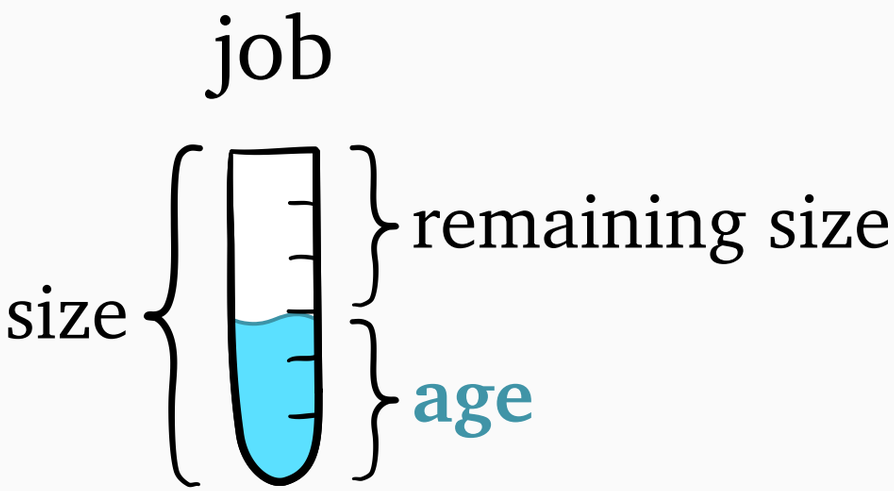




Single-server queueing system

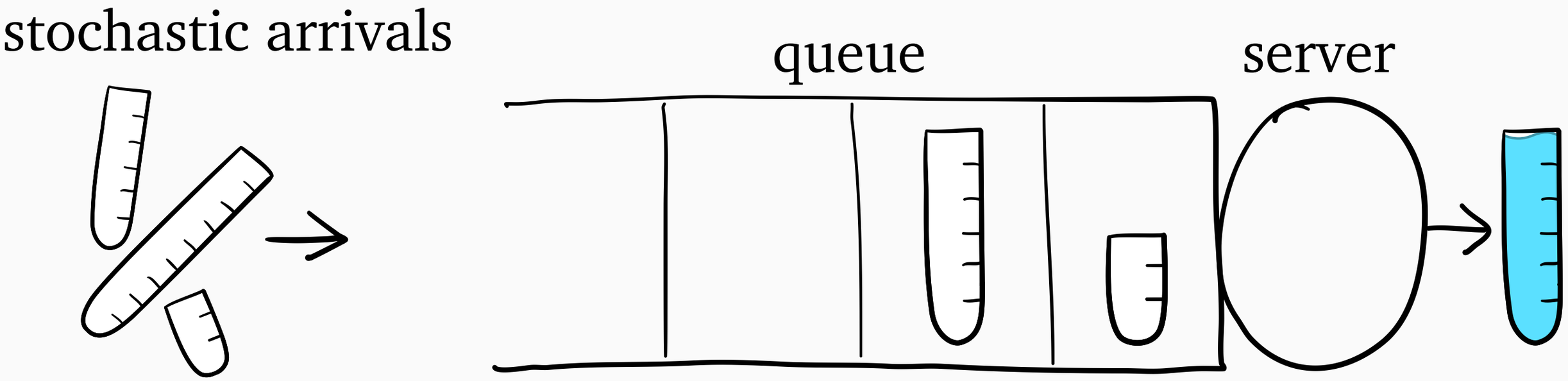
M/G/1

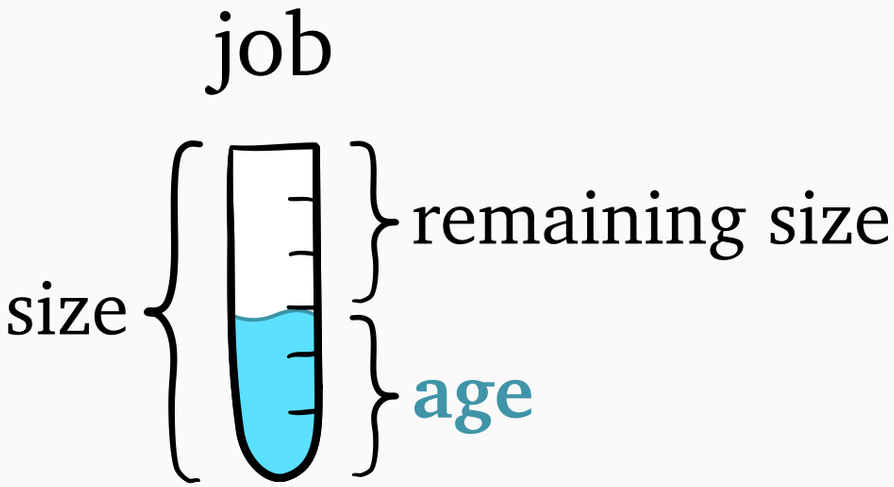




Single-server queueing system

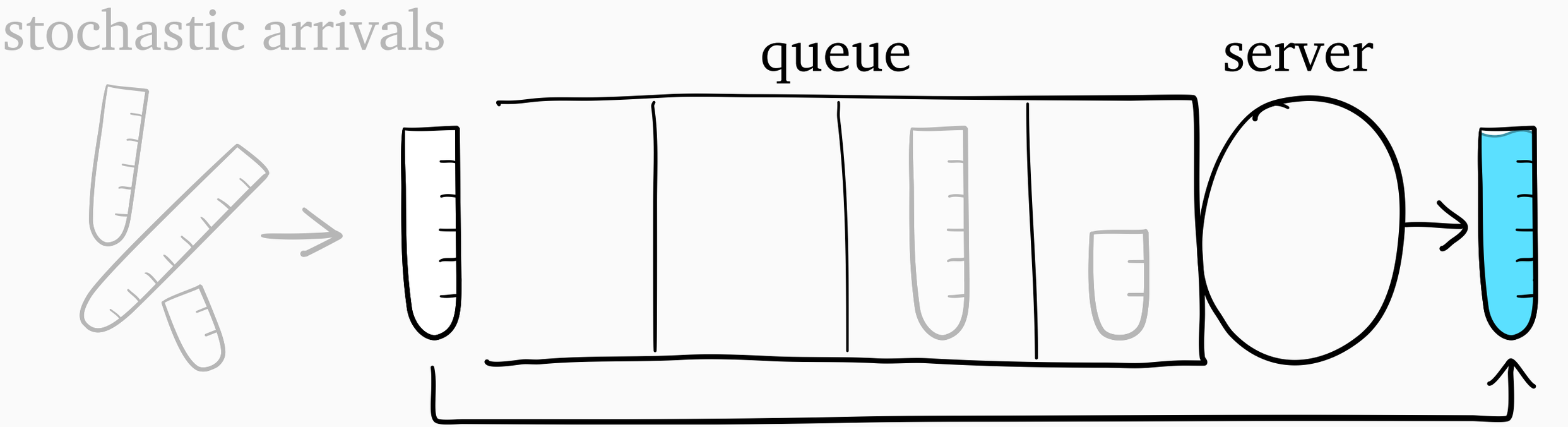
M/G/1



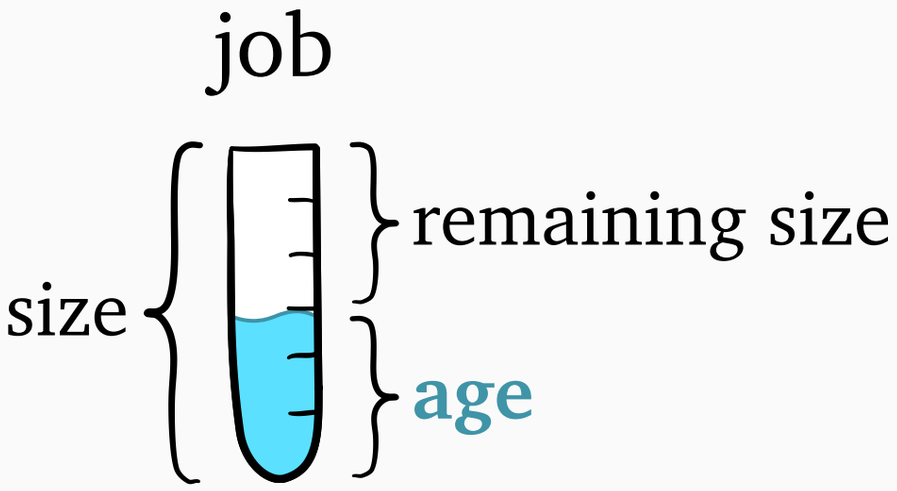


Single-server queueing system

M/G/1

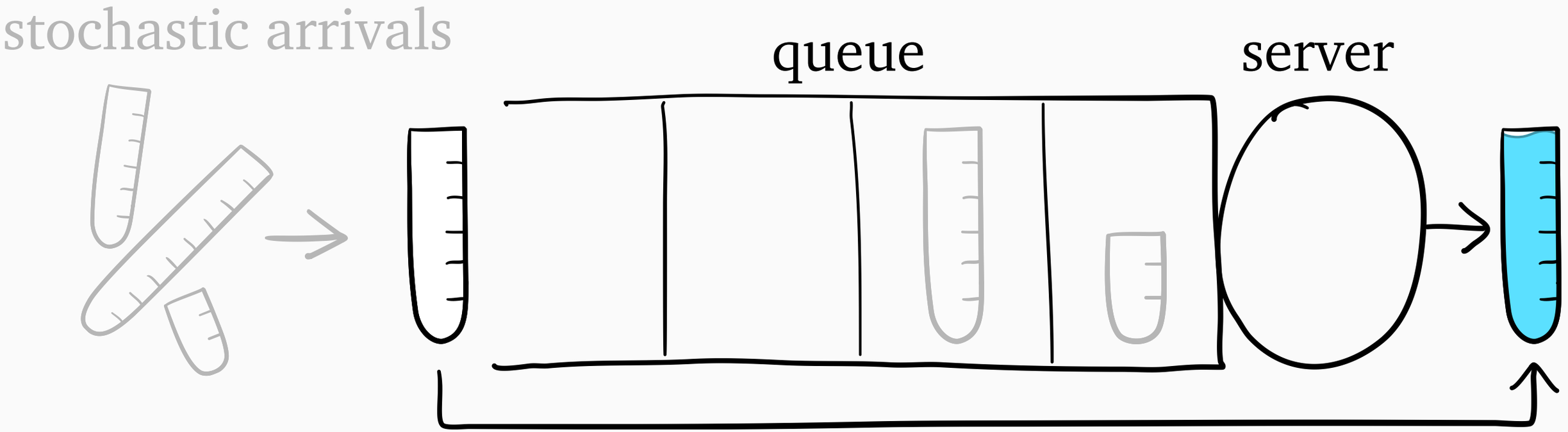


response time



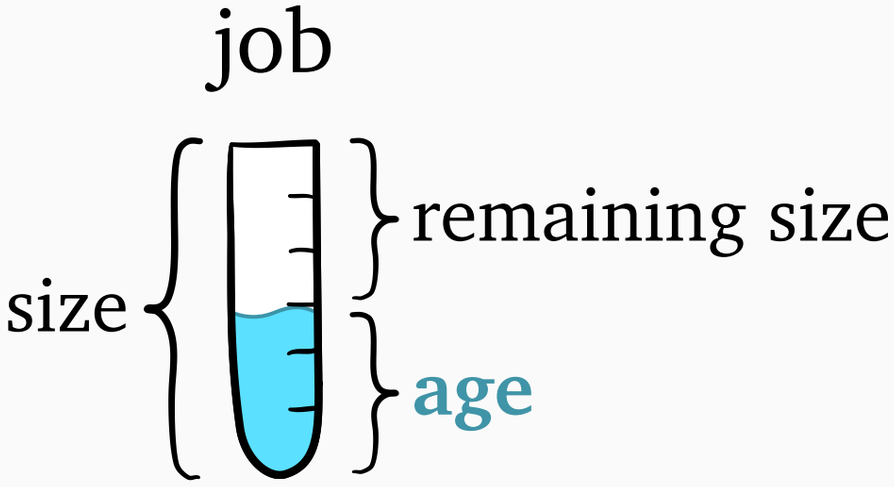
Single-server queueing system

M/G/1



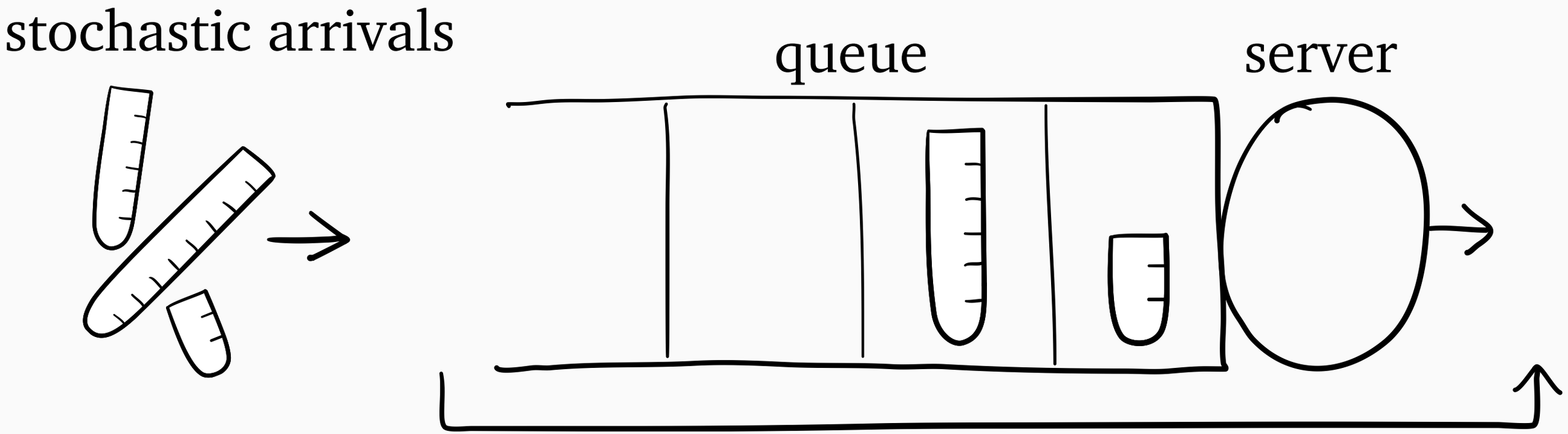
response time

a.k.a. delay, latency

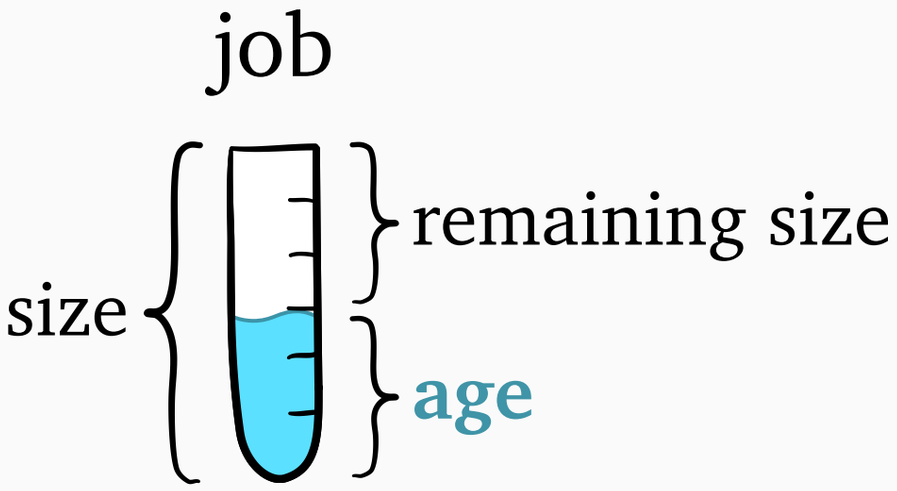


Single-server queueing system

M/G/1

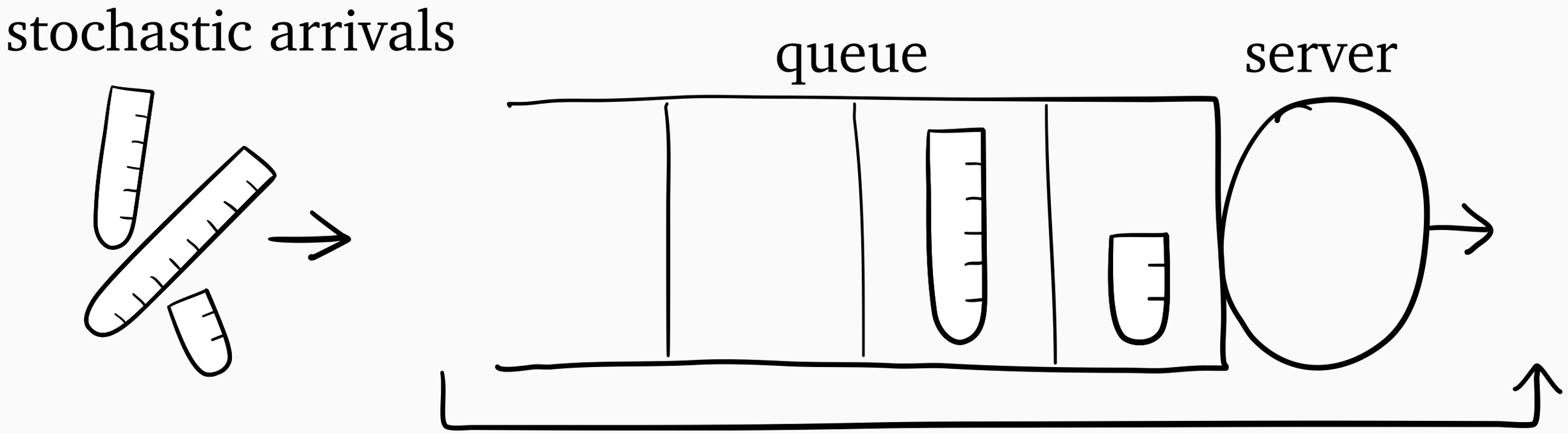


response time

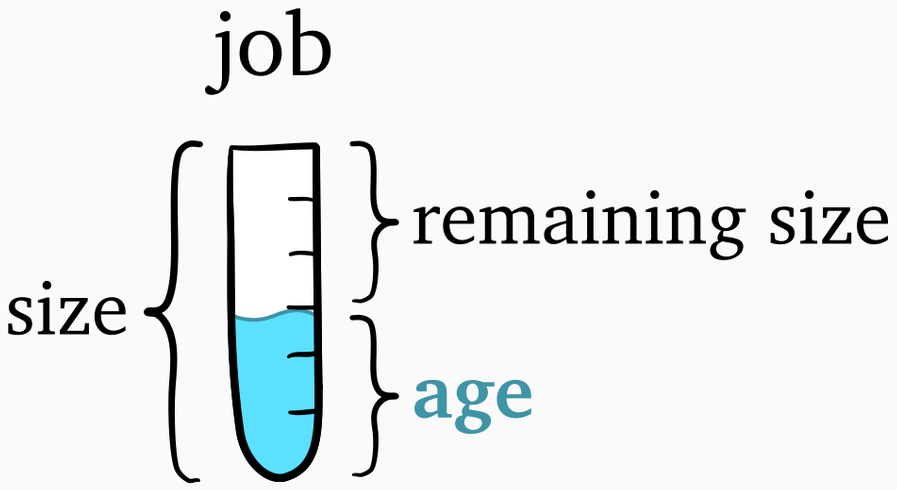


Single-server queueing system

M/G/1



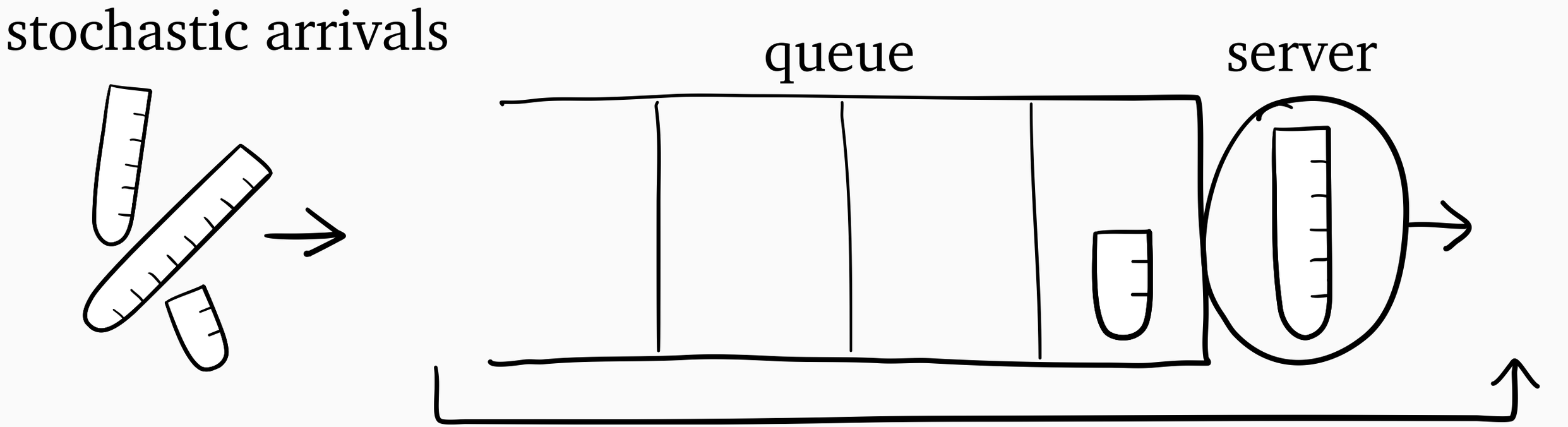
response time



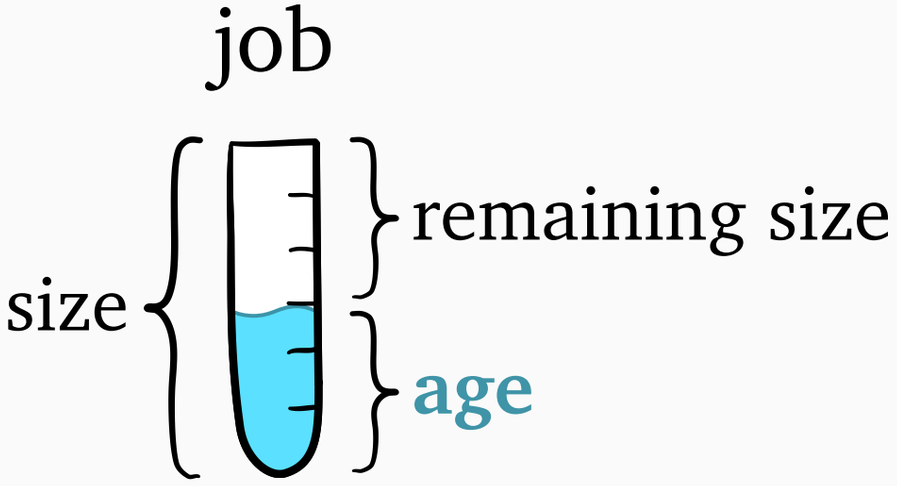
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



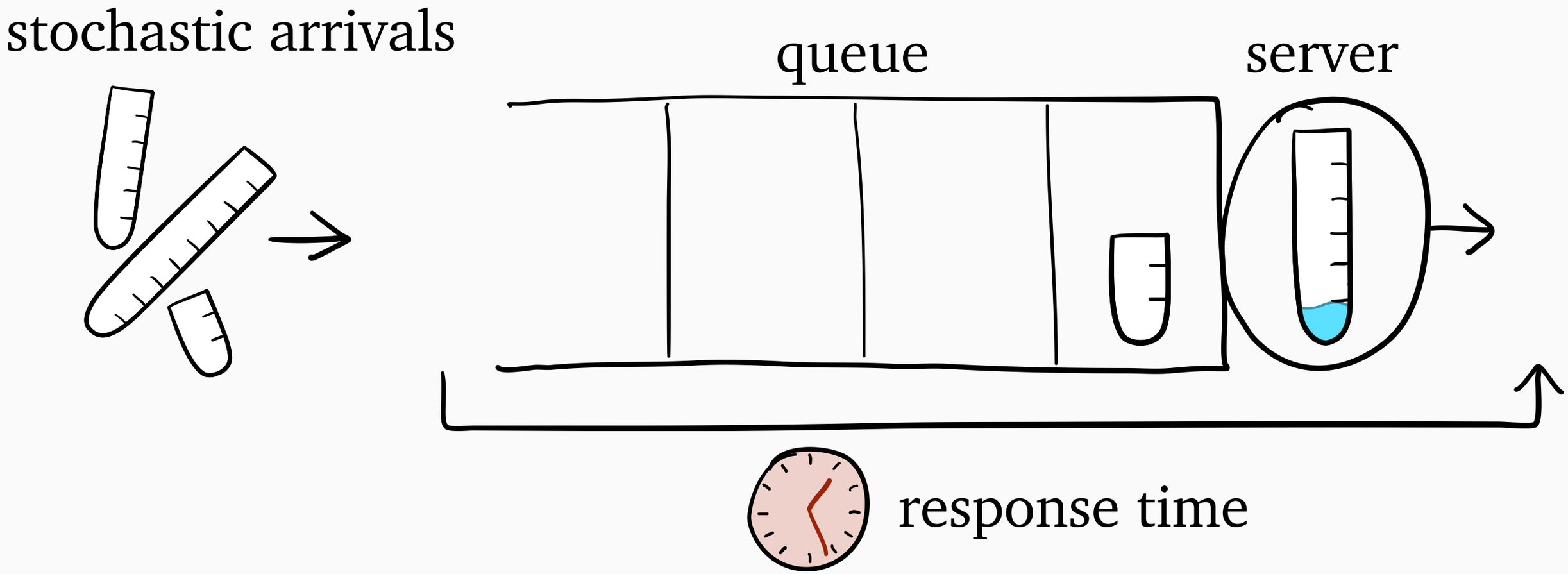
response time



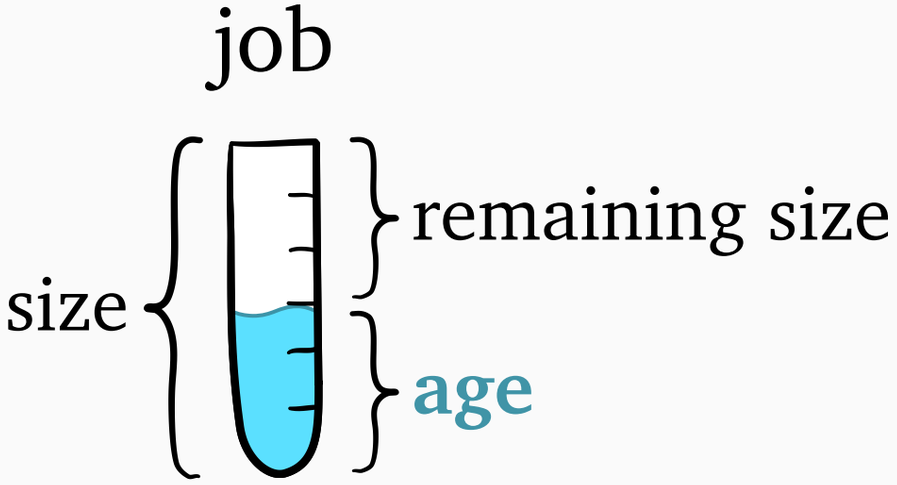
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



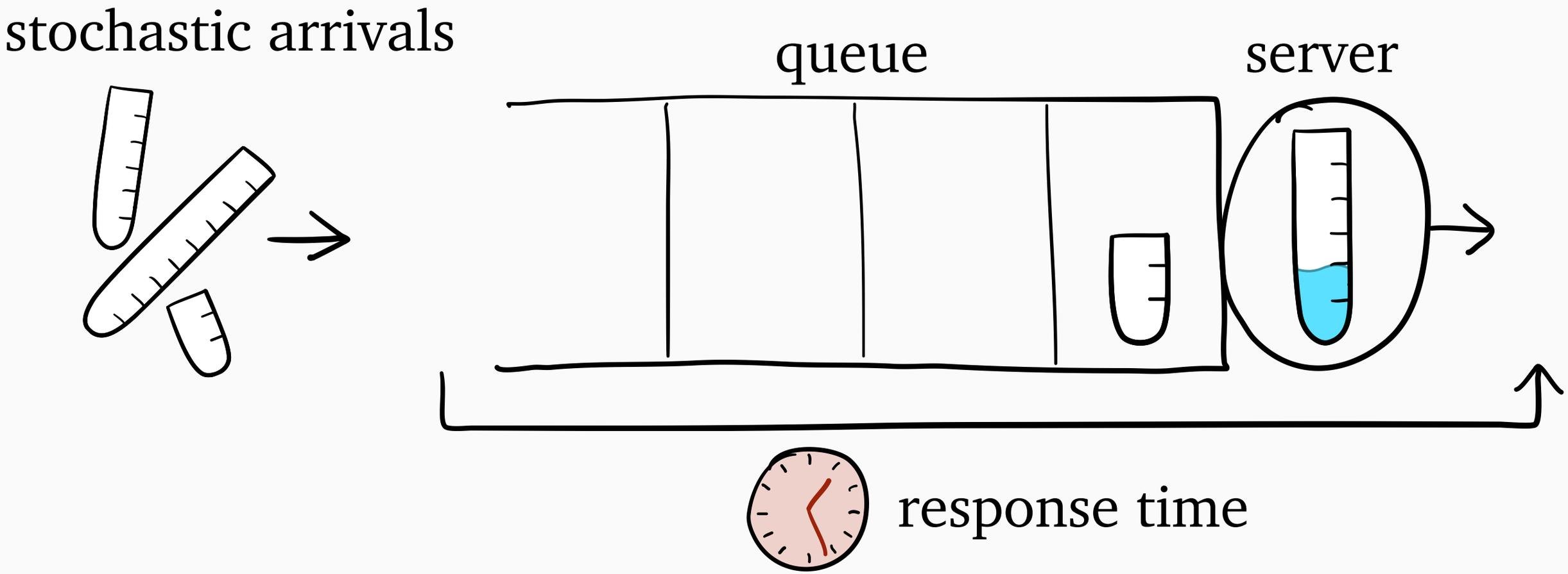
response time



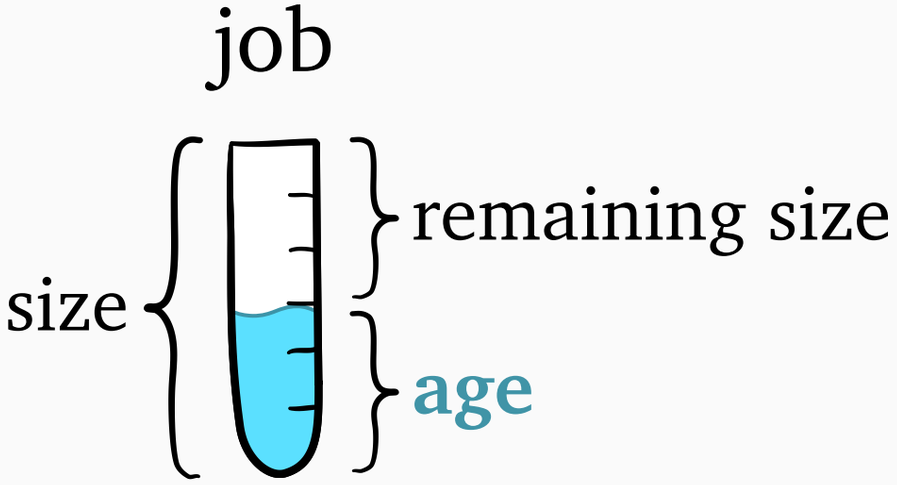
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



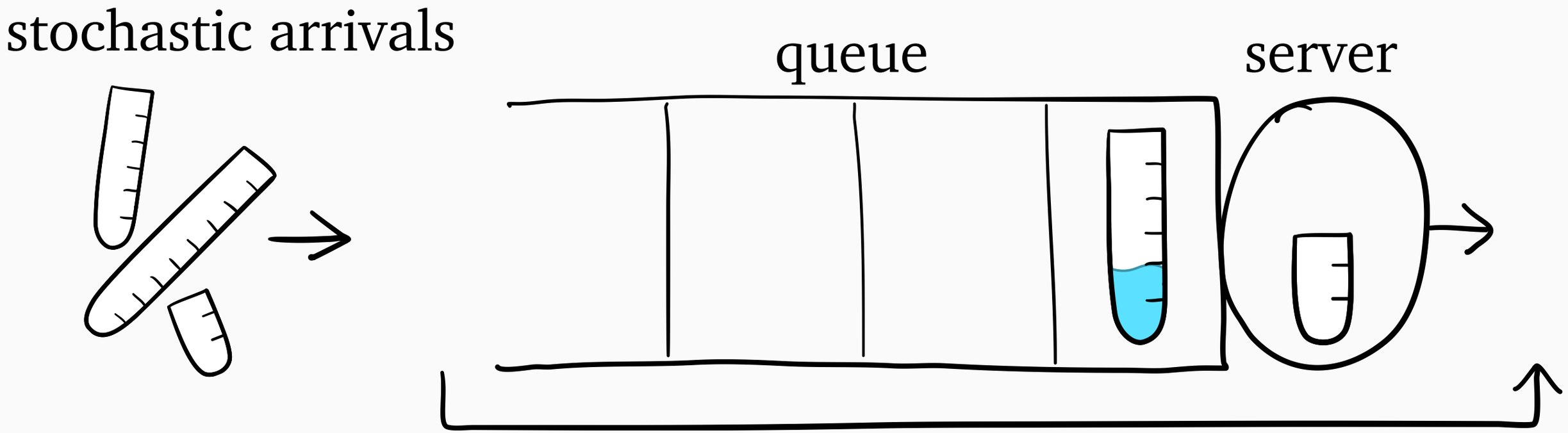
response time



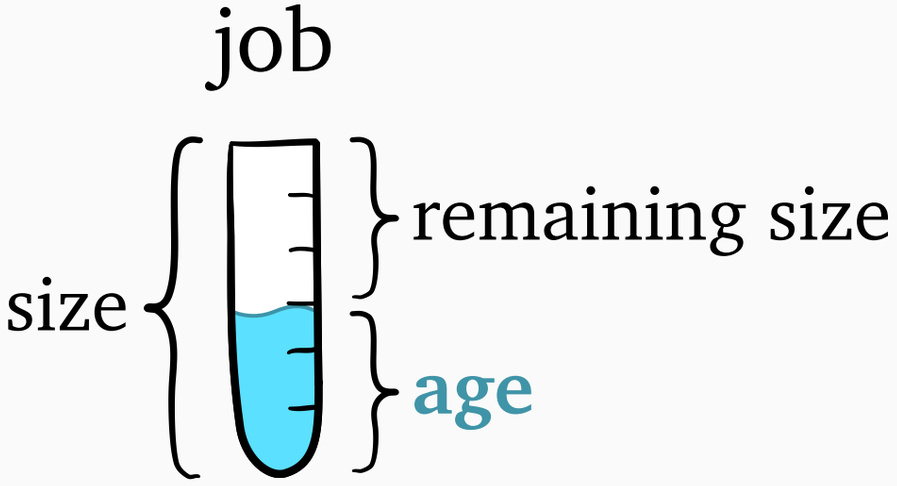
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



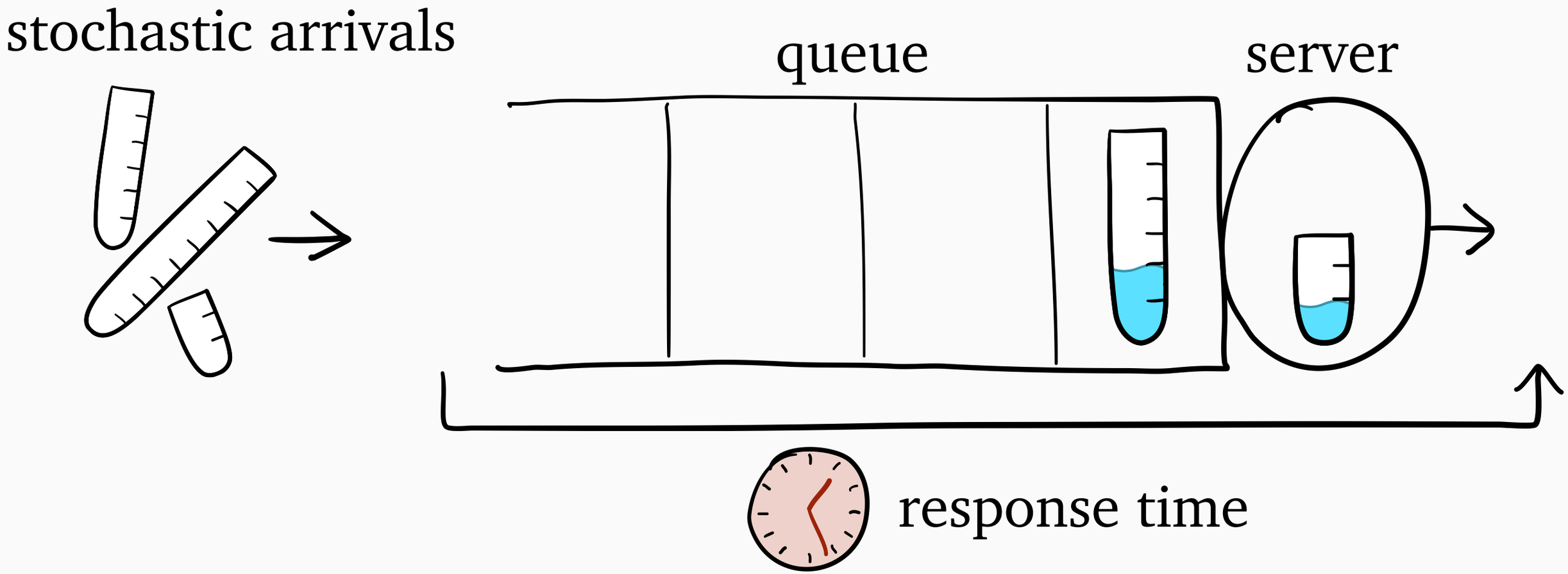
response time



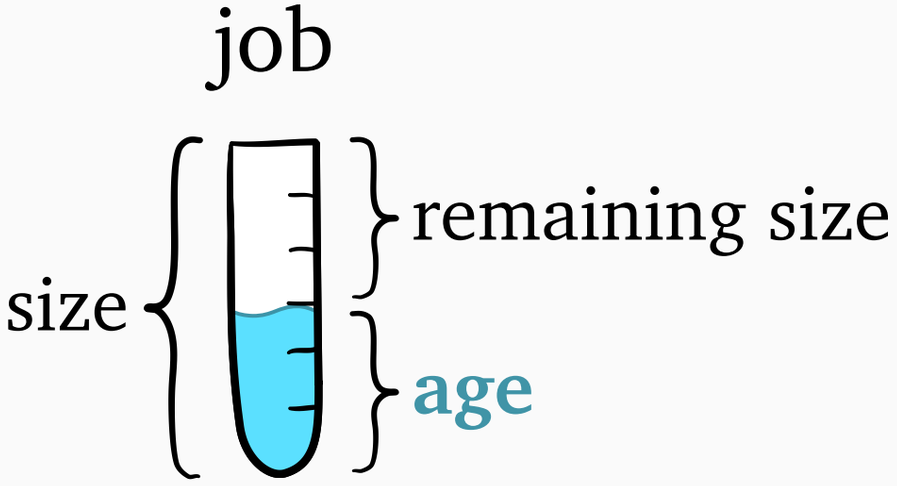
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



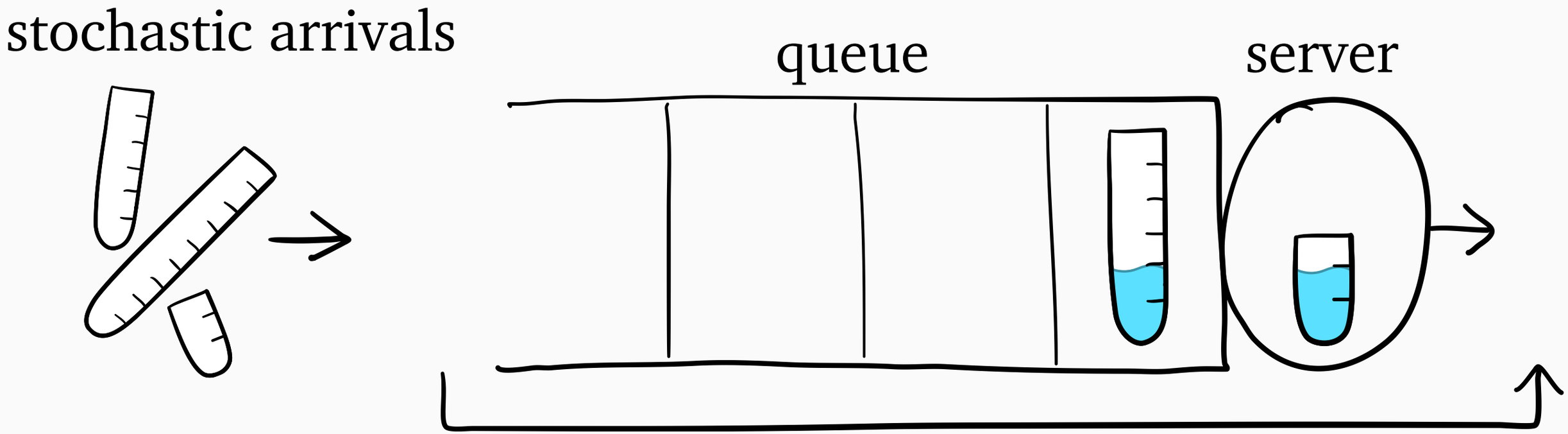
response time



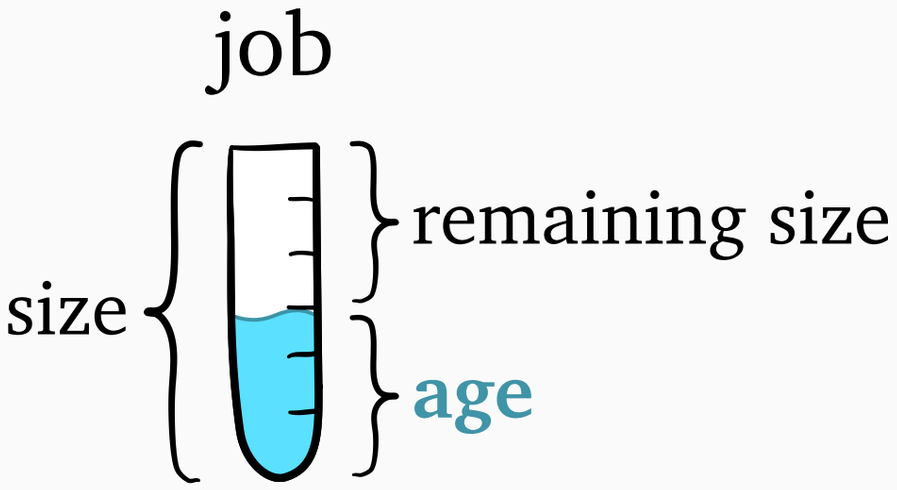
Scheduling policy:
decides which job to serve

Single-server queueing system

M/G/1



response time

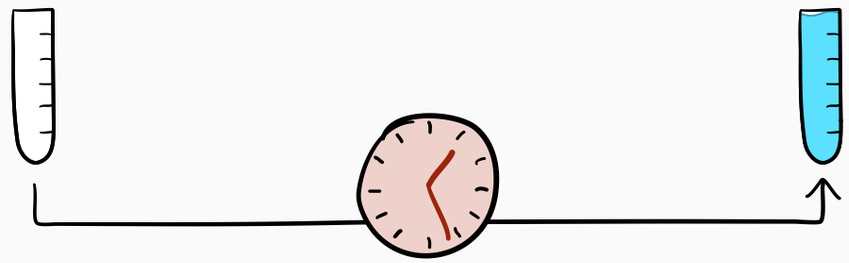


Scheduling policy:
decides which job to serve

Evaluating performance

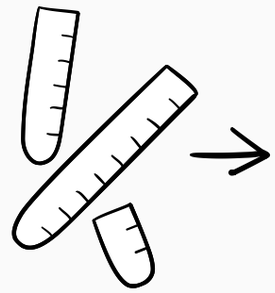
Evaluating performance

response time

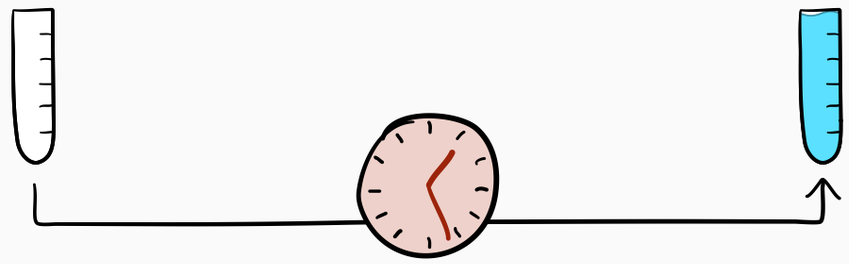


Evaluating performance

stochastic arrival process λ, S

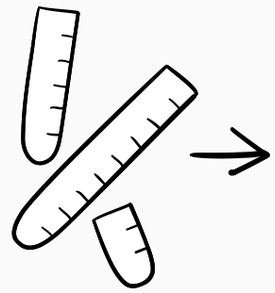


response time

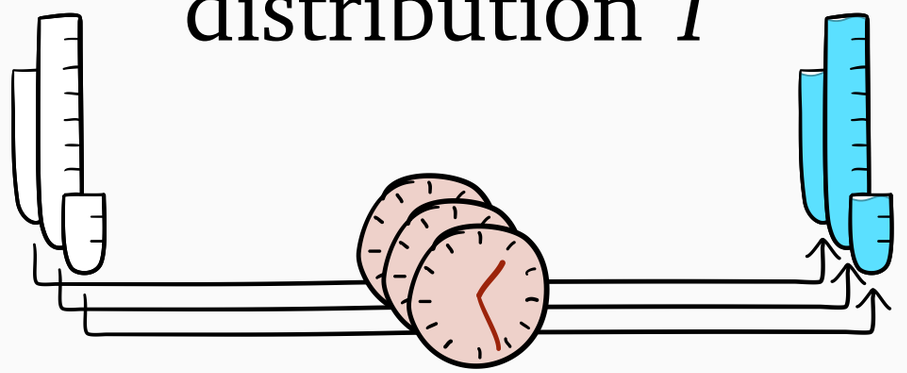


Evaluating performance

stochastic arrival process λ, S



response time distribution T



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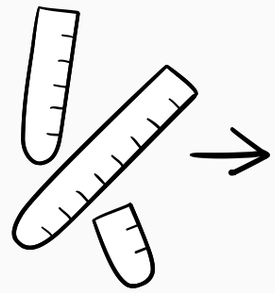
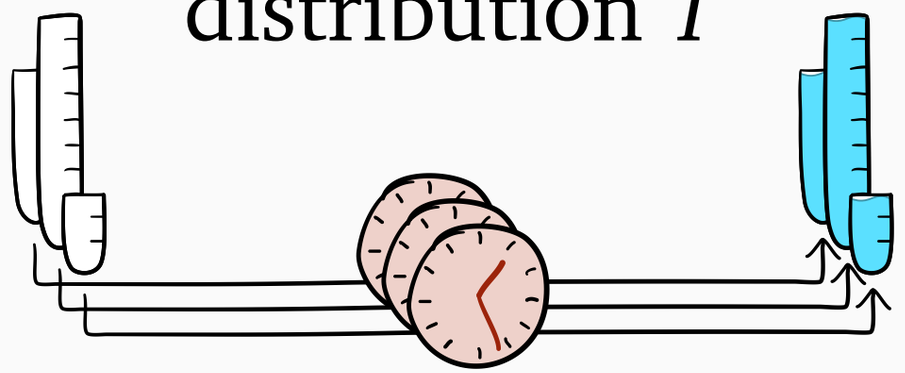


scheduling policy

response time distribution T



Evaluating performance

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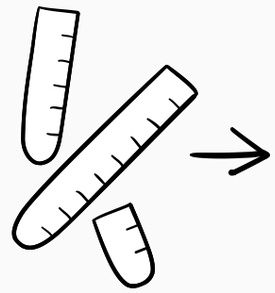

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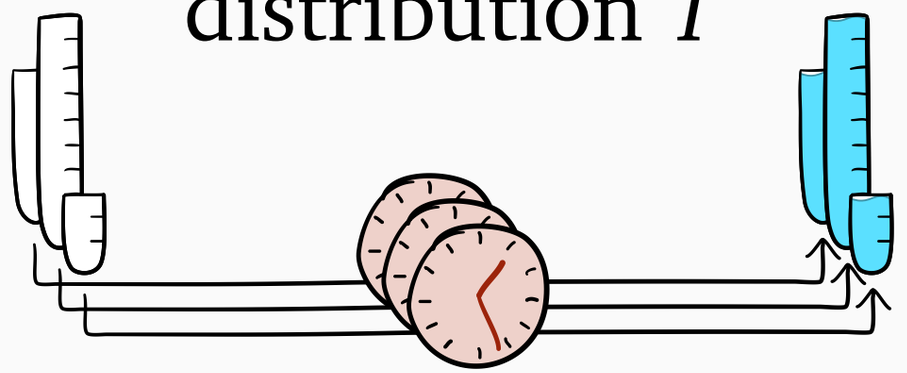
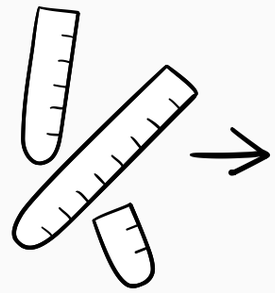


Metrics:

- mean $\mathbf{E}[T]$
- tail probability $\mathbf{P}[T > t]$
- percentiles, e.g. T_{99}

Evaluating performance

stochastic arrival process λ, S



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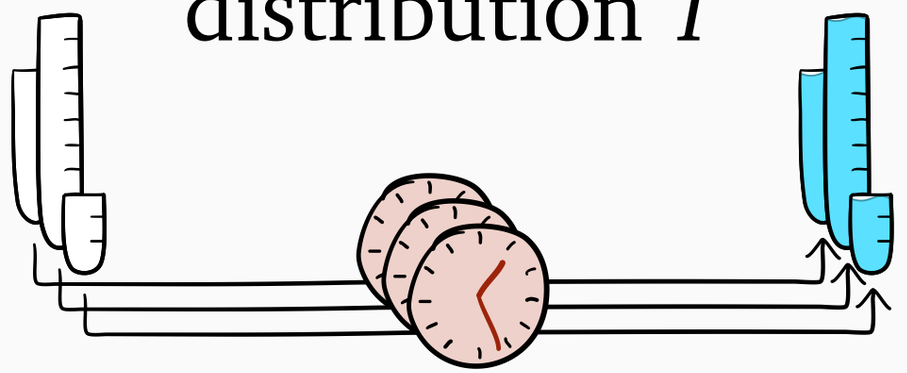

Goals:

- *analyze* **policy**

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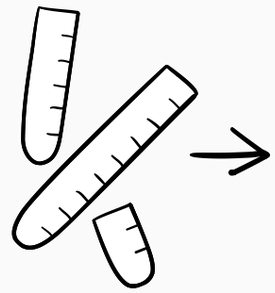
Evaluating performance

stochastic arrival process λ, S



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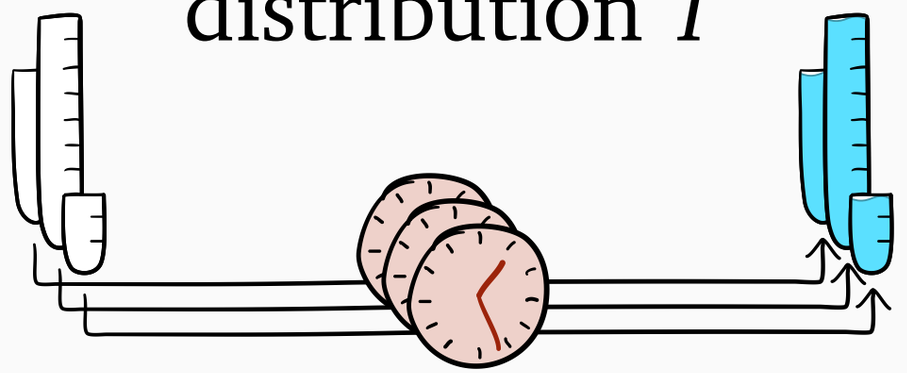

Goals:

- *analyze* **policy**
- *optimize* **policy**

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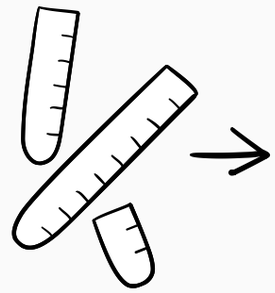

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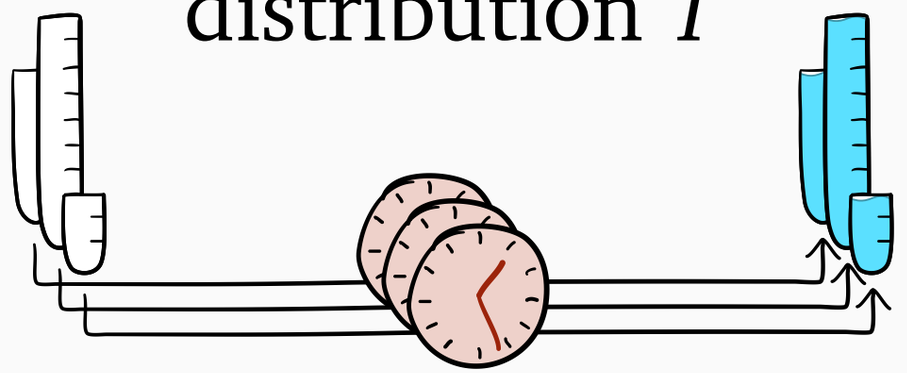

Goals:

- *analyze* **policy**
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our focus

Metrics:

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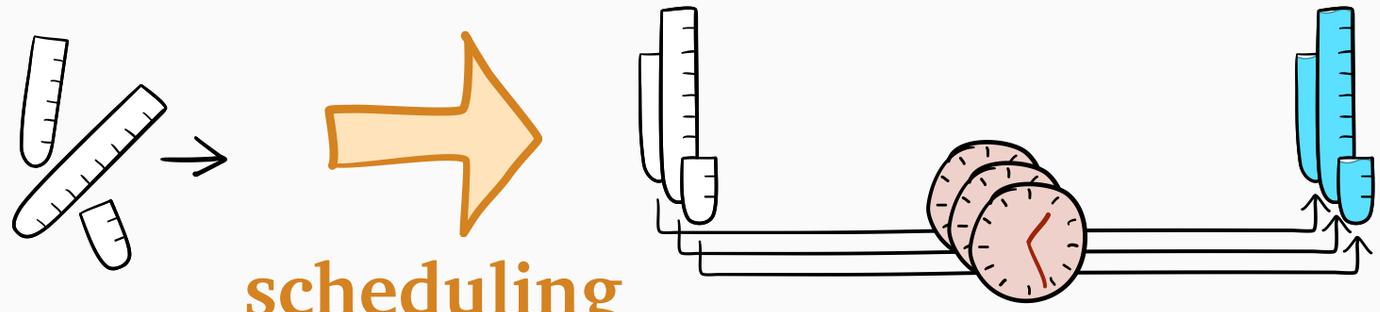

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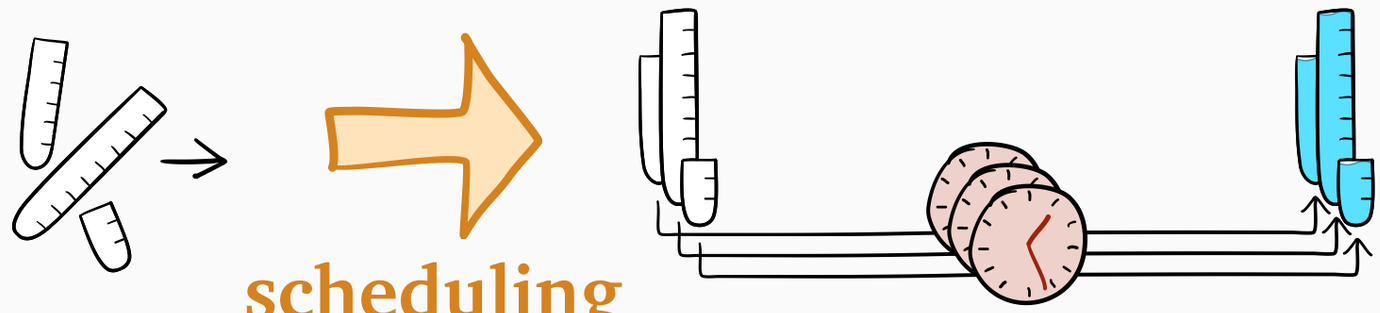
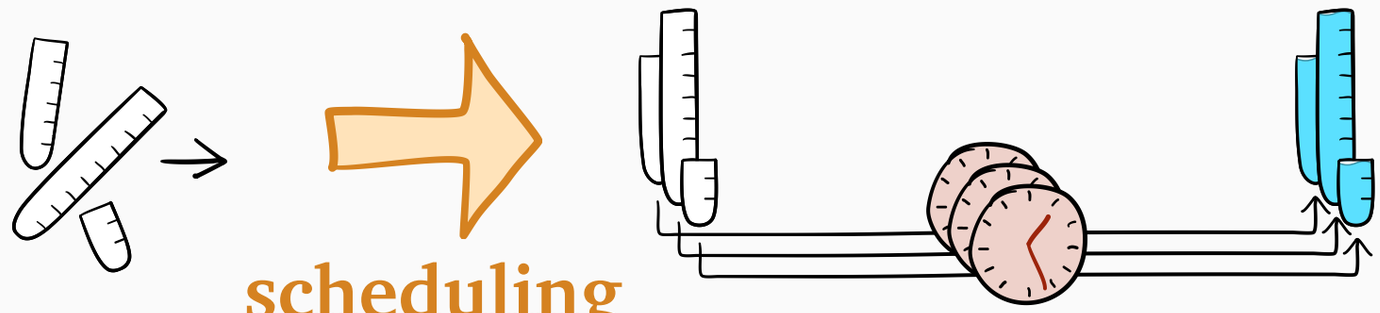


Problem:

- Theory limited to “simple” **policies**

Goal: analyze impact of **policy**



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- In practice, “complex” **policies** matter

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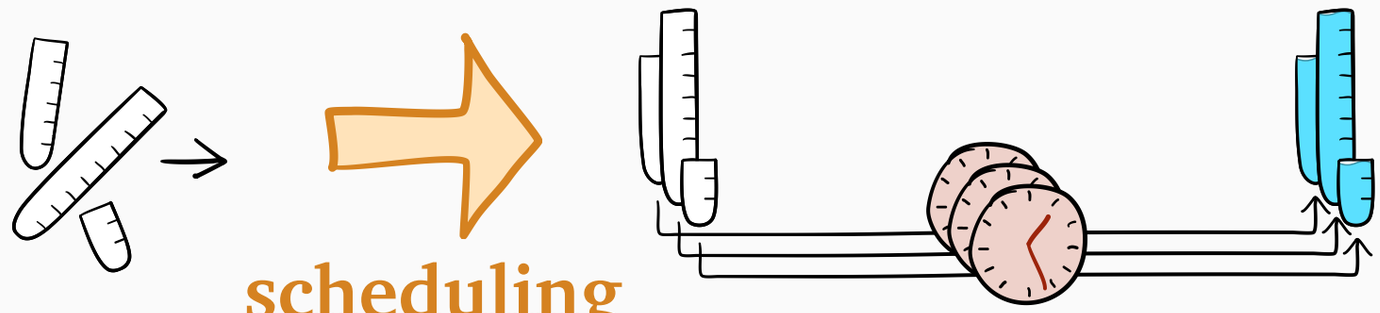

Problem:

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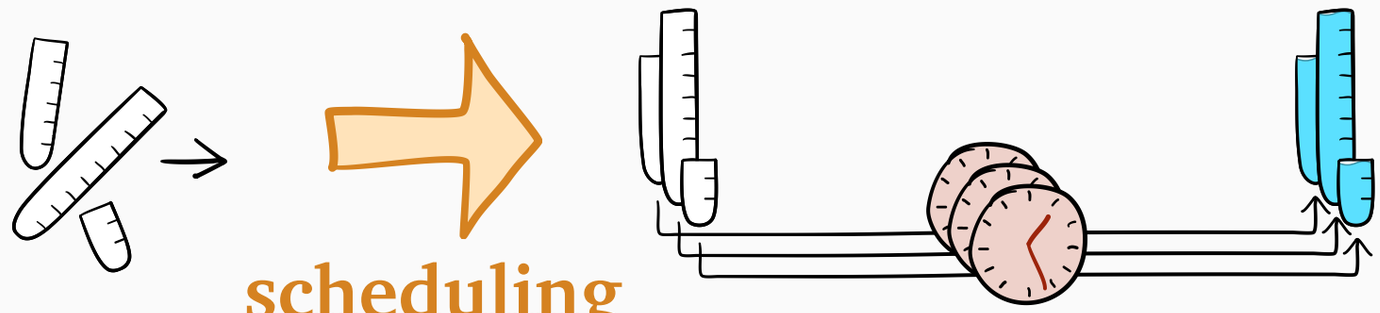
What makes a **policy** “simple” or “complex”?

Goal: analyze impact of **policy**



Problem:

- Theory limited to “simple” **policies**
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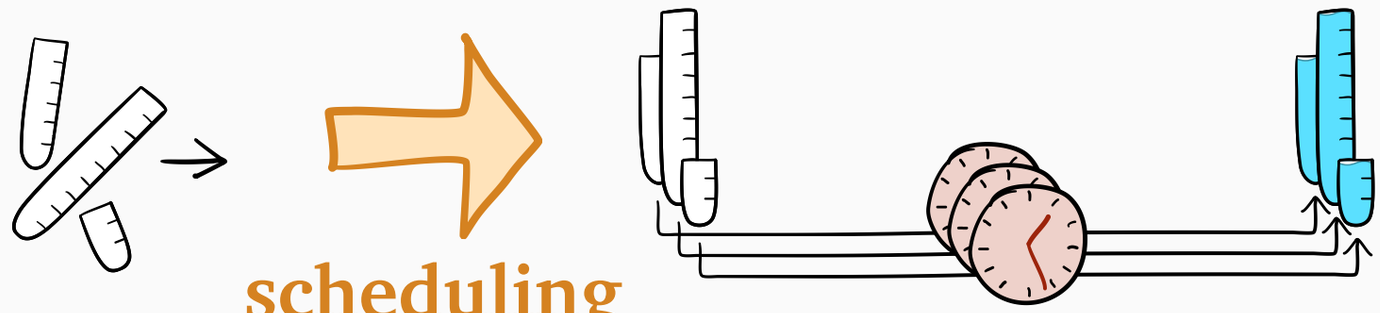
What makes a **policy** “simple” or “complex”?

What makes “complex” **policies** hard to analyze?

Goal: analyze impact of **policy**



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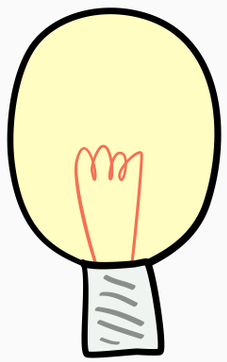

SOAP mindset:
unifying *language* to describe **policies**

What makes a **policy** “simple” or “complex”?

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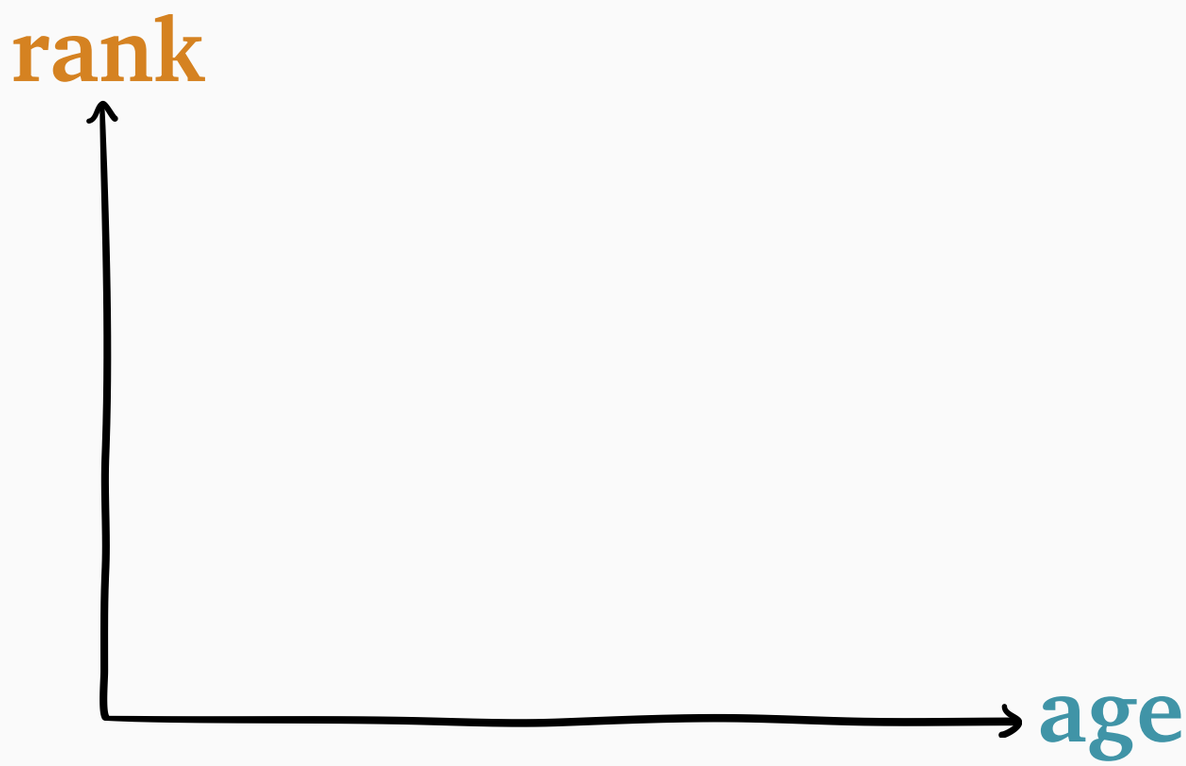
Unifying language: **rank** functions



Unifying language: **rank** functions

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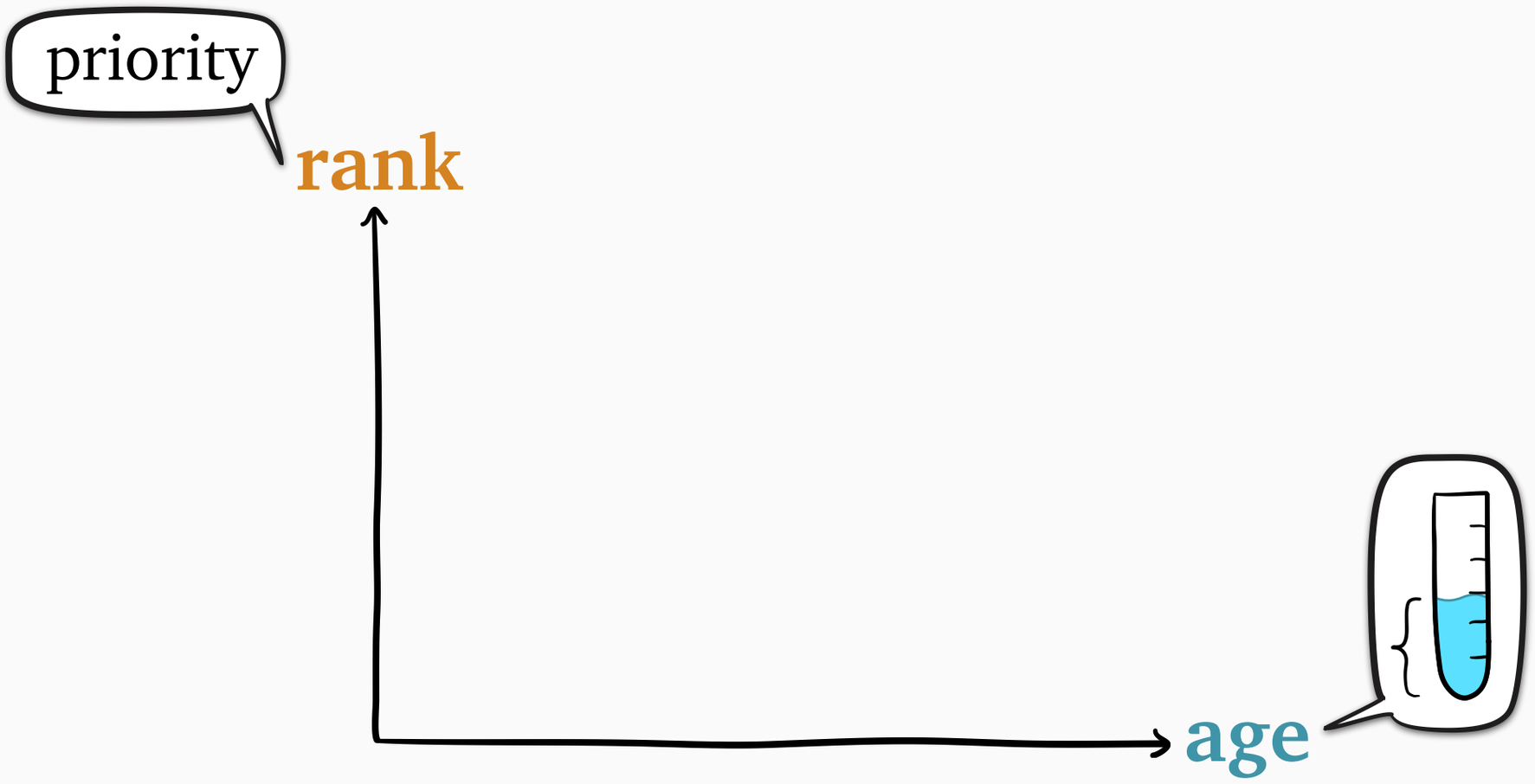
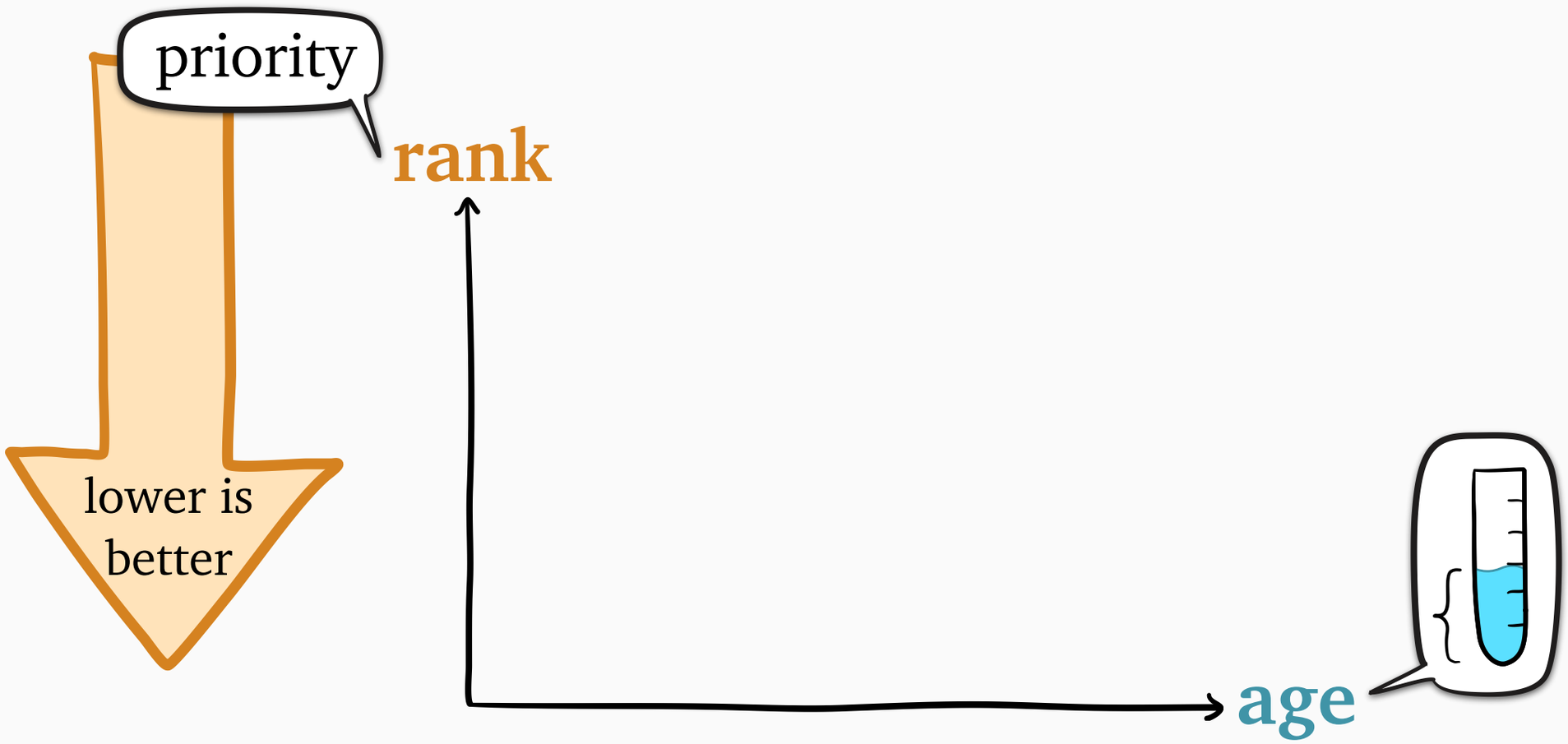


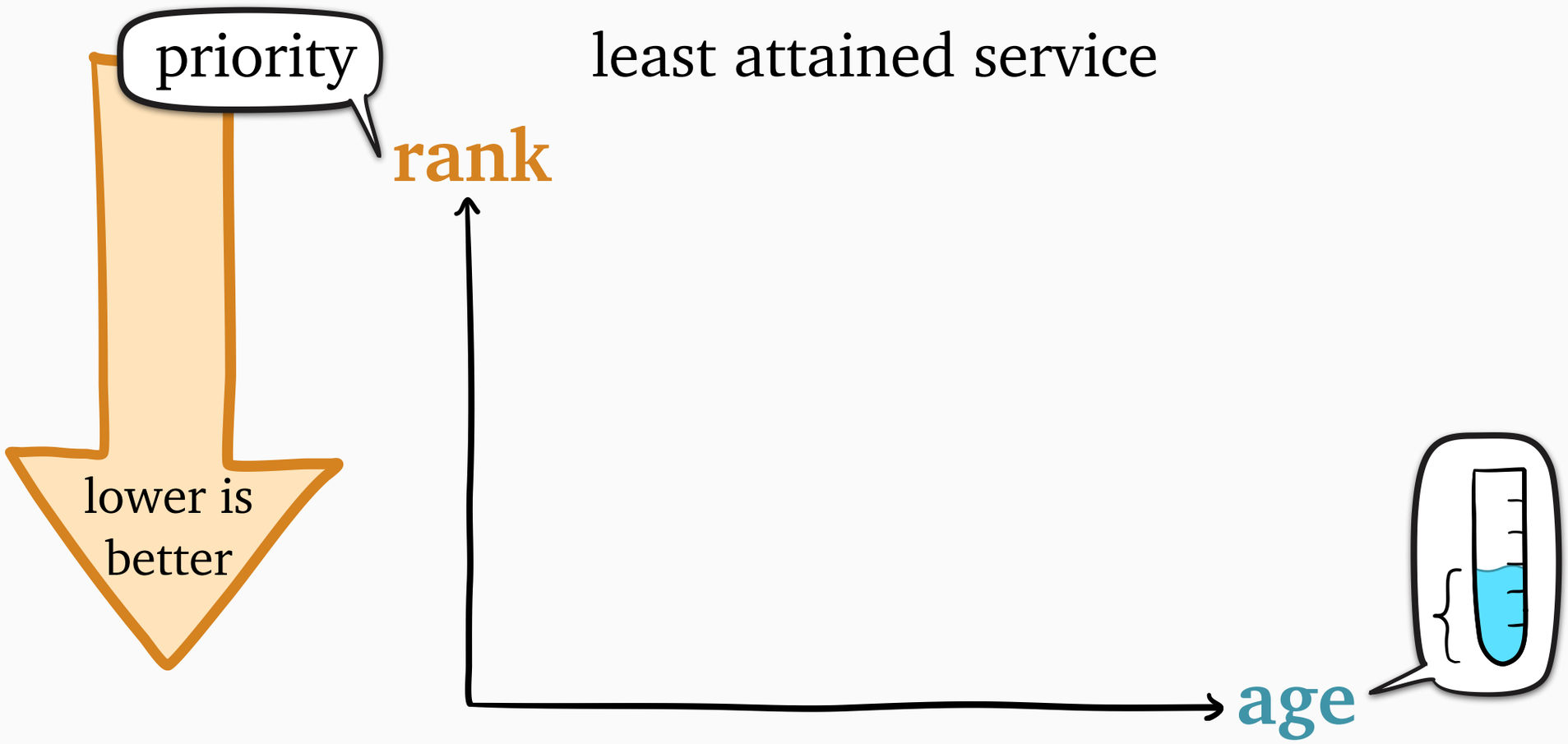
Unifying language: **rank** functions



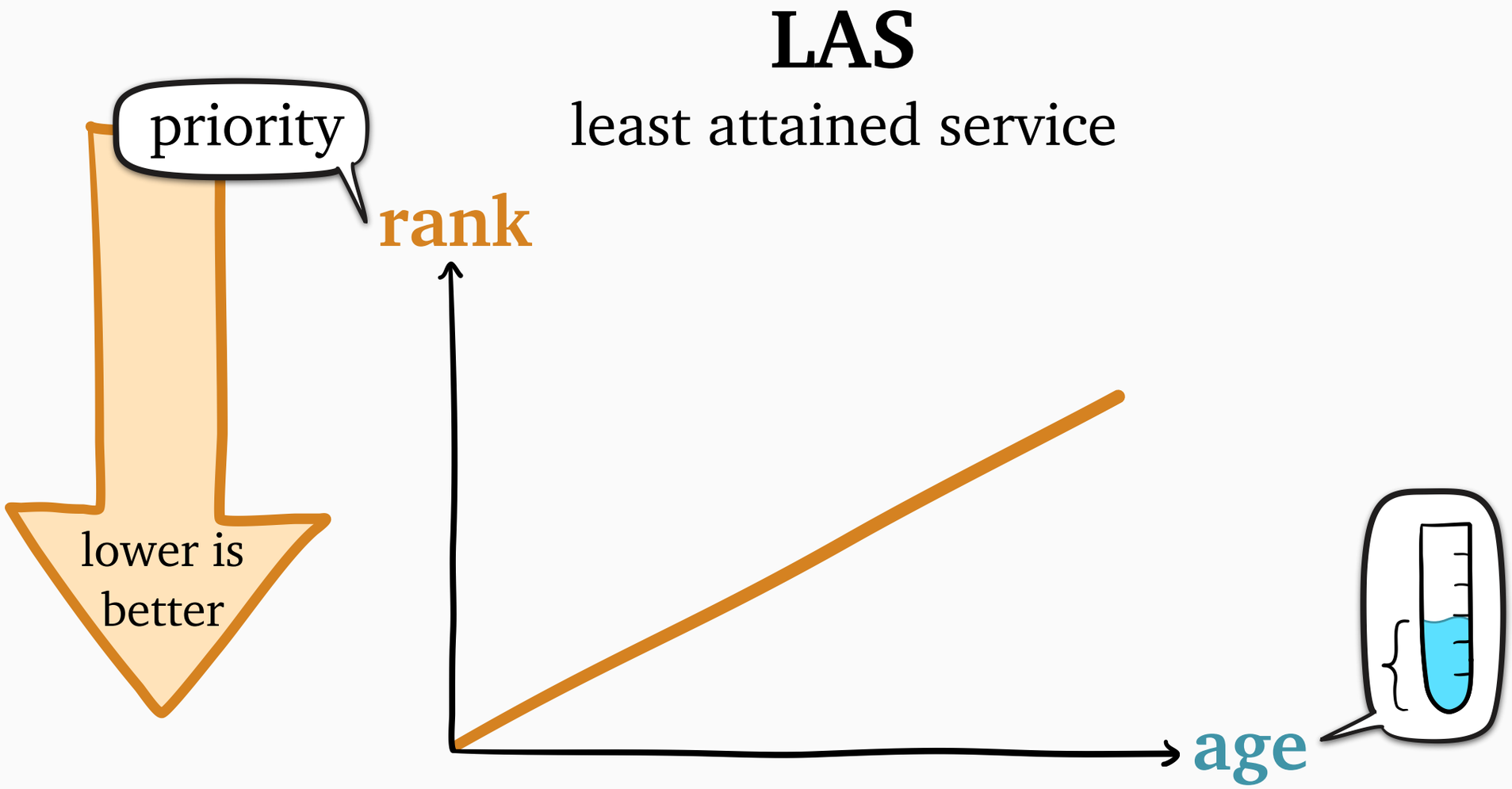
Unifying language: **rank** functions

LAS

least attained service



Unifying language: rank functions



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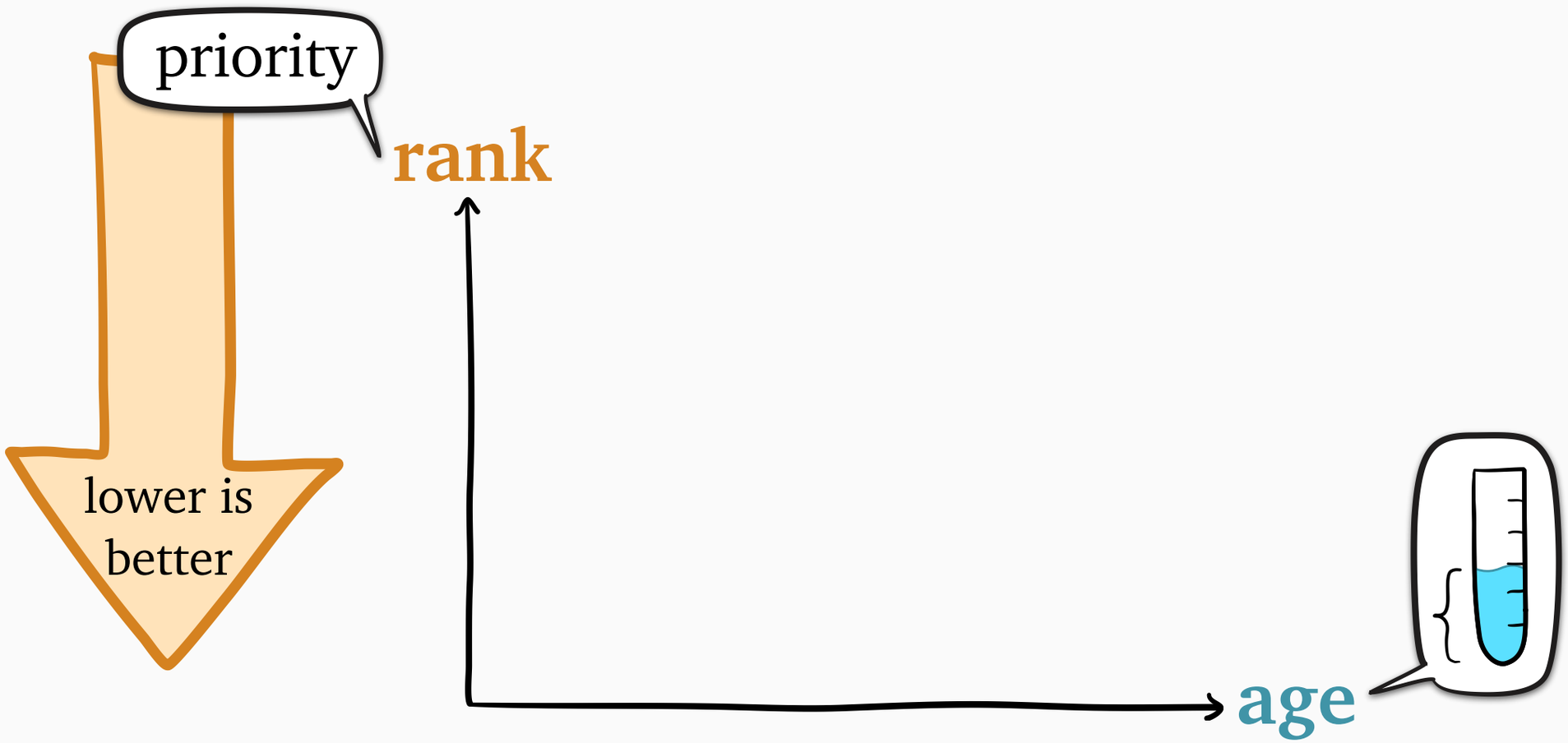
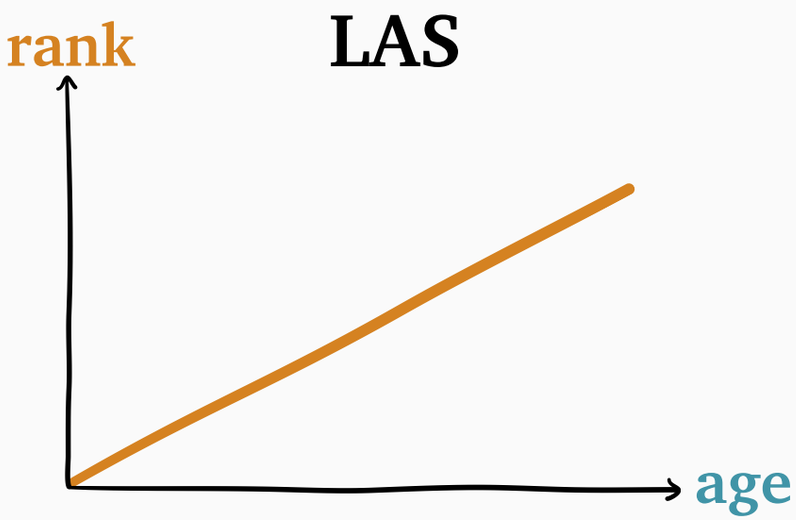
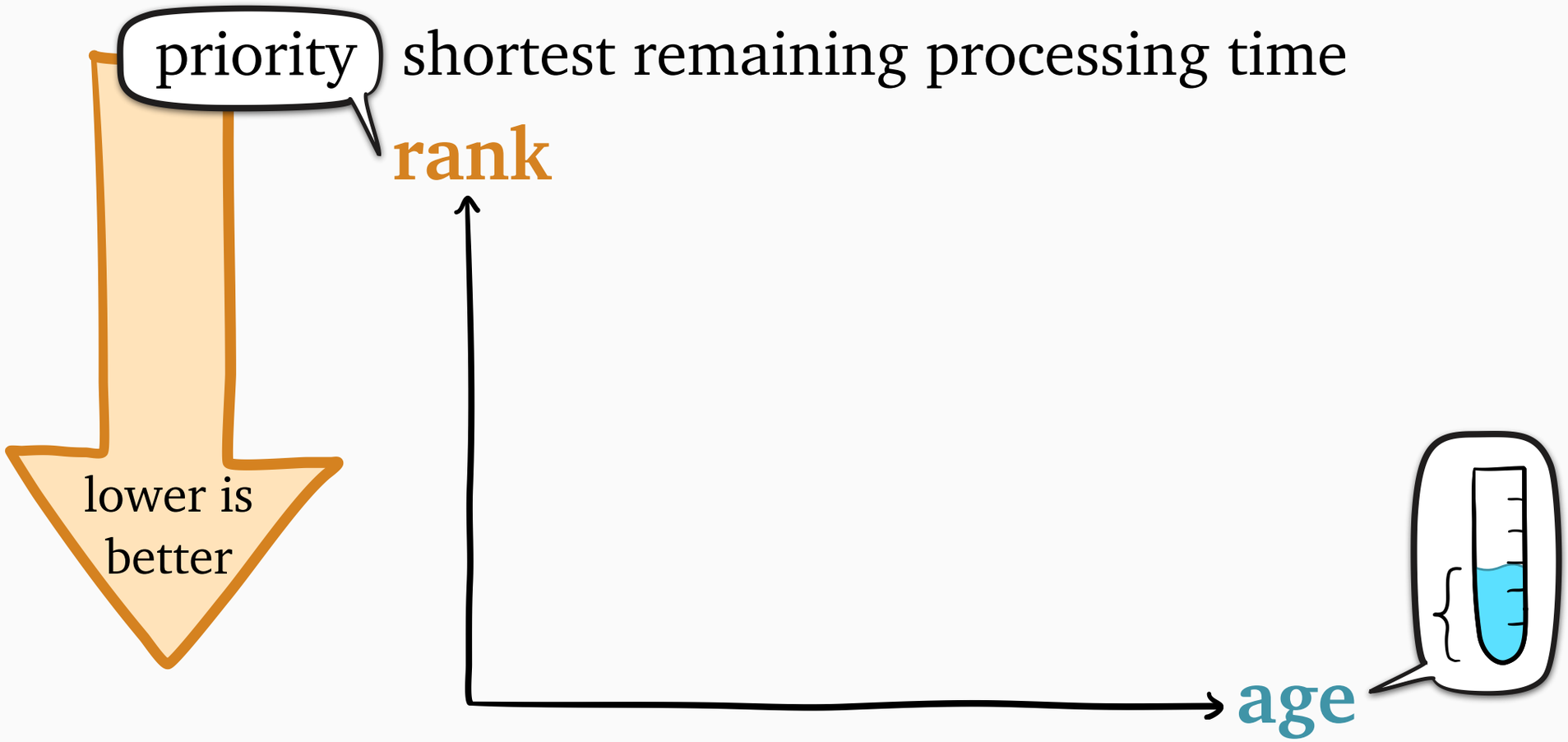


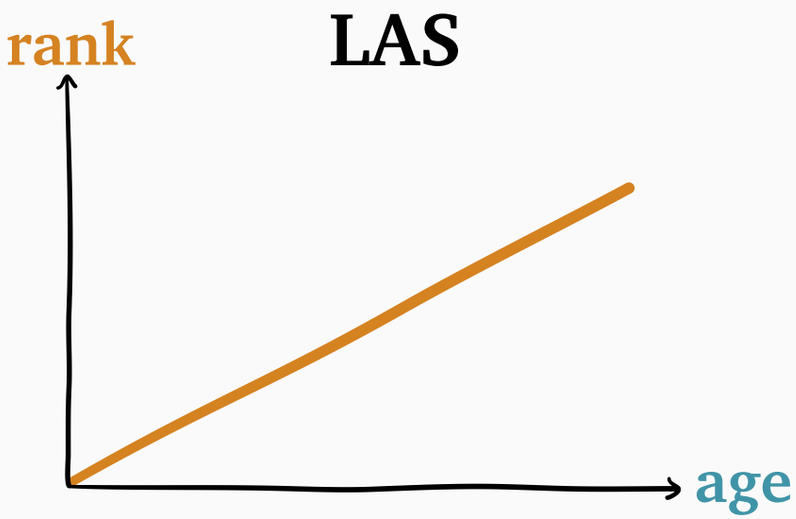


Unifying language: **rank** functions

SRPT



LAS



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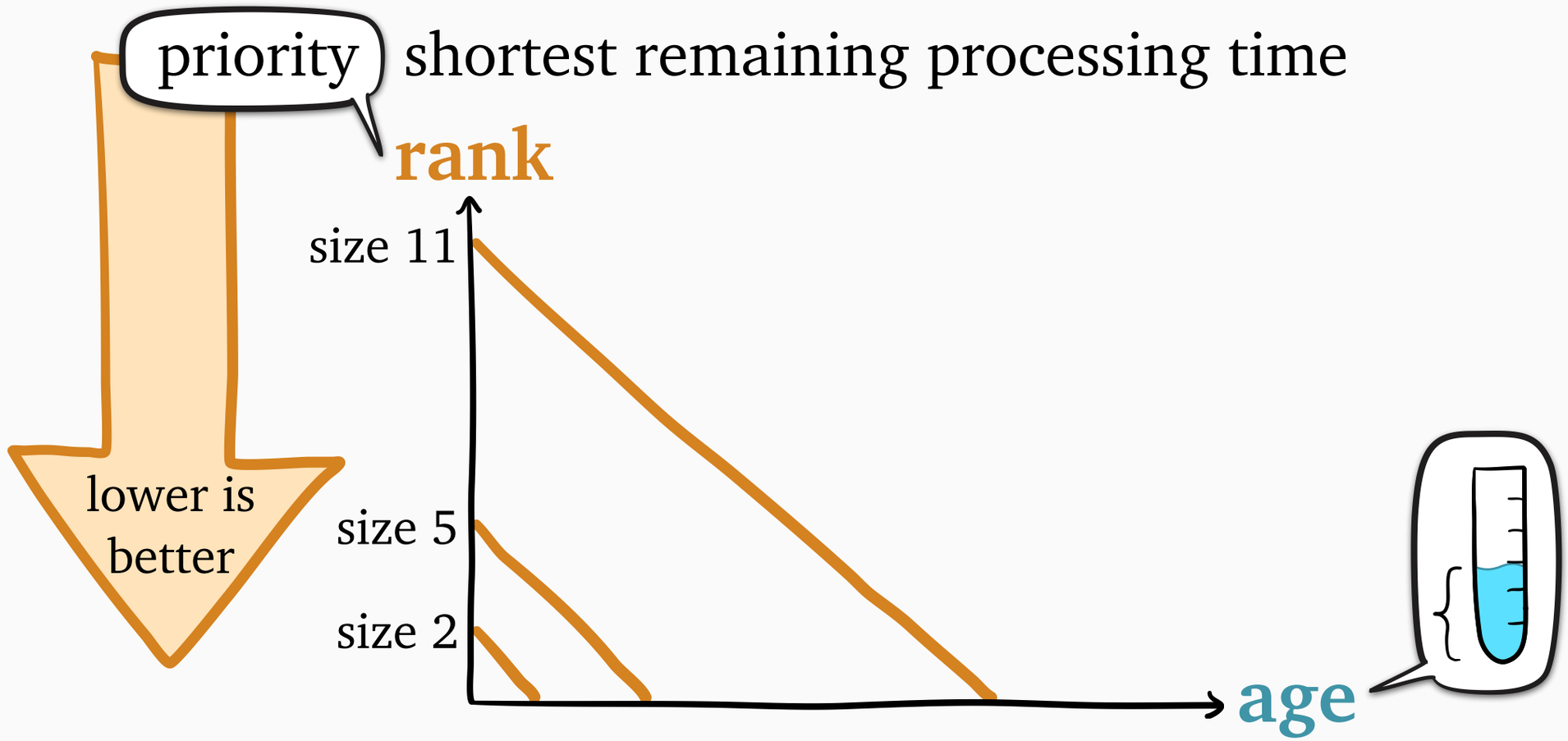

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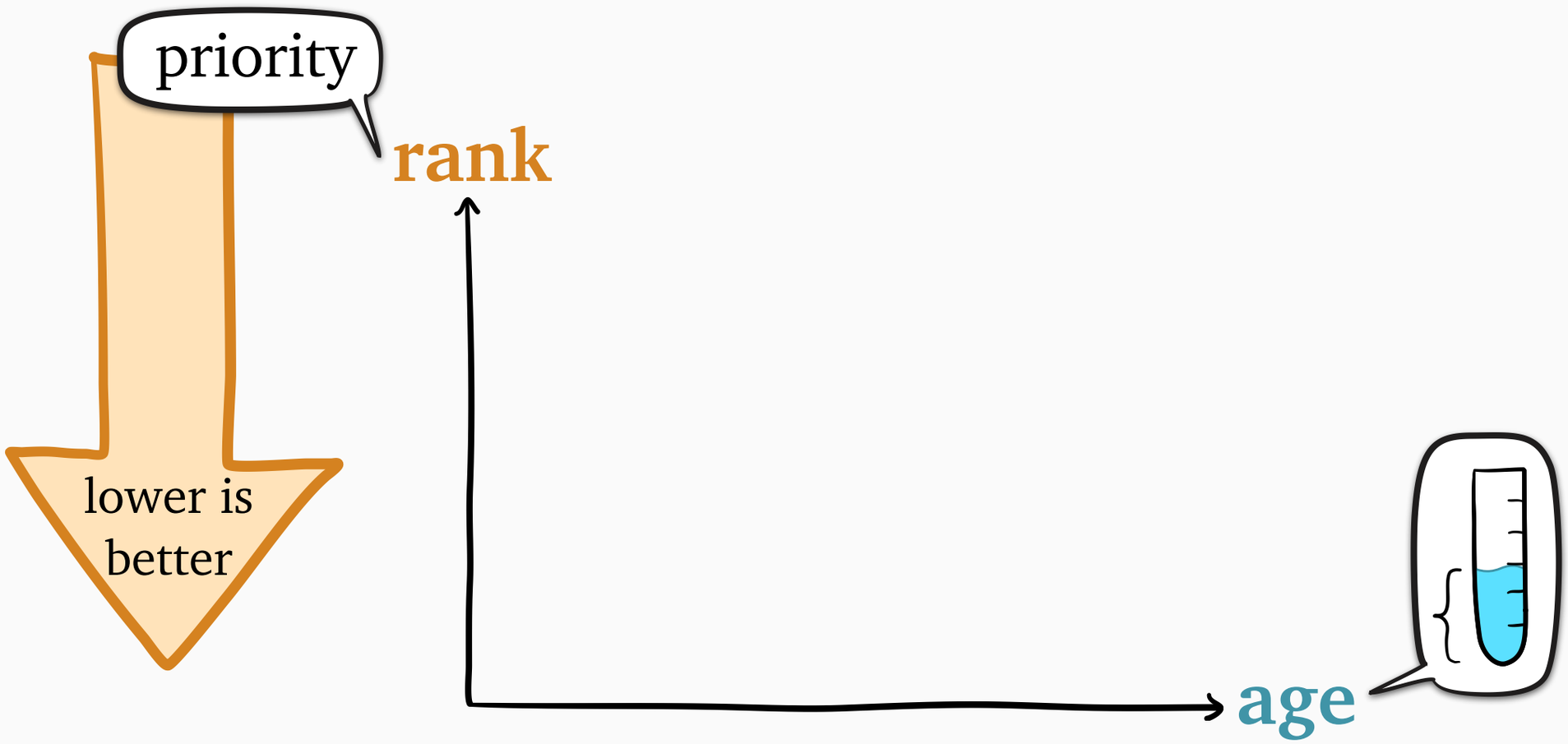
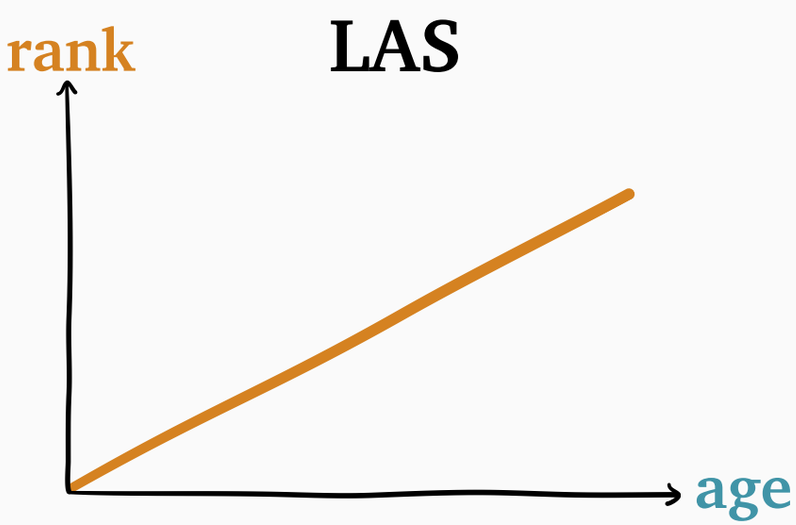
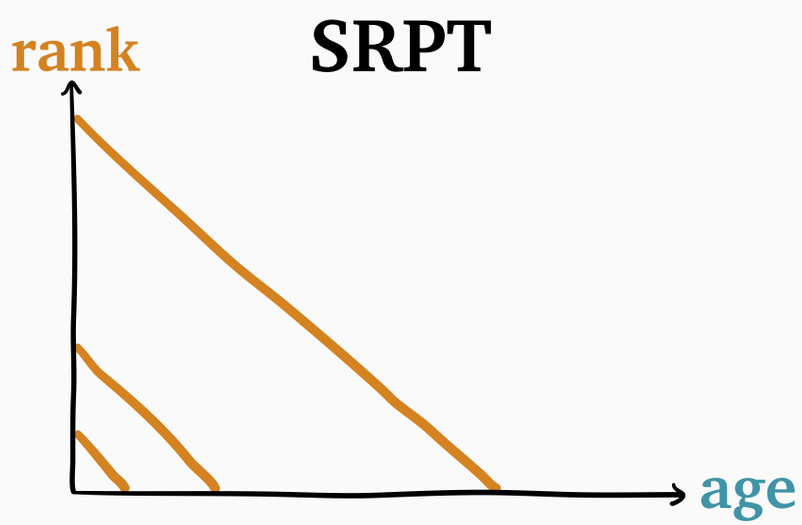


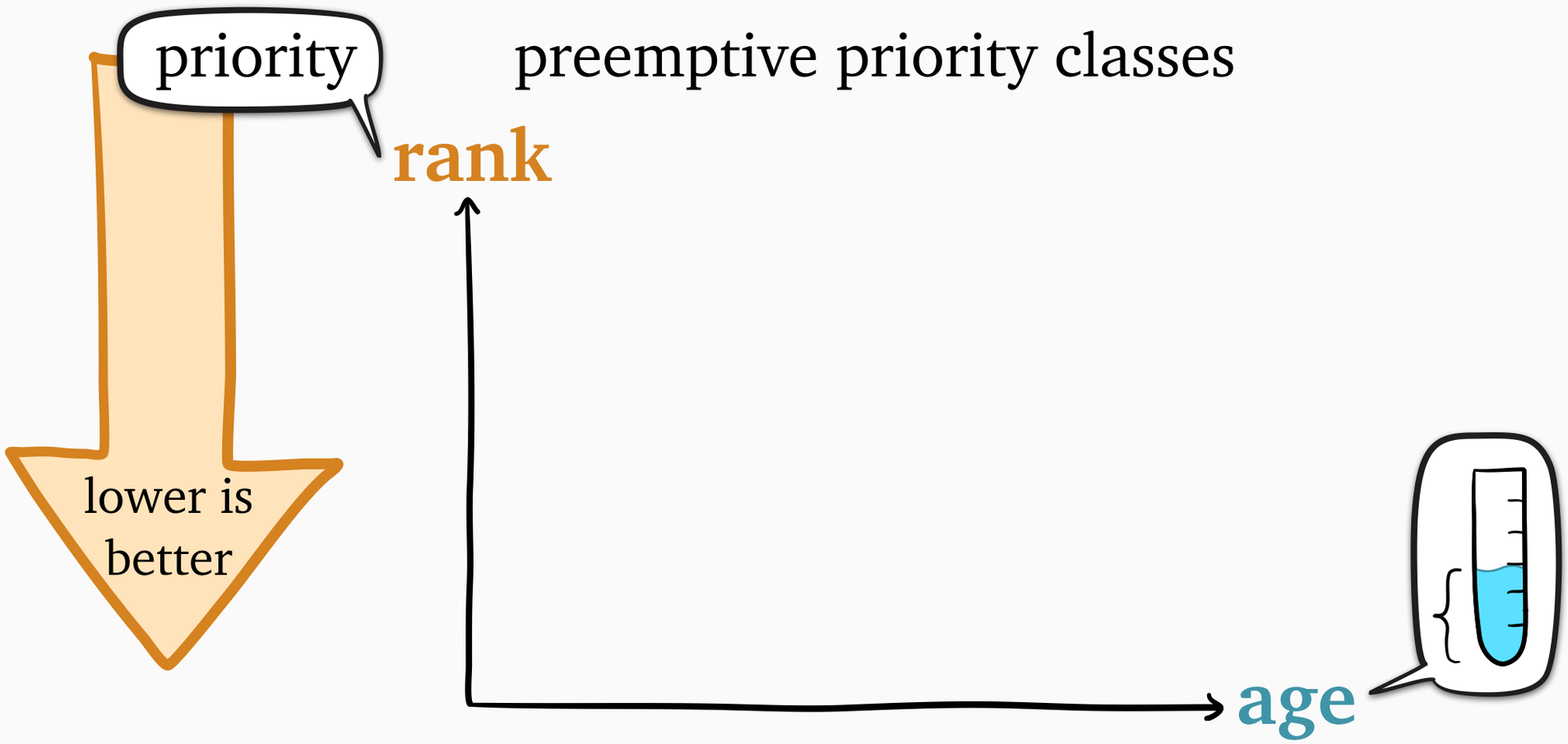




Unifying language: rank functions

P-Prio

preemptive priority classes



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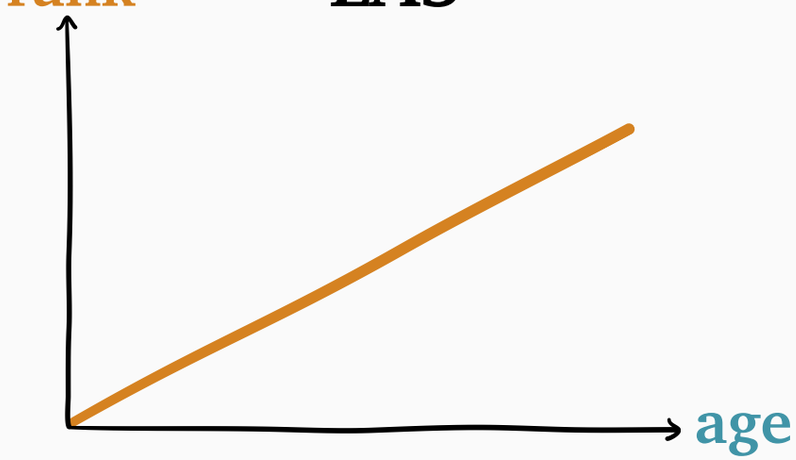

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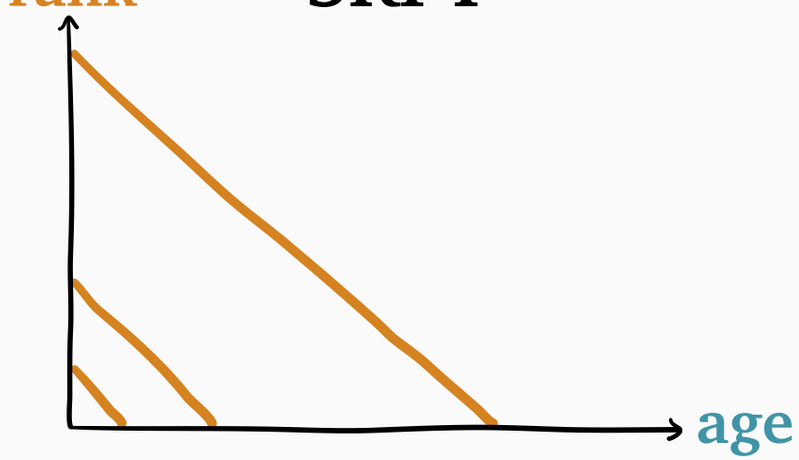

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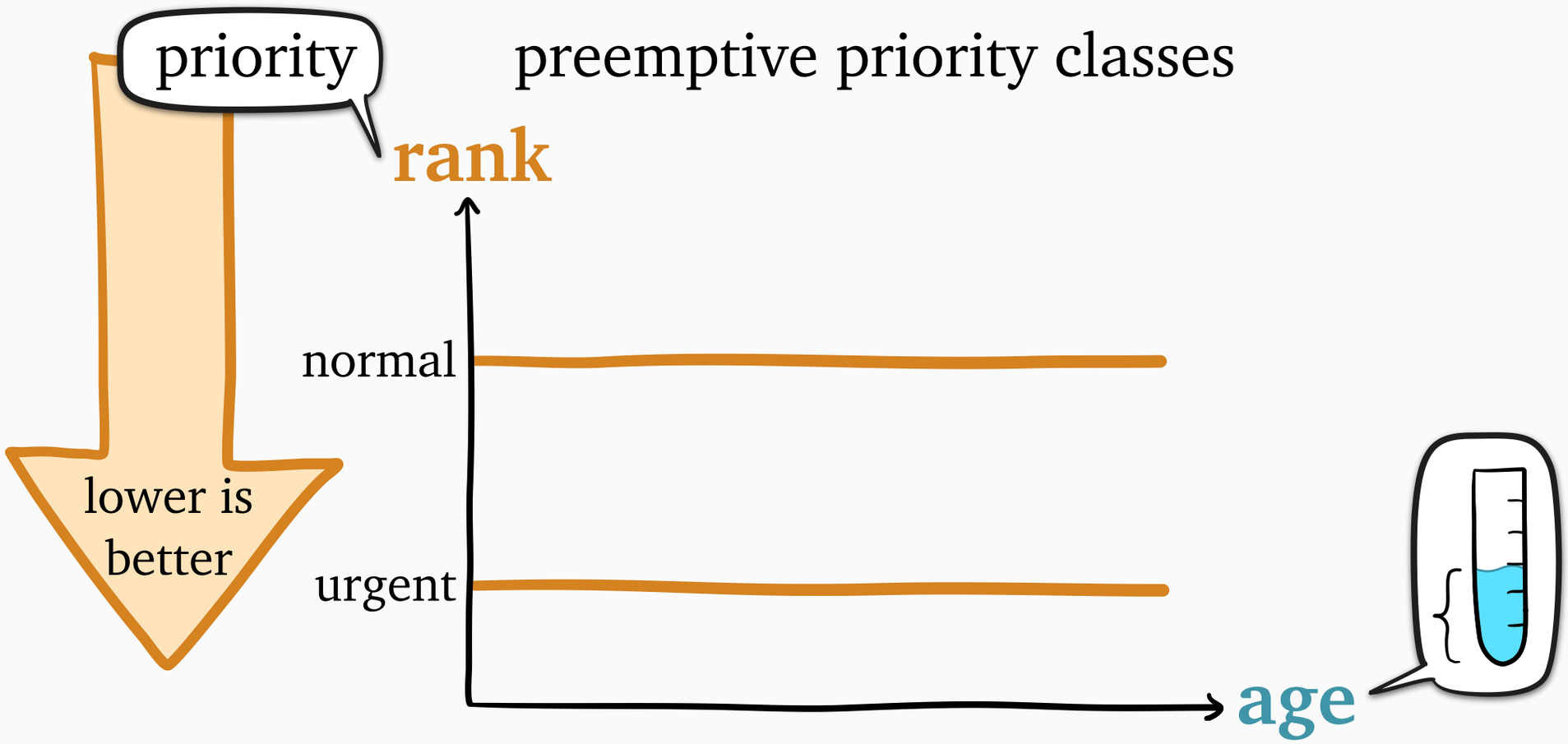

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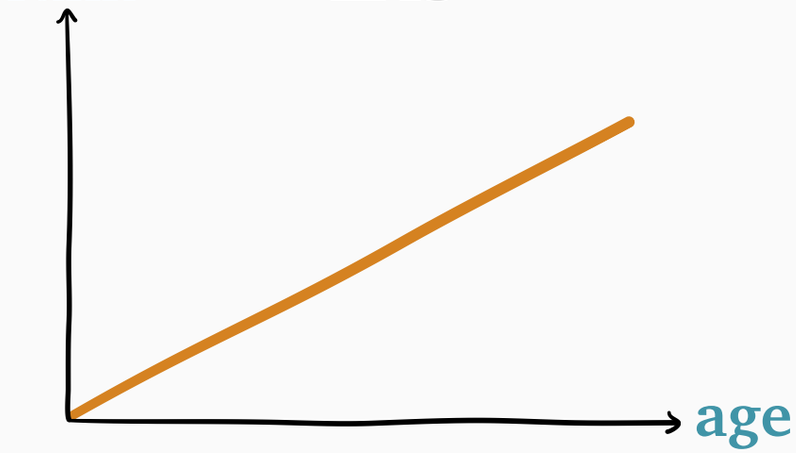

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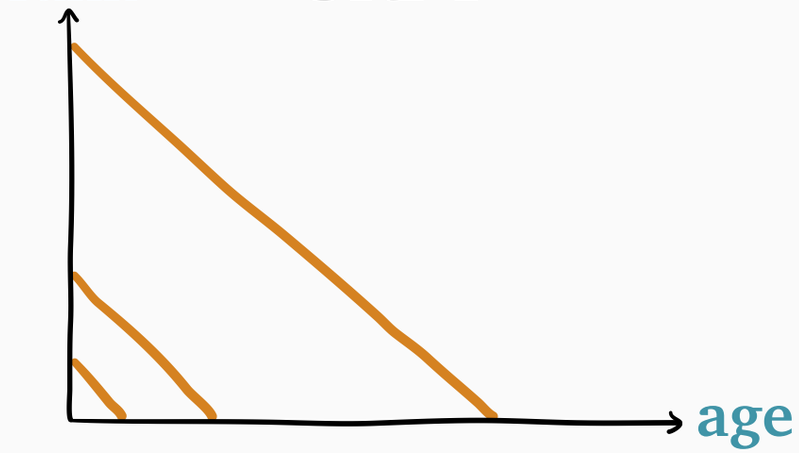
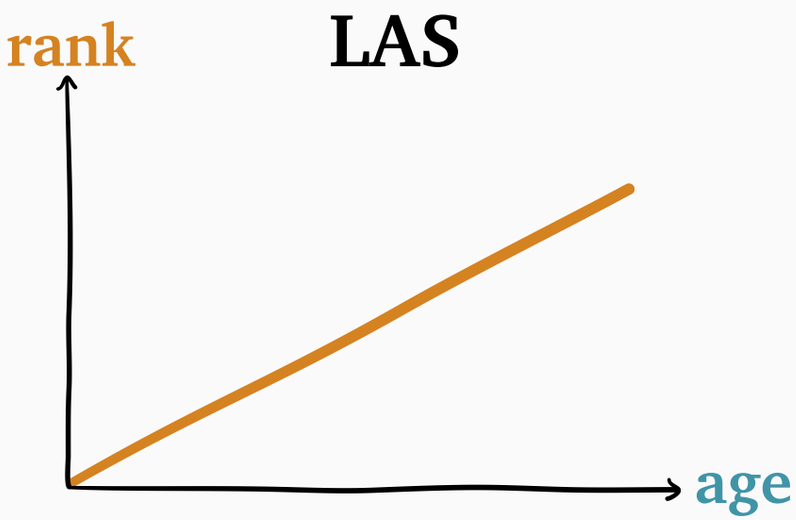
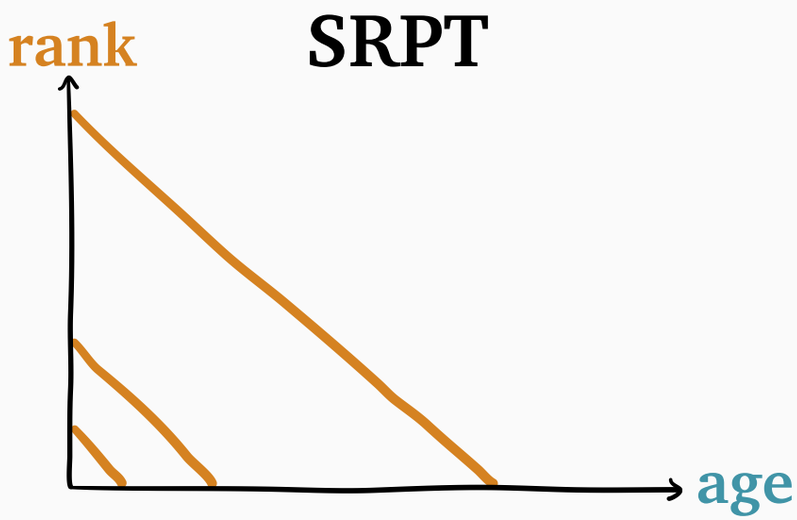
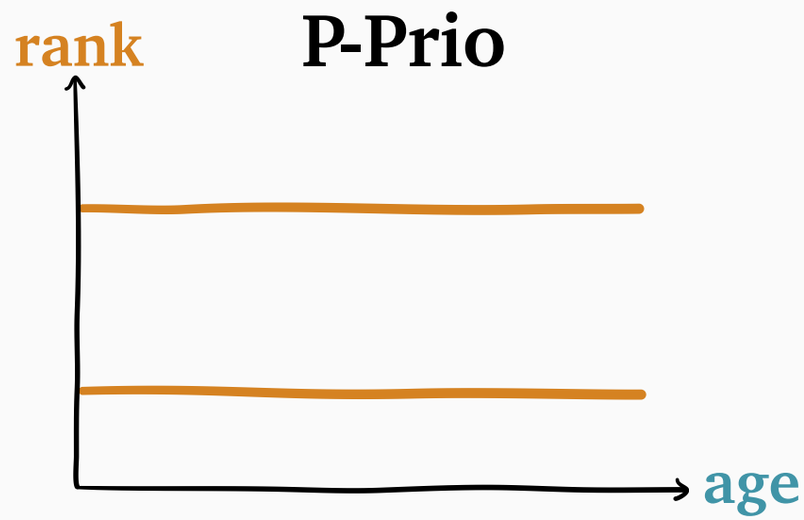


Unifying language: **rank** functions







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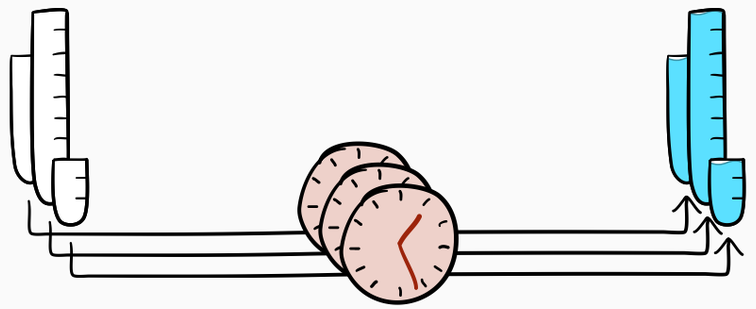
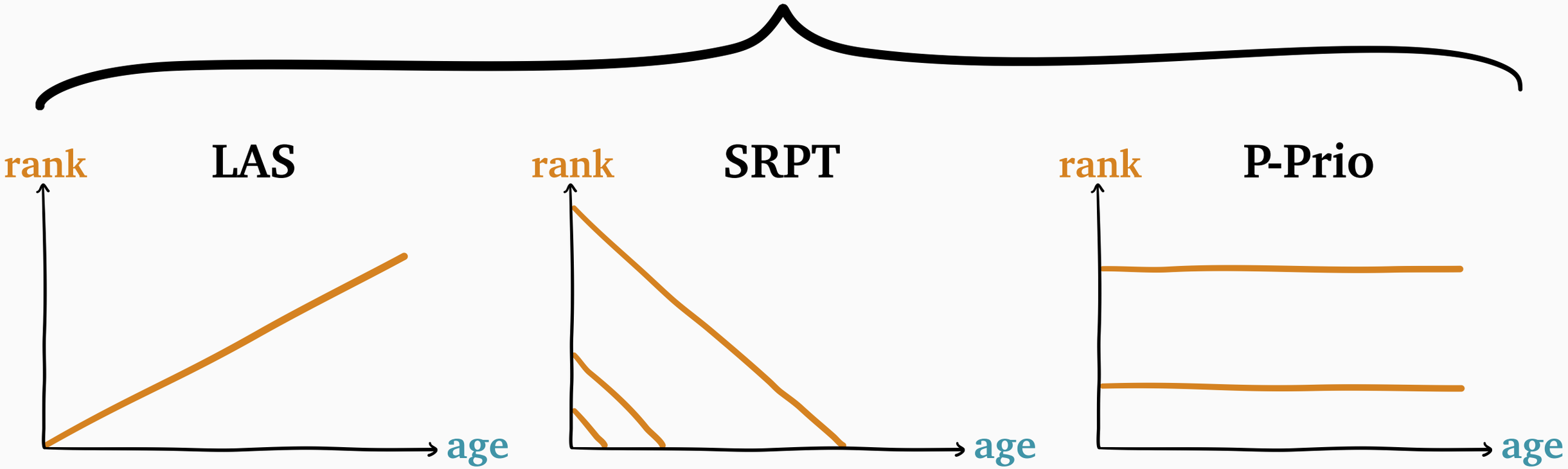
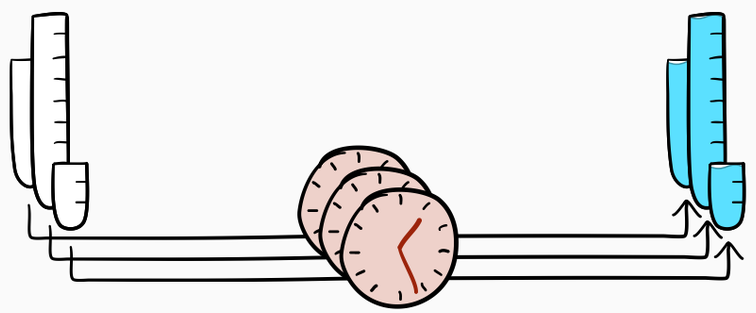


response time T known

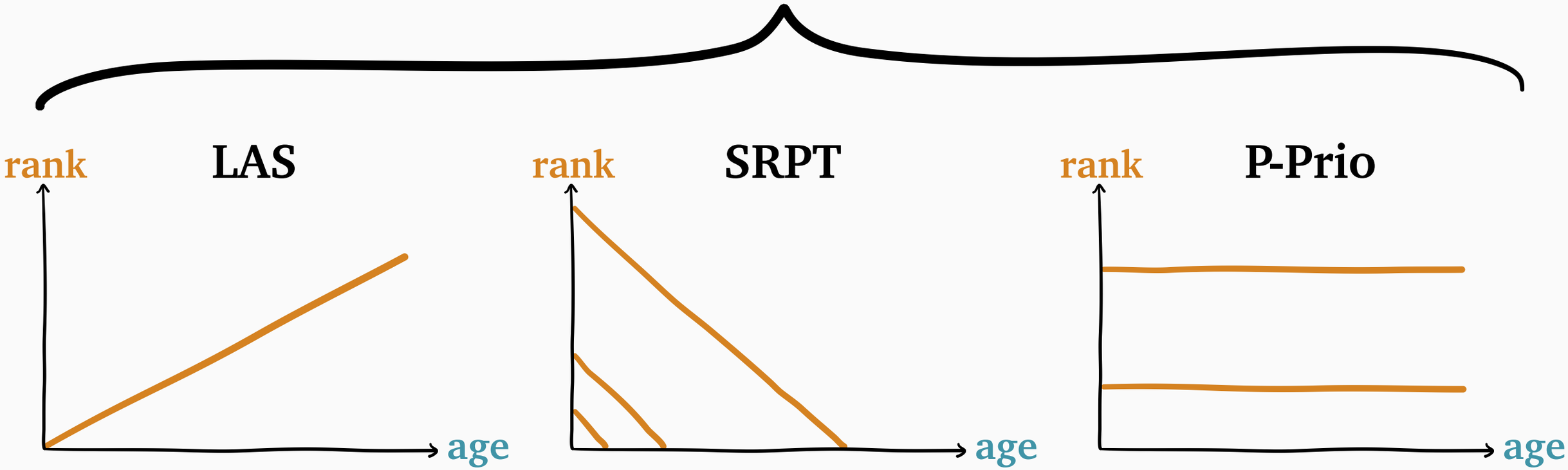


Unifying language: **rank** functions

"Simple" **policy** = ? **rank** function

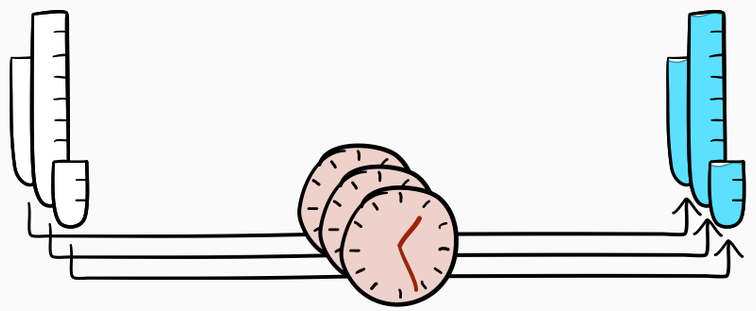


response time T known

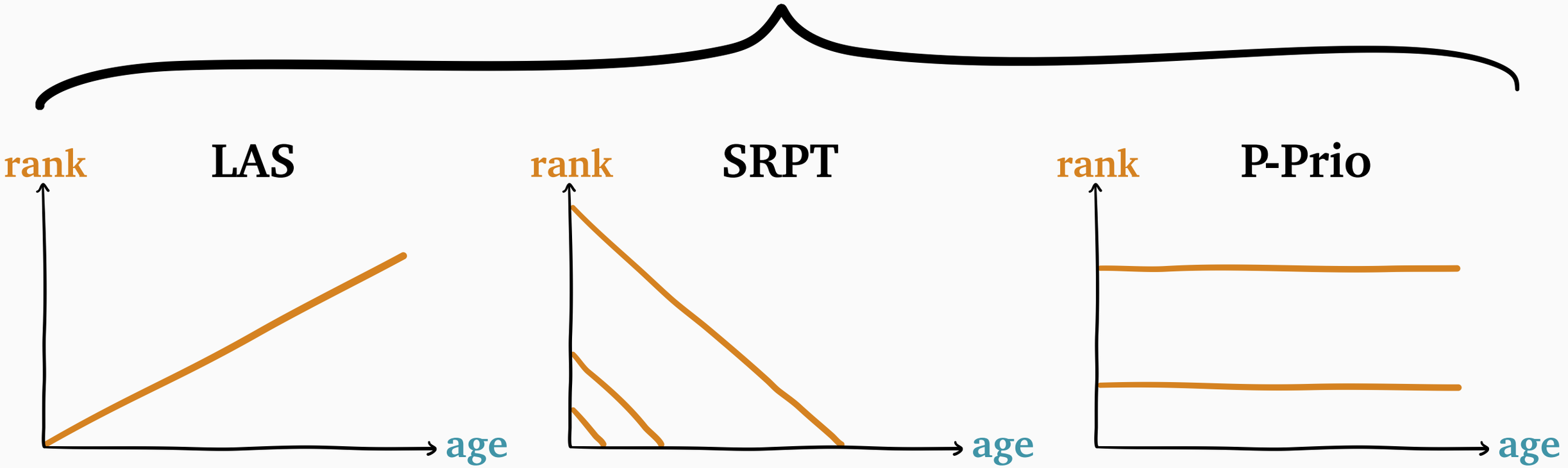


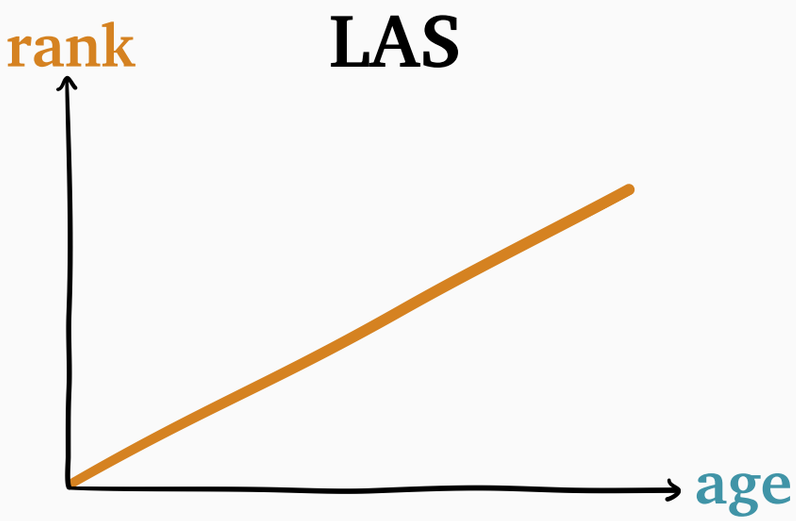
Unifying language: **rank** functions

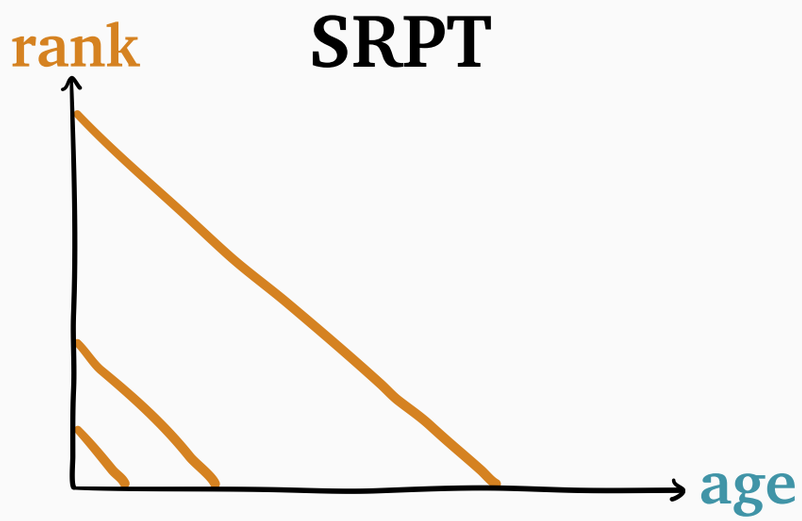
"Simple" **policy** = *monotonic* **rank** function

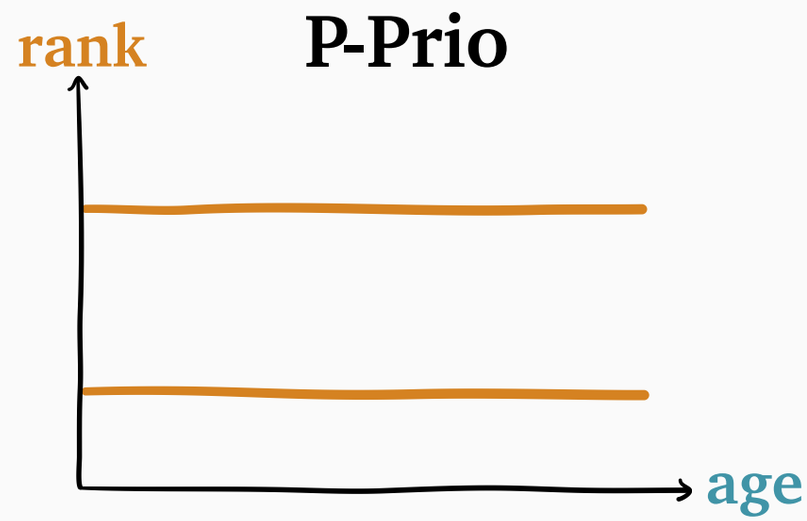


response time T known

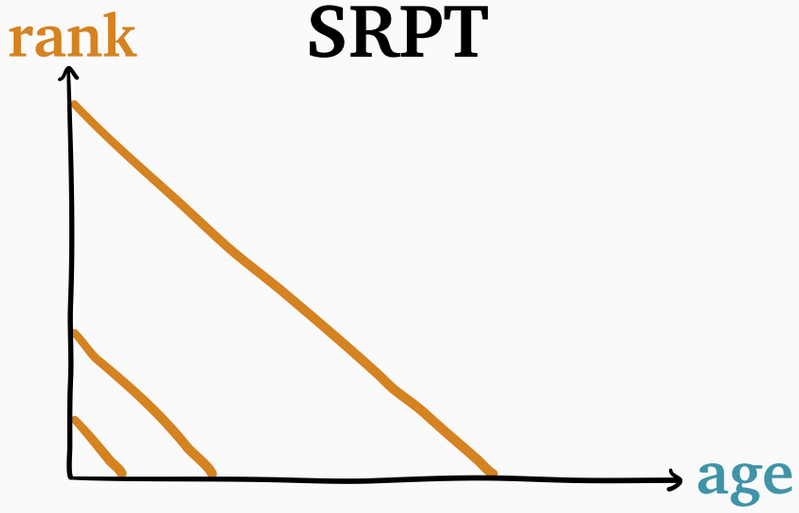


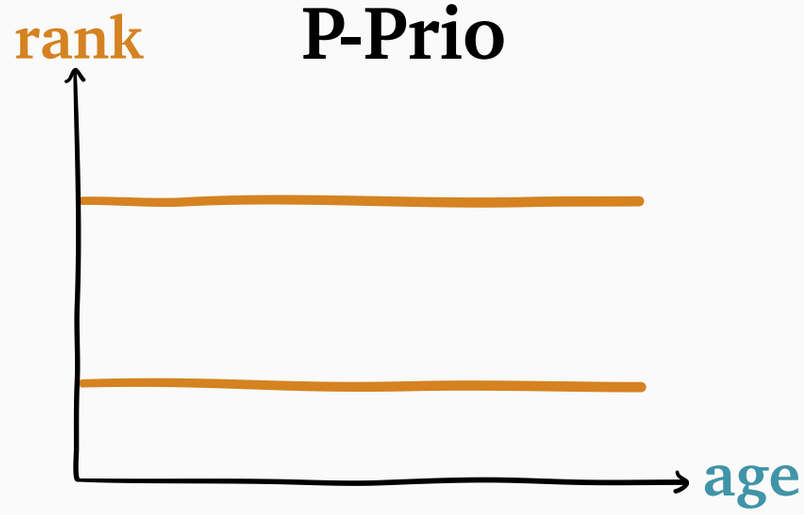






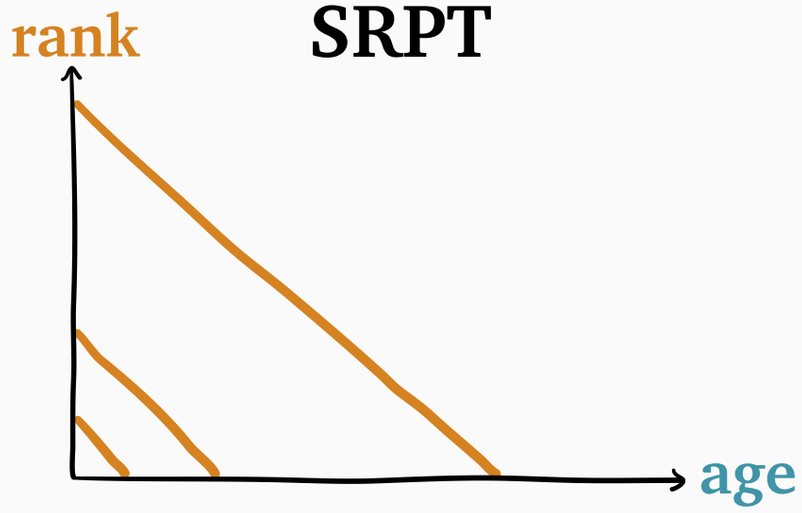
"Simple" = *monotonic* **rank** function

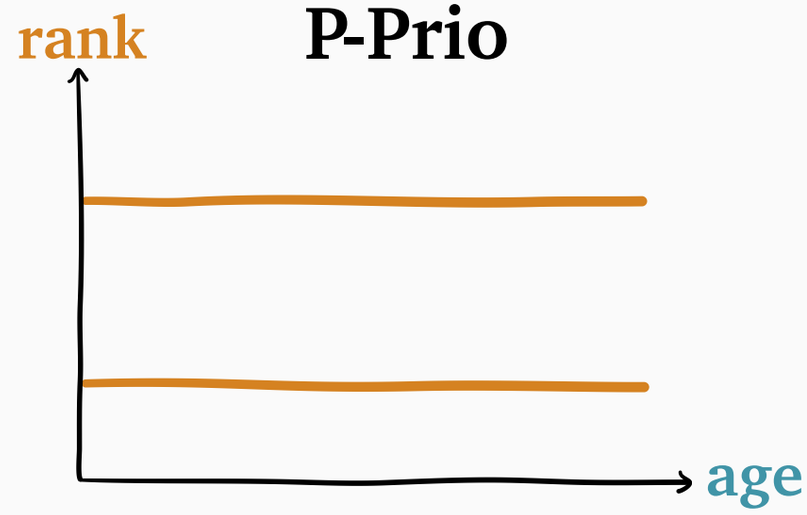




"Simple" = *monotonic* **rank** function

tractable before **SOAP**

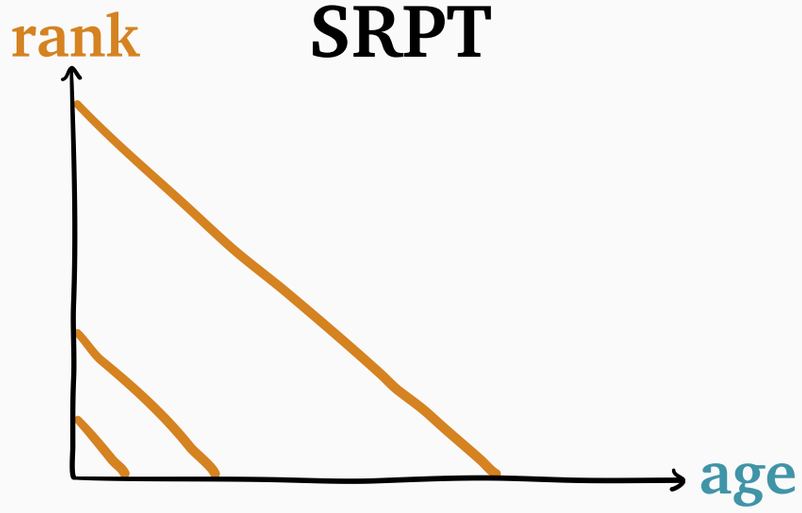


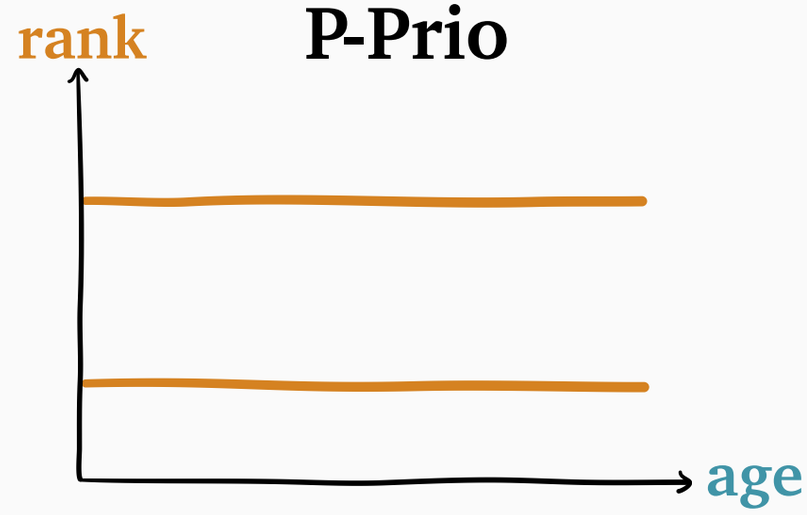


"Simple" = *monotonic* **rank** function

tractable before **SOAP**

"Complex" = *nonmonotonic* **rank** function





"Simple" = *monotonic* **rank** function

tractable before **SOAP**

"Complex" = *nonmonotonic* **rank** function

intractable before **SOAP**

“Complex” = *nonmonotonic* **rank**

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Motivation:
unknown job sizes

“Complex” = *nonmonotonic* **rank**

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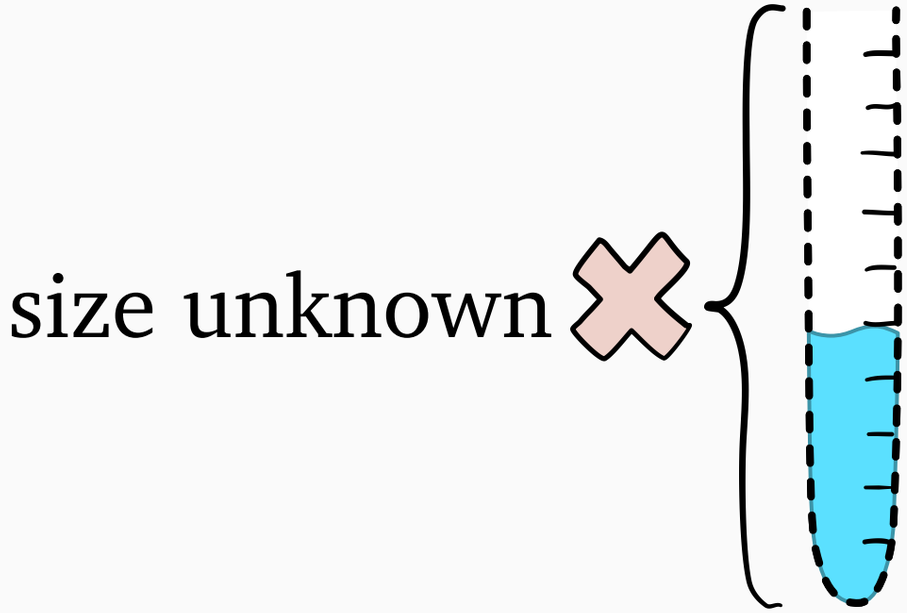

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Motivation:
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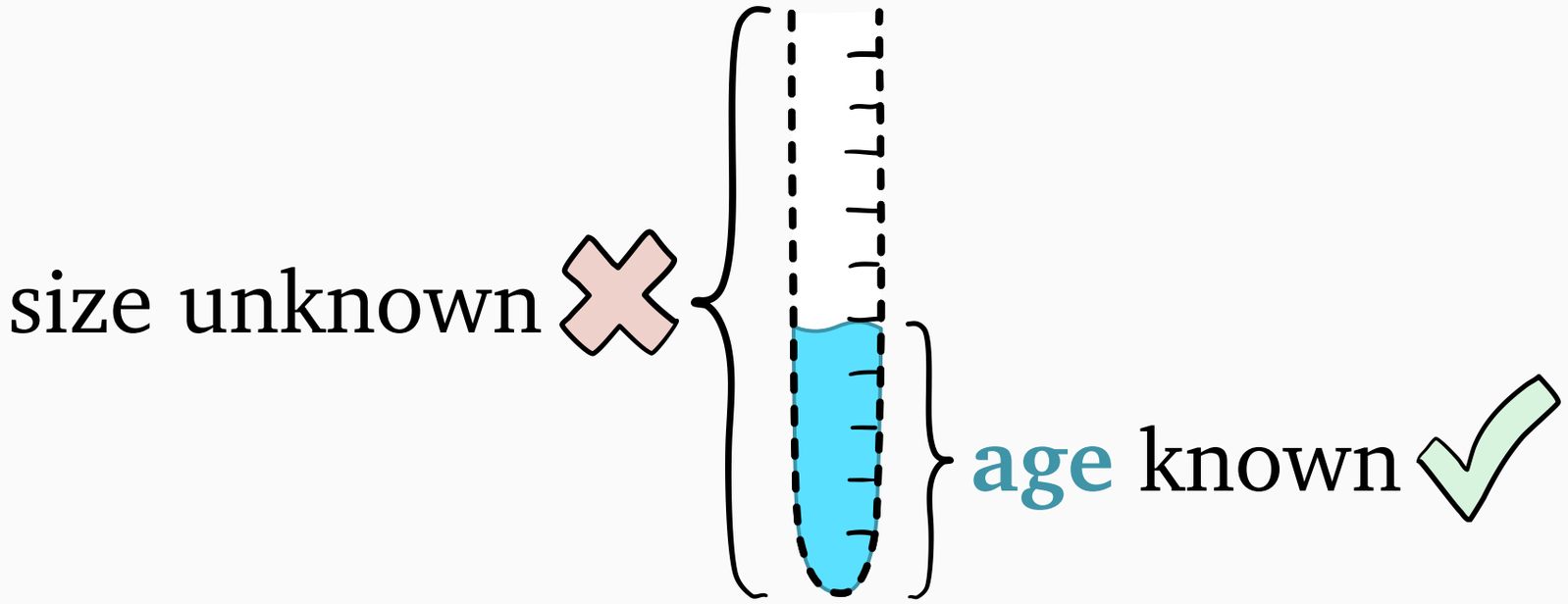

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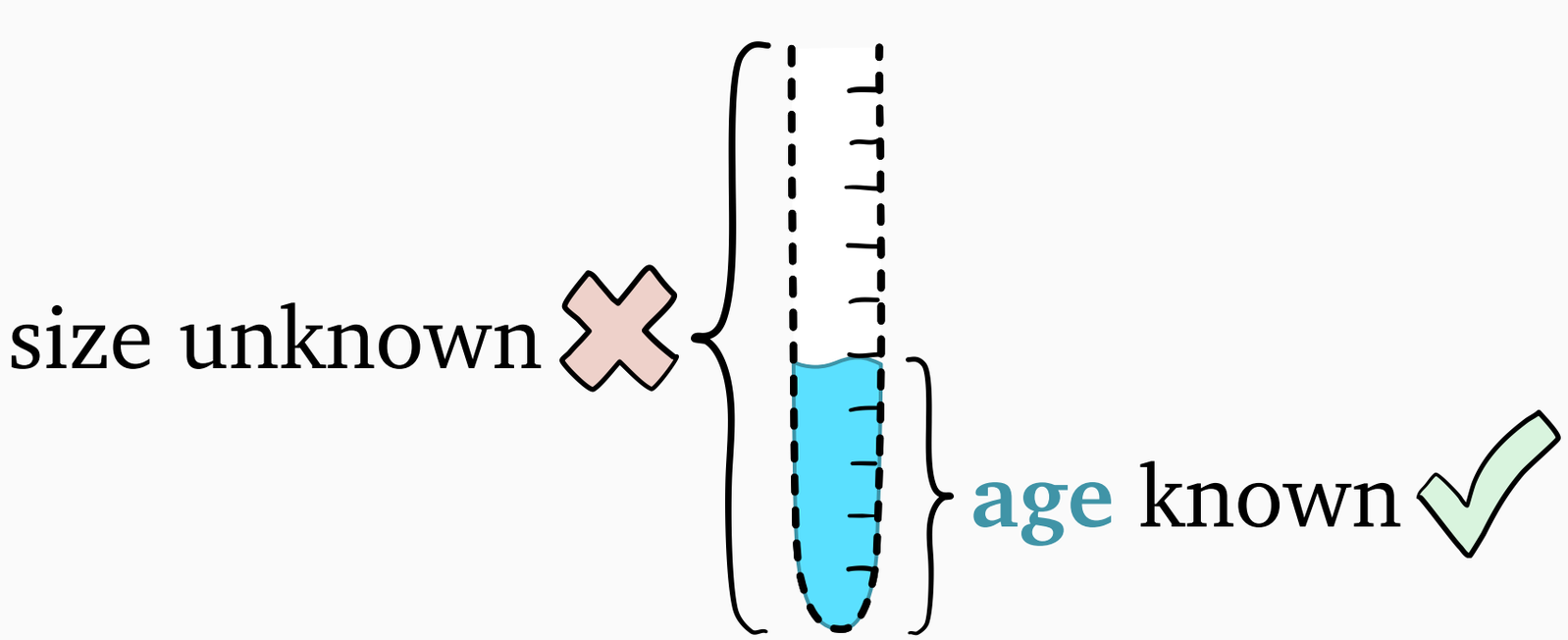
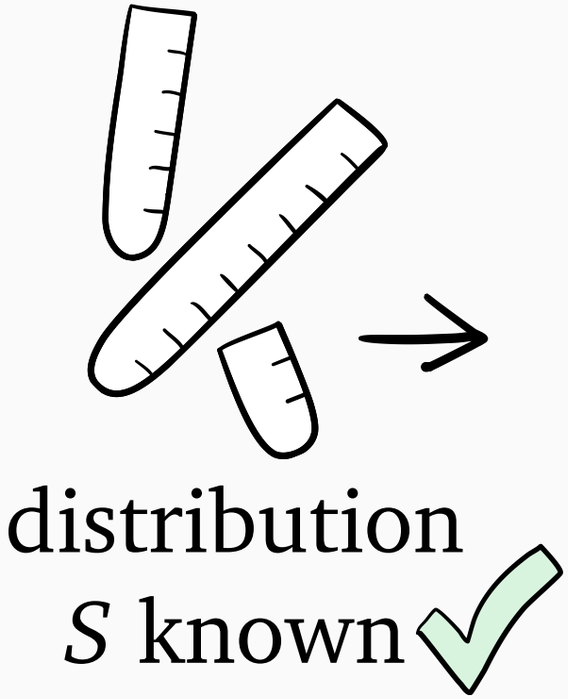


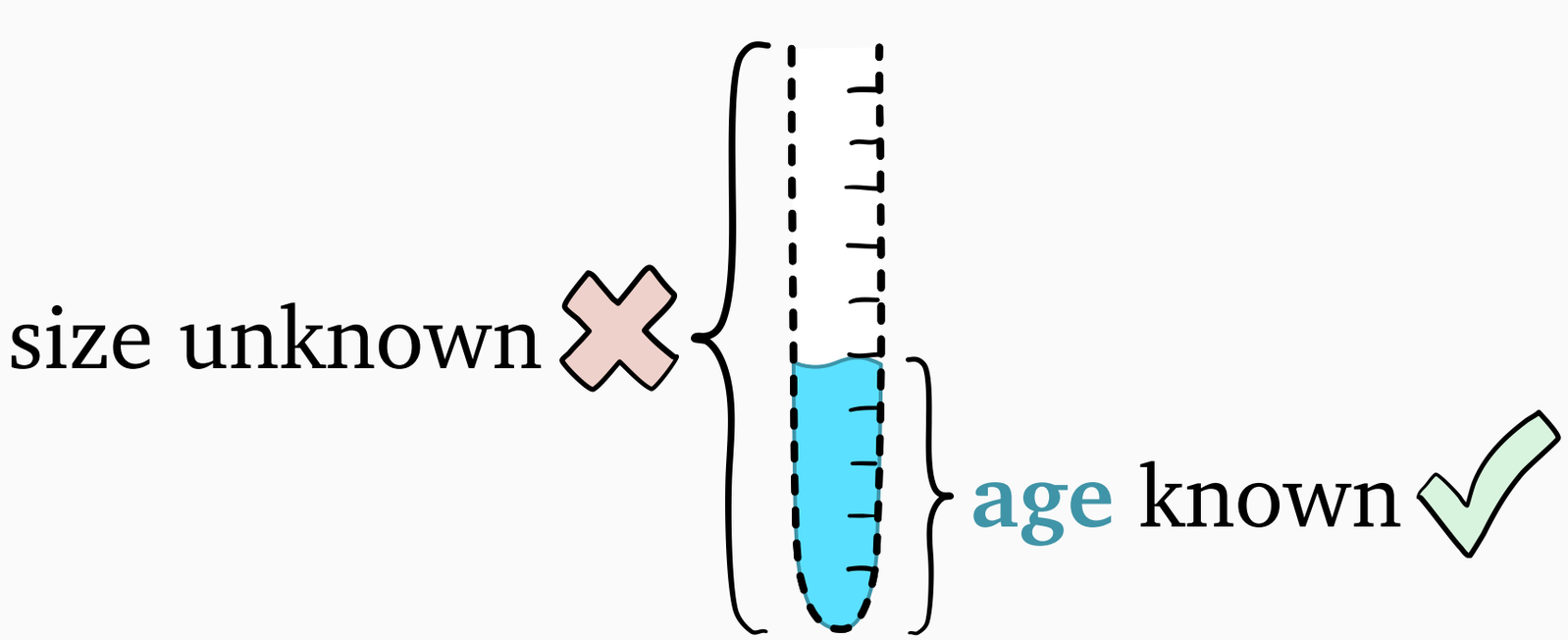


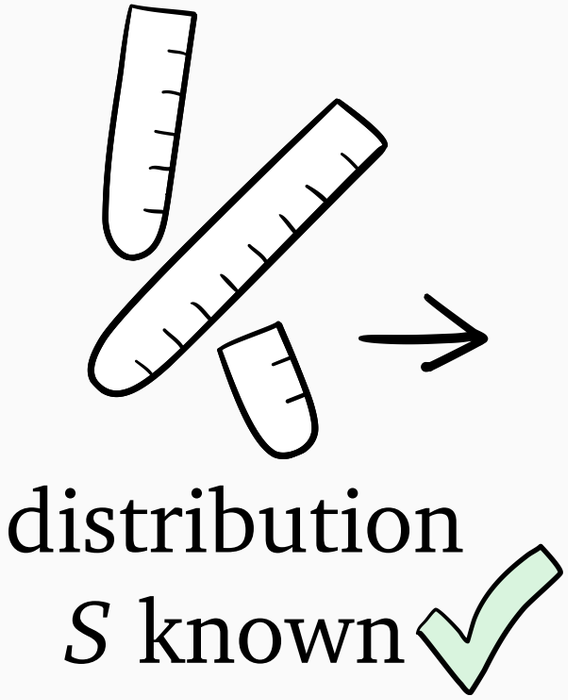
“Complex” = *nonmonotonic* **rank**

Motivation:
unknown job sizes

SERPT

shortest *expected* remaining processing time





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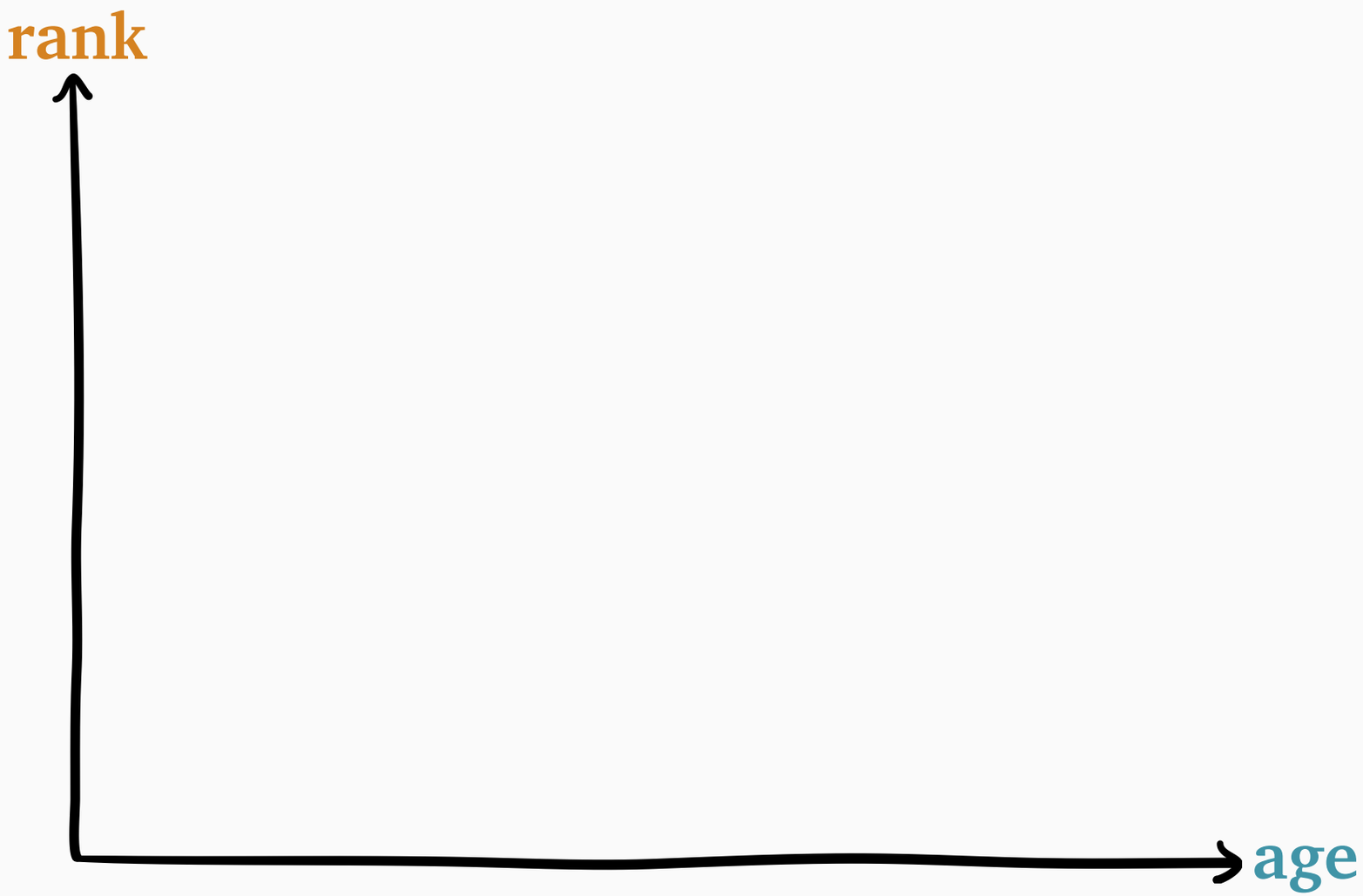


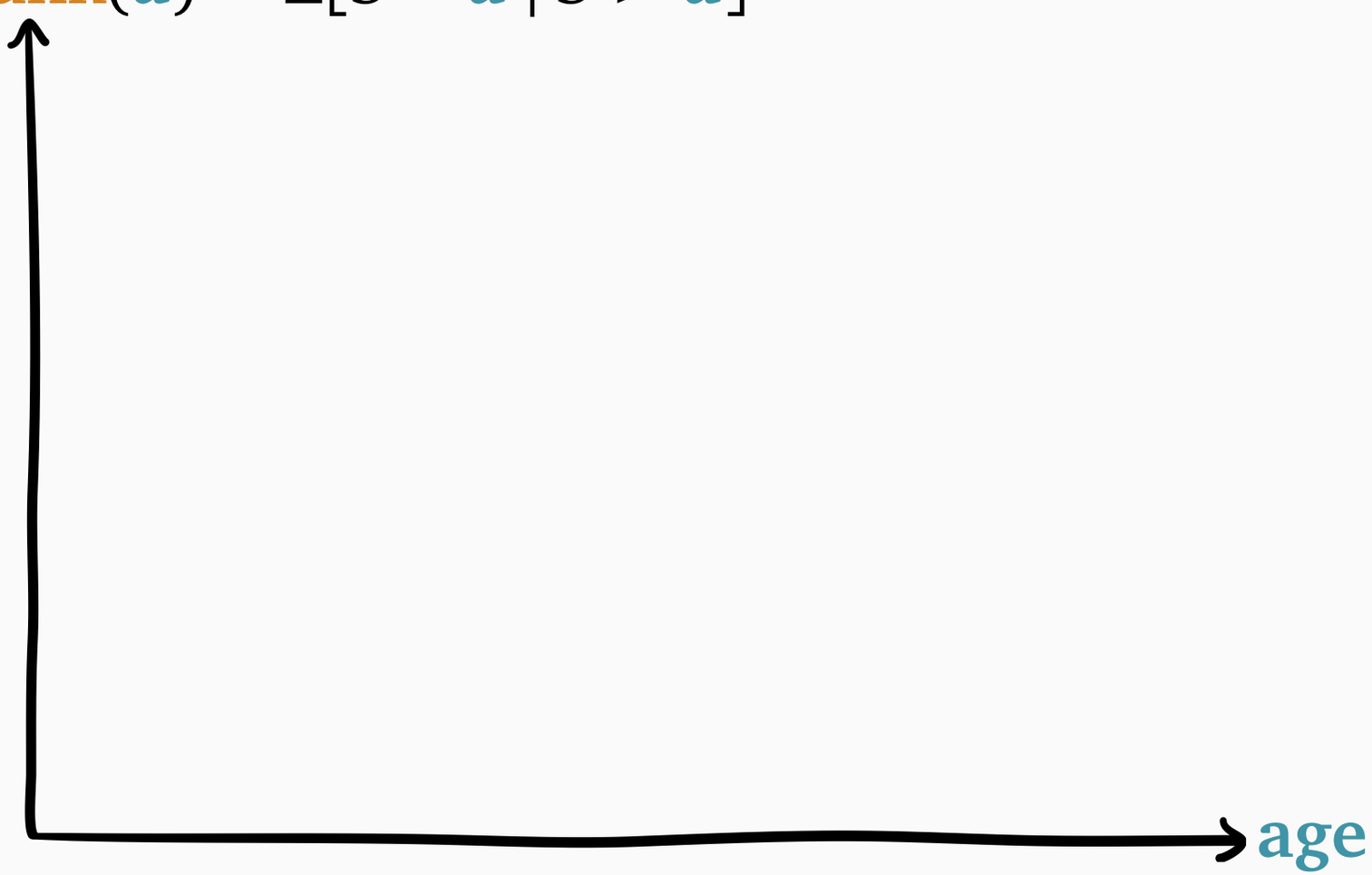
"Complex" = *nonmonotonic* **rank**

Motivation:
unknown job sizes

SERPT

shortest *expected* remaining processing time

$$\textbf{rank}(a) = \mathbf{E}[S - a \mid S > a]$$



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SERPT

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Job size distribution:

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$

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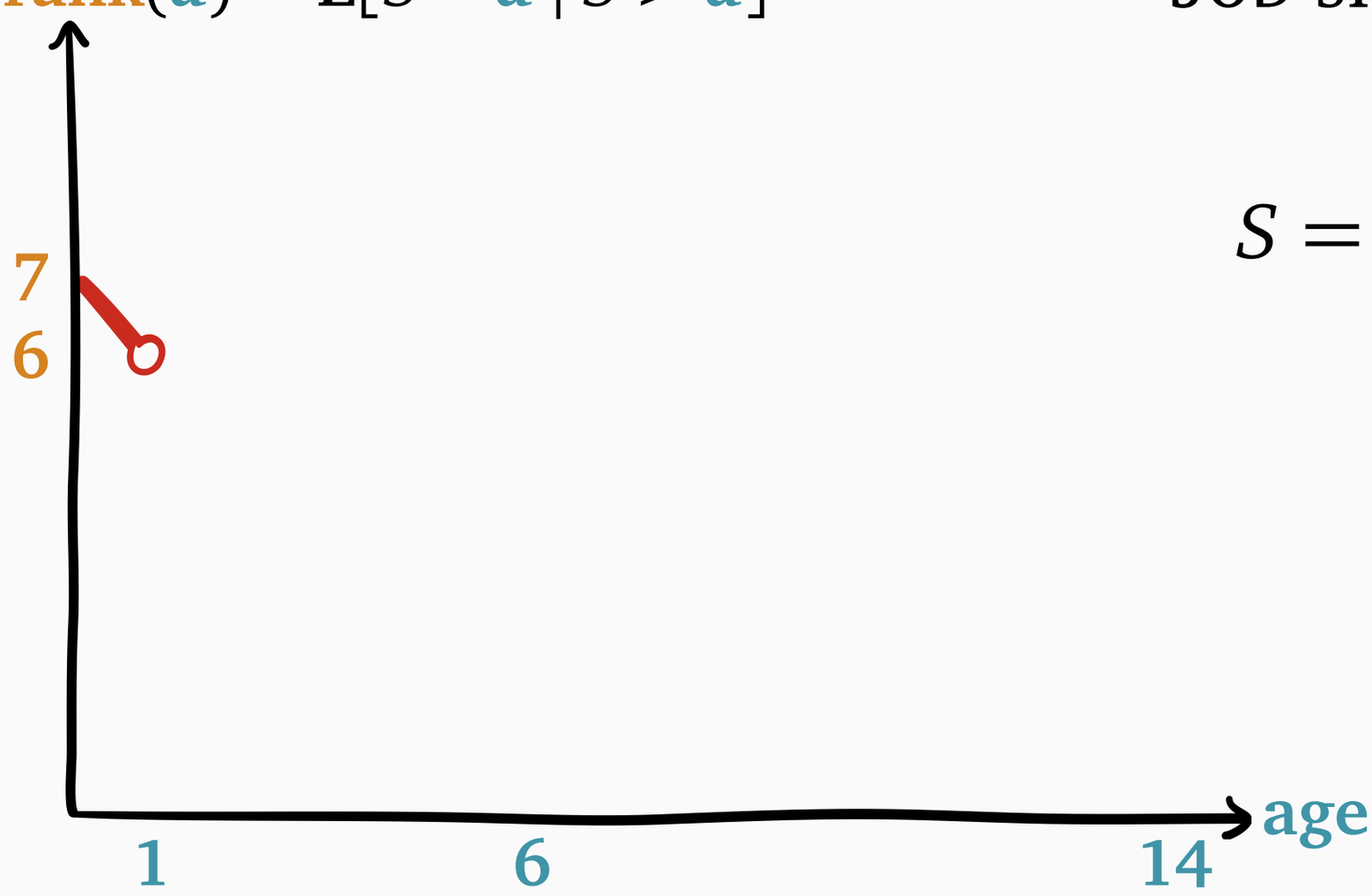

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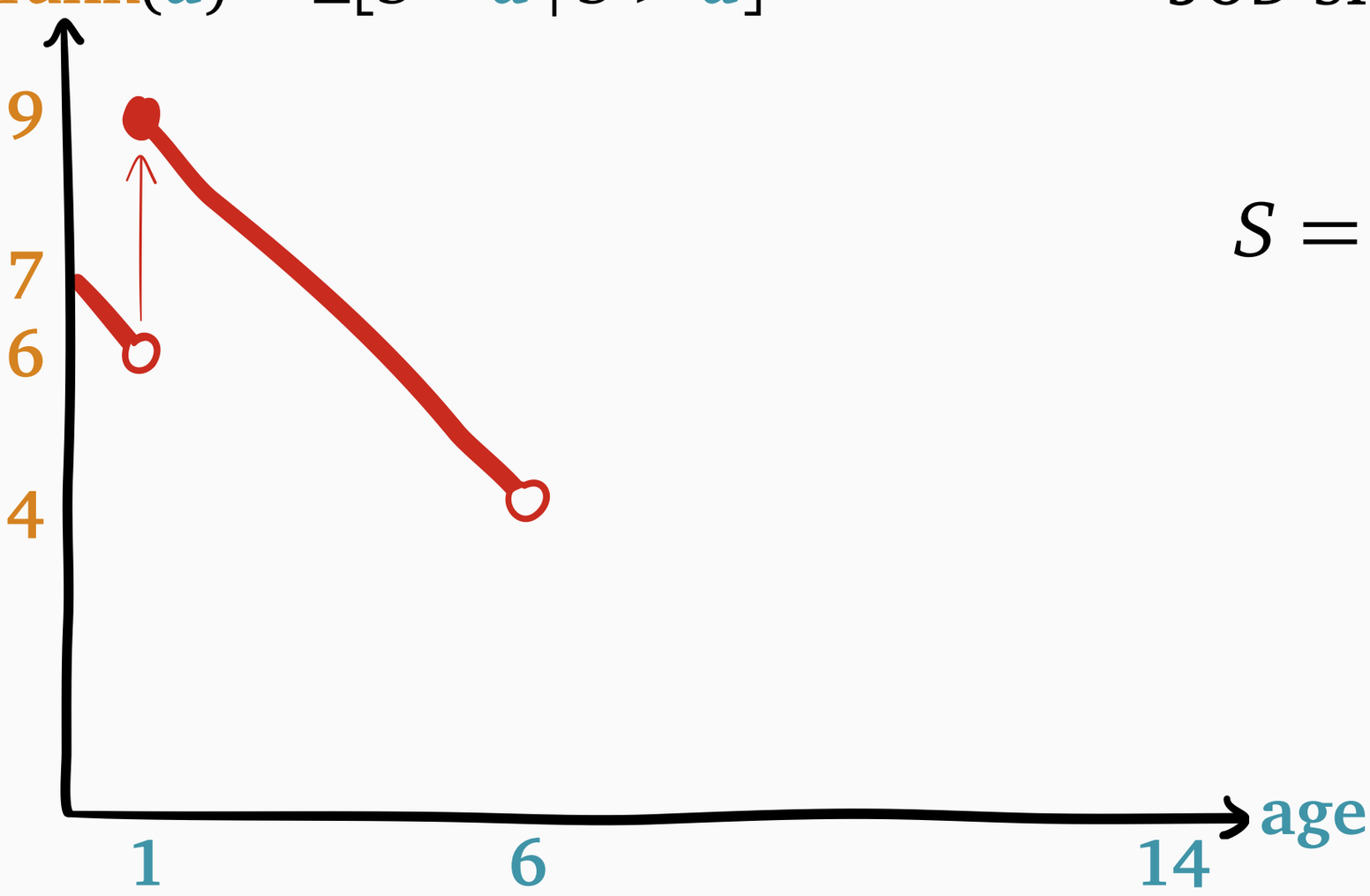

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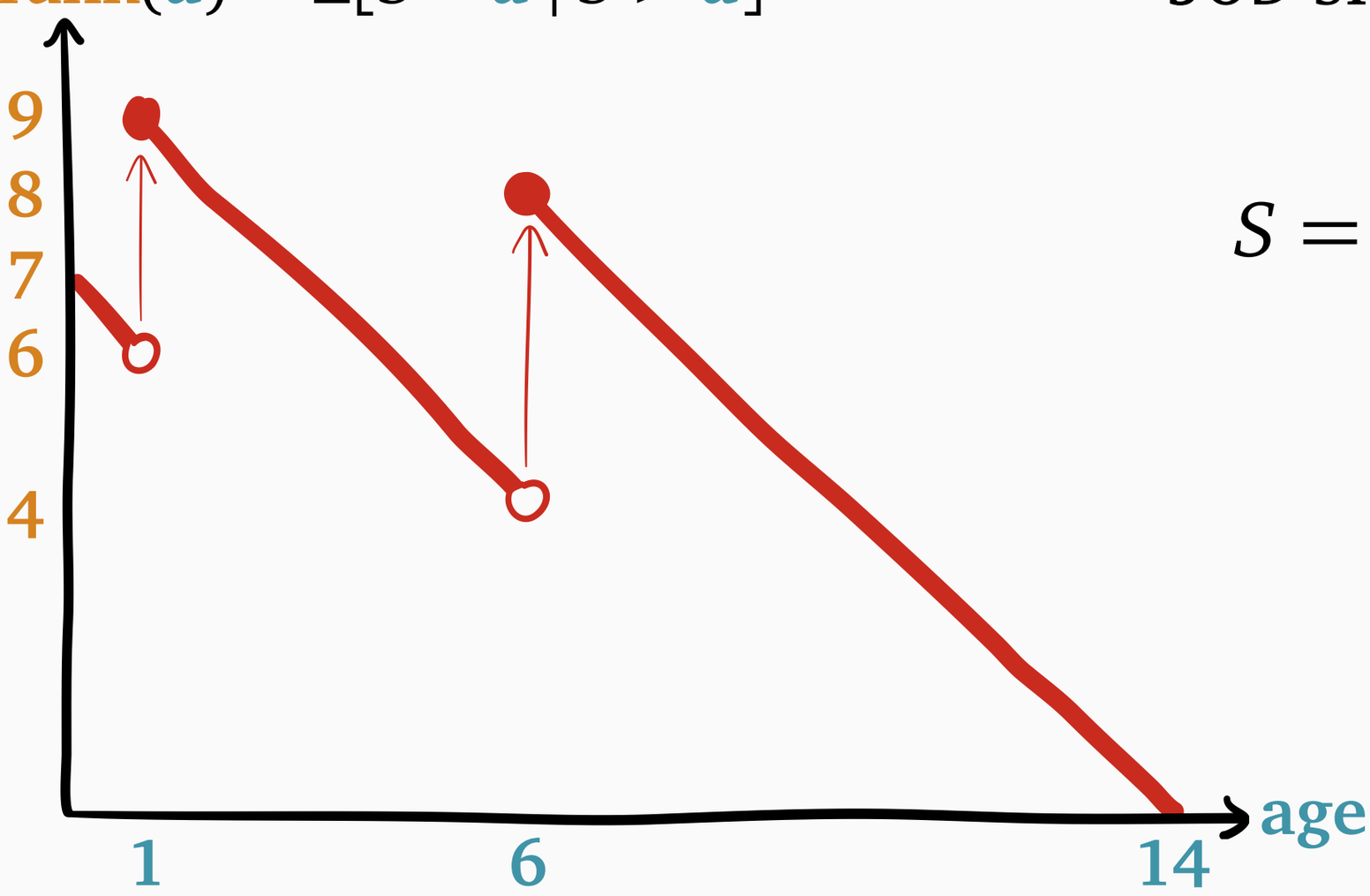

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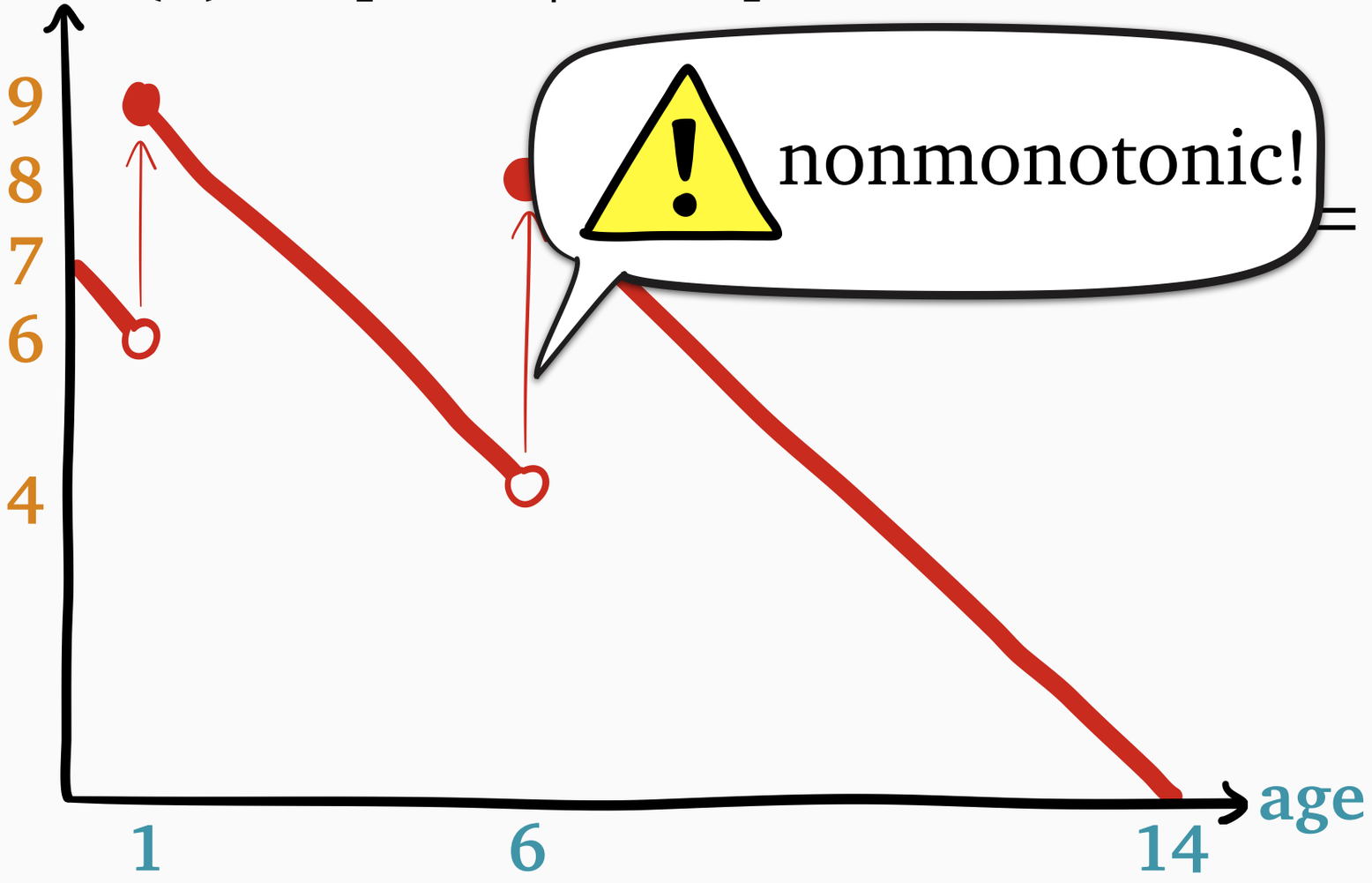

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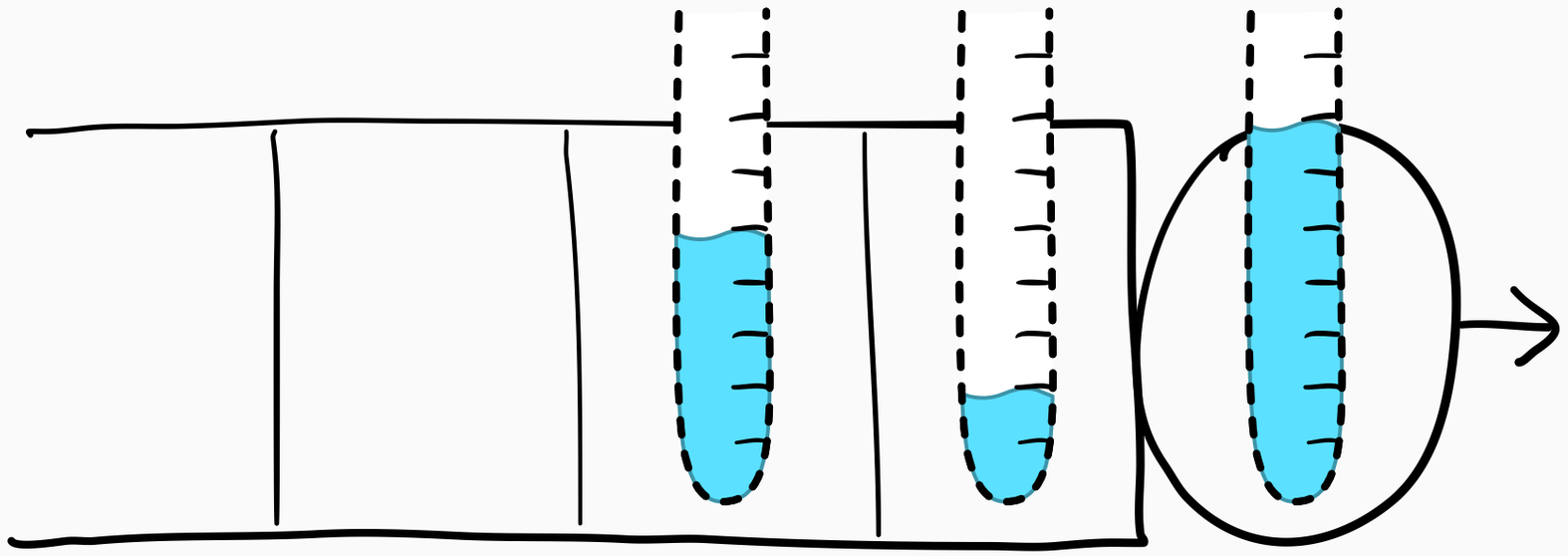
Nonmonotonic **rank** is hard



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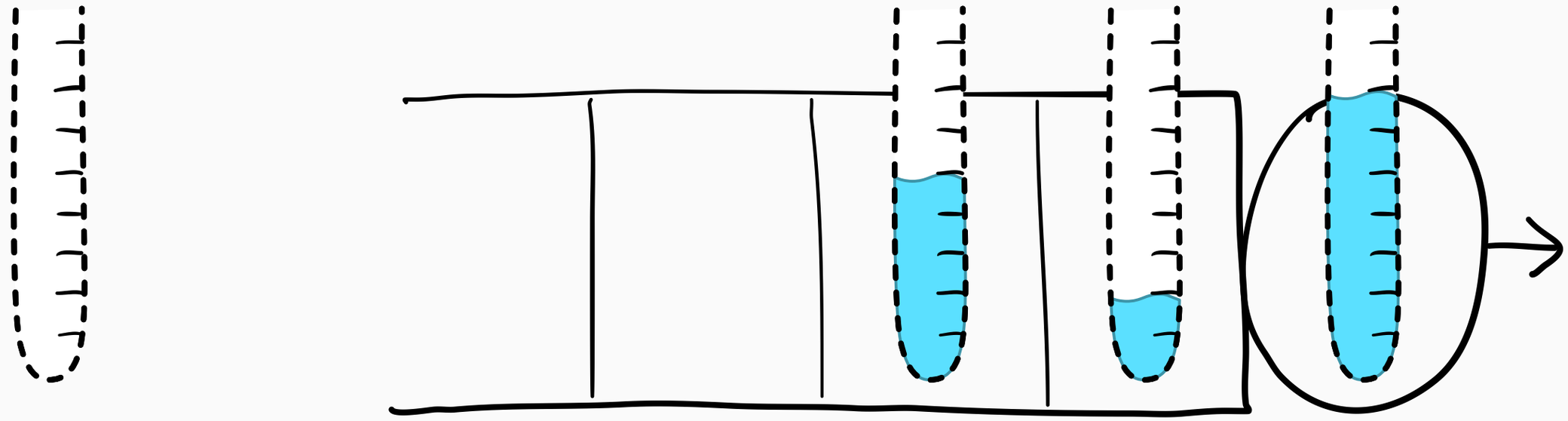
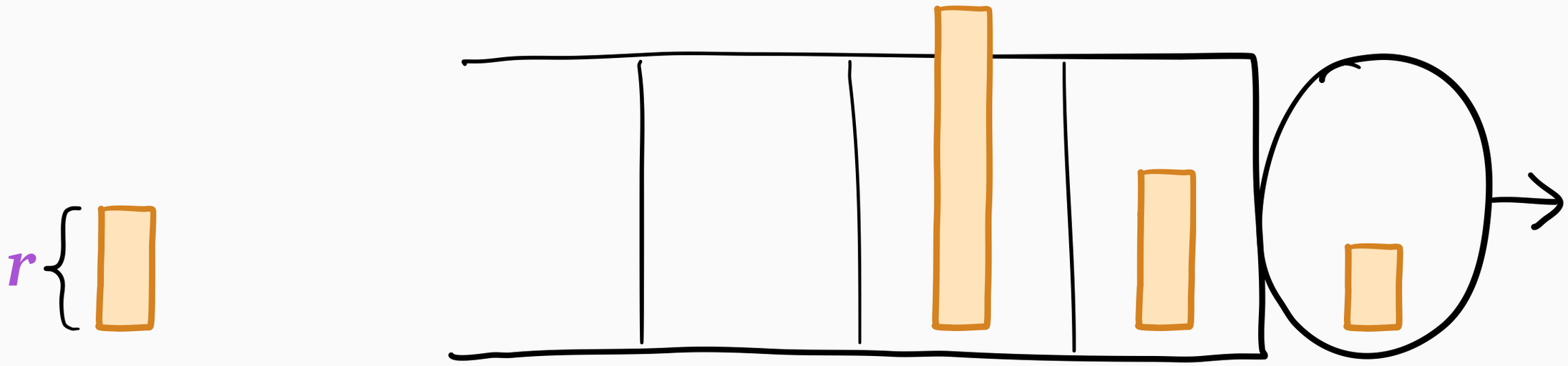


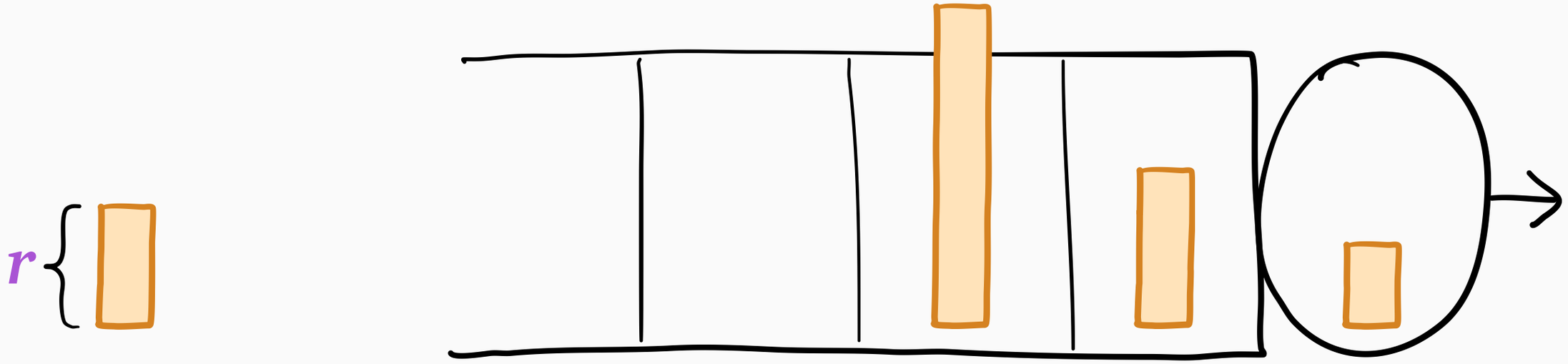
“me”

Nonmonotonic **rank** is hard



“me” = **rank**

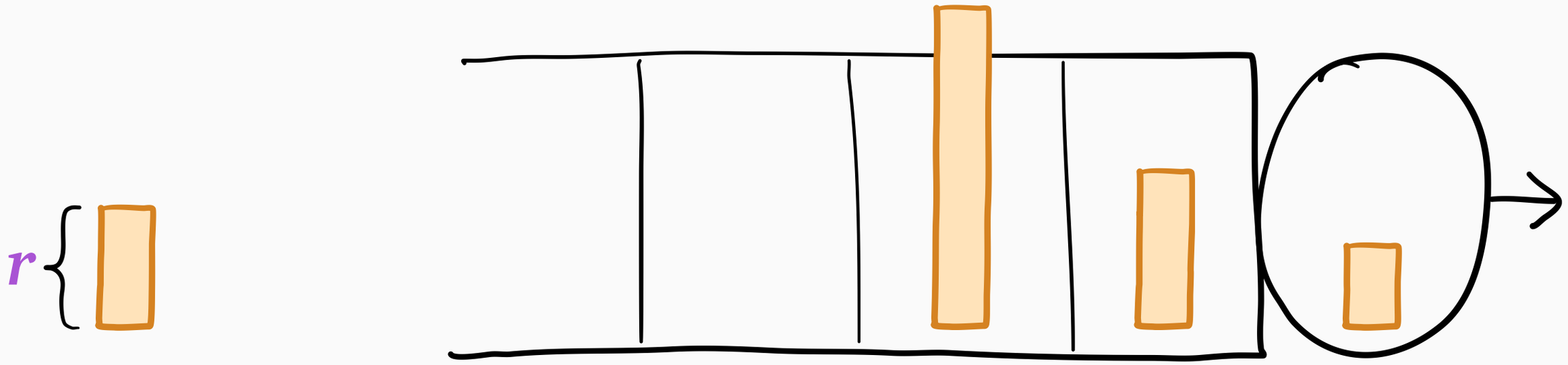
Nonmonotonic **rank** is hard



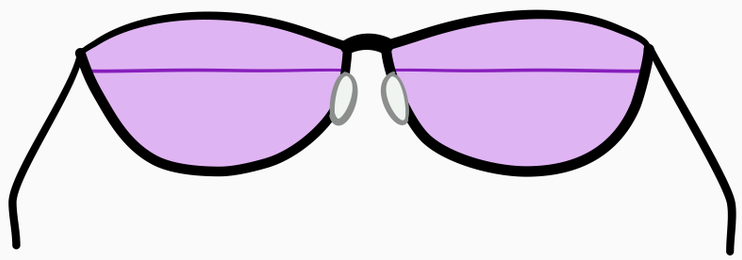
Key quantity: observed ***r*-*work*** $W(r)$

$W(r)$ = work relevant to job of **rank** r

Nonmonotonic **rank** is hard



“me” = **rank**



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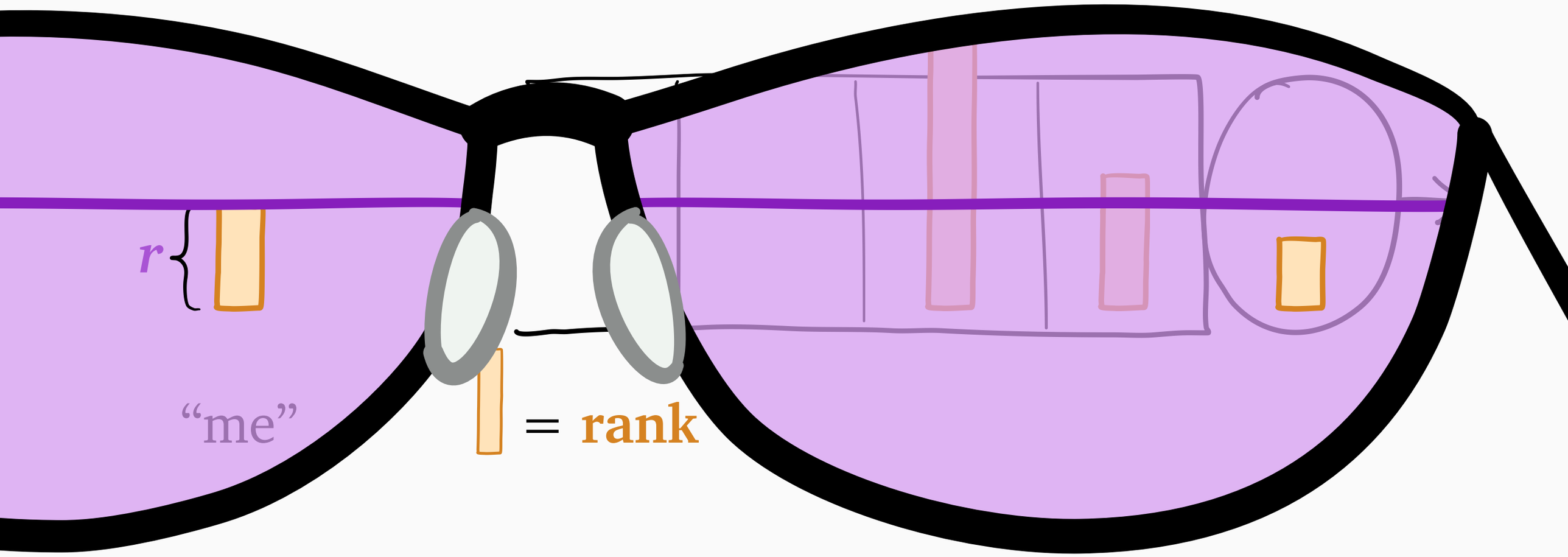

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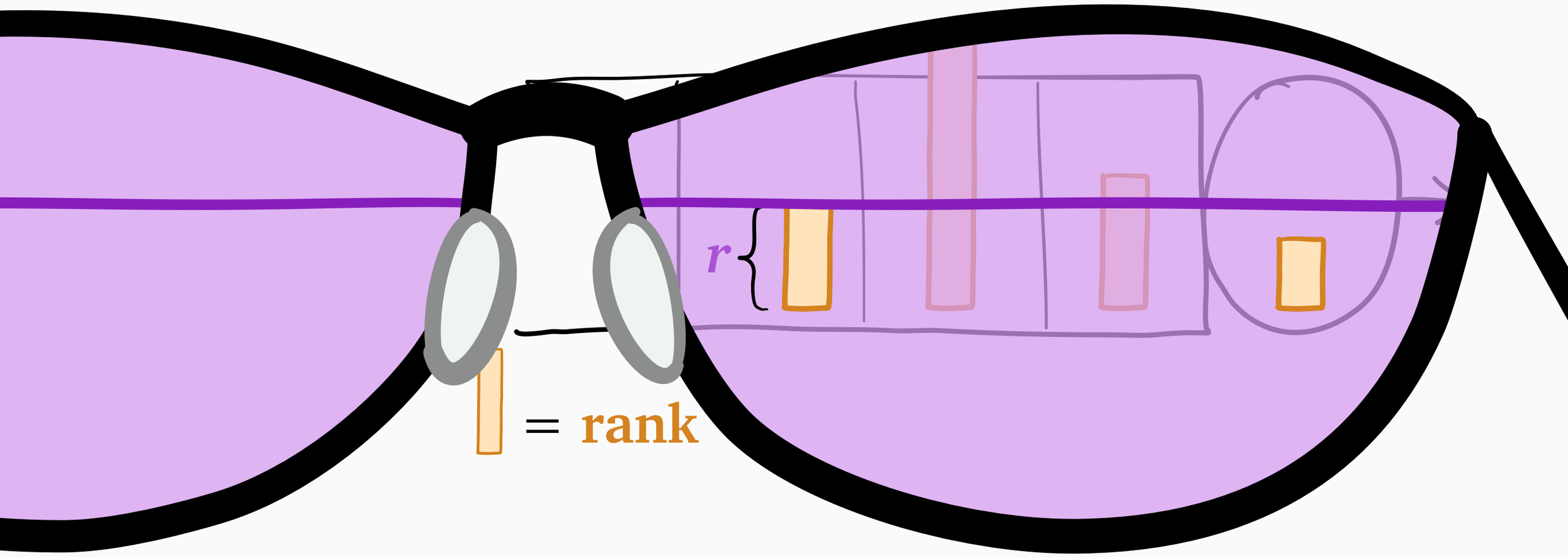

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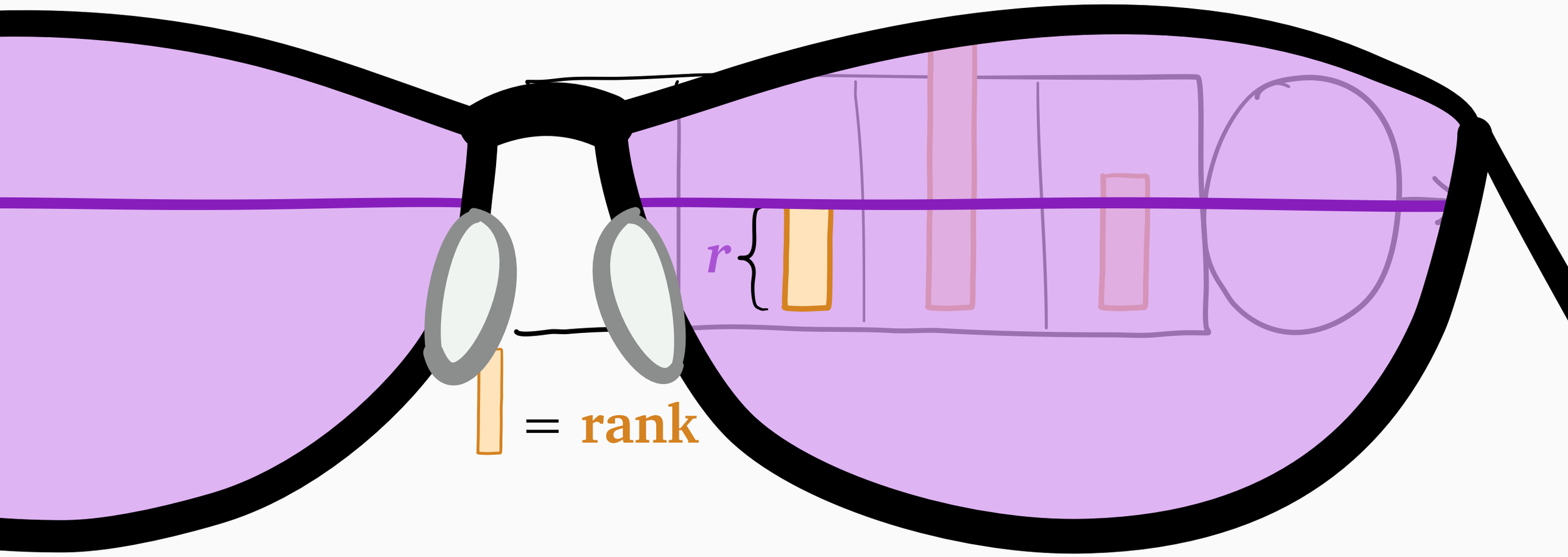

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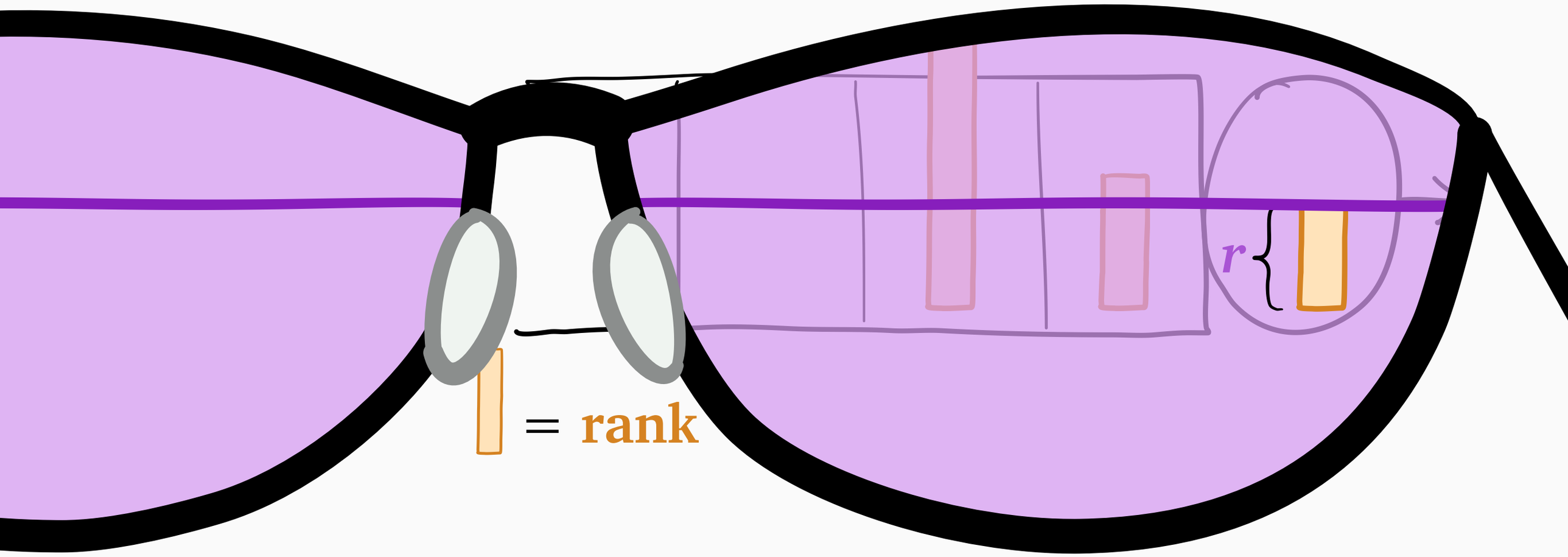

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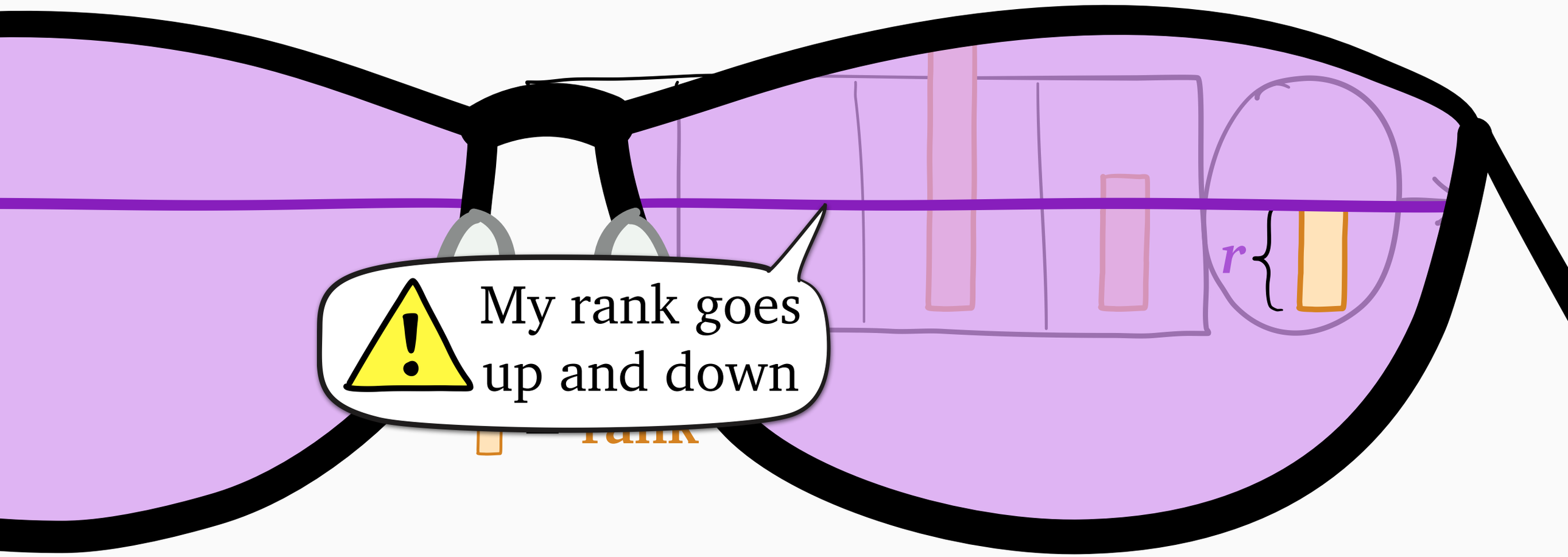

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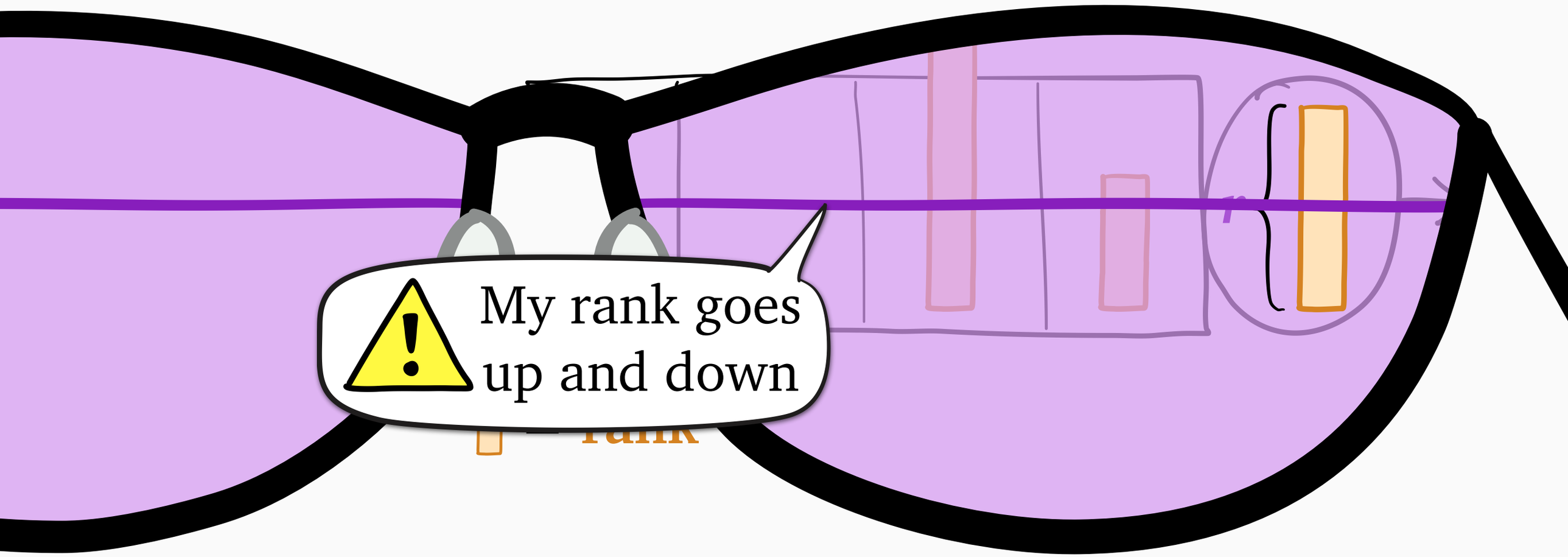

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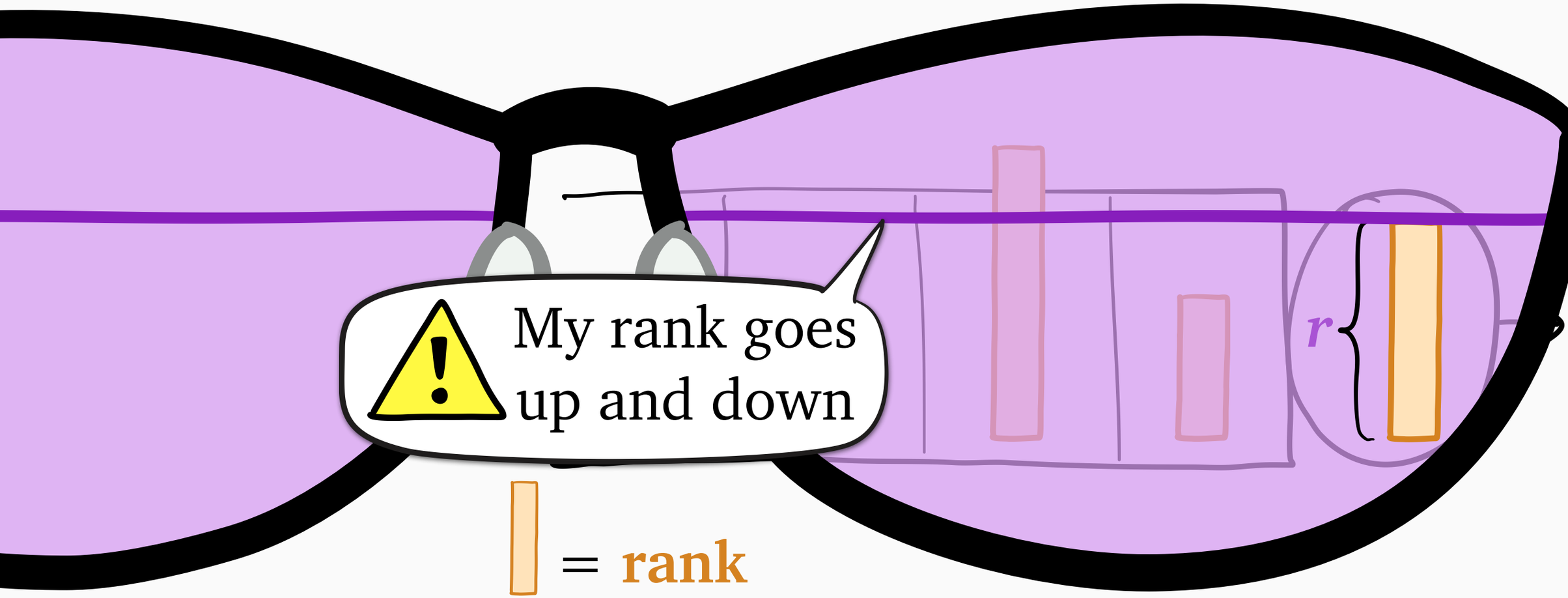

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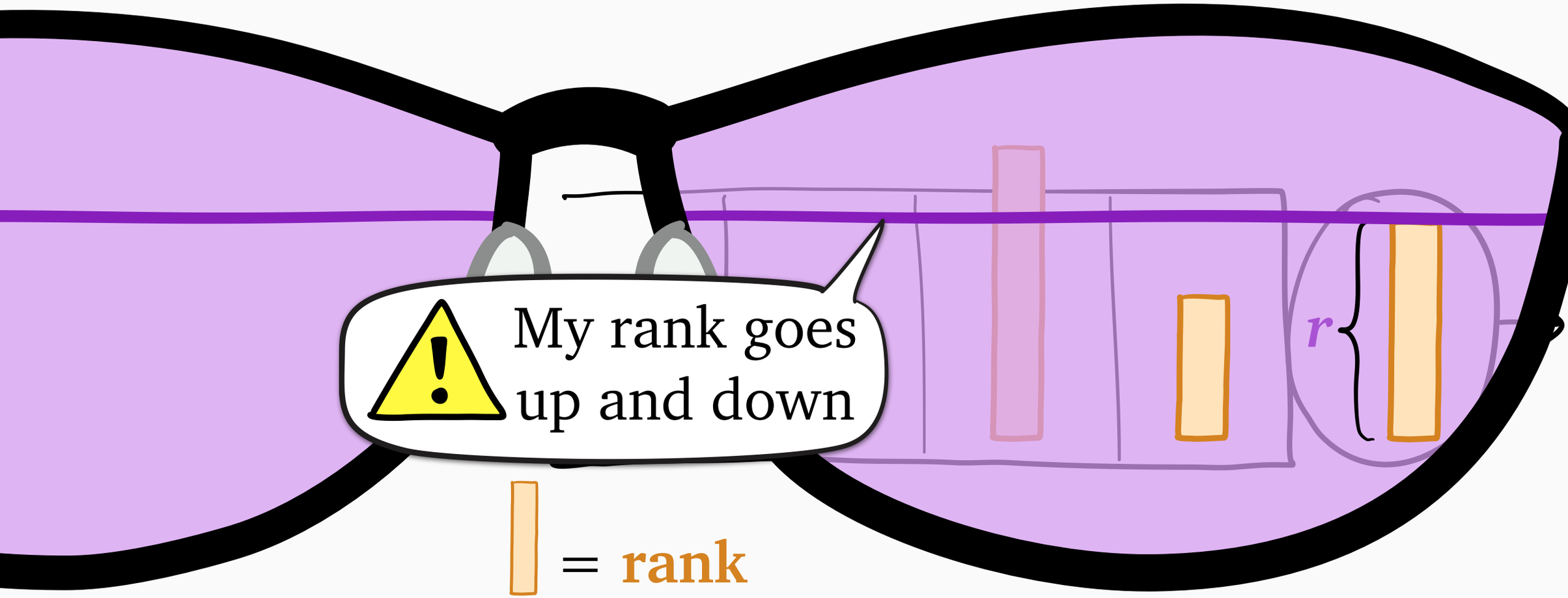

Key quantity: observed r-*work* $W(r)$

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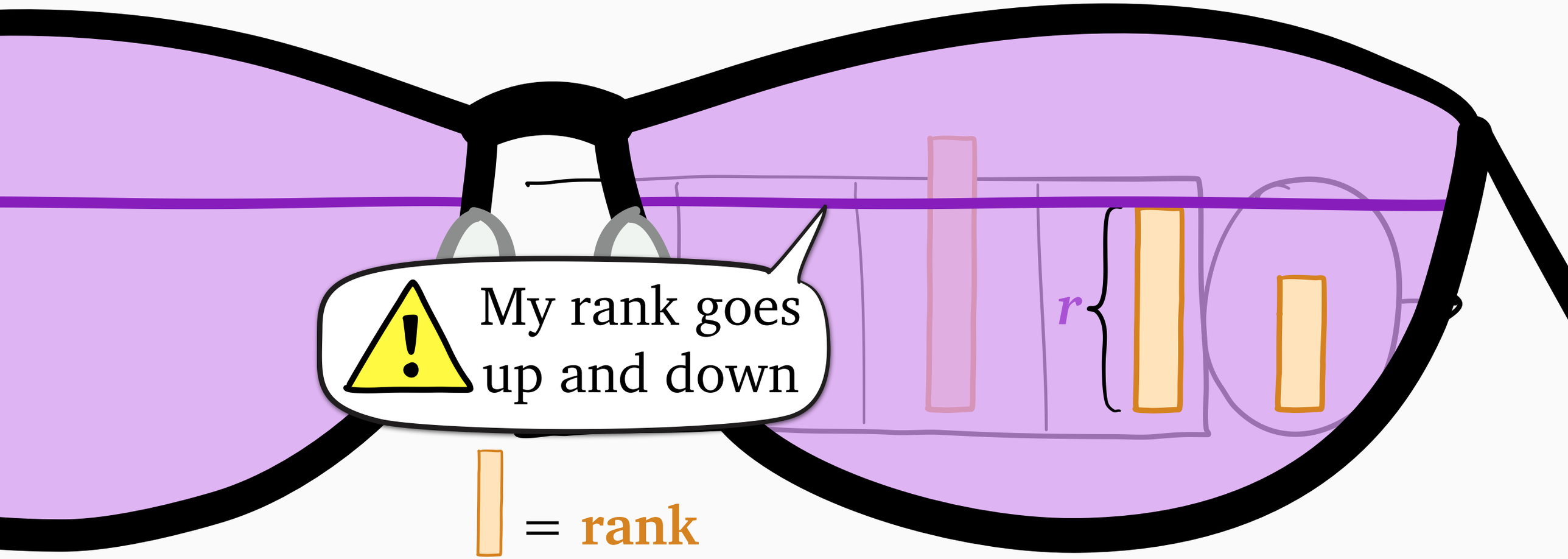
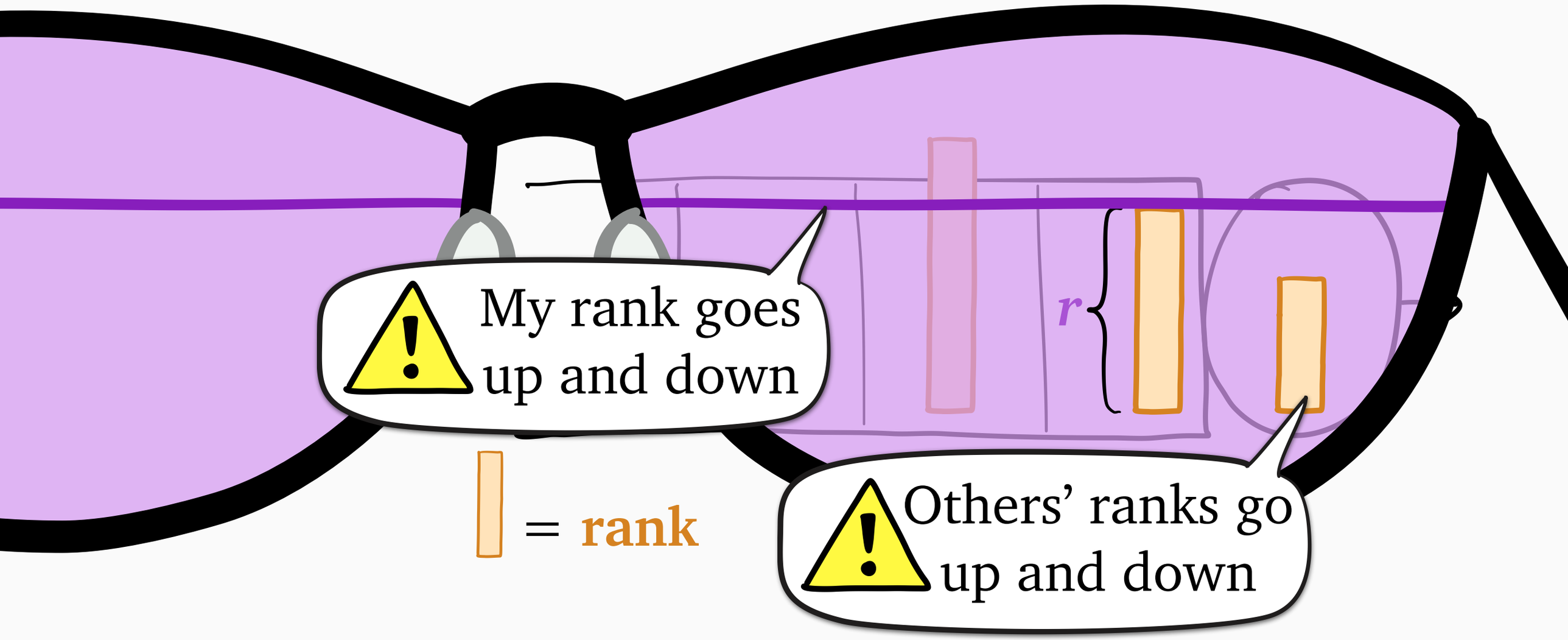
Nonmonotonic **rank** is hard



Key quantity: observed *r-work* $W(r)$

$W(r)$ = work relevant to job of **rank** r

Nonmonotonic **rank** is hard



Key quantity: observed *r-work* $W(r)$

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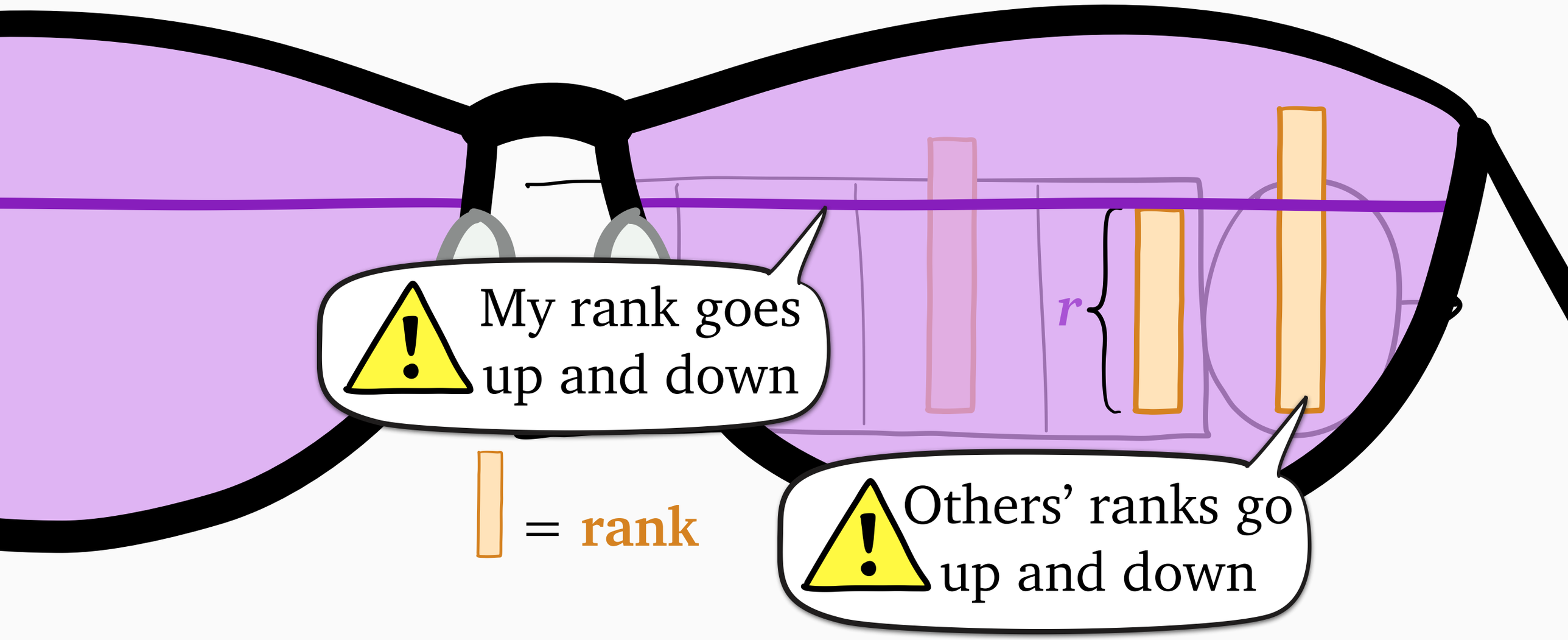

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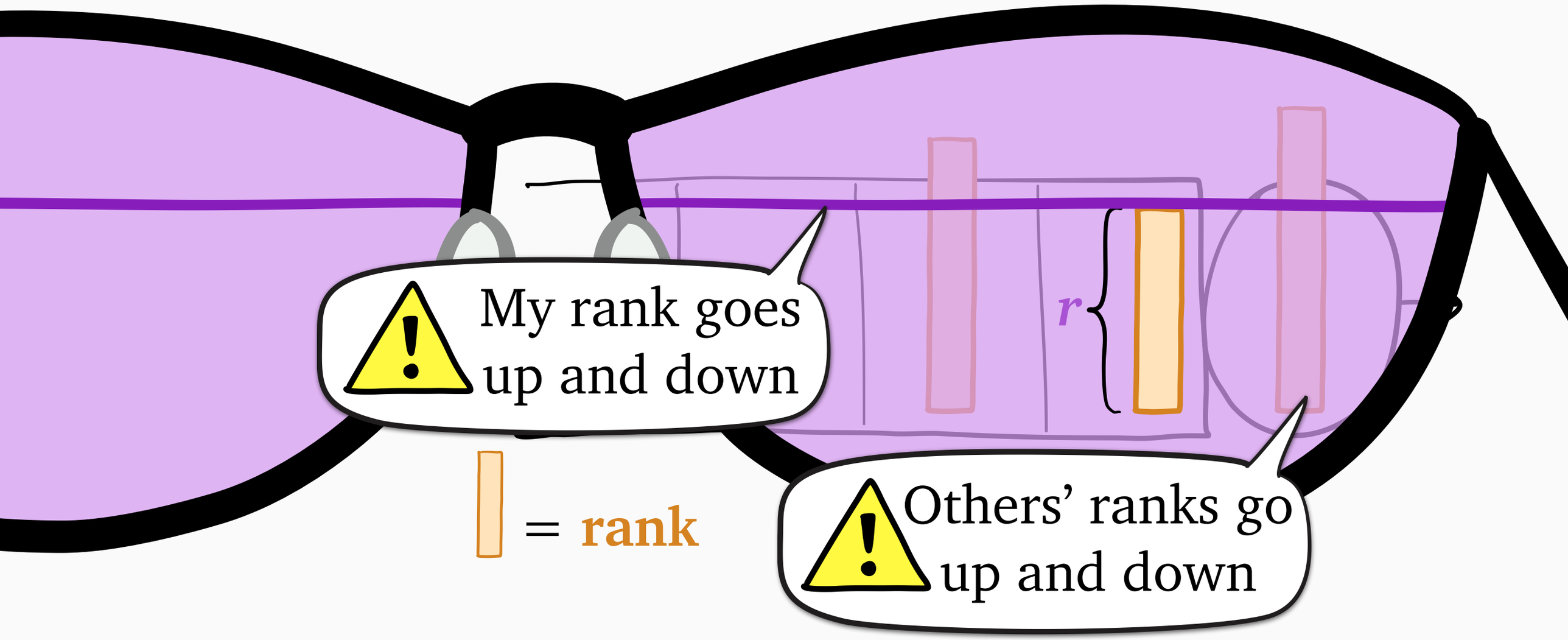

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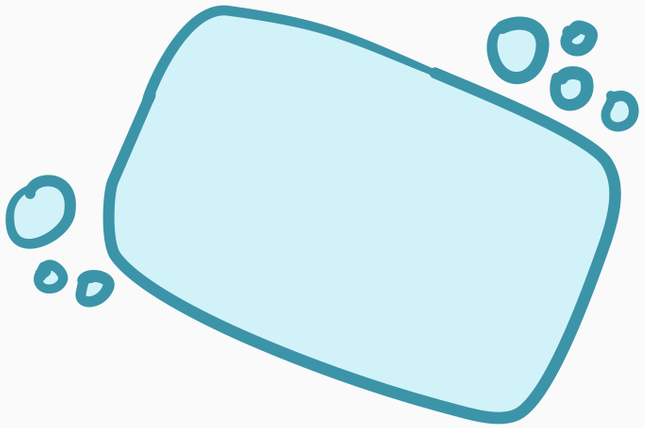
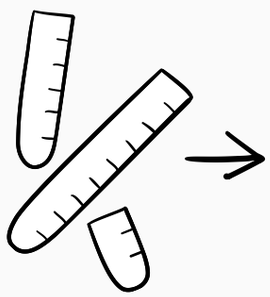


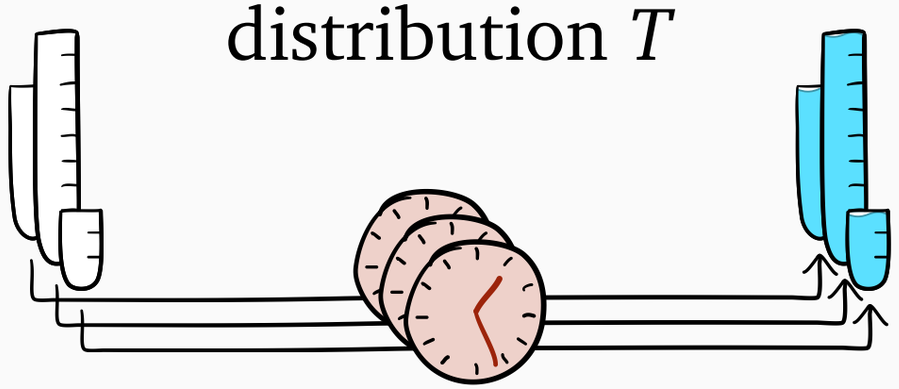
SOAP

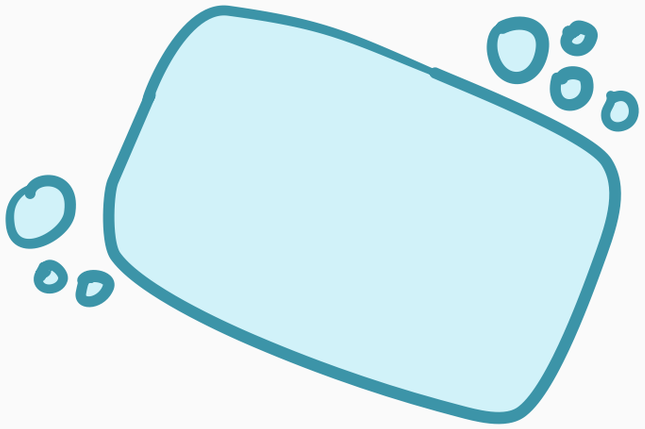
Schedule **O**rdered by **A**ge-based **P**riority

stochastic arrival process λ, S



response time distribution T





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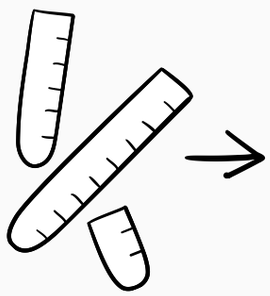
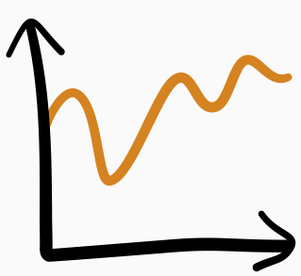


rank function



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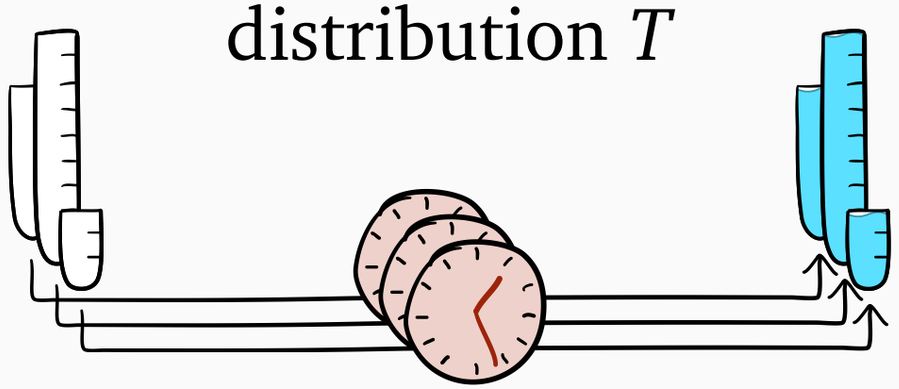

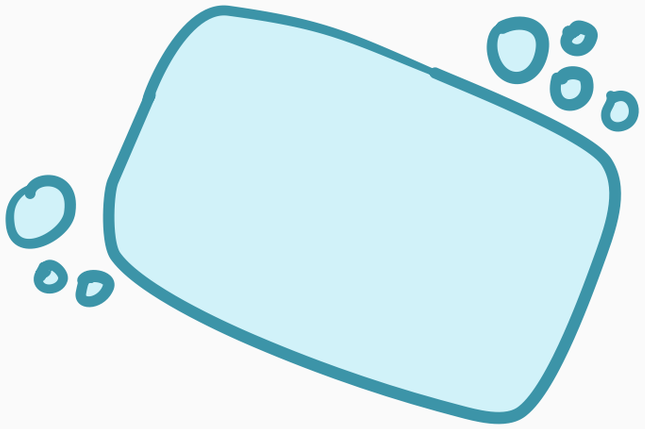

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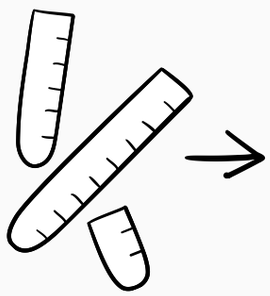
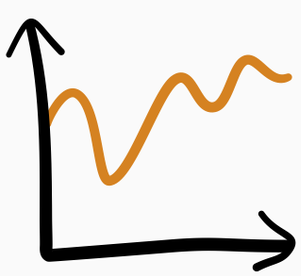


rank function



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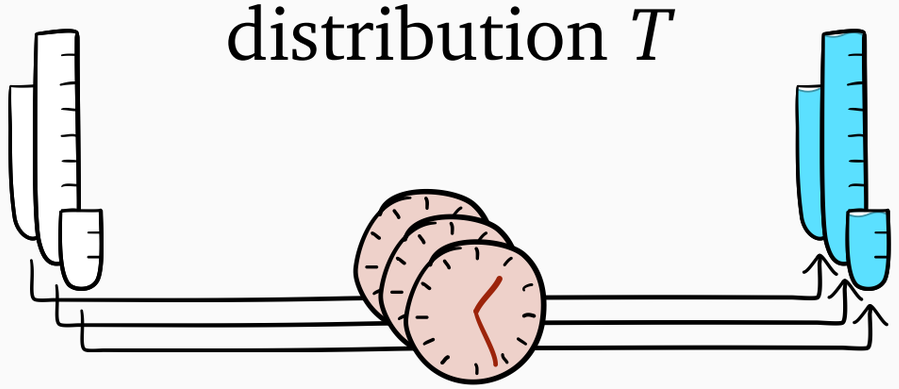

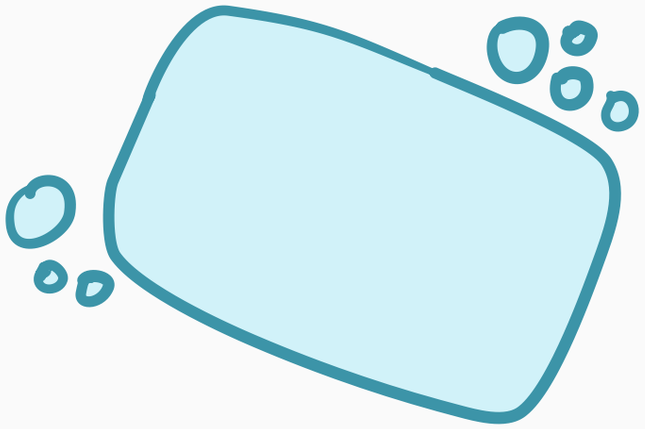

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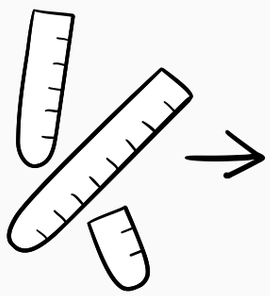

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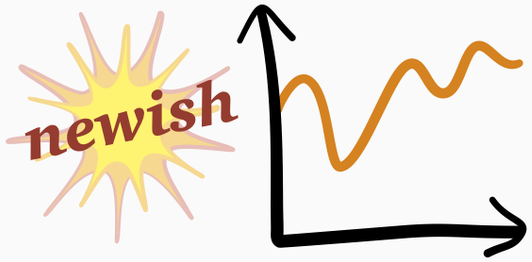

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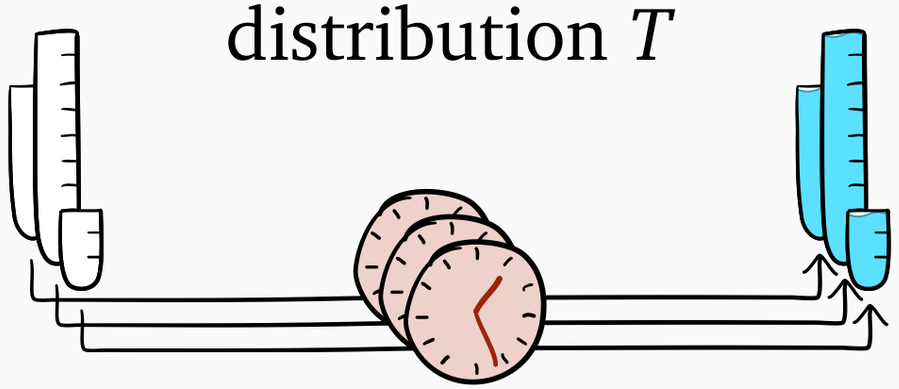
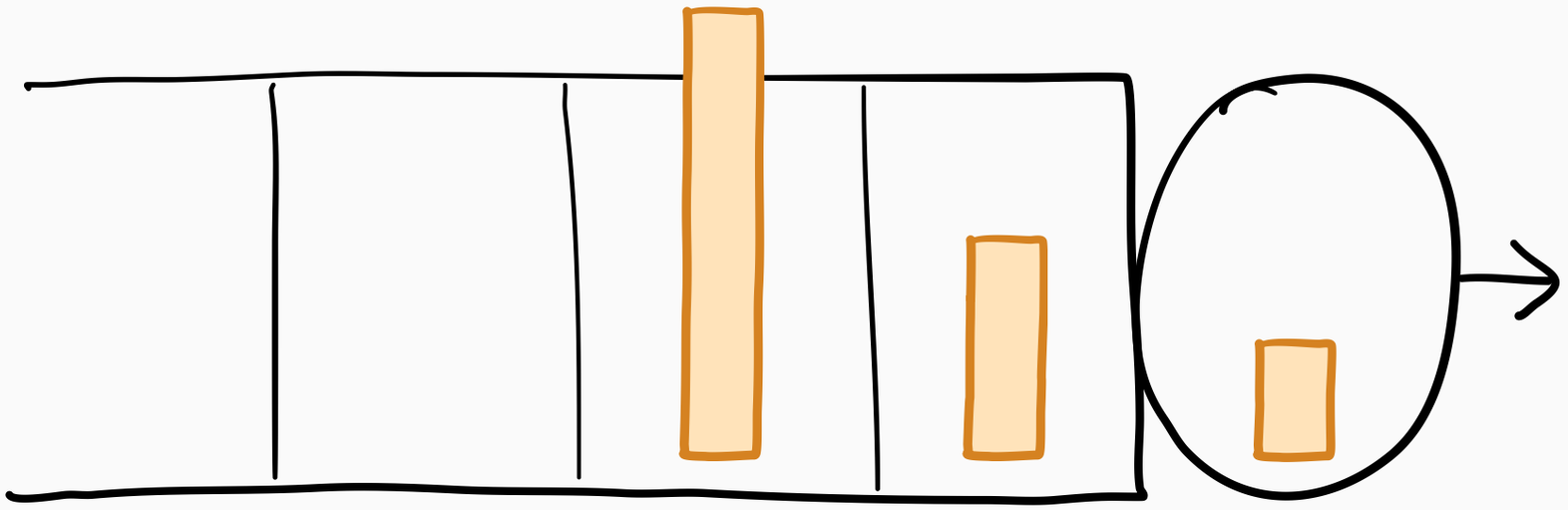
response time distribution T



Key **SOAP** insight



= **rank**

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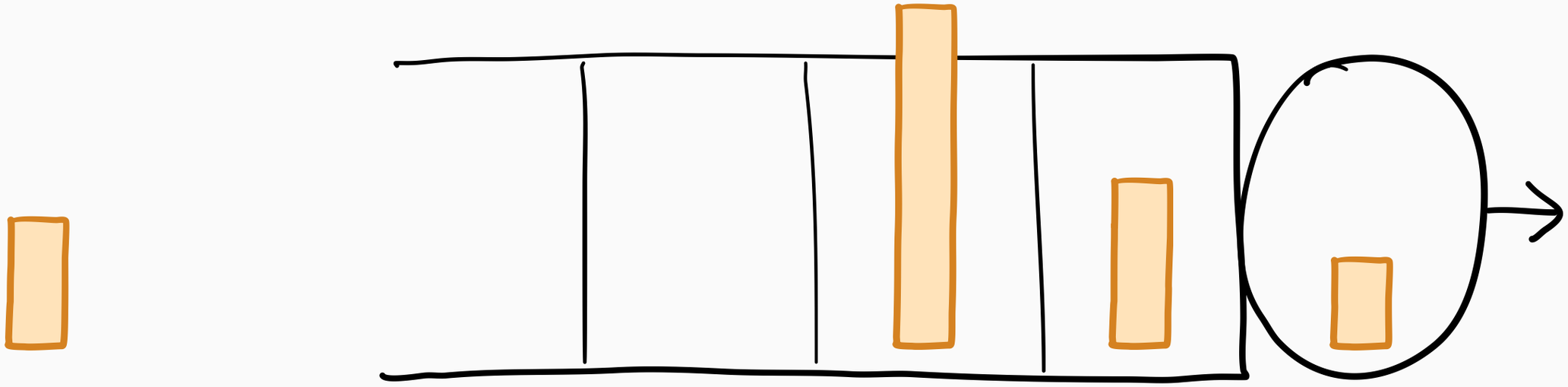
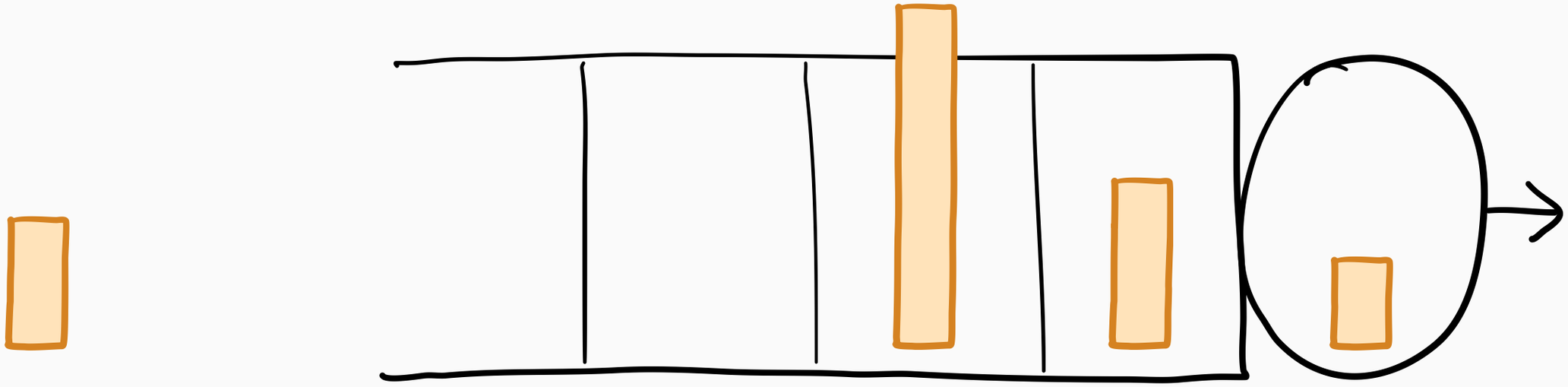


“me” = **rank**

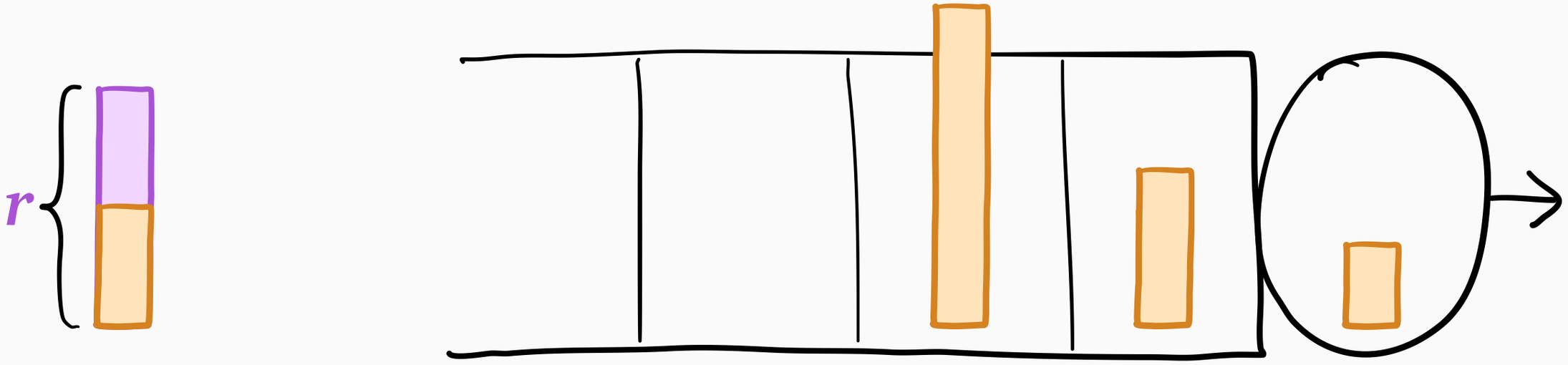
Key **SOAP** insight



“me” = **rank**

Pessimism Priniciple: compute $W(r)$
with r = my *worst future* ***rank***

Key **SOAP** insight



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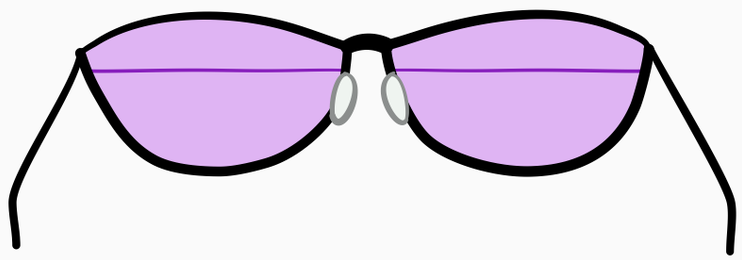

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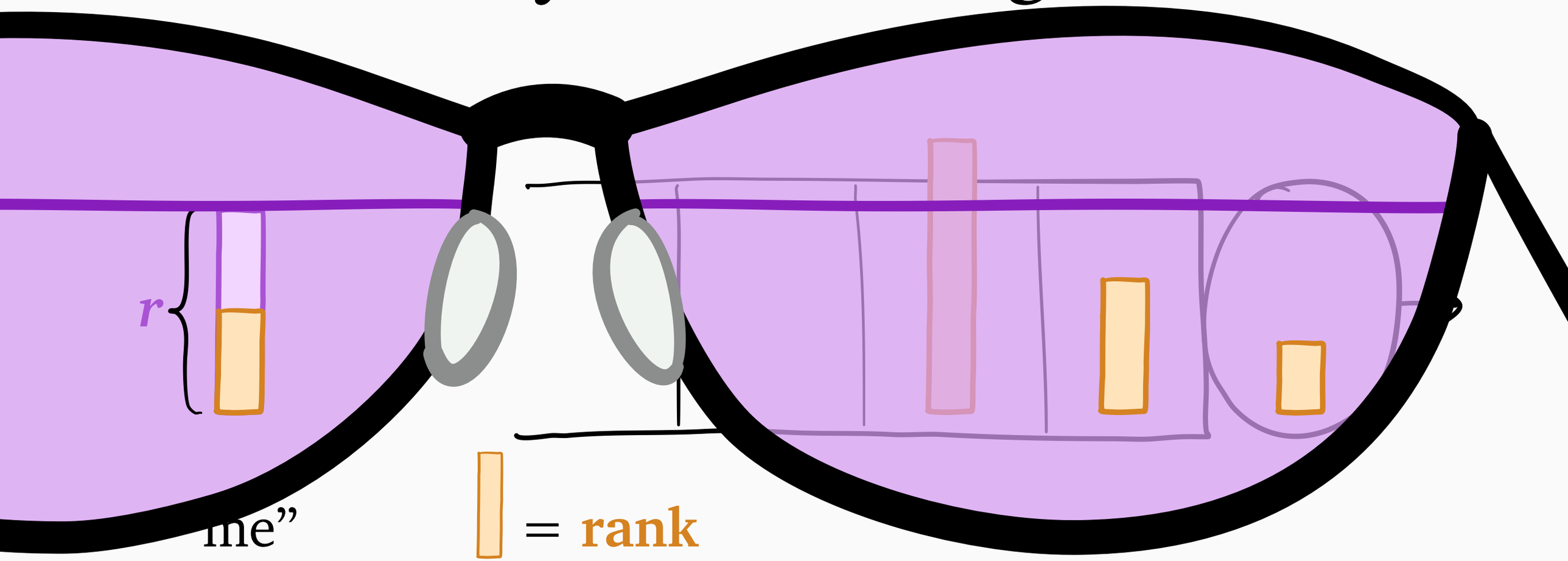

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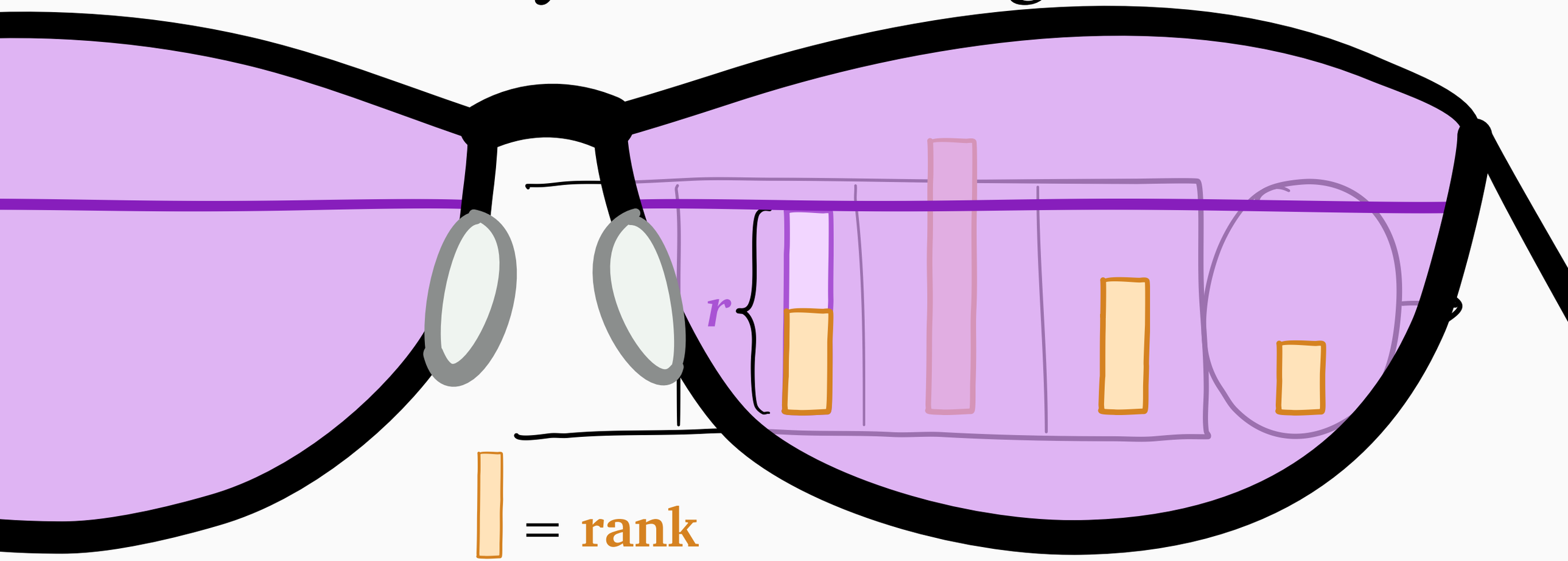

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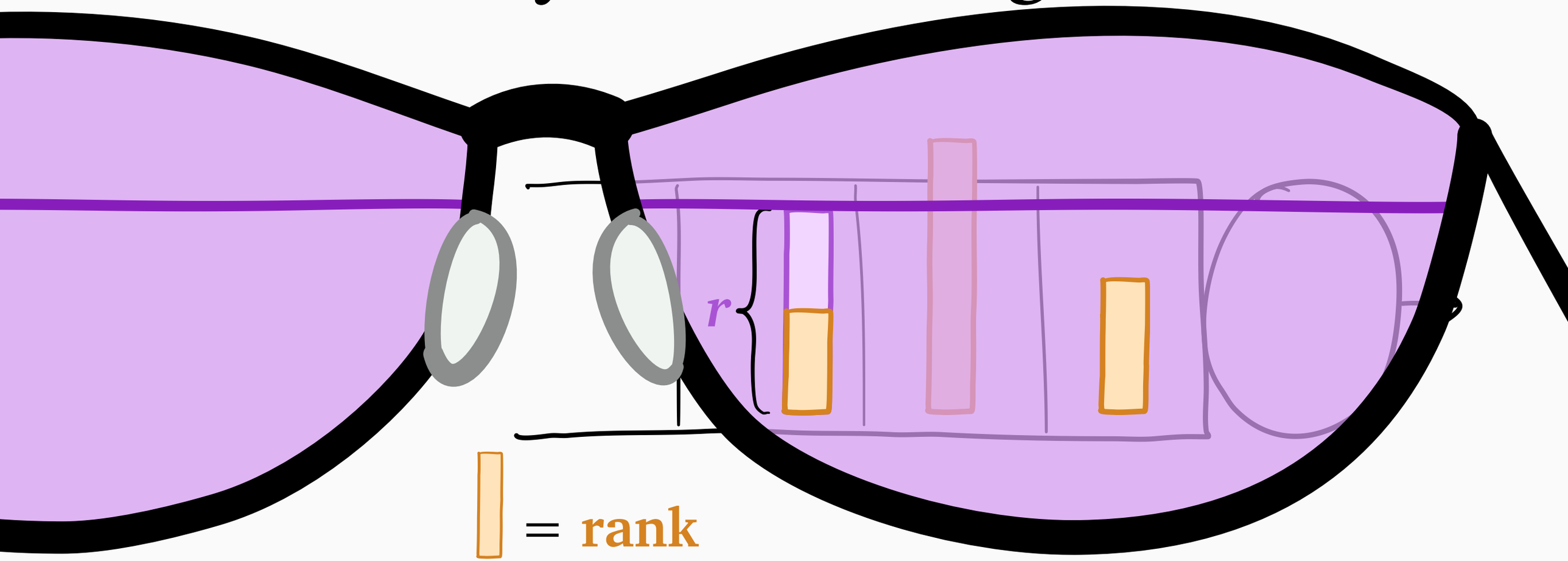

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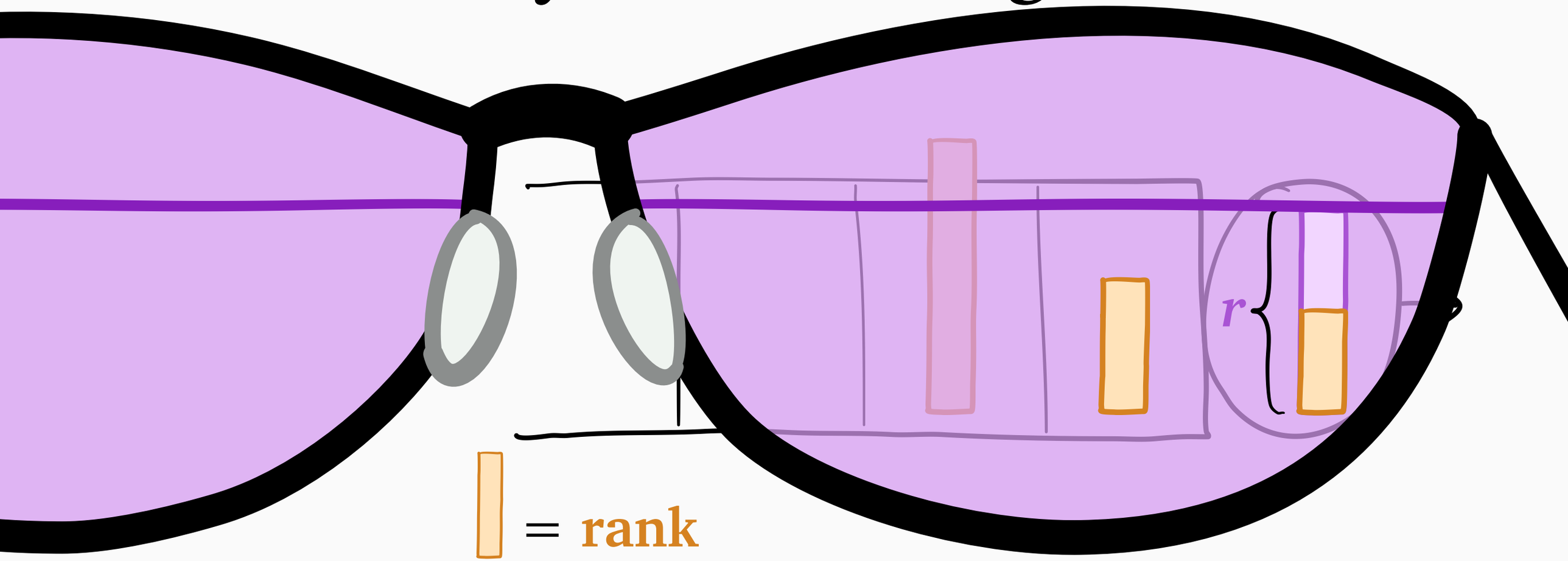

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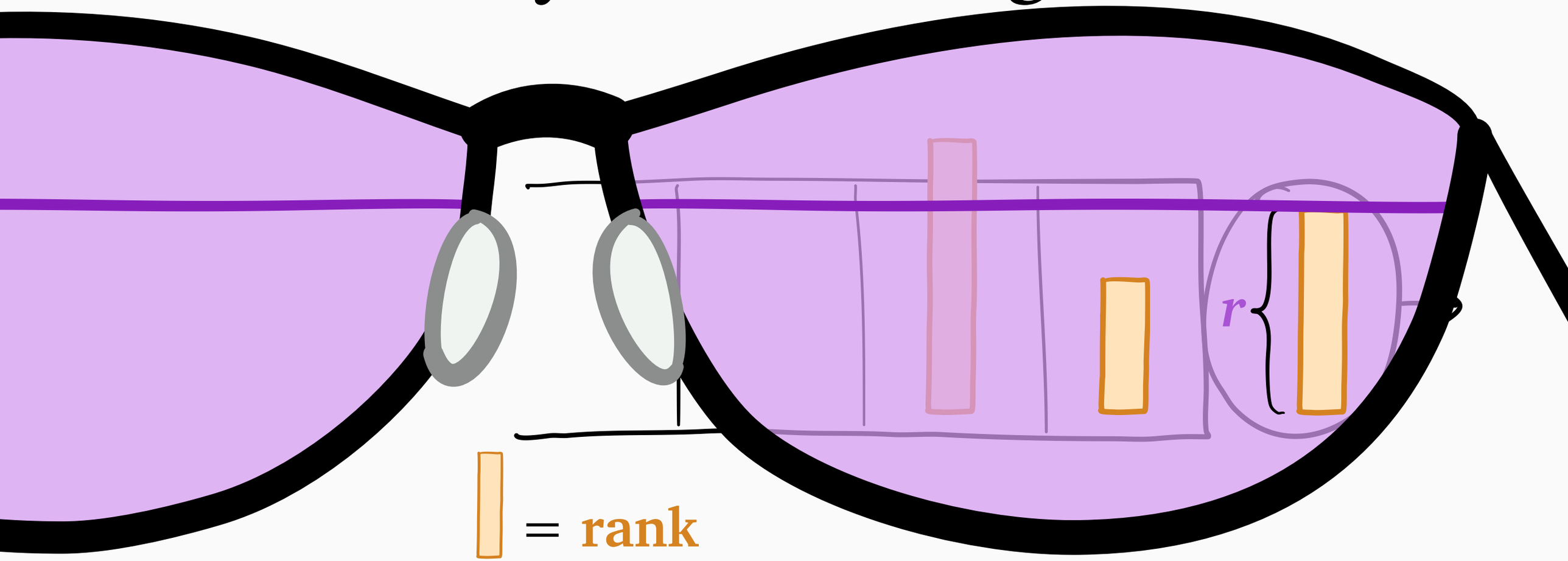

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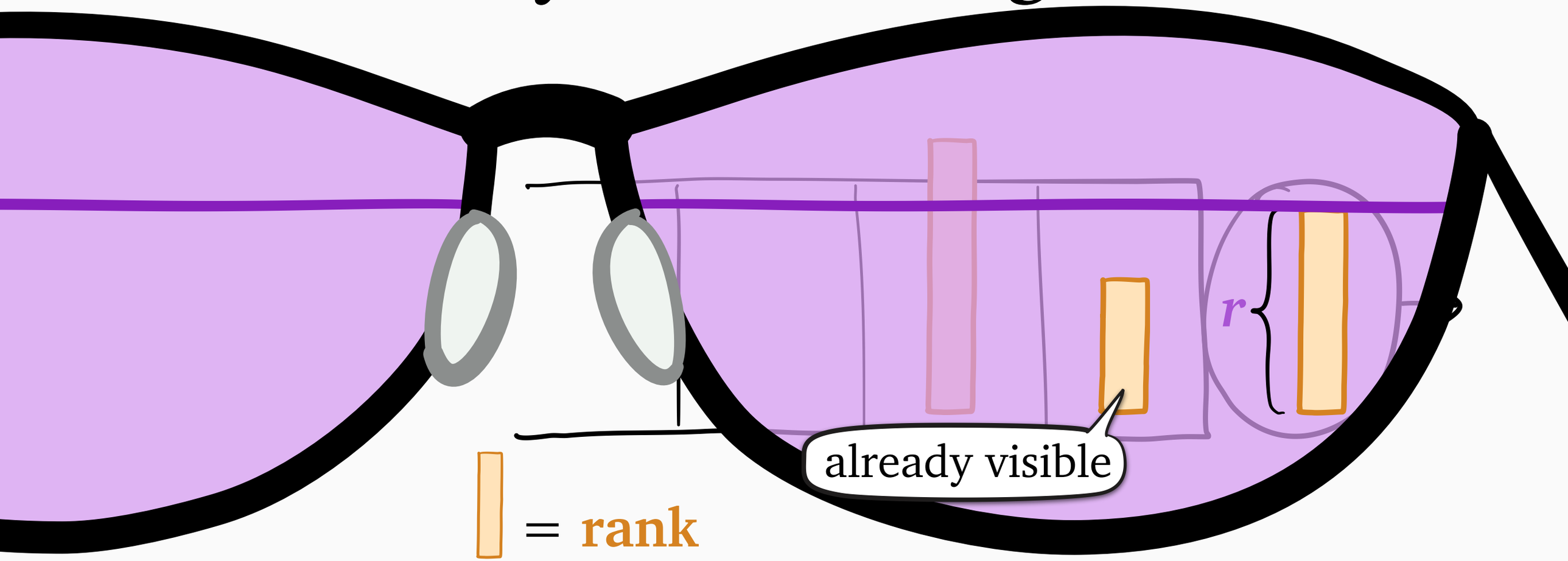

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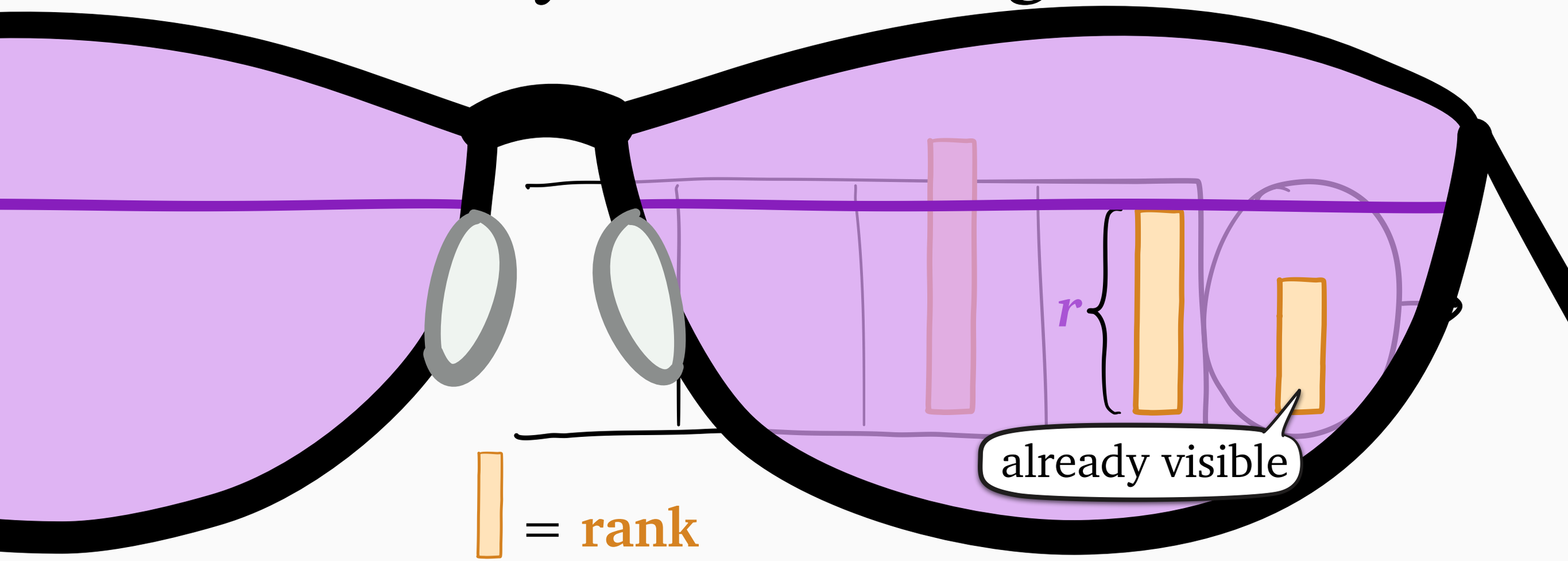

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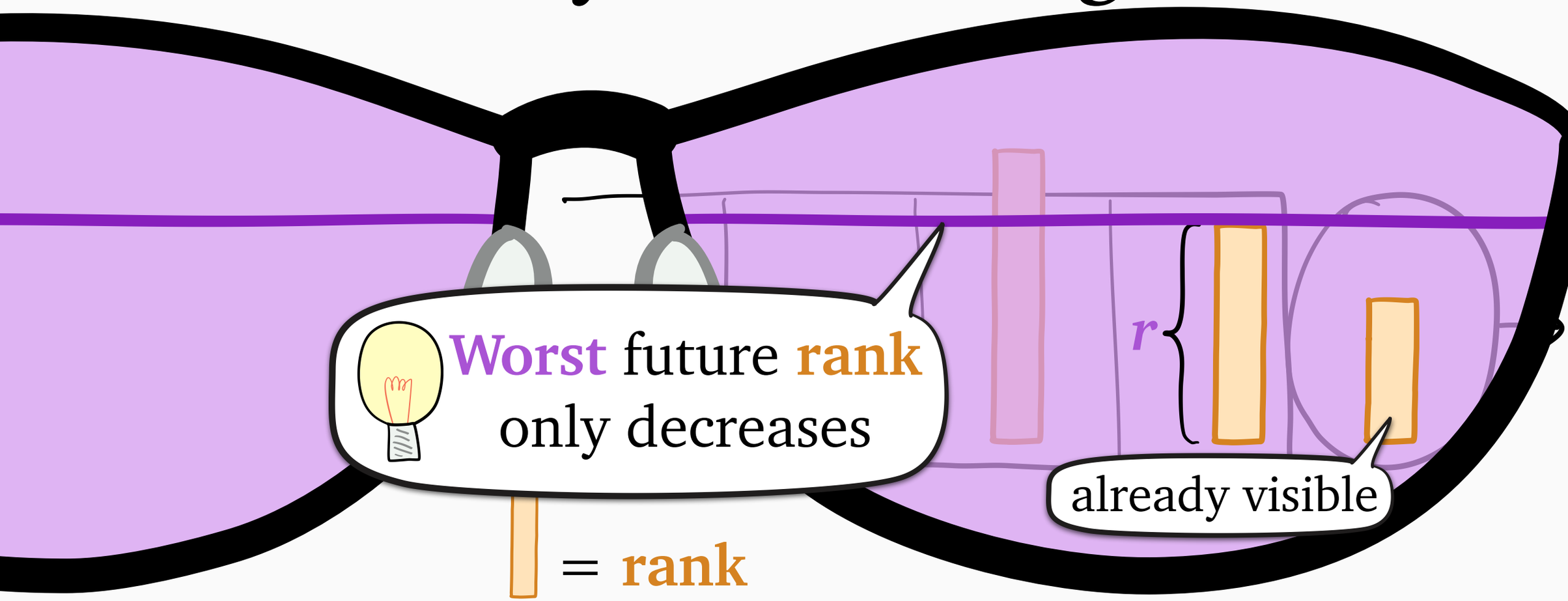

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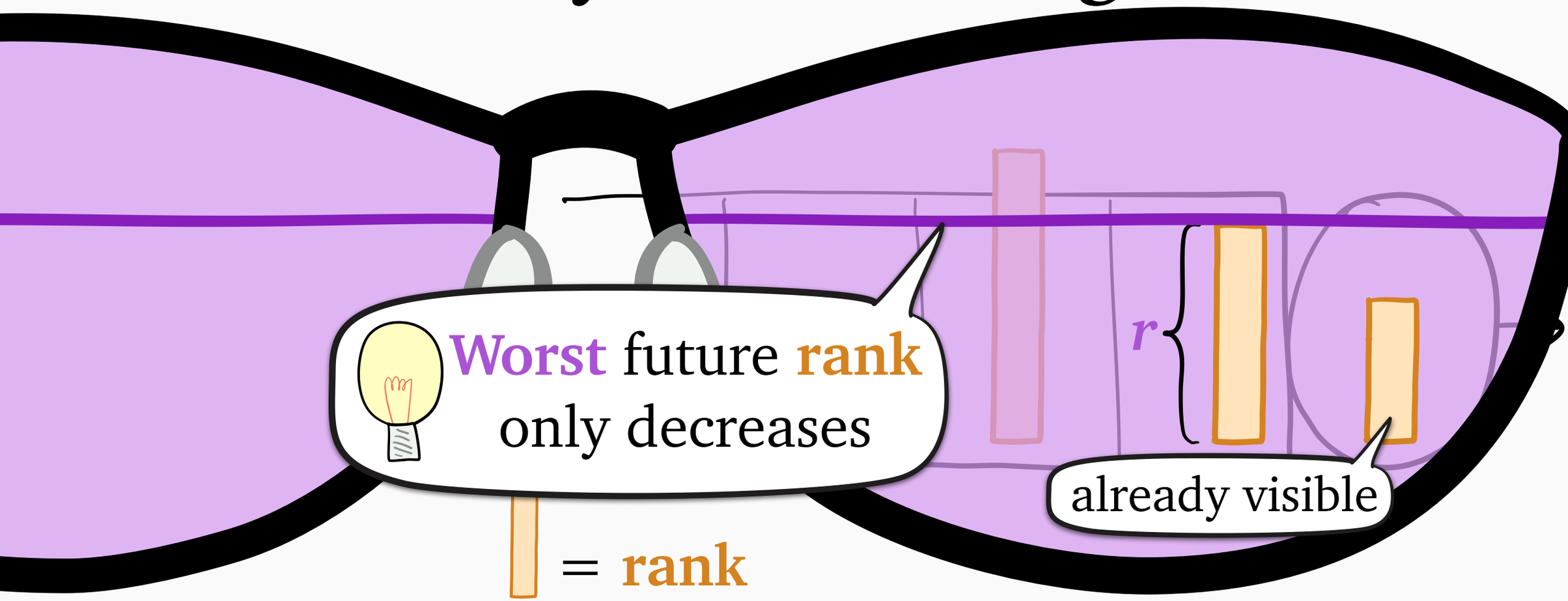
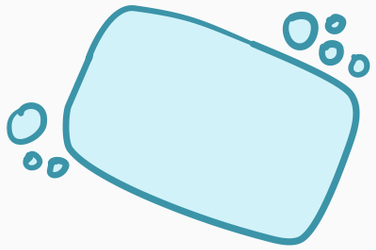
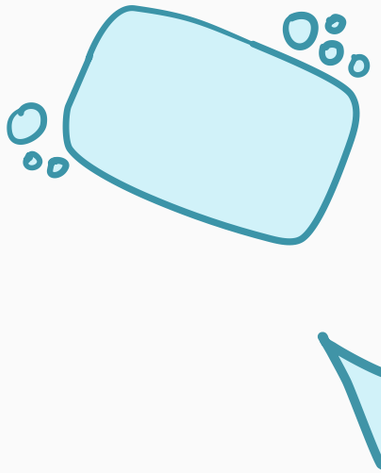


exact!

Pessimism Priniciple: compute $W(r)$ with r = my *worst future rank*



Impact of **SOAP**



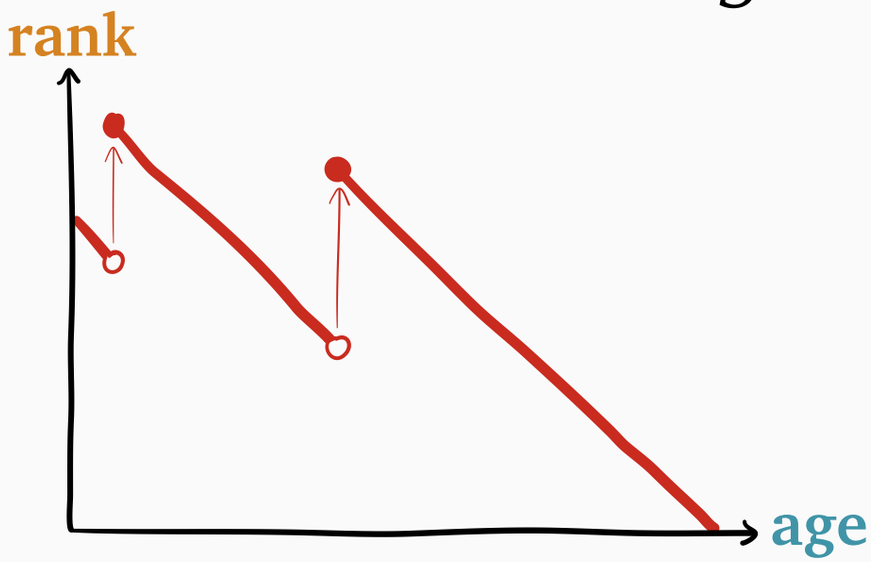
Impact of **SOAP**

unknown sizes

First analyses via **SOAP**

SERPT

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$



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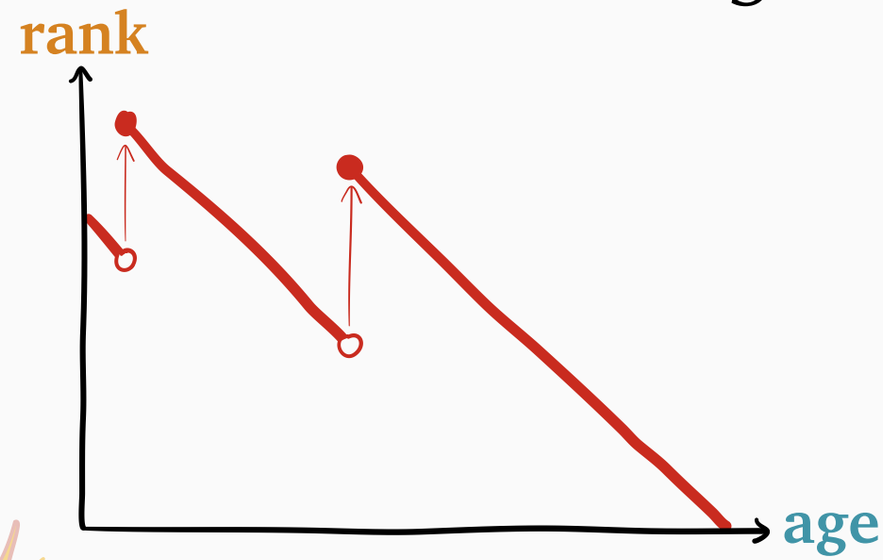

NEW! *first analysis* of **SERPT**

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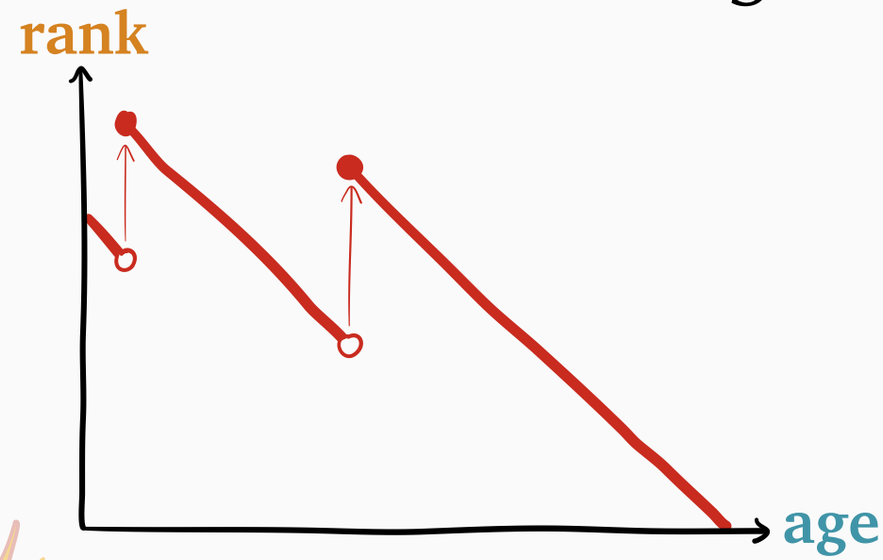
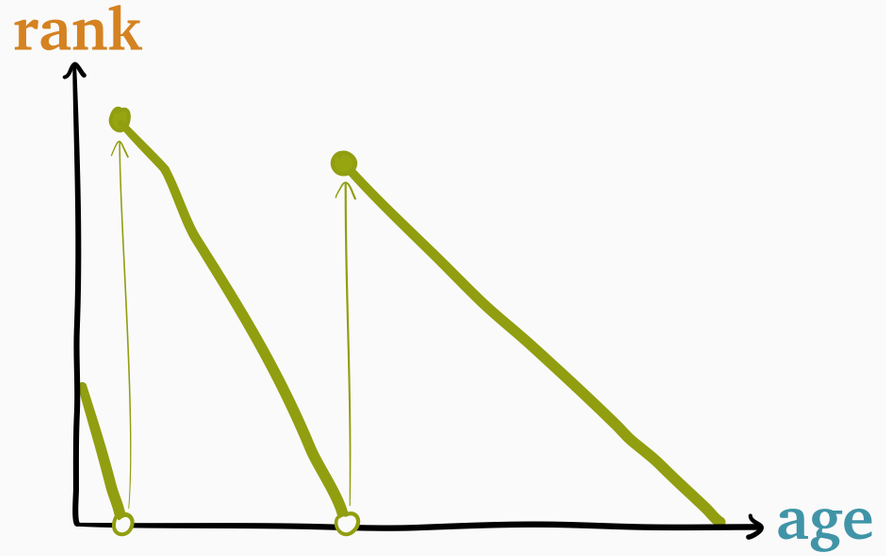


Gittins

minimizes $\mathbf{E}[T]$



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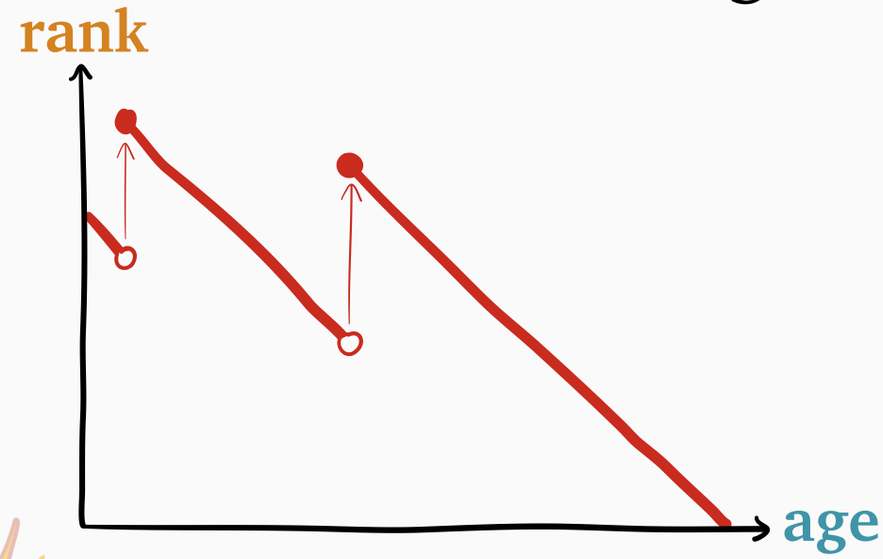

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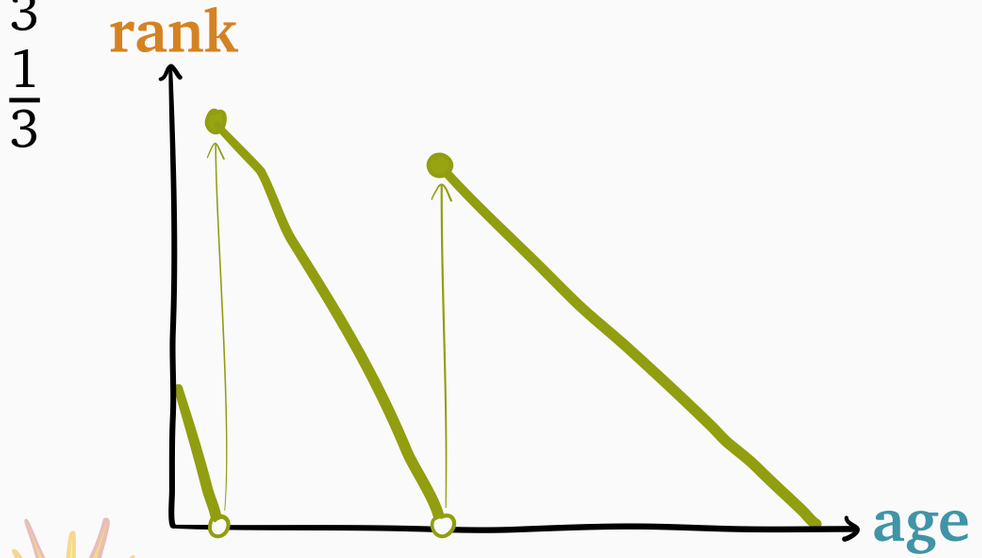

NEW! *first analysis* of **Gittins**

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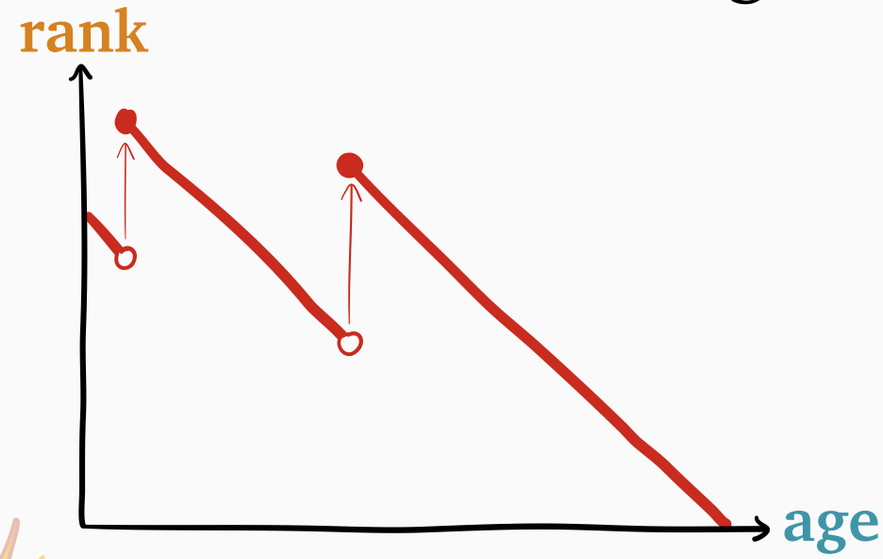

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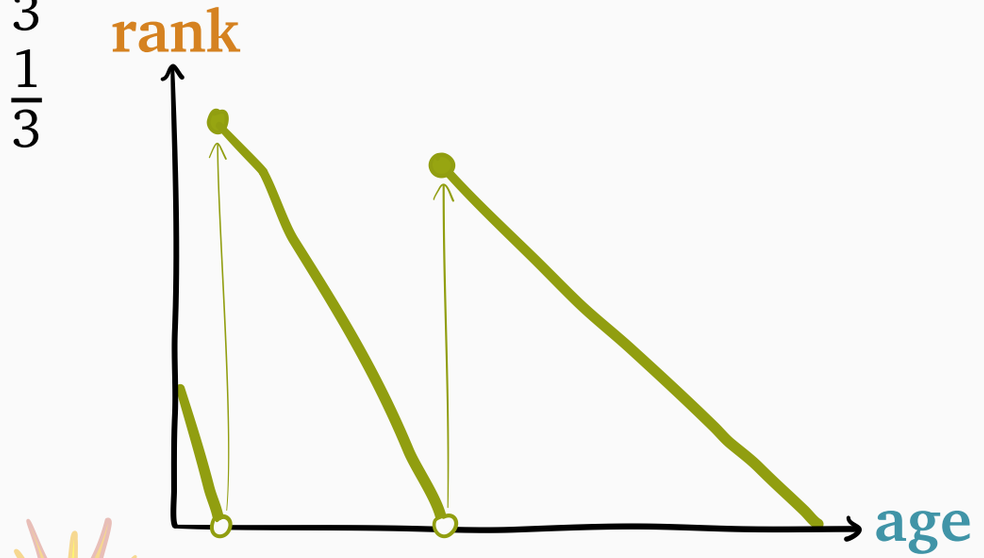


NEW! *first analysis* of **Gittins**

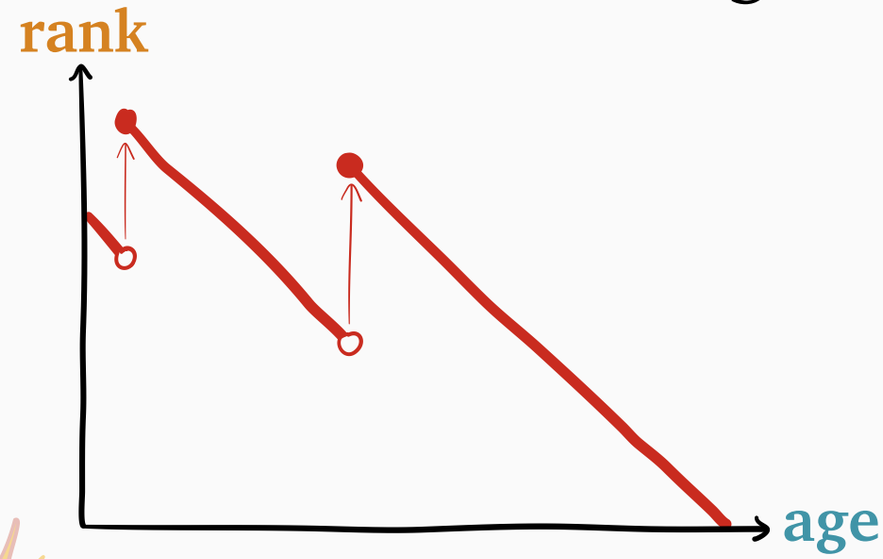
NEW! **Sample result:** for heavy-tailed S, **SERPT** and **Gittins** have *optimal tail decay* in large-t limit:

$$\mathbf{P}[T > t] \sim \mathbf{P}[S > (1-\rho)t]$$

First analyses via **SOAP**

SERPT

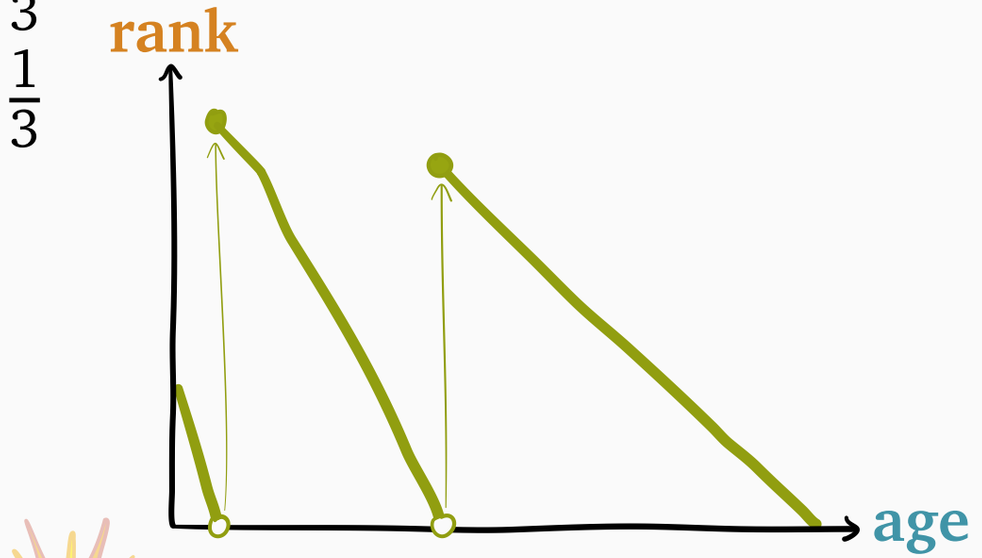
$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$



NEW! *first analysis* of **SERPT**

Gittins

minimizes $\mathbf{E}[T]$

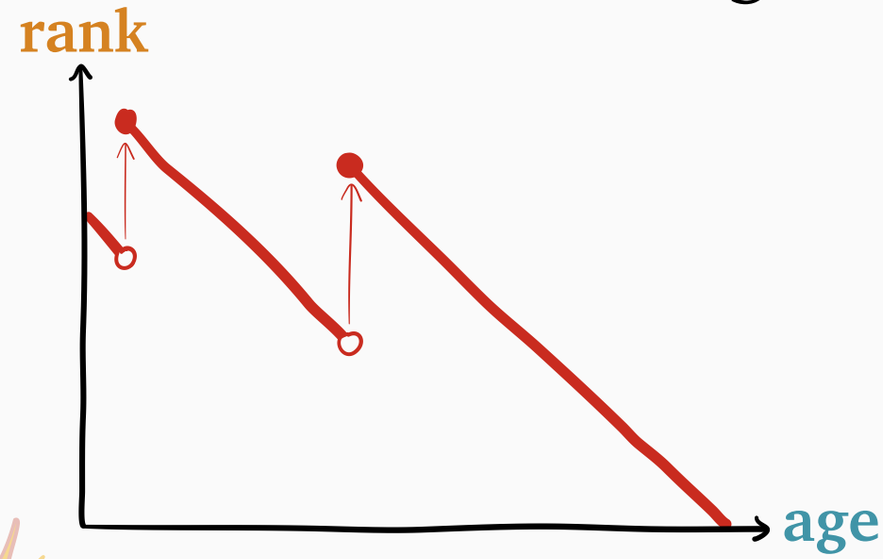


NEW! *first analysis* of **Gittins**

First analyses via **SOAP**

SERPT

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$



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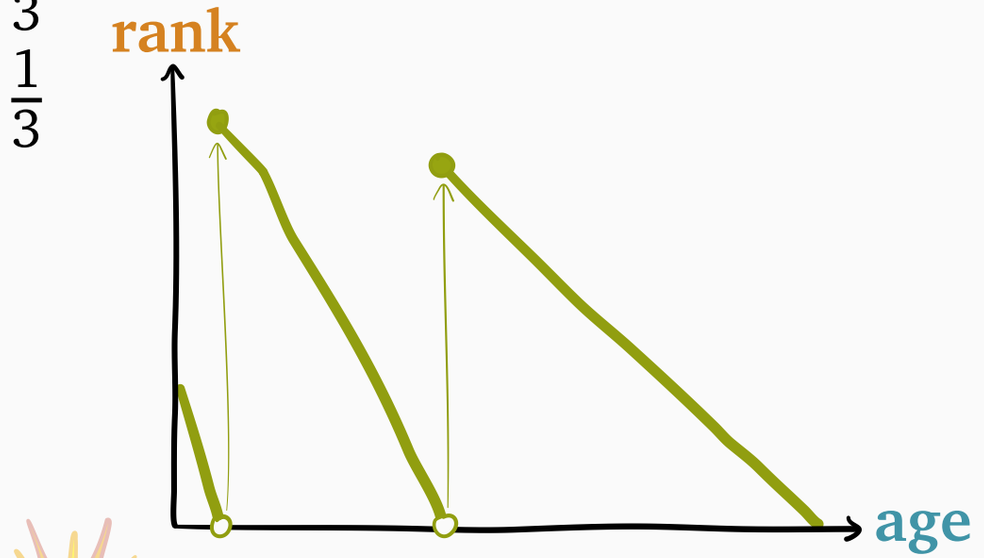


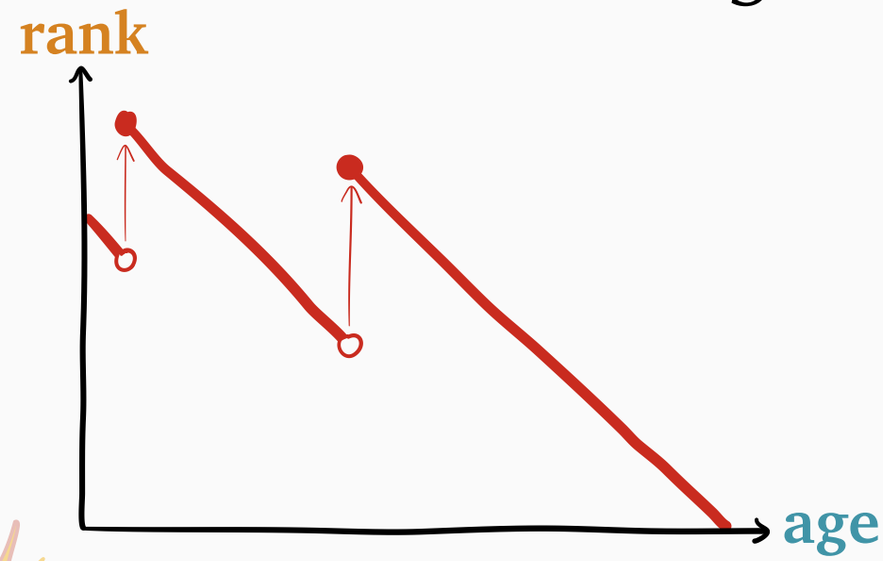
NEW! *first analysis* of **Gittins**

Observation: **SERPT**'s $\mathbf{E}[T]$ "usually" within 10% of **Gittins**'s

First analyses via **SOAP**

SERPT

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$



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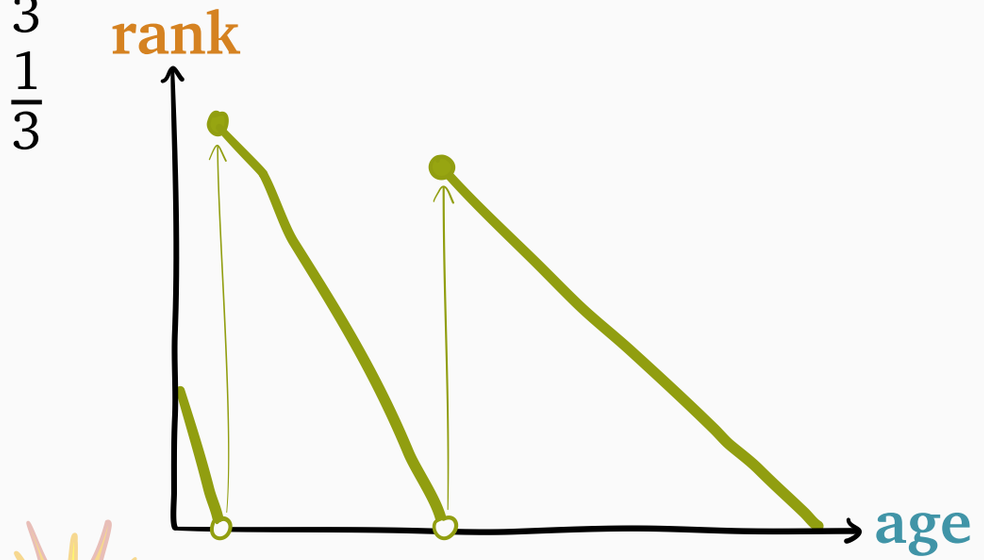
minimizes $\mathbf{E}[T]$



NEW! *first analysis* of **Gittins**

Observation: **SERPT**'s $\mathbf{E}[T]$ "usually" within 10% of **Gittins**'s

First analyses via SOAP

M-SERPT

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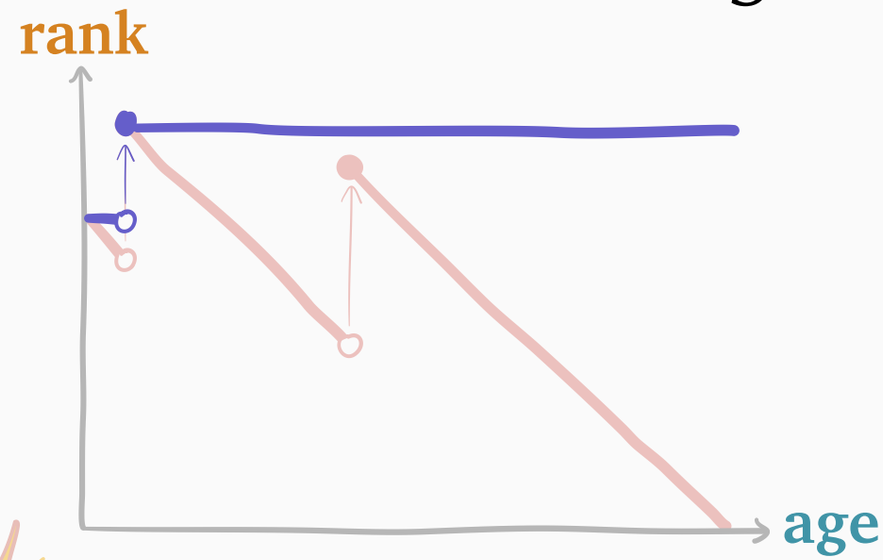

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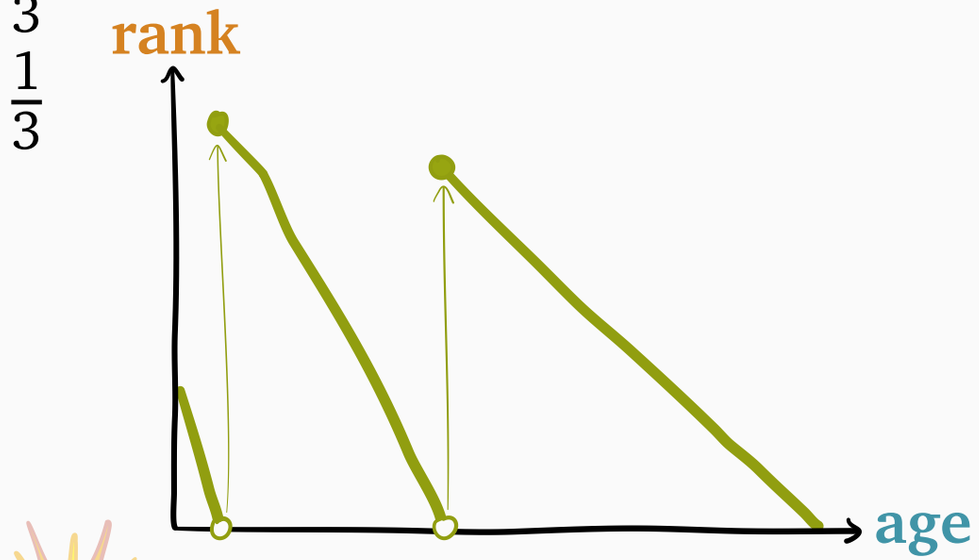
Gittins

minimizes $\mathbf{E}[T]$



NEW! *first analysis* of **Gittins**

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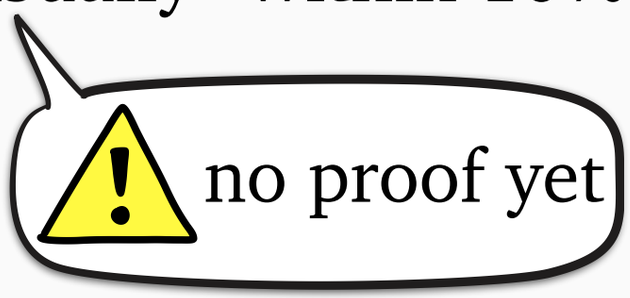

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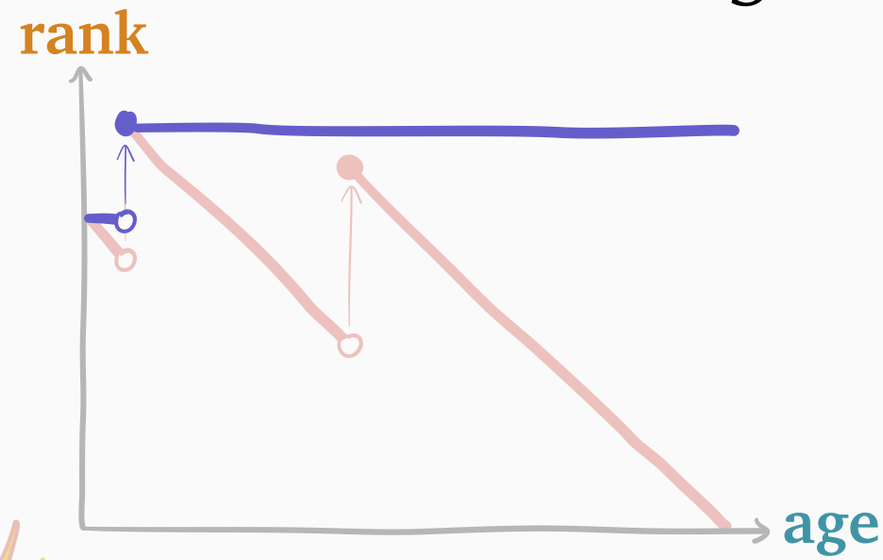

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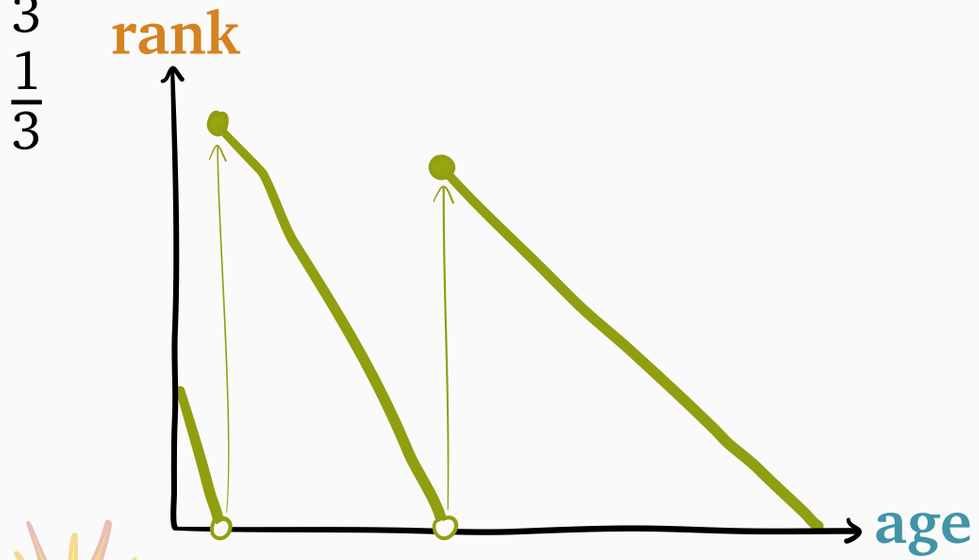
Gittins

minimizes $\mathbf{E}[T]$



NEW! *first analysis* of **Gittins**

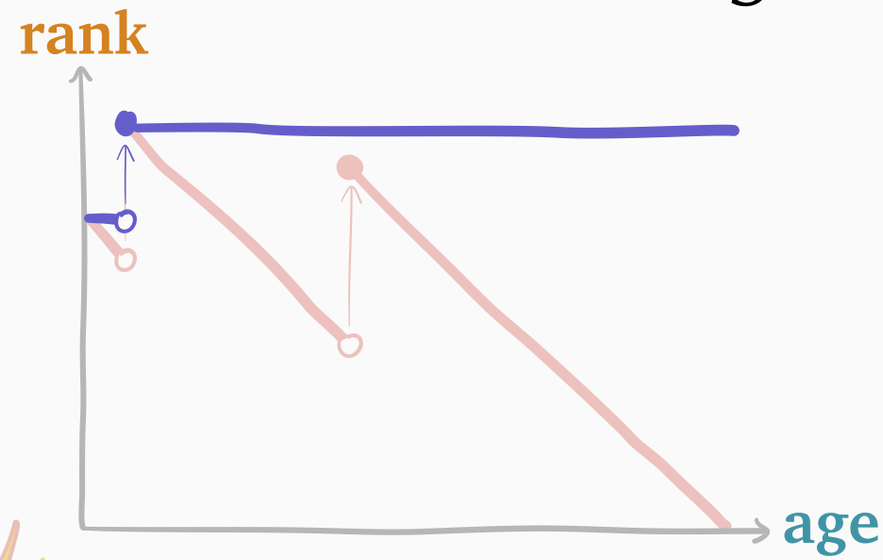
Observation: **SERPT**'s $\mathbf{E}[T]$ "usually" within 10% of **Gittins**'s

Theorem: **M-SERPT** is a 5-approximation for $\mathbf{E}[T]$

First analyses via SOAP

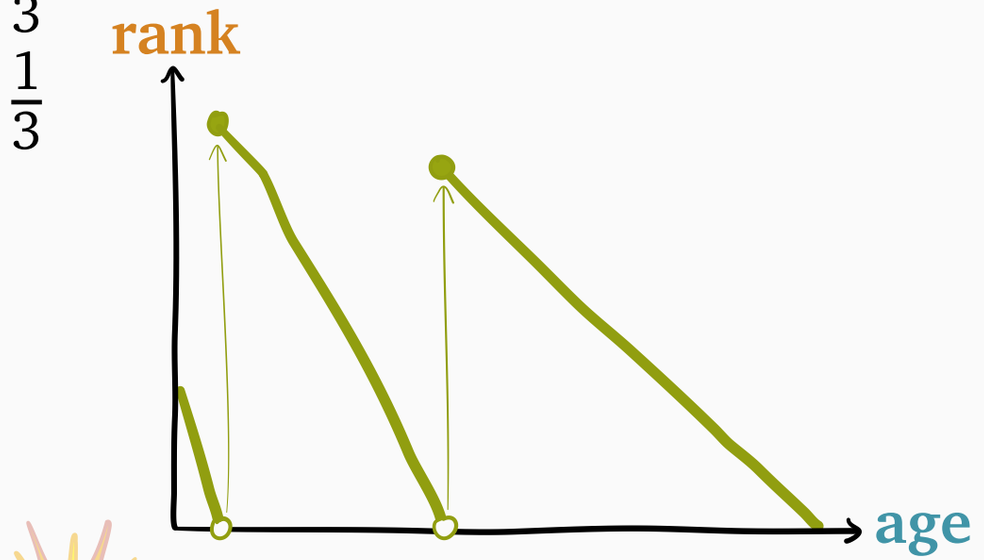
M-SERPT

$$S = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 6 & \text{w.p. } \frac{1}{3} \\ 14 & \text{w.p. } \frac{1}{3} \end{cases}$$



Gittins

minimizes $\mathbf{E}[T]$



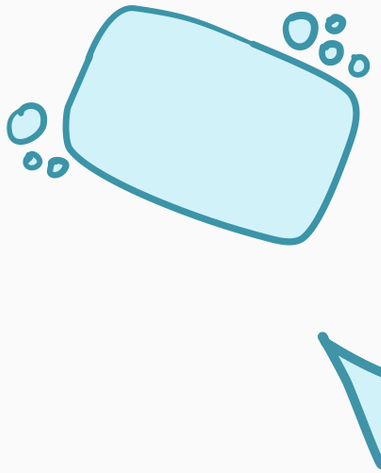
NEW! *first analysis* of **SERPT**

NEW! *first analysis* of **Gittins**

Lesson: carefully plan how **rank** gets worse

Obs [1] "usually" within 10% of **Gittins**'s

Theorem: **M-SERPT** is a 5-approximation for $\mathbf{E}[T]$



Impact of **SOAP**

unknown sizes

Impact of **SOAP**

unknown sizes

NEW! *First analysis* of

- mean
- tail decay

Impact of **SOAP**

unknown sizes

NEW!

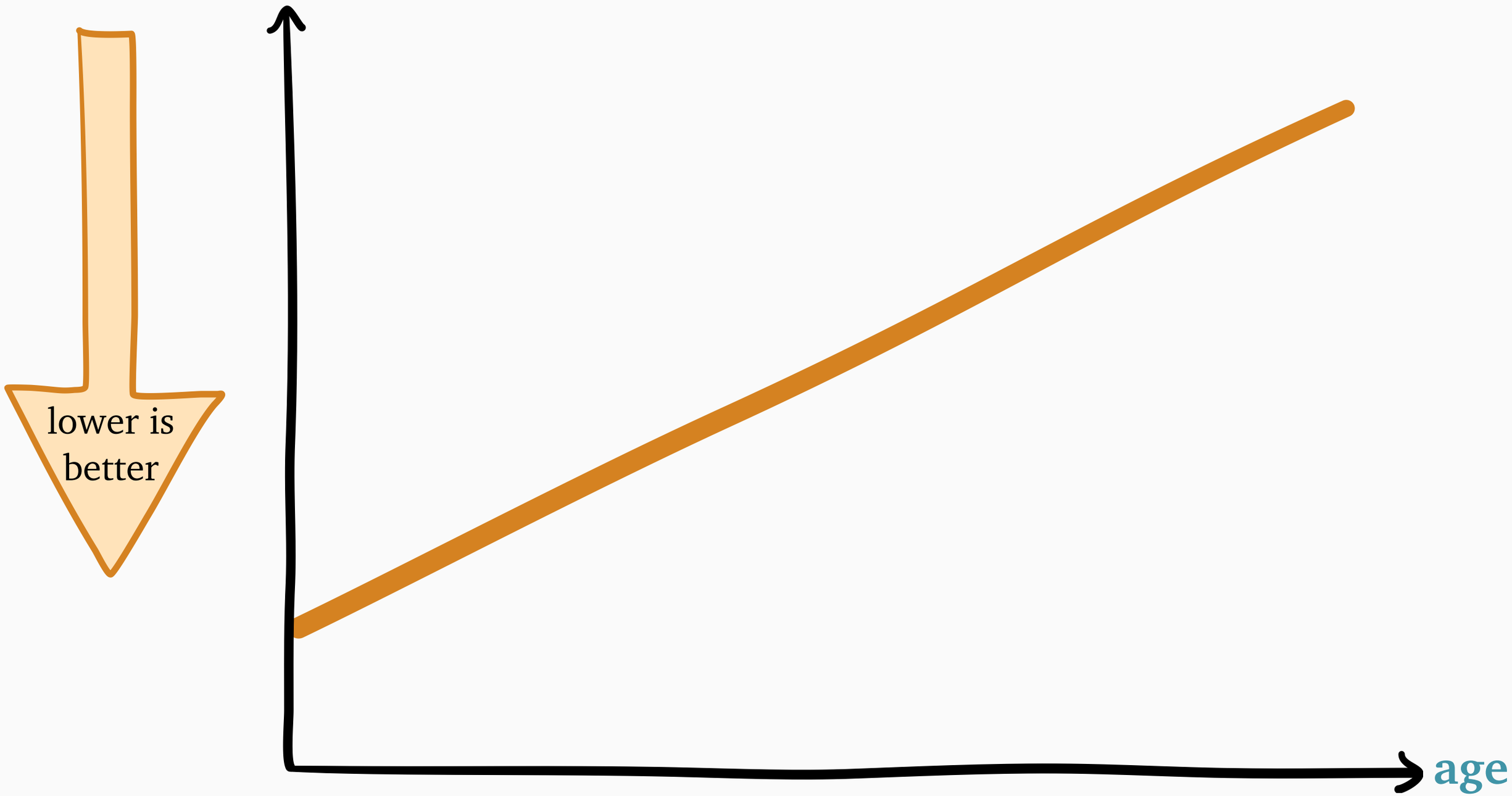
First analysis of

- mean
- tail decay

preemption limitations

Preemption checkpoints

Can only preempt only at *checkpoint* ***ages*** a_i



Preemption checkpoints

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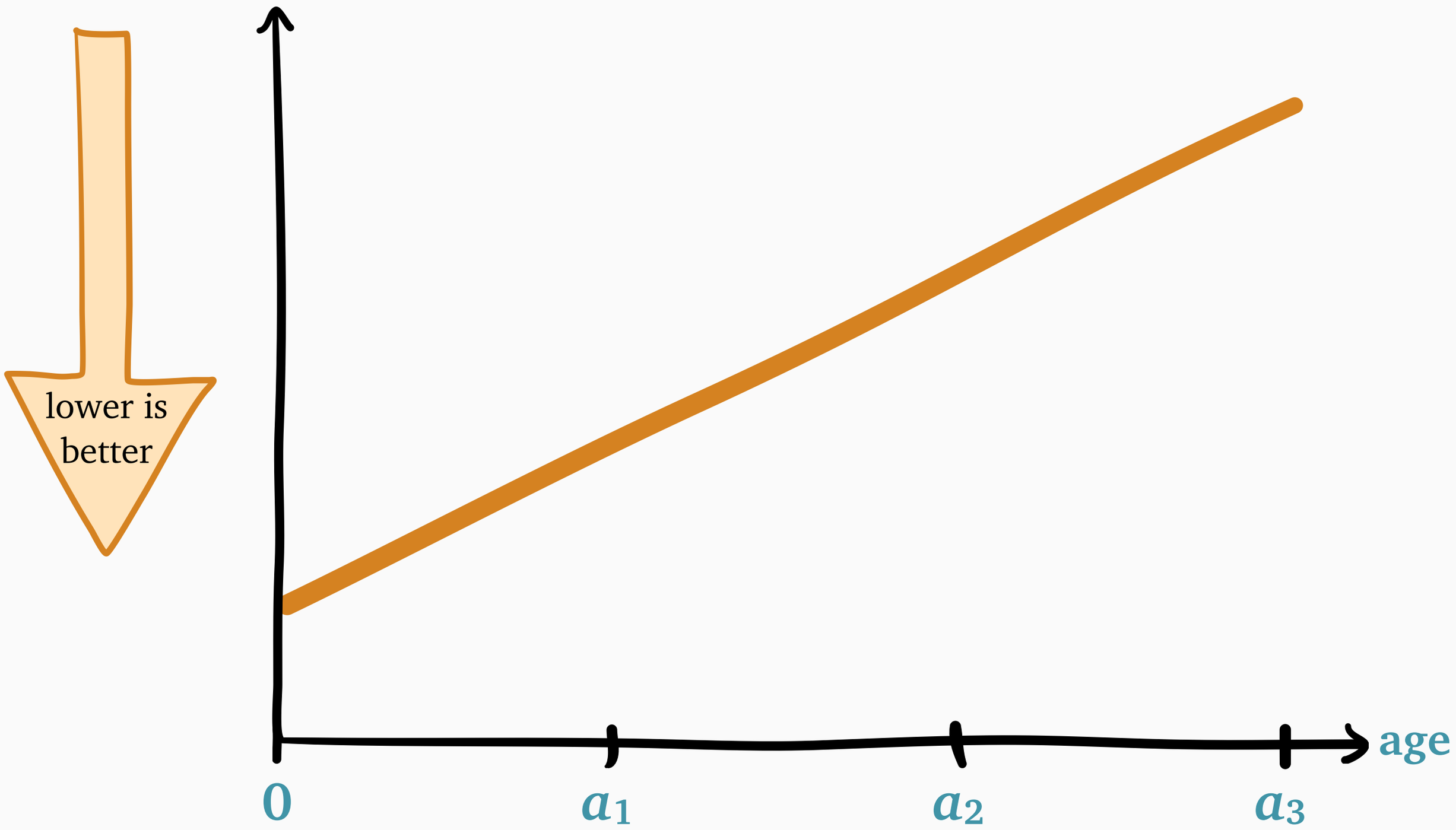

Preemption checkpoints

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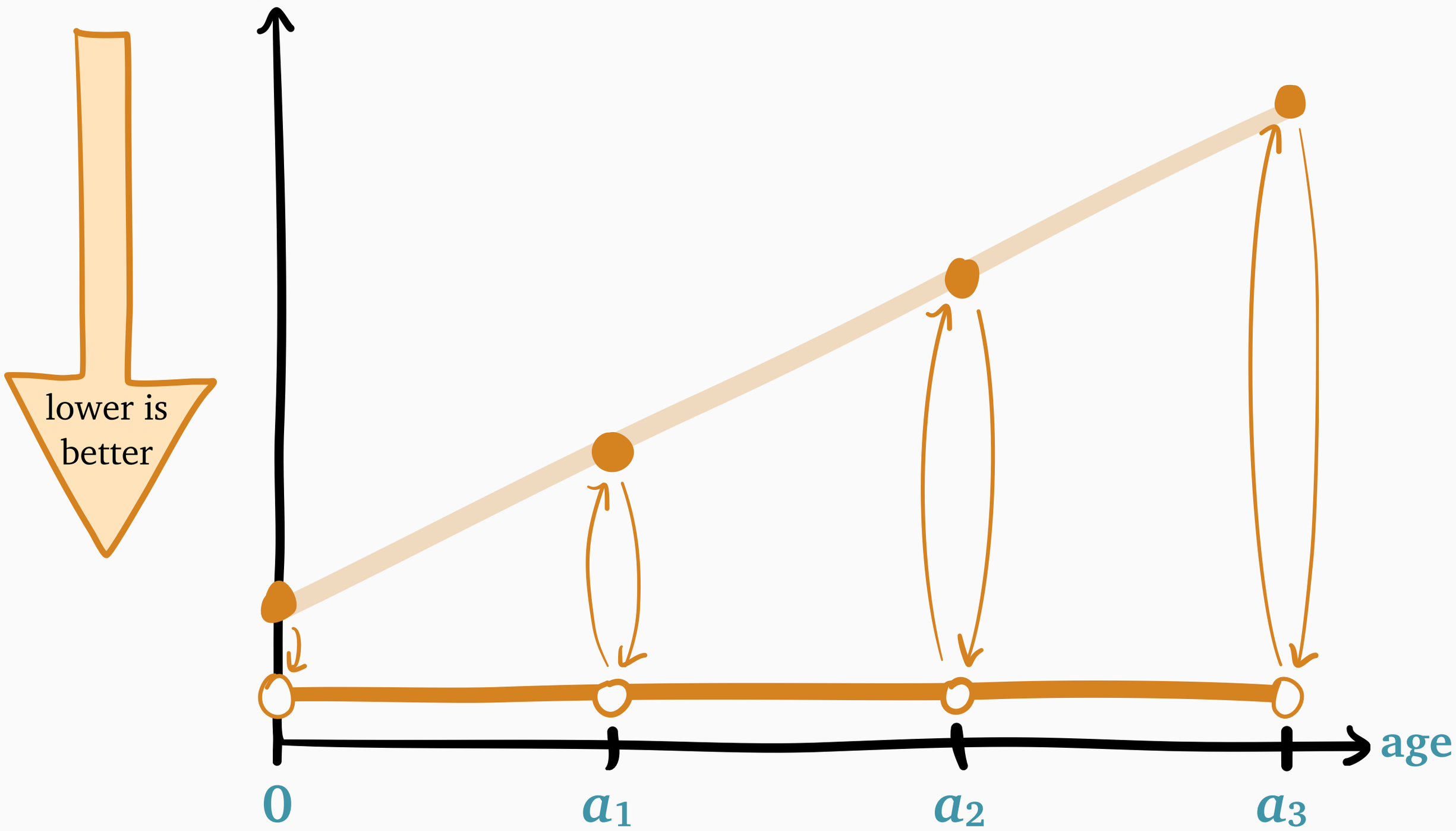

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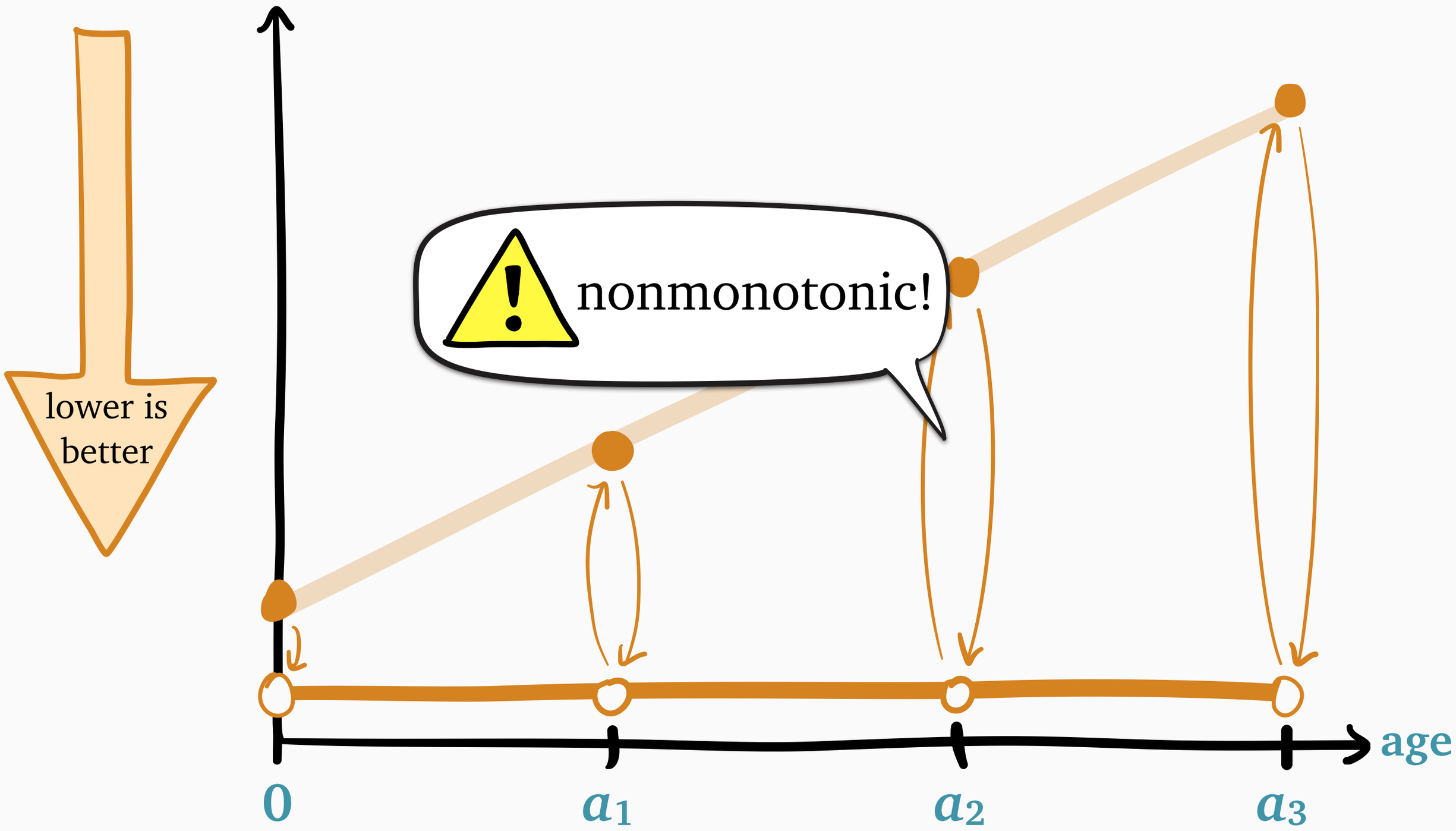

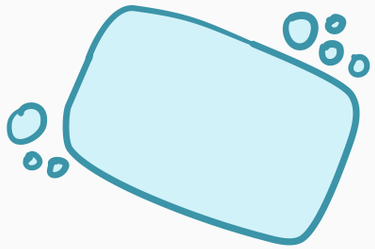

Impact of **SOAP**

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First analysis of

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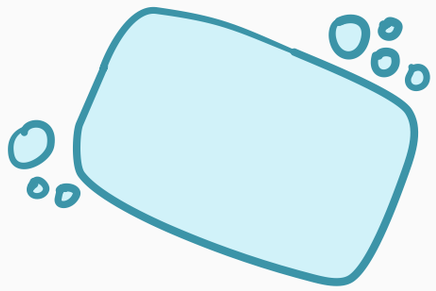
case studies

Outline: two new tools



SOAP

new unifying theory of single-server scheduling

→ unknown sizes

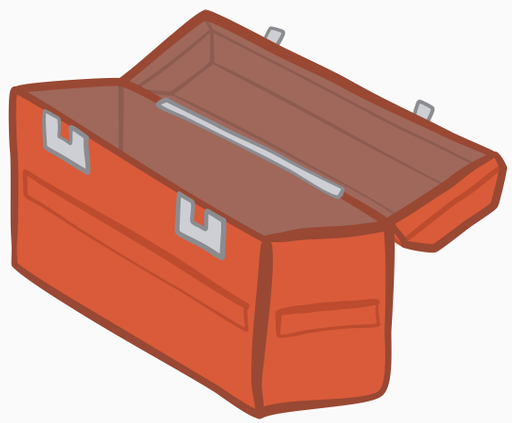
→ preemption limitations

WINE

new queueing identity to complement Little's Law

→ multiserver systems

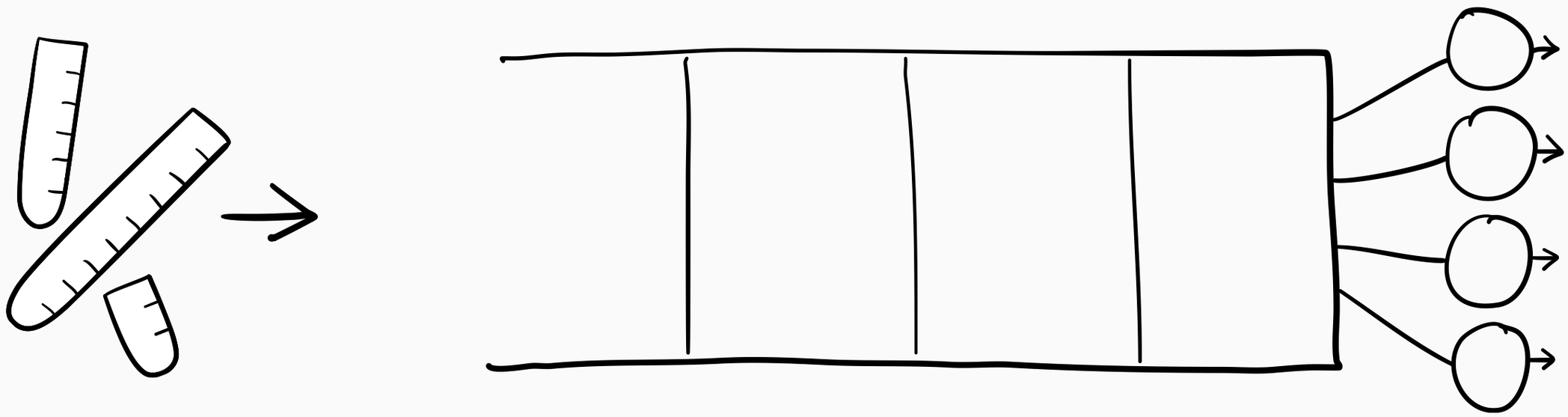
→ noisy size estimates



Part 2

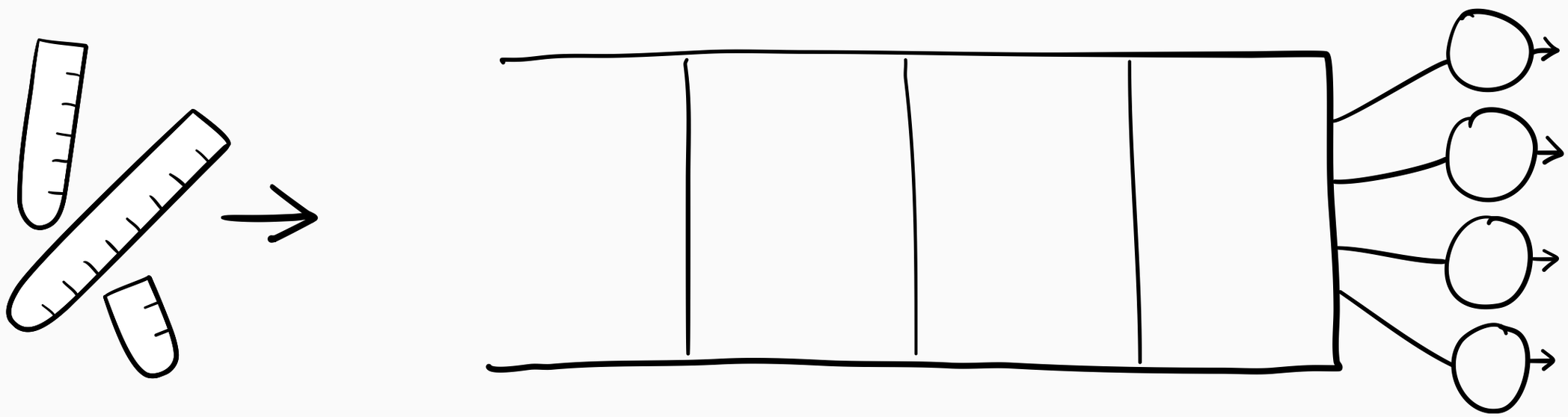
WINE

Multiserver queueing system



Multiserver queueing system

M/G/*k*



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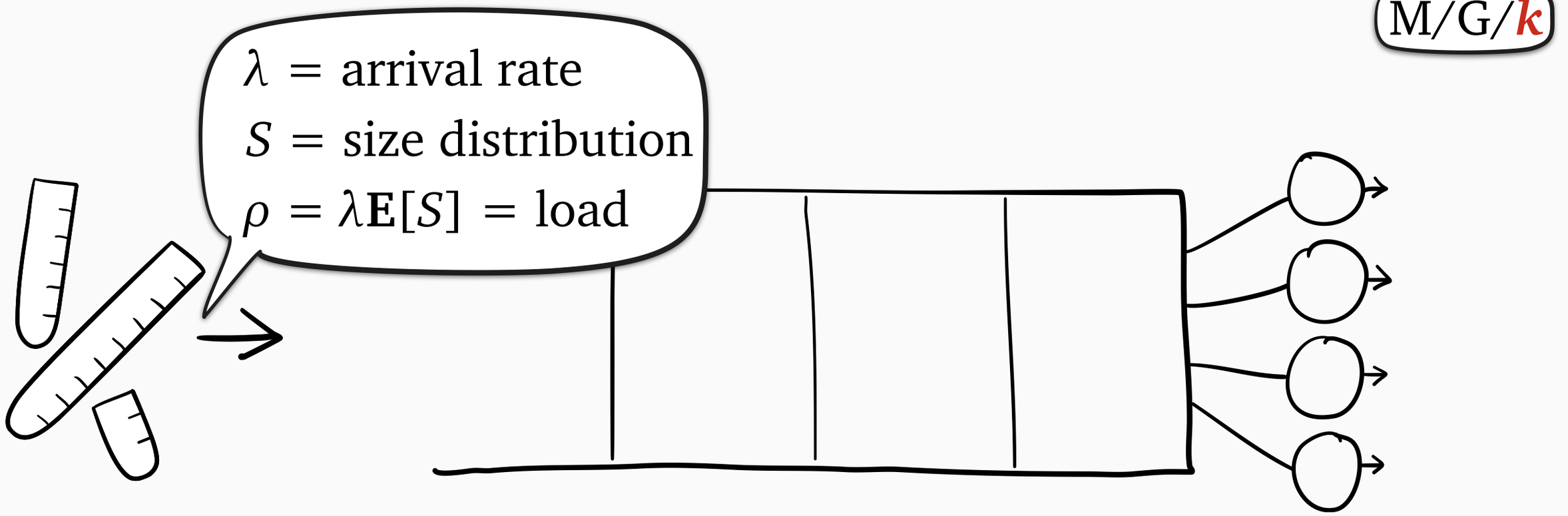


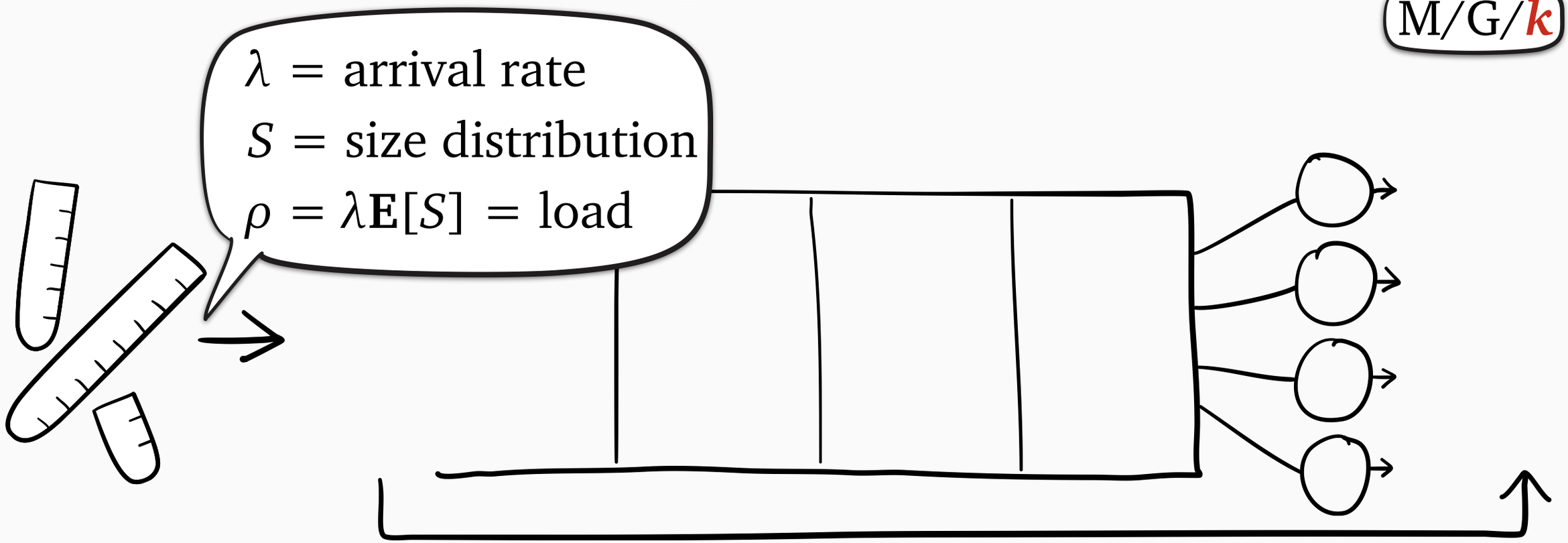
Multiserver queueing system

M/G/k

λ = arrival rate

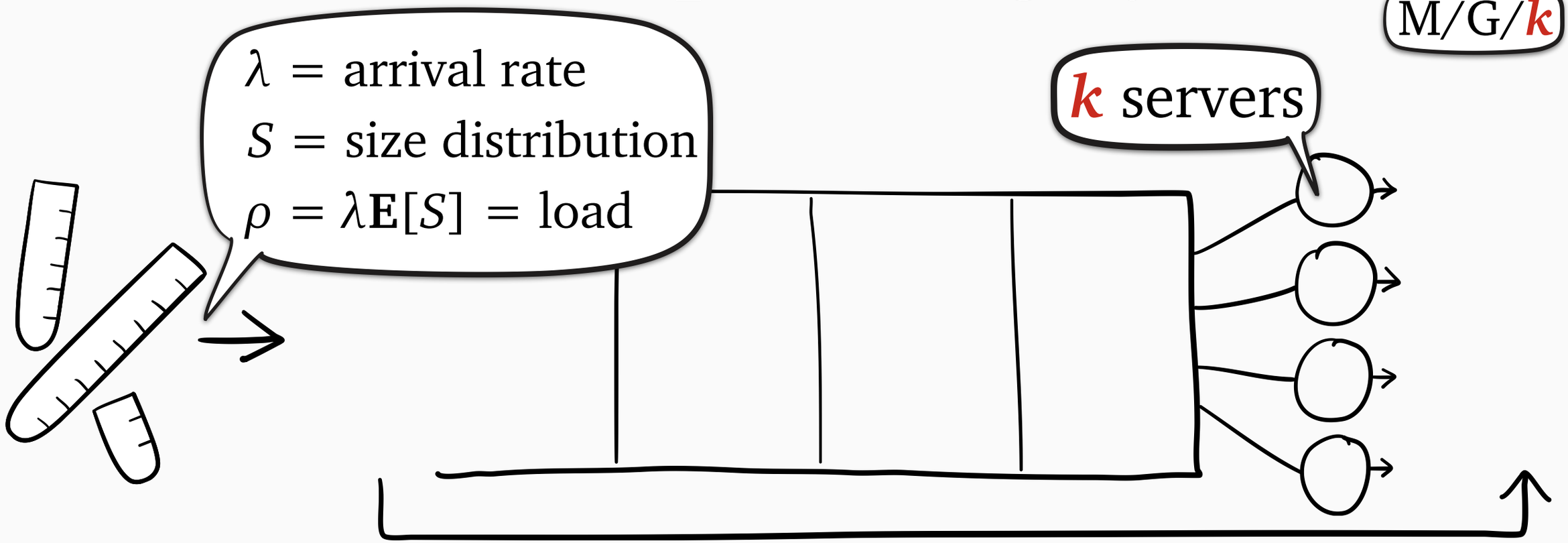
S = size distribution

$\rho = \lambda \mathbf{E}[S]$ = load



response time distribution T

Multiserver queueing system



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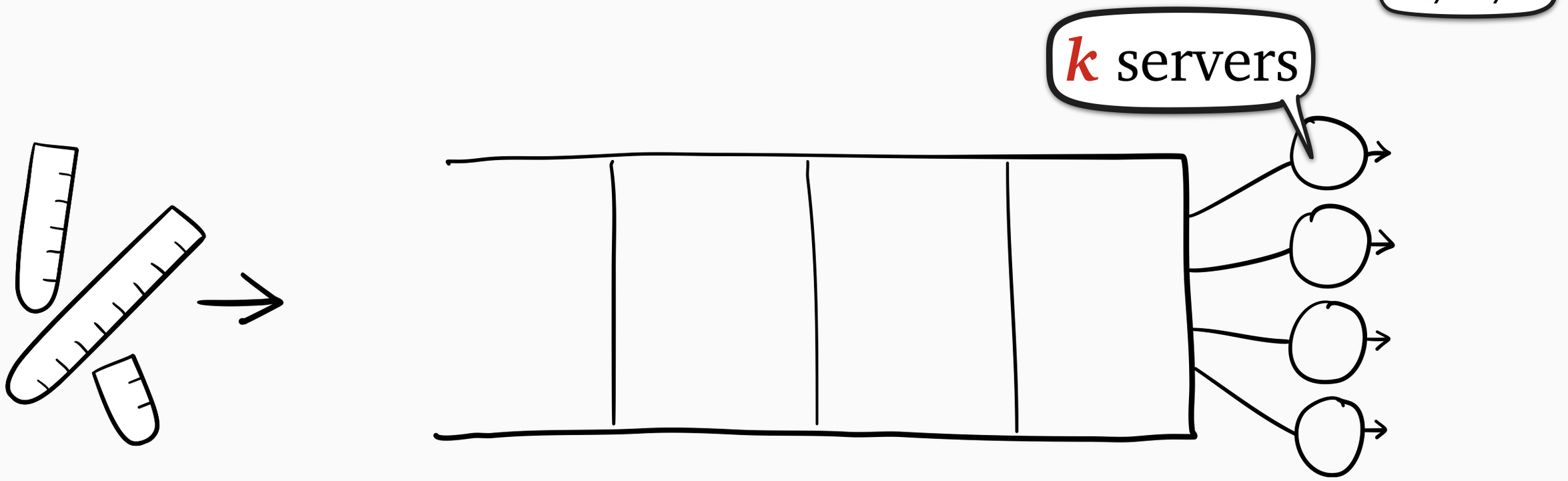
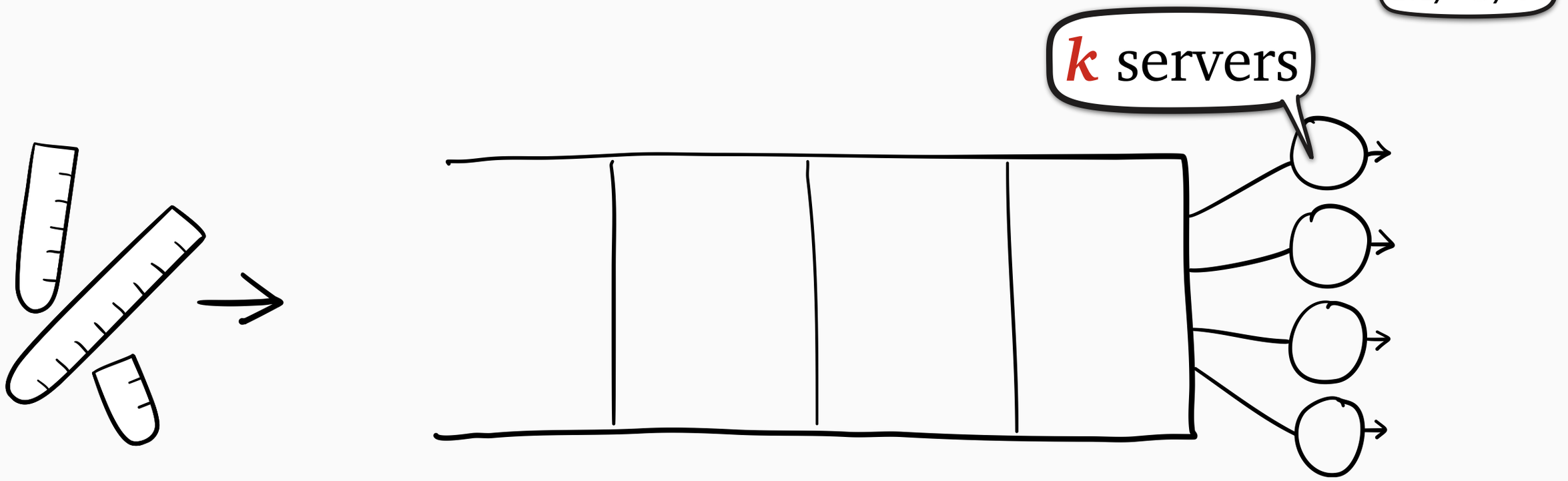


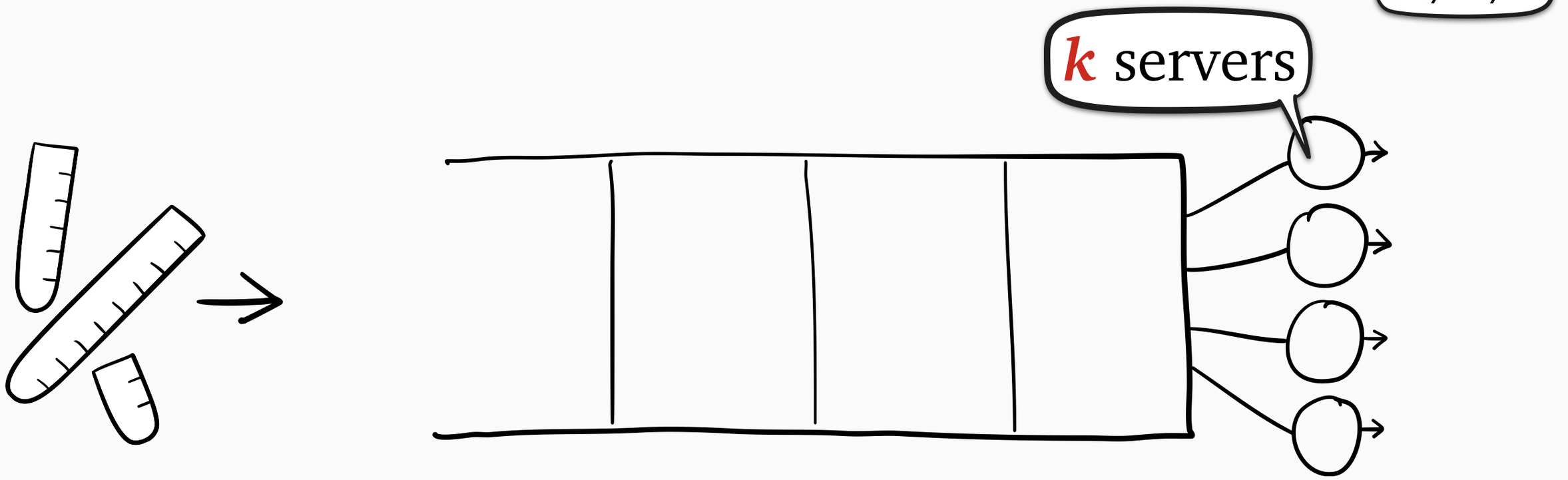
Multiserver queueing system



SRPT: **rank** = remaining size

Multiserver queueing system

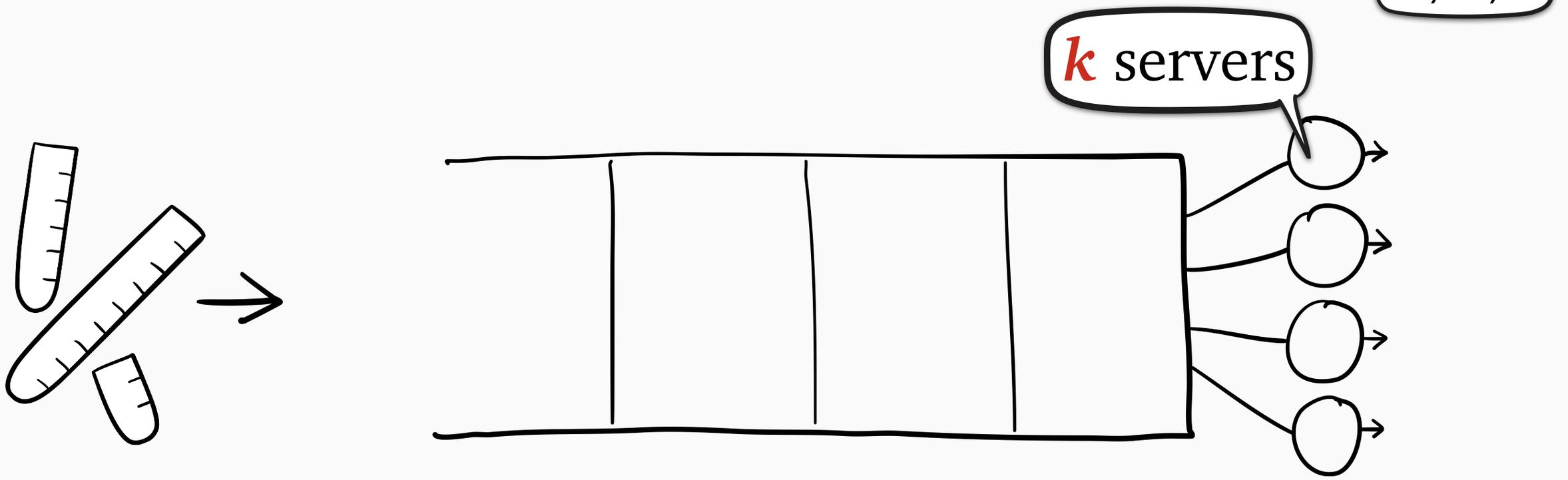
M/G/k



SRPT-1 (single-server): serves the job of least **rank**

SRPT: **rank** = remaining size

Multiserver queueing system

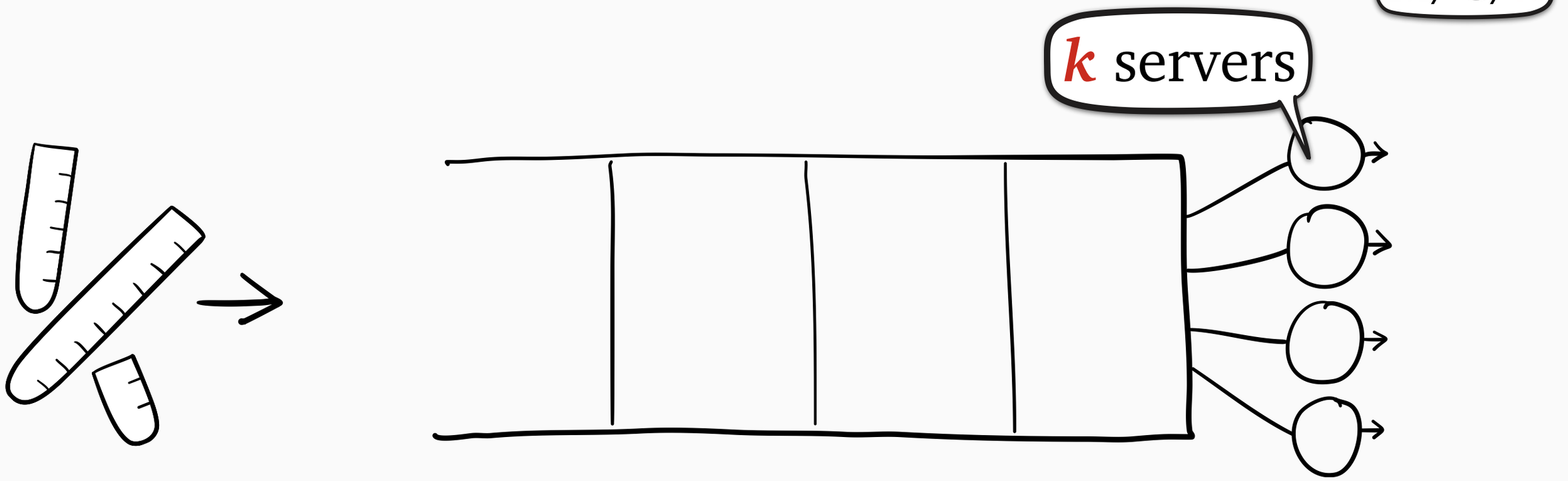


SRPT-1 (single-server): serves the job of least **rank**

SRPT: **rank** = remaining size

SRPT-*k* (multiserver): serves the ***k*** jobs of least **rank**

Multiserver queueing system



SRPT-1 (single-server): serves the job of least **rank**

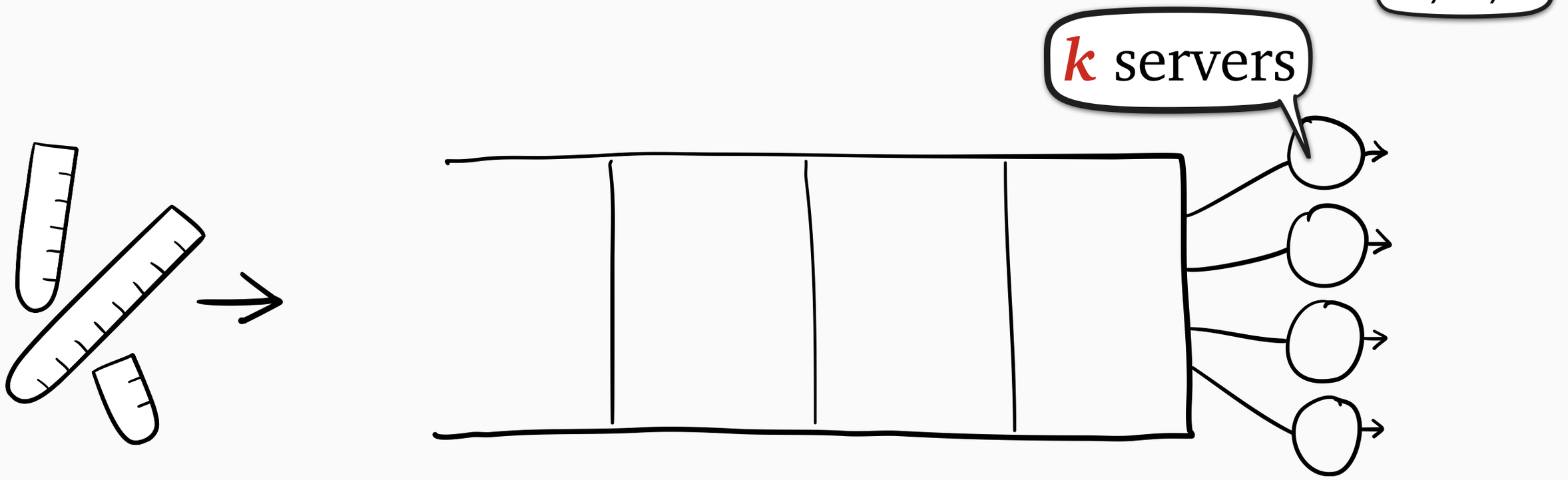
similarly for **Gittins-*k***, etc.

PT: **rank** = remaining size

SRPT-*k* (multiserver): serves the ***k*** jobs of least **rank**

Multiserver queueing system

M/G/k



analyzed

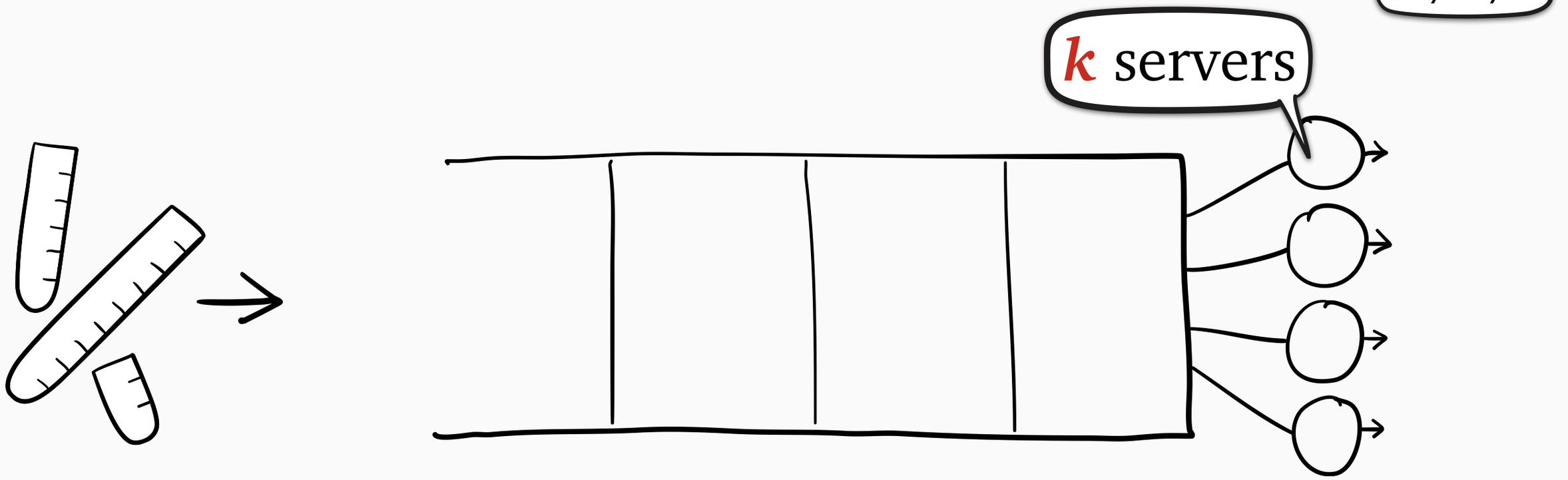
SRPT-1 (single-server): serves the job of least **rank**

SRPT: **rank** = remaining size

SRPT-k (multiserver): serves the k jobs of least **rank**

Multiserver queueing system

M/G/k



analyzed

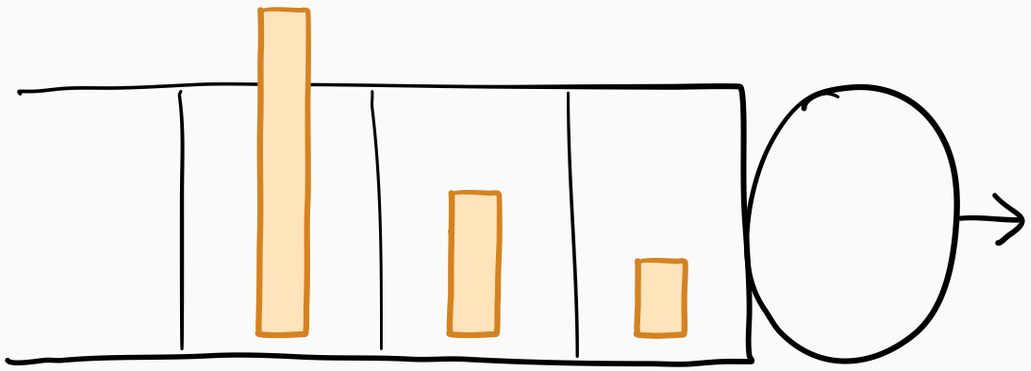
SRPT-1 (single-server): serves the job of least **rank**

analysis open

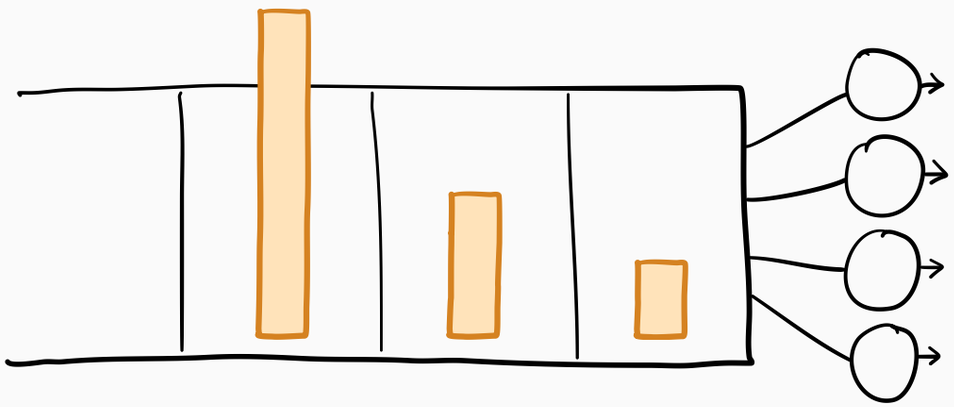
rank = remaining size

SRPT-k (multiserver): serves the k jobs of least **rank**

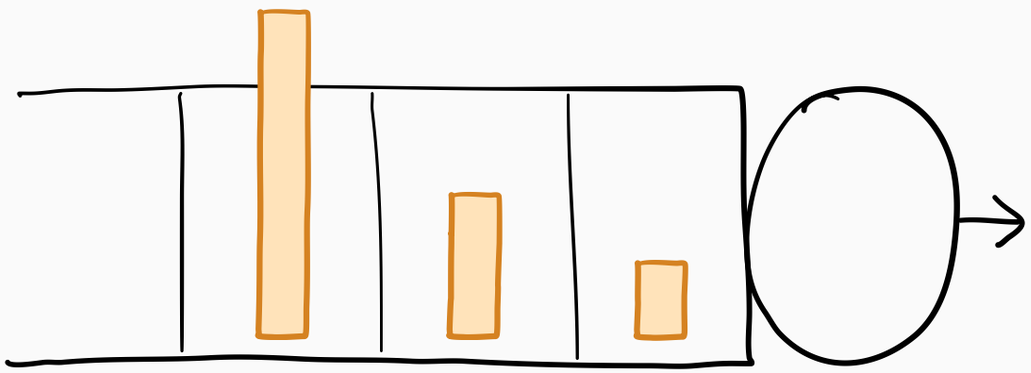
Single-server system



Multiserver system

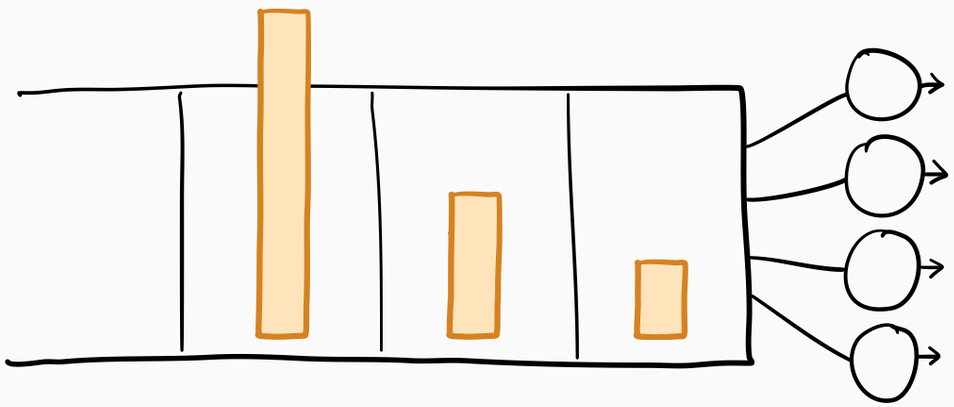


Single-server system

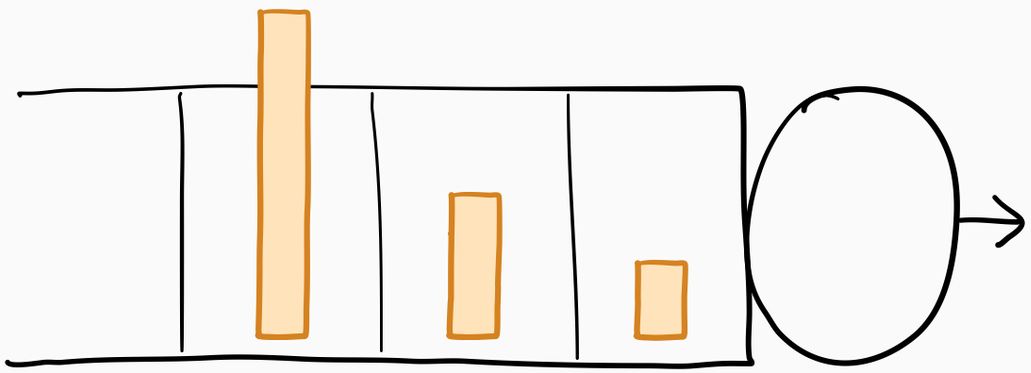


server is “choke point”

Multiserver system



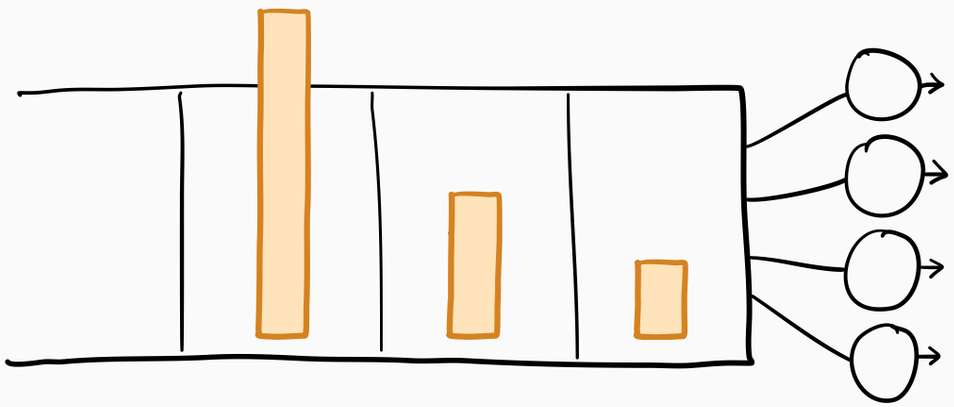
Single-server system



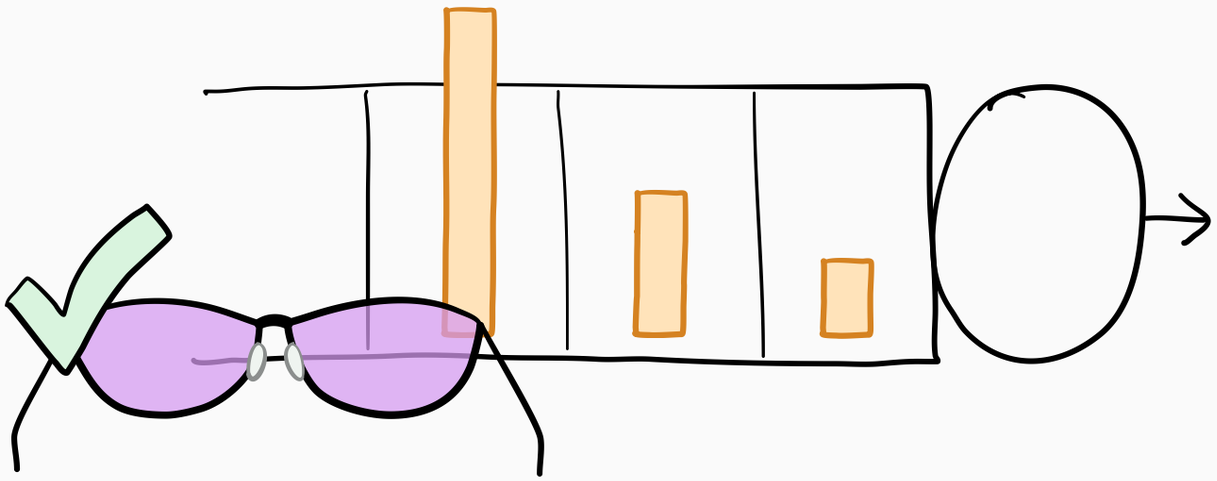
server is “choke point”

rank ordering absolute

Multiserver system



Single-server system

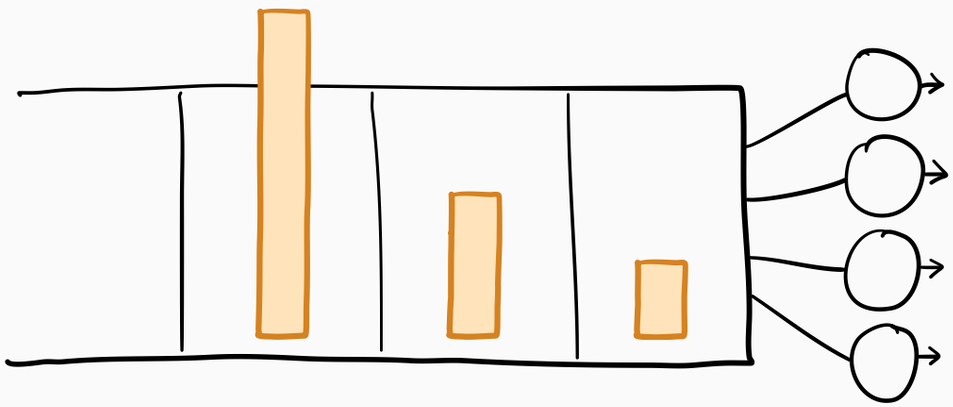


server is “choke point”

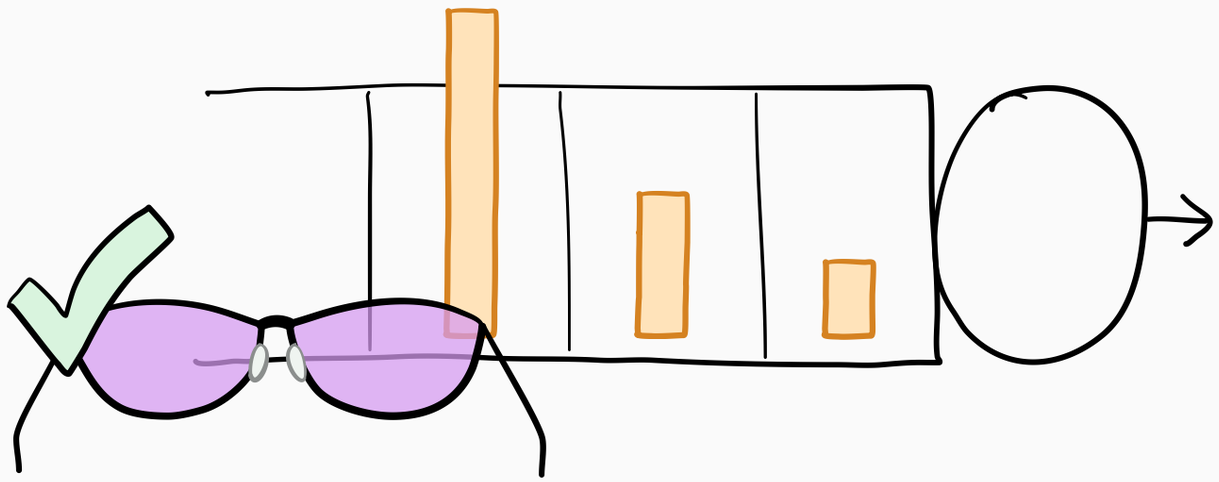
rank ordering absolute

observed ***r***-work determines T

Multiserver system



Single-server system



server is “choke point”

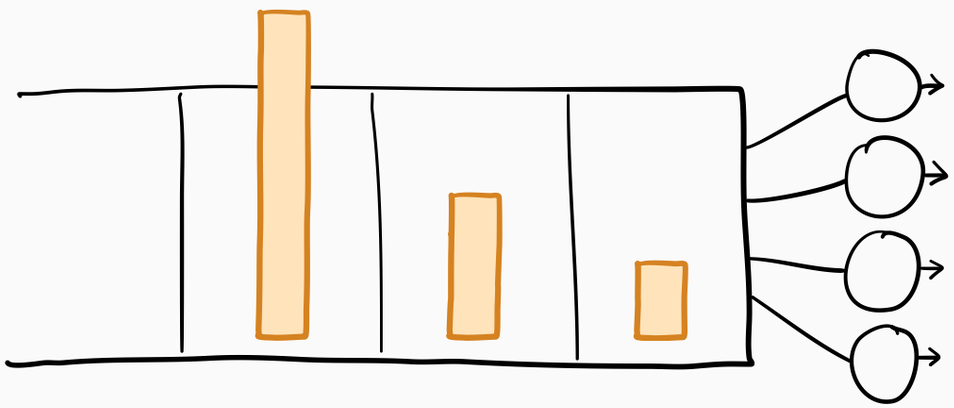
⇓

rank ordering absolute

⇓

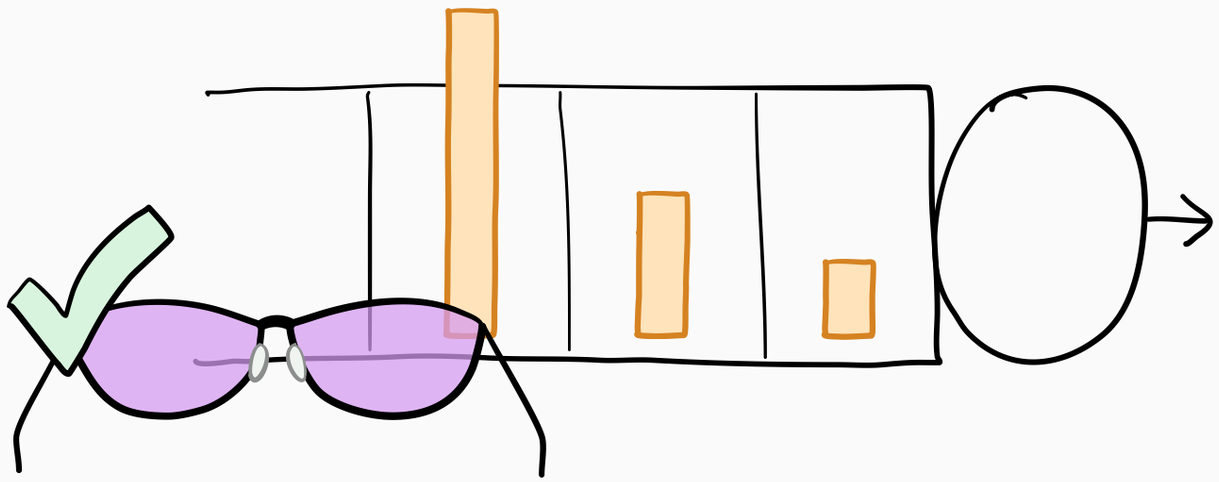
observed r-work determines T

Multiserver system



no single “choke point”

Single-server system



server is “choke point”

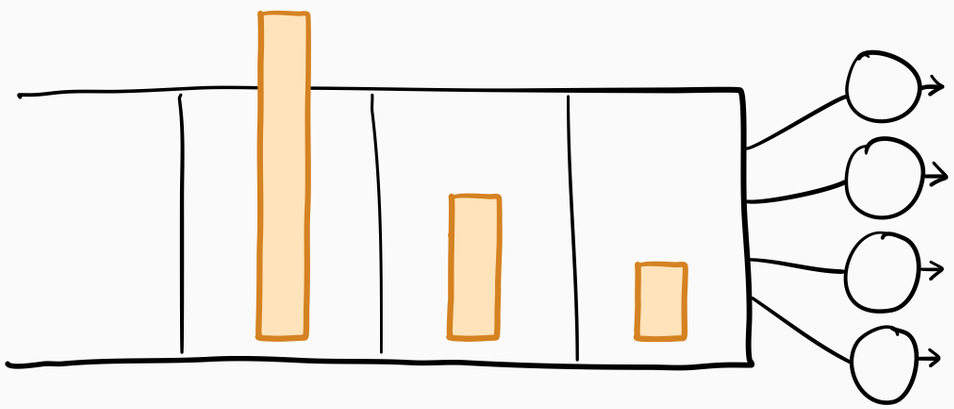
⇓

rank ordering absolute

⇓

observed r-work determines T

Multiserver system

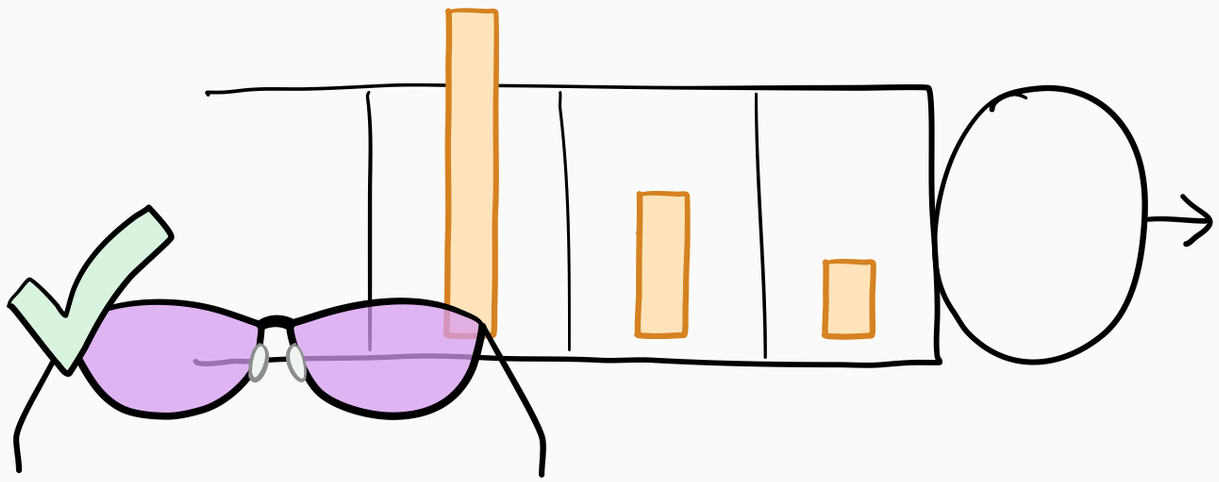


no single “choke point”

⇓

rank ordering *not* absolute

Single-server system



server is “choke point”

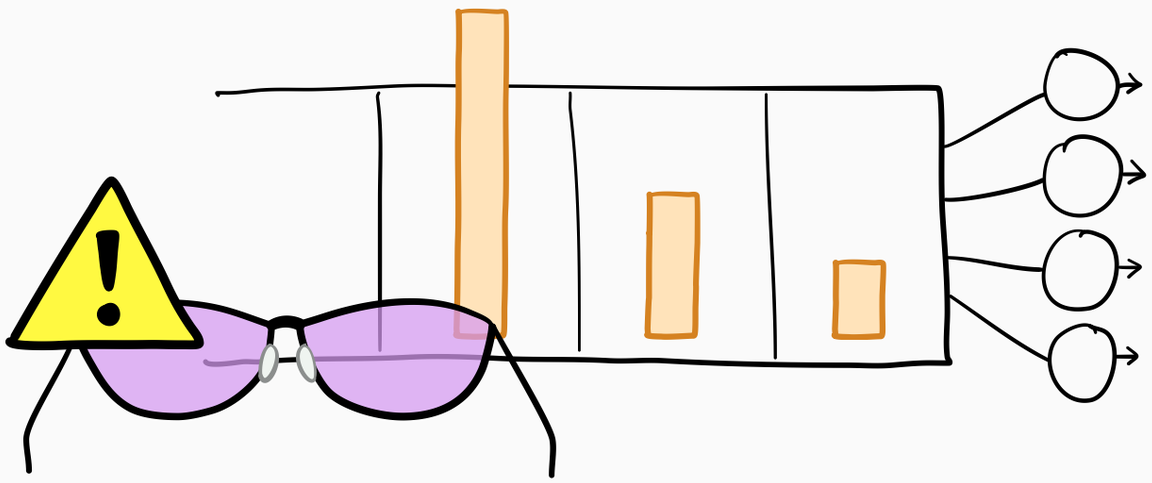
⇓

rank ordering absolute

⇓

observed ***r***-work determines T

Multiserver system



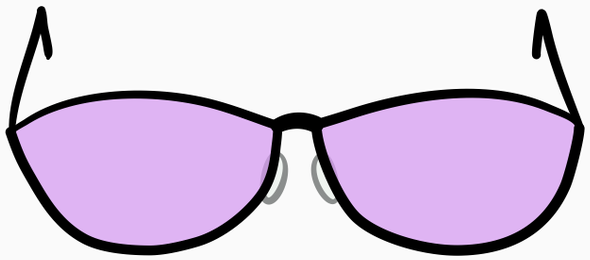
no single “choke point”

⇓

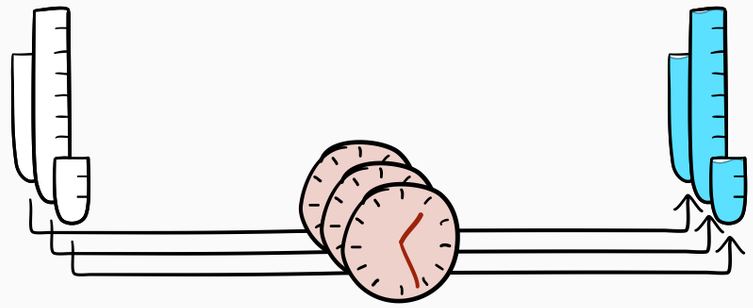
rank ordering *not* absolute

⇓

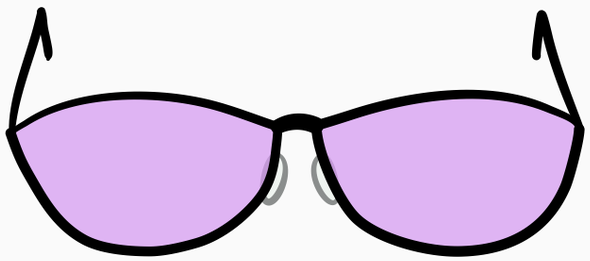
observed ***r***-work *not enough!*



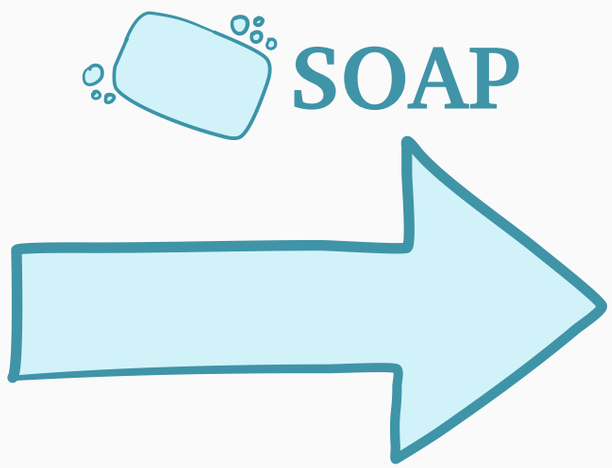
r-work $W(r)$

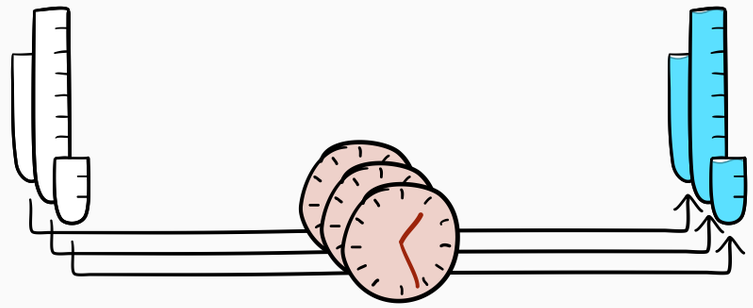


response time T

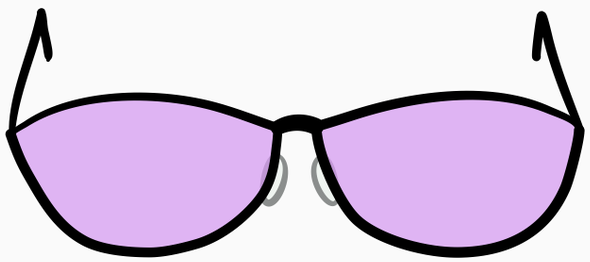


r-work $W(r)$

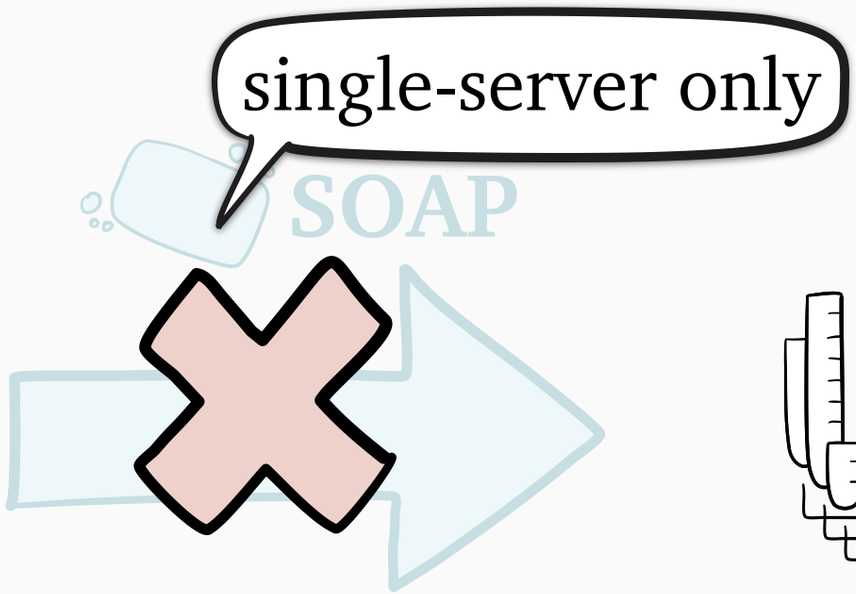


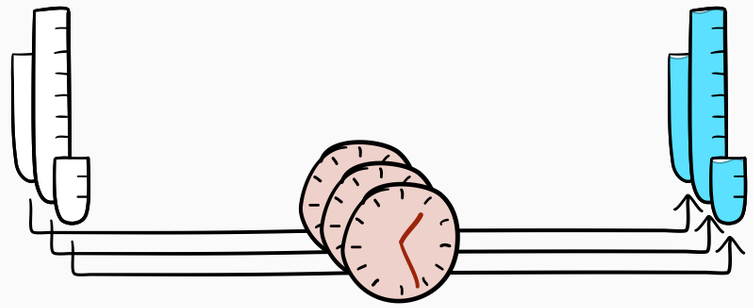


response time T



r-work $W(r)$

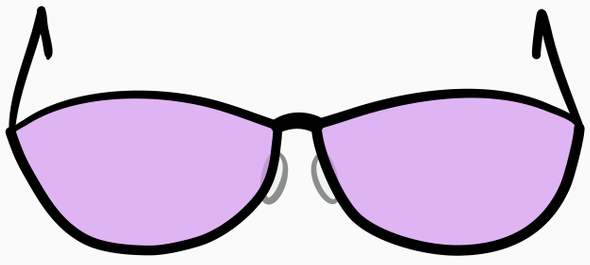




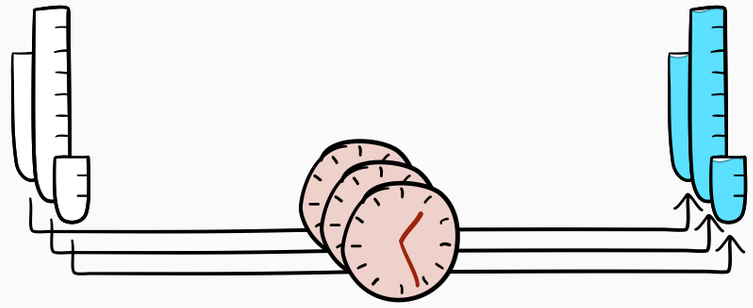
response time T

single-server only

SOAP

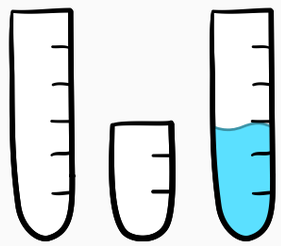


r-work $W(r)$

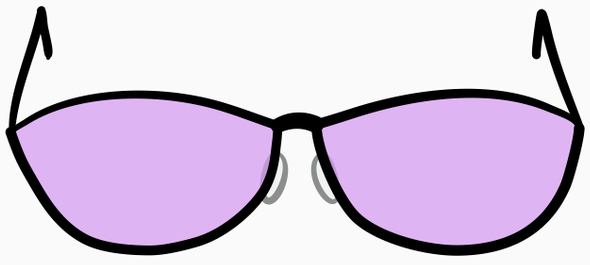


response time T

Little's law



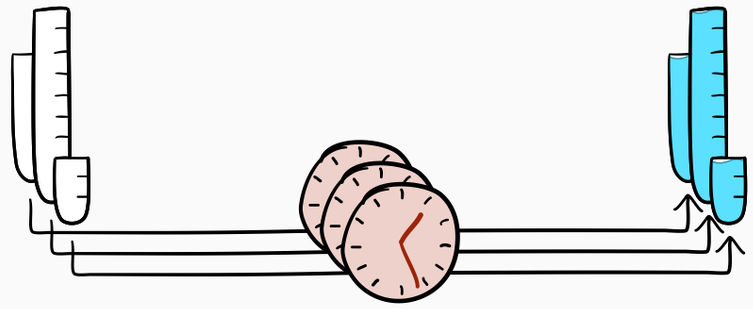
number of jobs N



r-work $W(r)$

single-server only

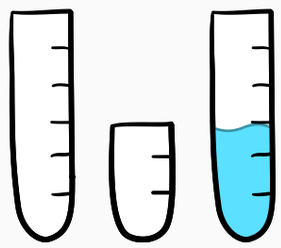
SOAP



response time T

$$\mathbf{E}[N] = \lambda \mathbf{E}[T]$$

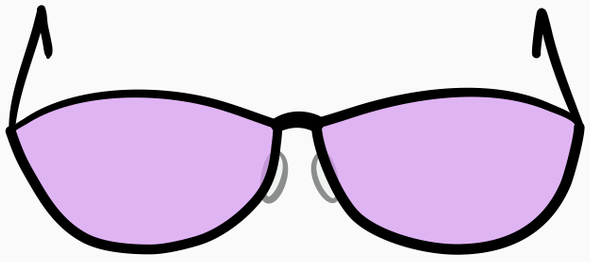
Little's law



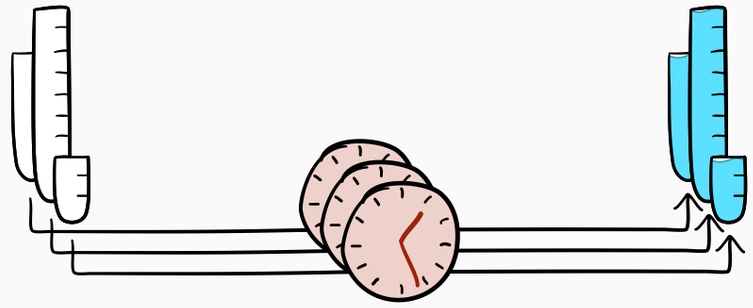
number of jobs N

single-server only

SOAP



r-work $W(r)$

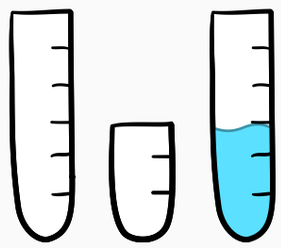


response time T

$\mathbf{E}[N] = \lambda \mathbf{E}[T]$

Little's law

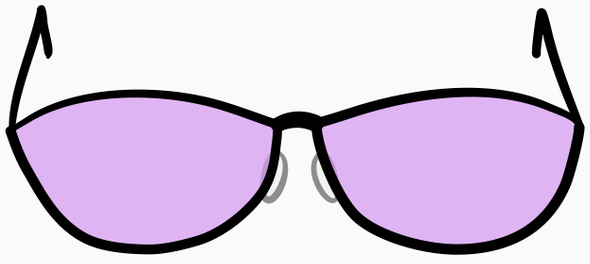
any number of servers



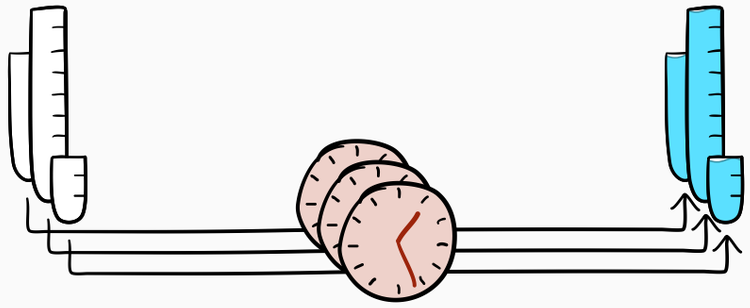
number of jobs N

single-server only

SOAP



r-work $W(r)$



response time T

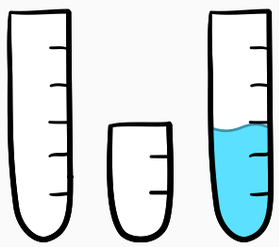
NEW!

WINE

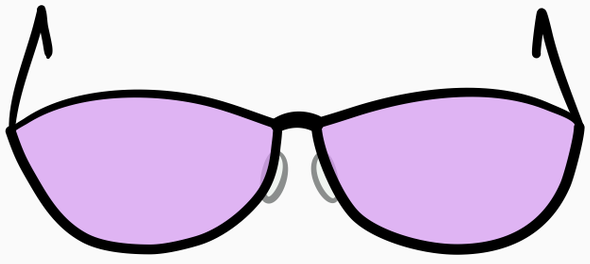
$$\mathbf{E}[N] = \lambda \mathbf{E}[T]$$

Little's law

any number of servers



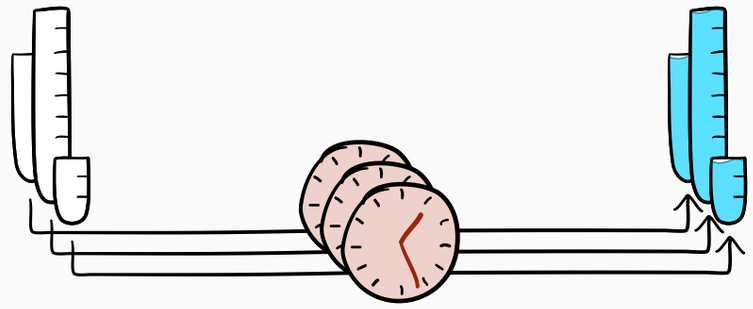
number of jobs N



r-work $W(r)$

single-server only

SOAP



response time T

NEW!

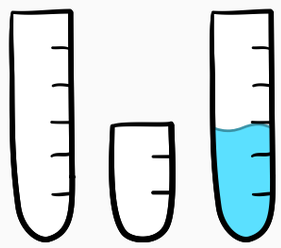
WINE

any number of servers

$\mathbf{E}[N] = \lambda \mathbf{E}[T]$

Little's law

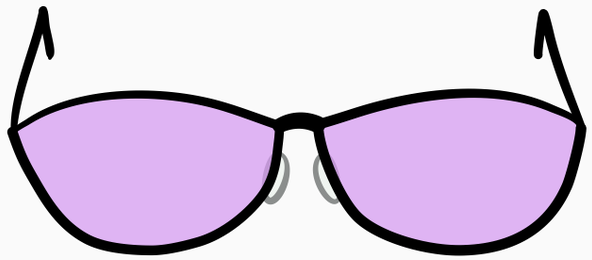
any number of servers



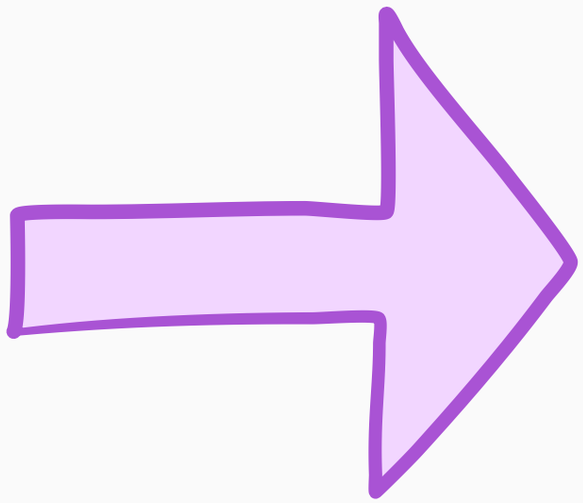
number of jobs N

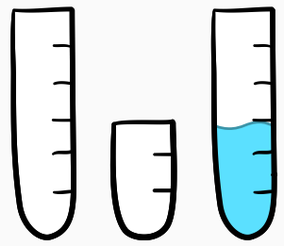
WINE

Work **I**ntegral **N**umber **E**quality



r-work $W(r)$

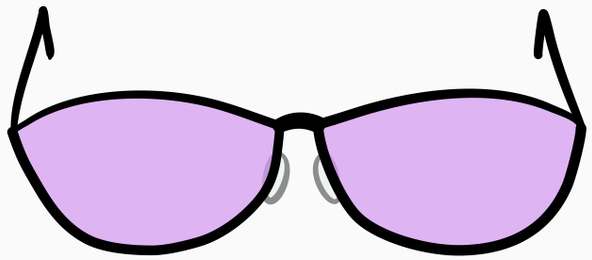




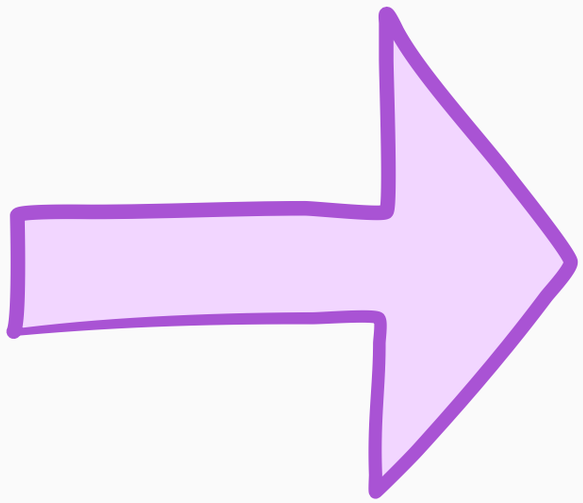
number of jobs N

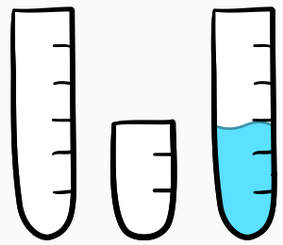
WINE

Work **I**ntegral **N**umber **E**quality

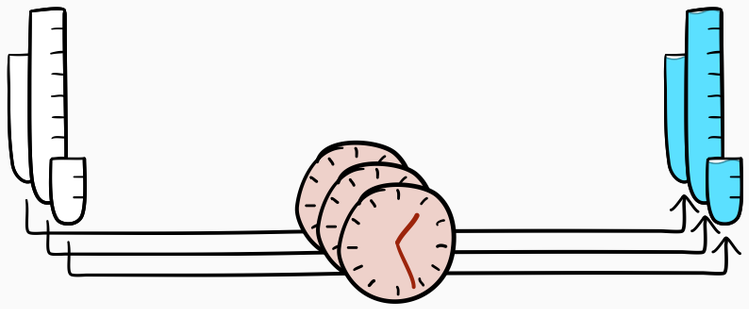


r-work $W(r)$





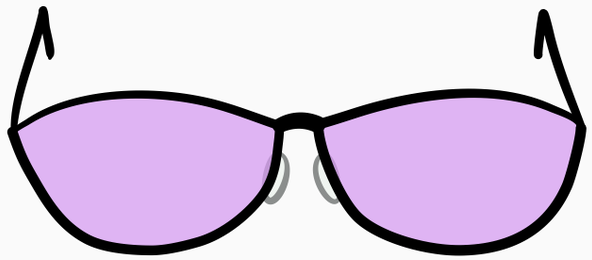
number of jobs N



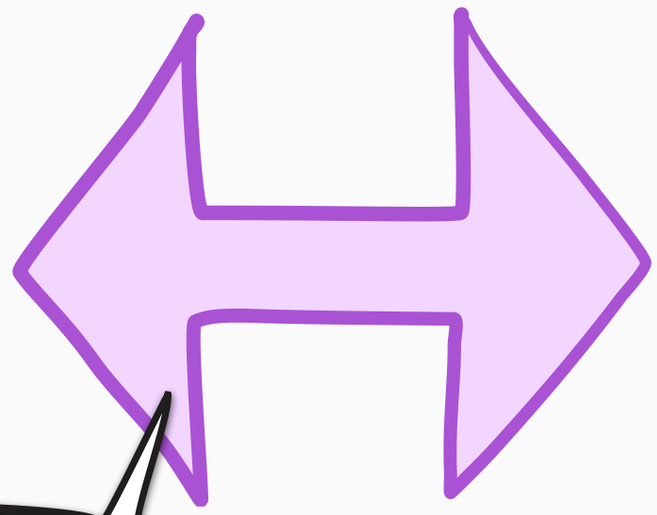
mean response time $\mathbf{E}[T]$

WINE

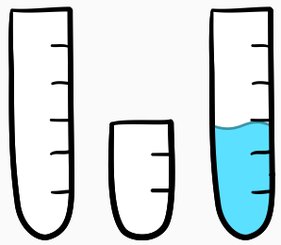
Work **I**ntegral **N**umber **E**quality



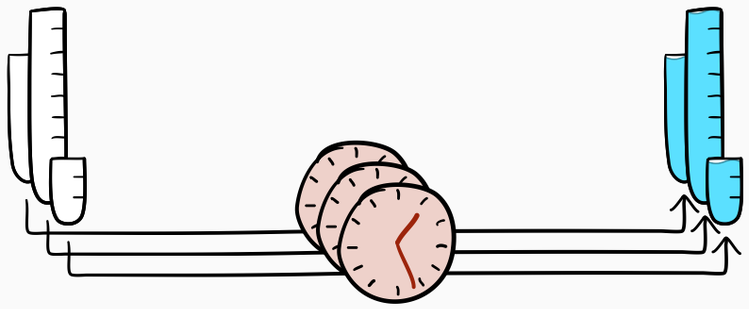
r-work $W(r)$



reduces $\mathbf{E}[T]$ to r-work



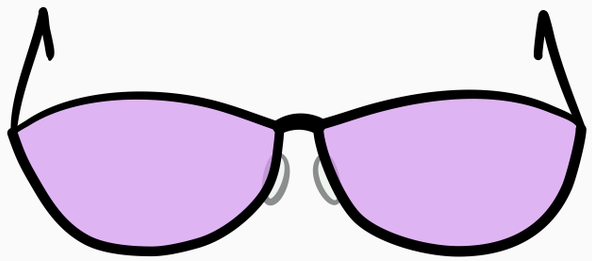
number of jobs N



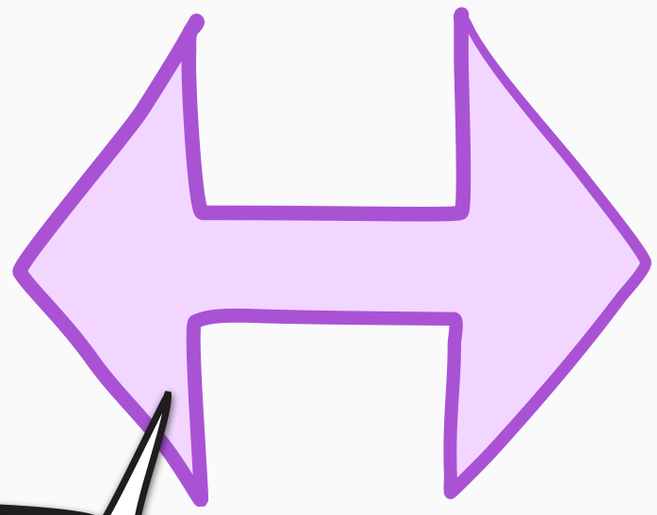
mean response time $\mathbf{E}[T]$

WINE

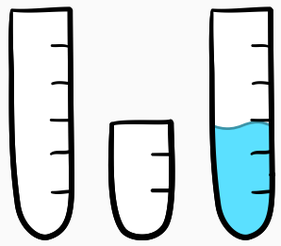
Work **I**ntegral **N**umber **E**quality



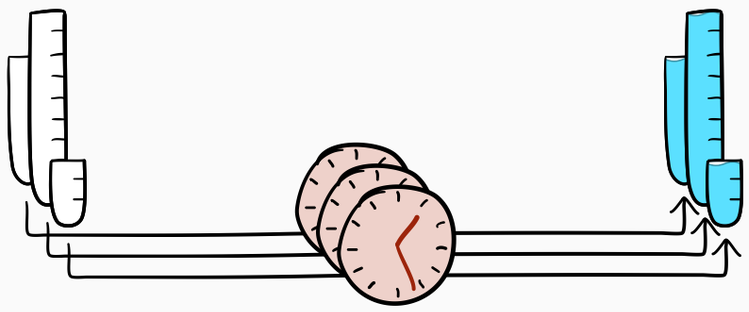
r-work $W(r)$



reduces $\mathbf{E}[T]$ to r-work

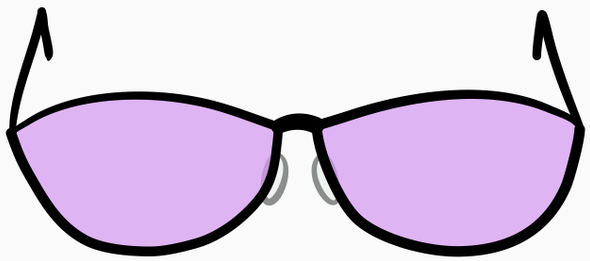


number of jobs N

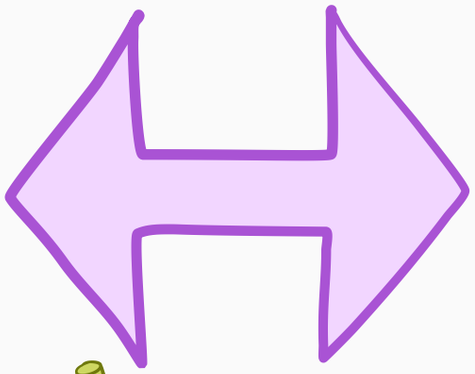


mean response time $\mathbf{E}[T]$

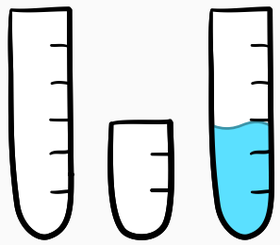
First analysis of SRPT-k, **Gittins**-k, **noisy** size estimates



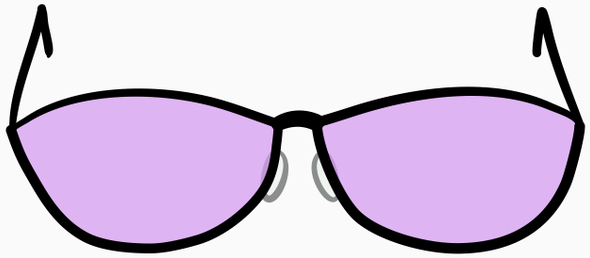
***r*-work $W(r)$**



WINE



number of jobs N



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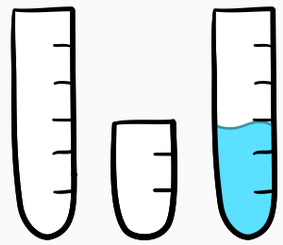
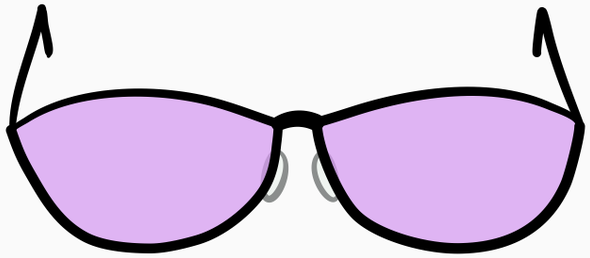


number of jobs N

What is *r*-work?



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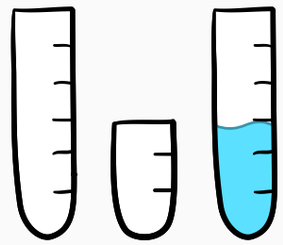
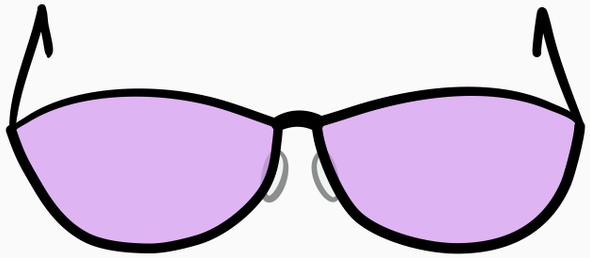


number of jobs N

What is ***r*-work?**

How do we get number of jobs from ***r*-work?**



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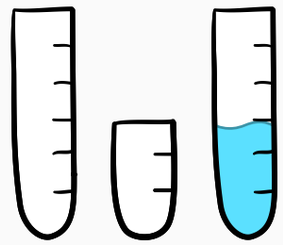
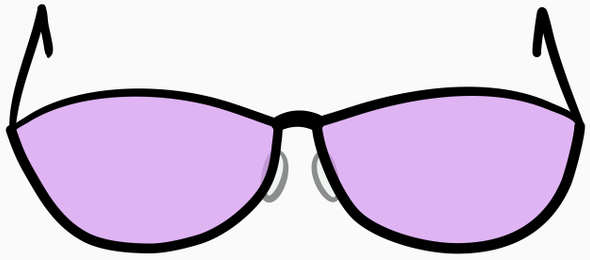


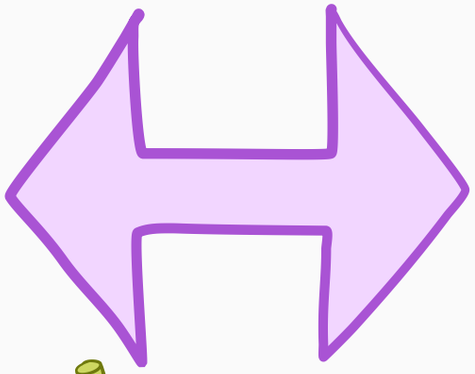
number of jobs N

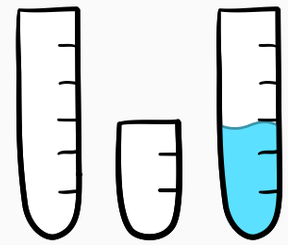
What is ***r*-work?**

How do we get number of jobs from ***r*-work?**

How do we analyze ***r*-work?**







***r*-work $W(r)$**

WINE

number of jobs N

This talk:
SRPT-***k***

What is ***r***-work?

How do we get number of jobs from ***r***-work?

How do we analyze ***r***-work?

Defining r-work

for SRPT

$W(r)$ = work relevant to **rank r**

Defining r-work

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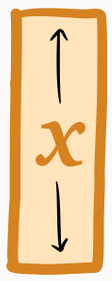
$w_x(r)$ = r-work of *single job* of rem. size x = $\begin{cases}$



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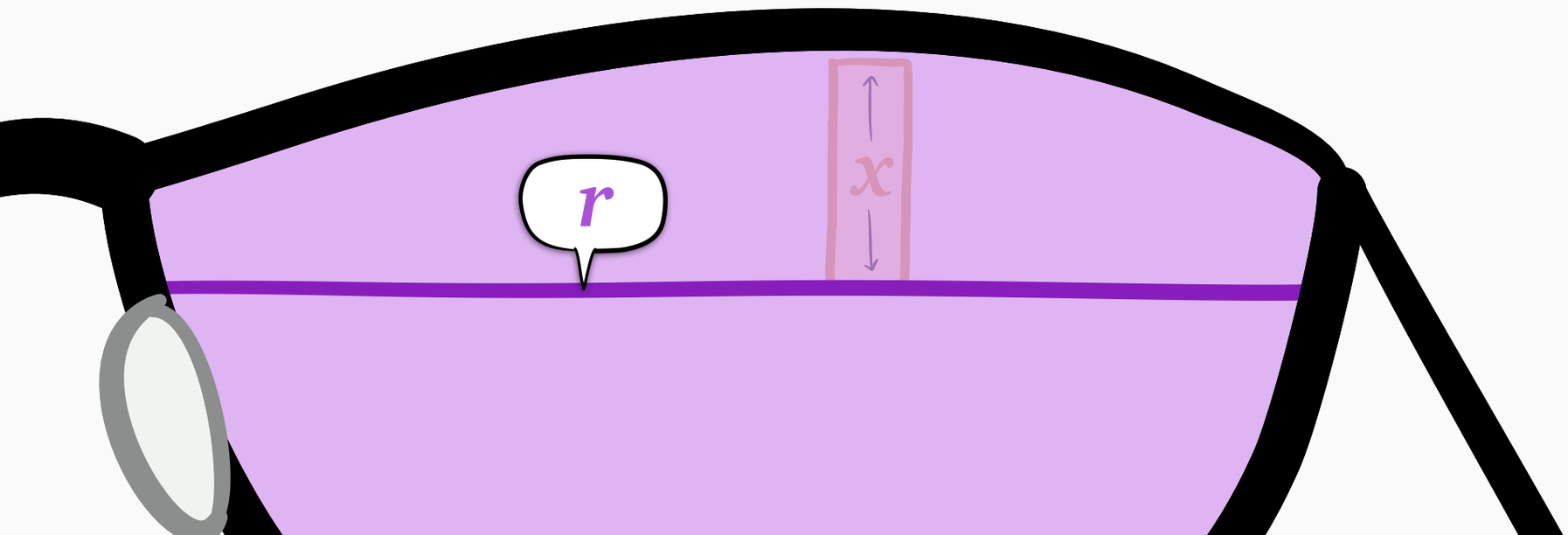

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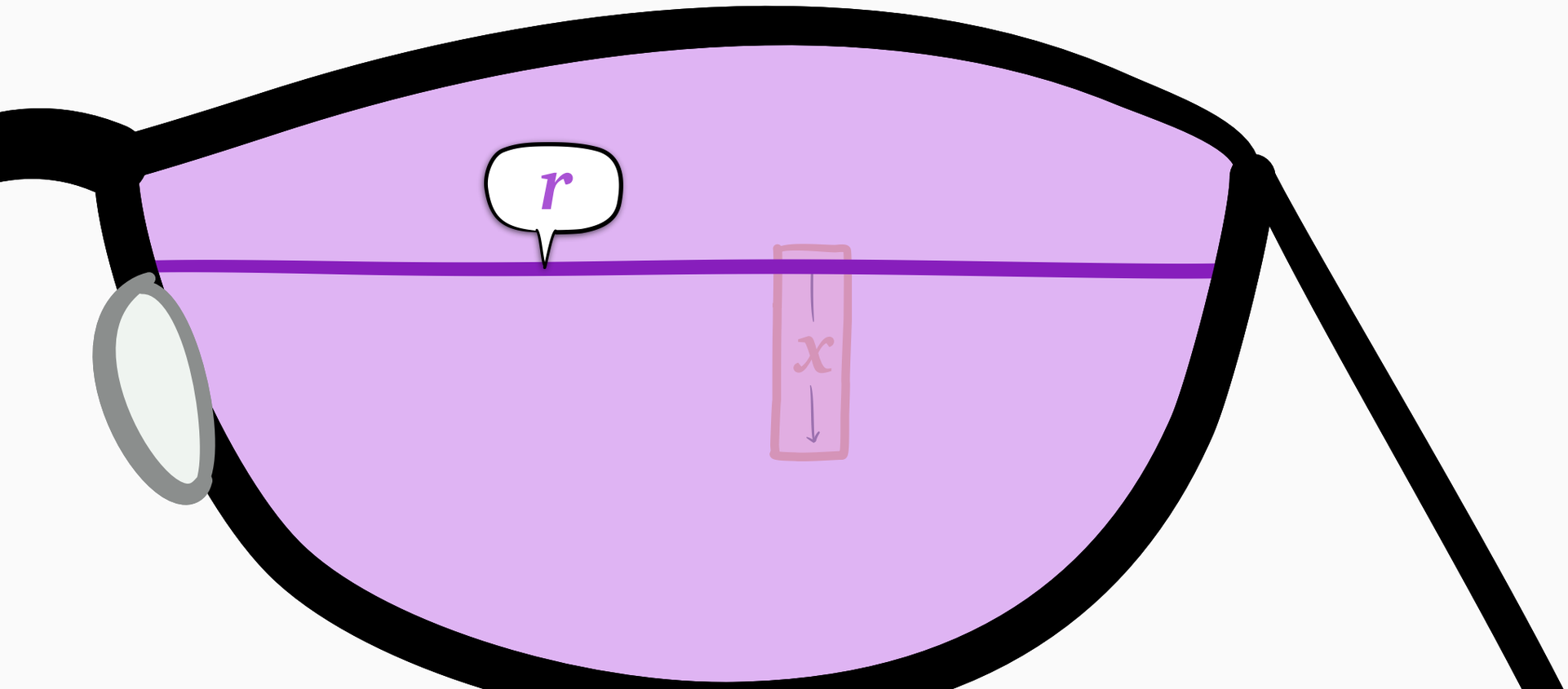

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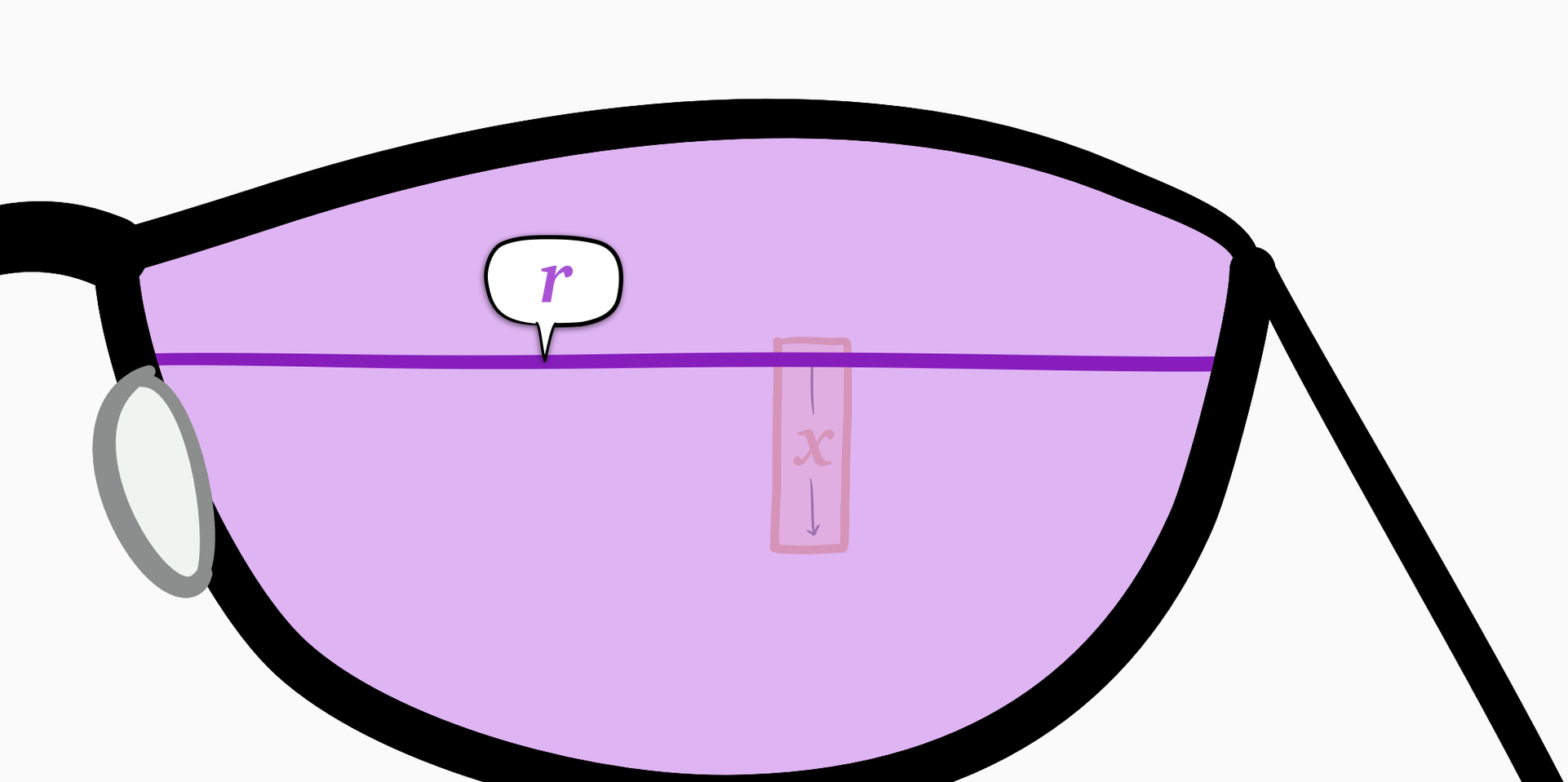

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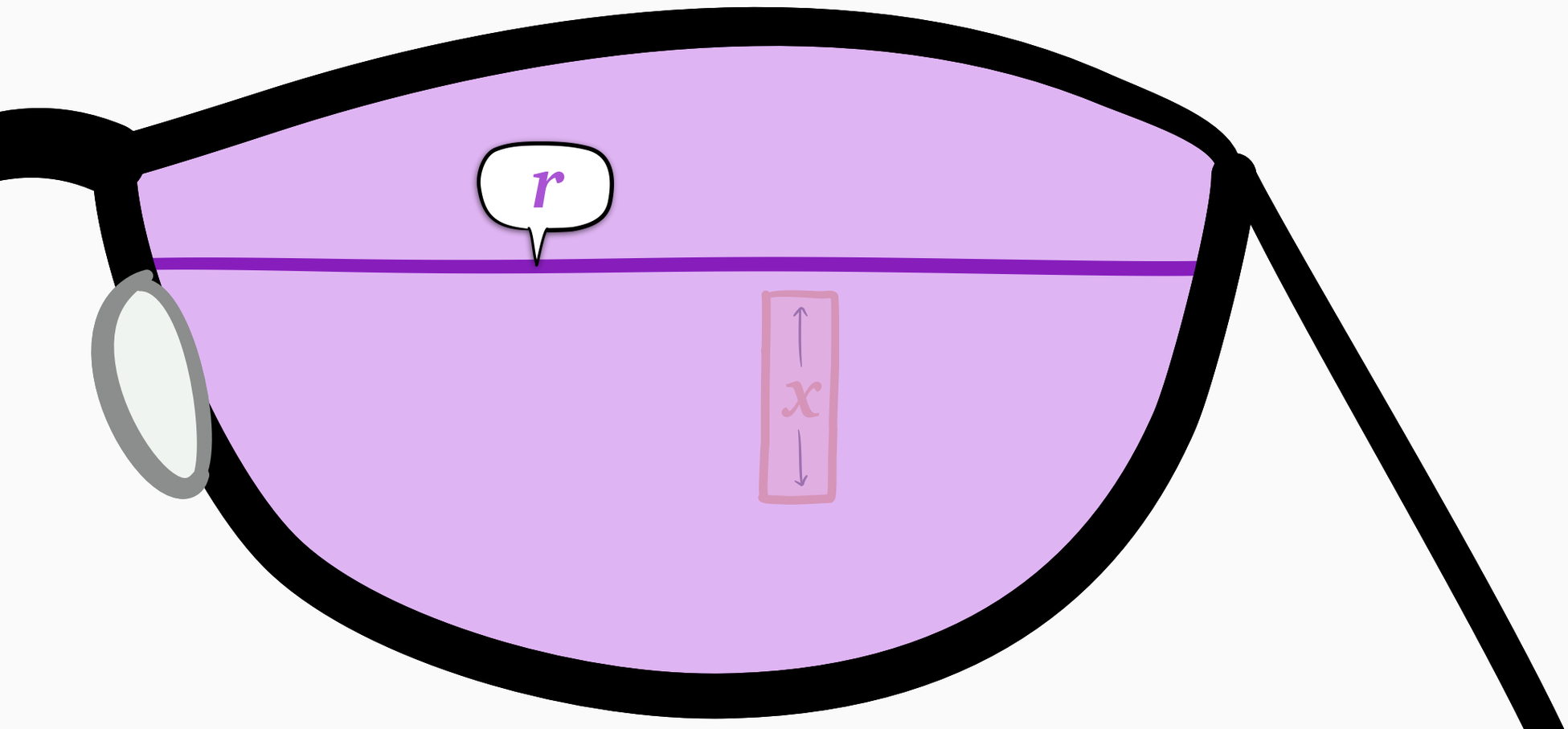

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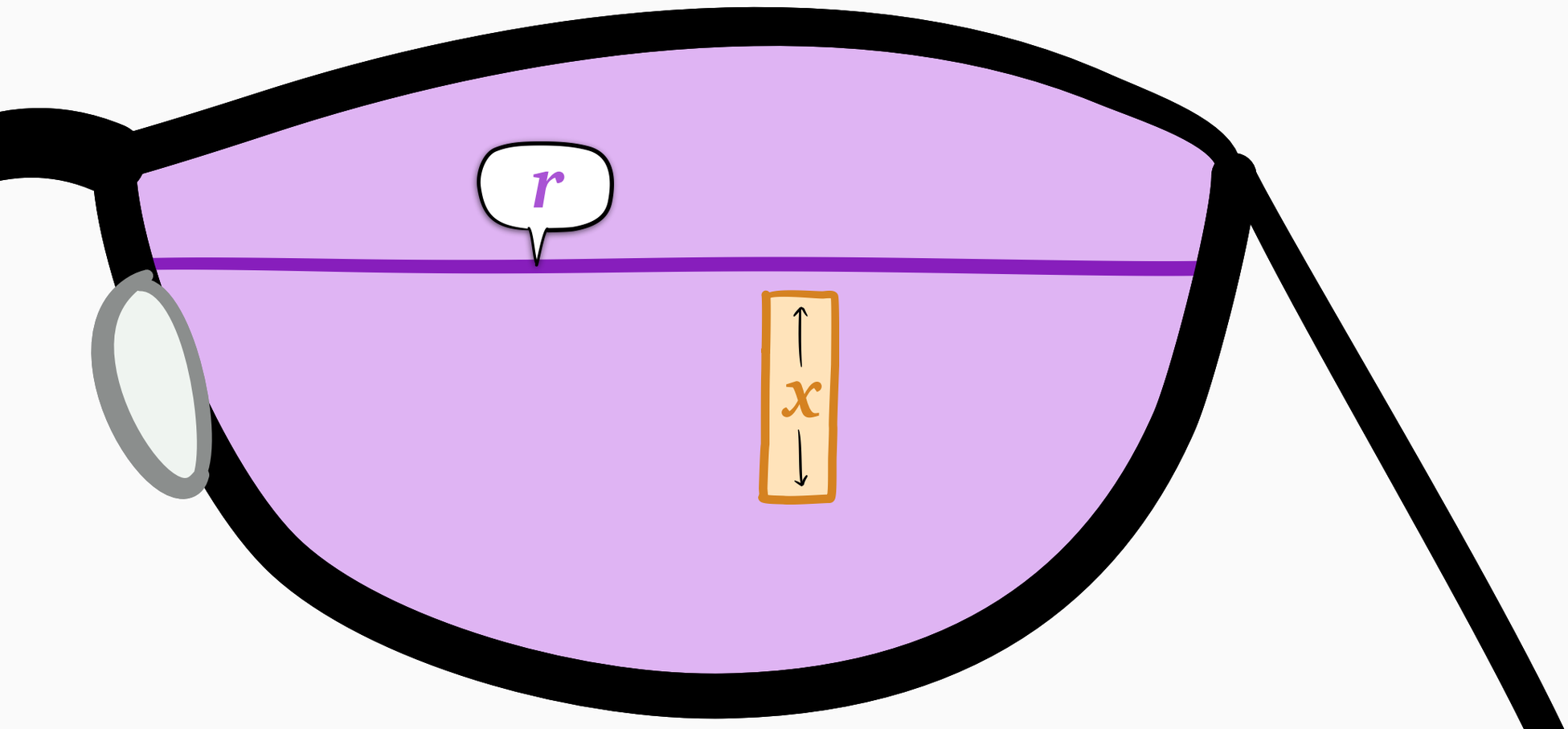

Defining r-work

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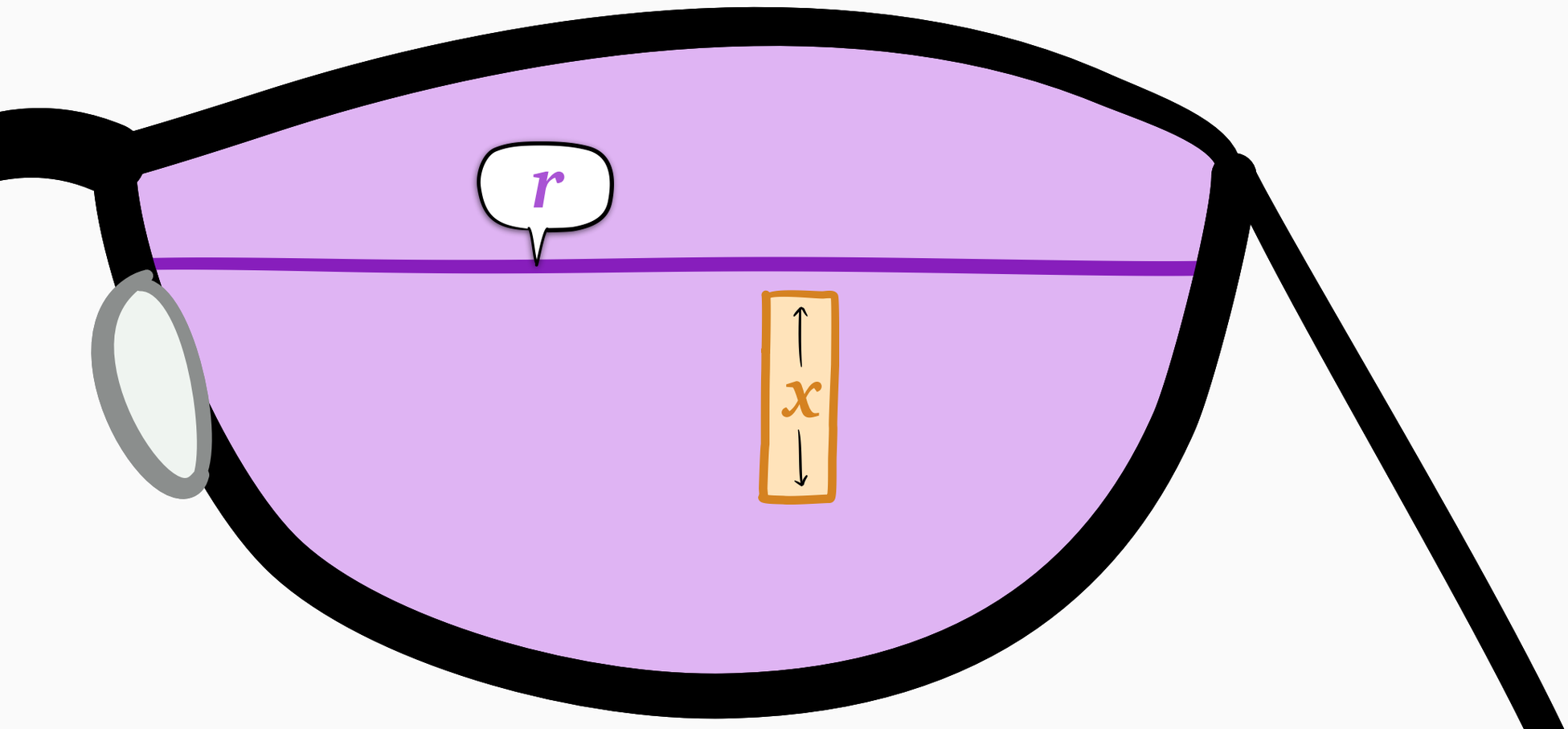

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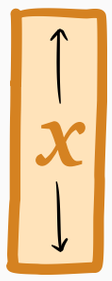
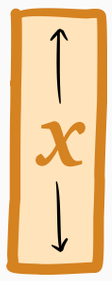


Defining r-work

for SRPT

$W(r)$ = work relevant to **rank r**
= total r-work of all jobs

$w_x(r)$ = r-work of *single job* of rem. size x = $\begin{cases} 0 & \text{if } r < x \\ x & \text{if } r \geq x \end{cases}$



From r-work to number of jobs N

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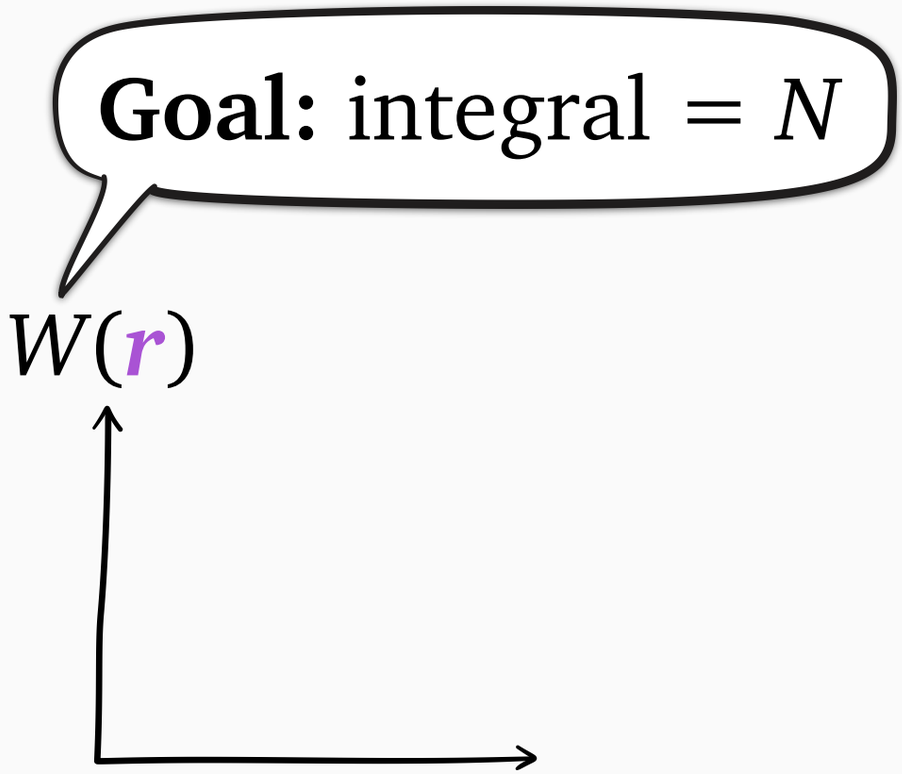

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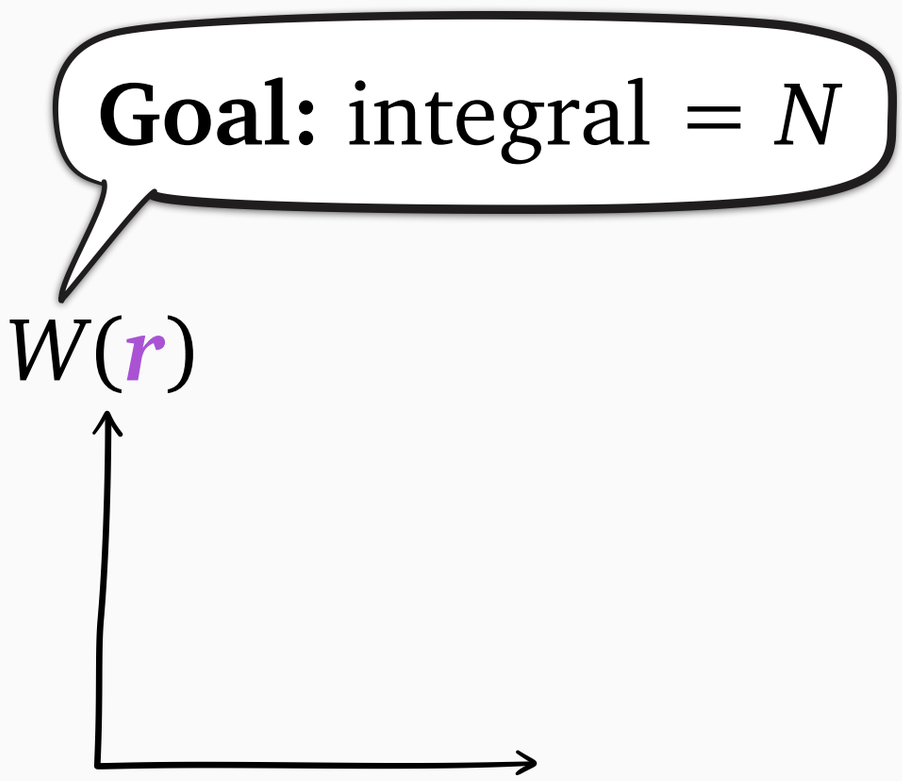
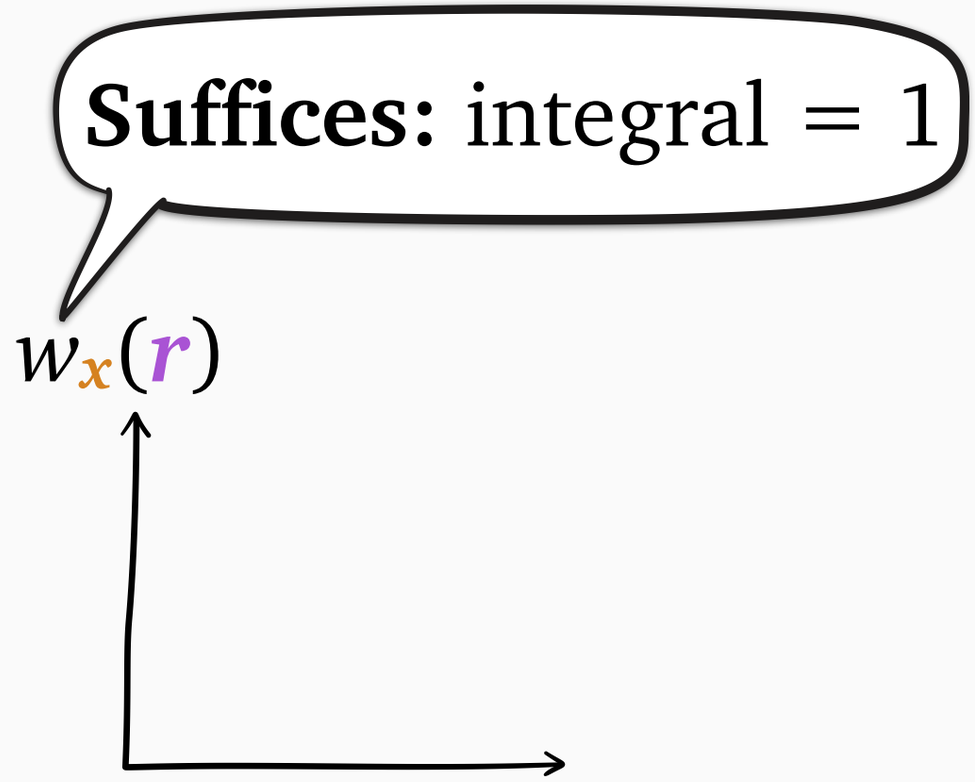
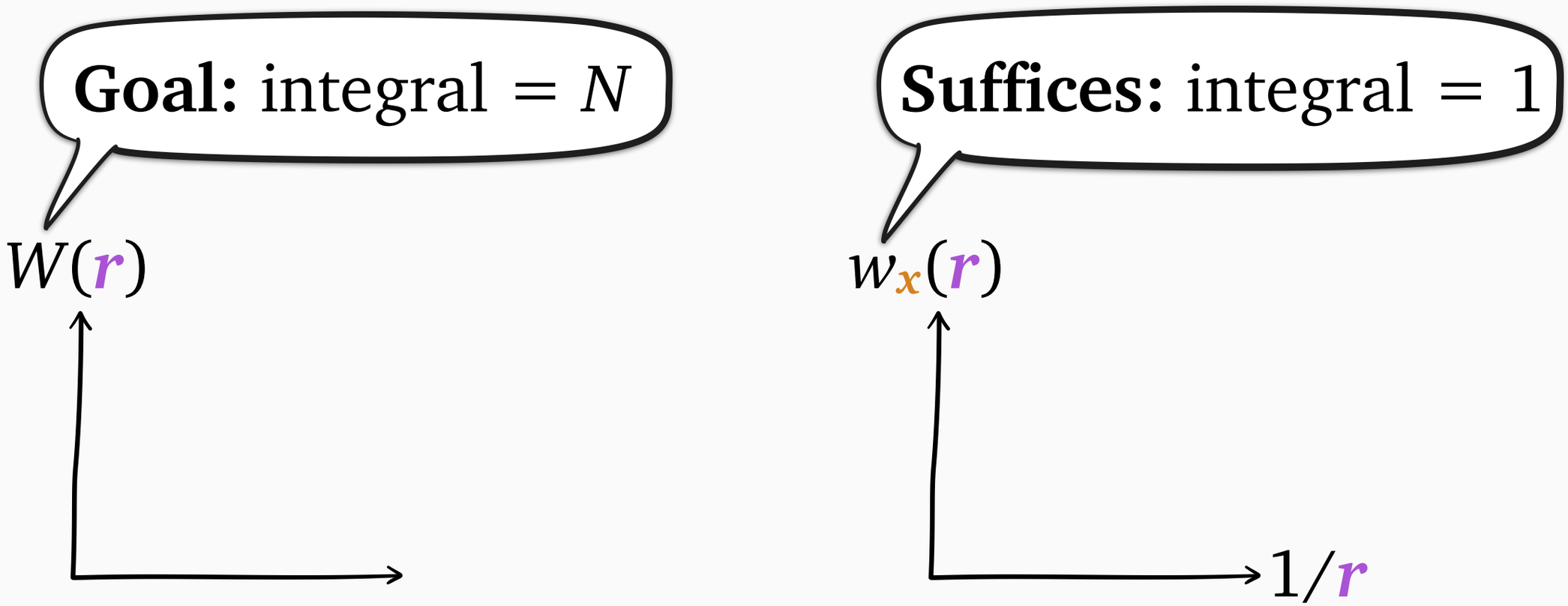




$$w_x(r) = r\text{-work of } \textit{job} \text{ of rem. size } x = \begin{cases} 0 & \text{if } r < x \\ x & \text{if } r \geq x \end{cases}$$

From r-work to number of jobs N



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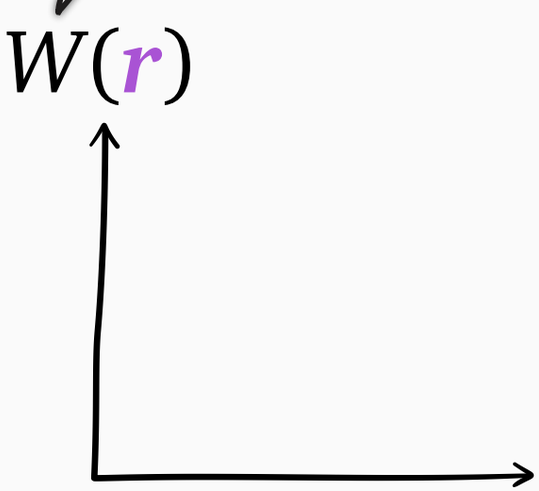
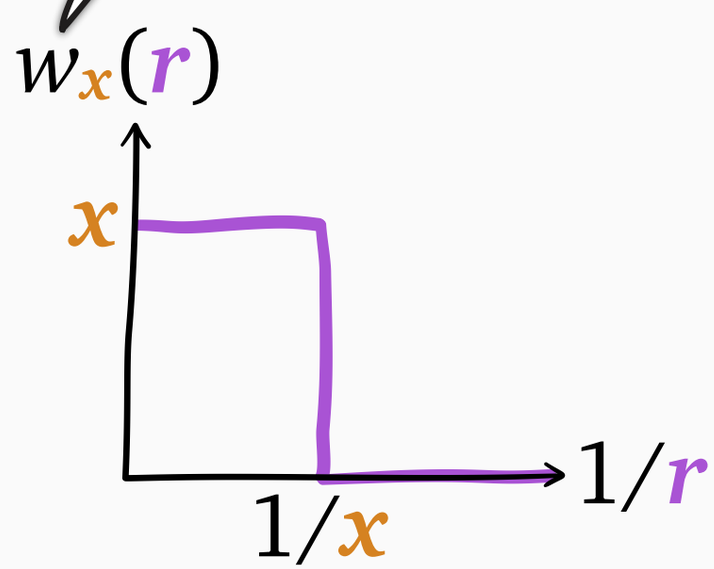
From r-work to number of jobs N

Goal: integral $= N$



Suffices: integral $= 1$



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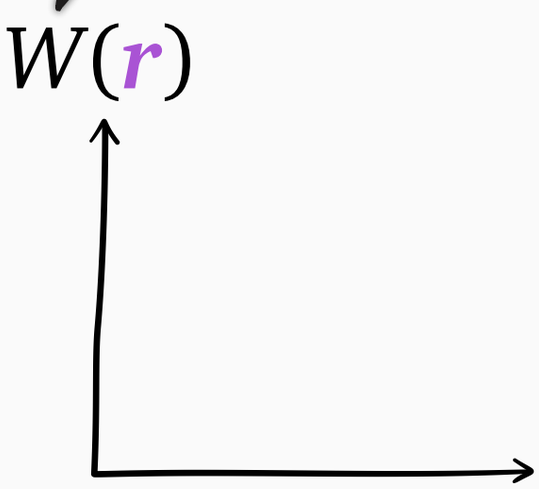

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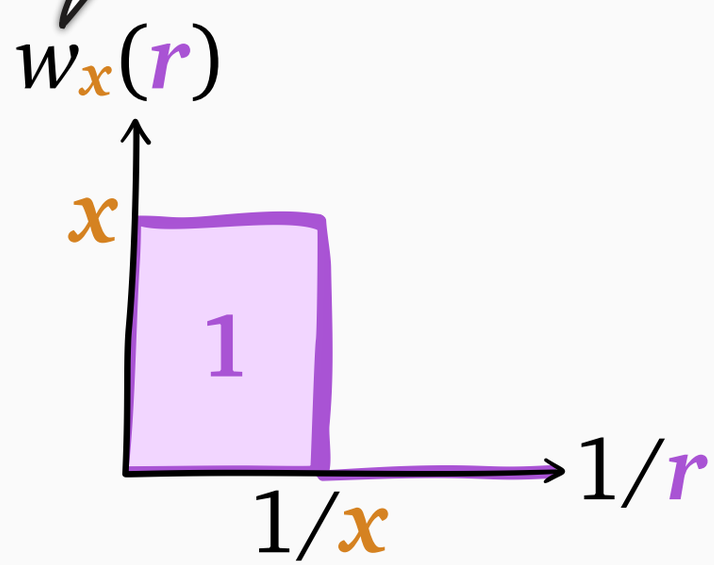

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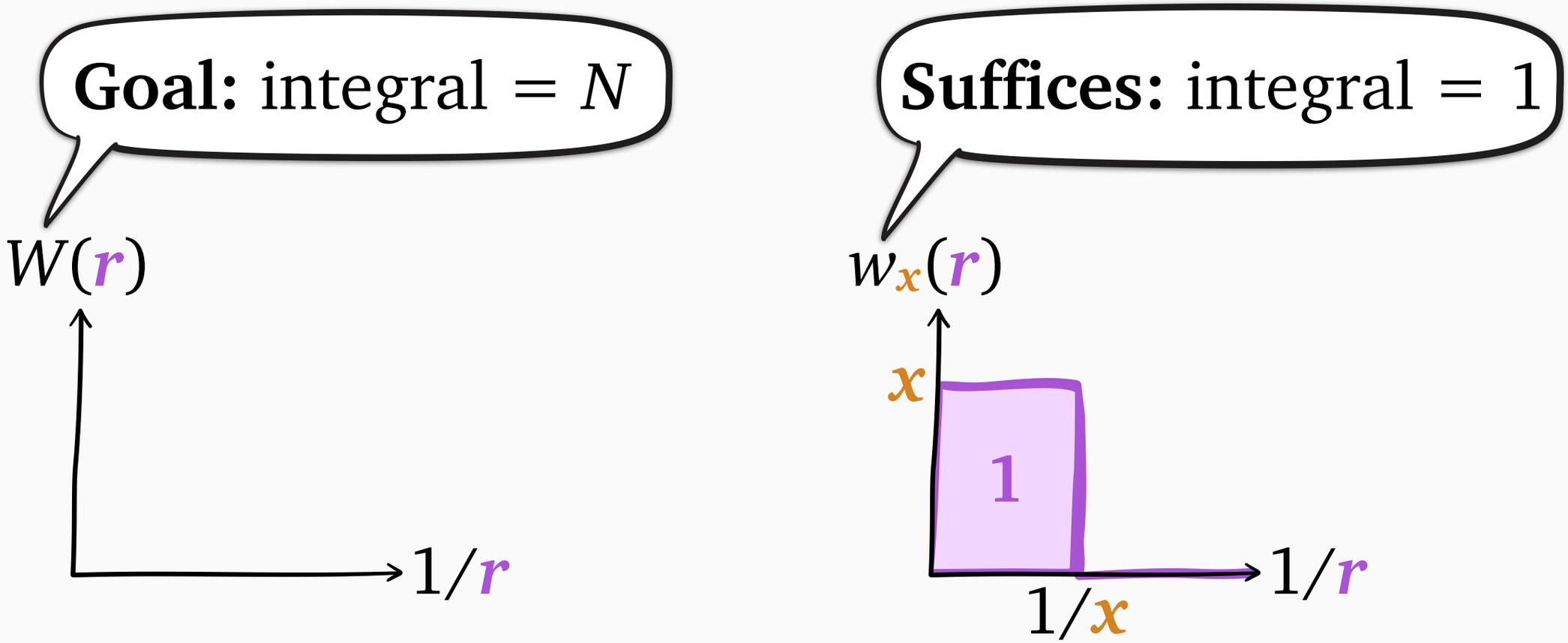

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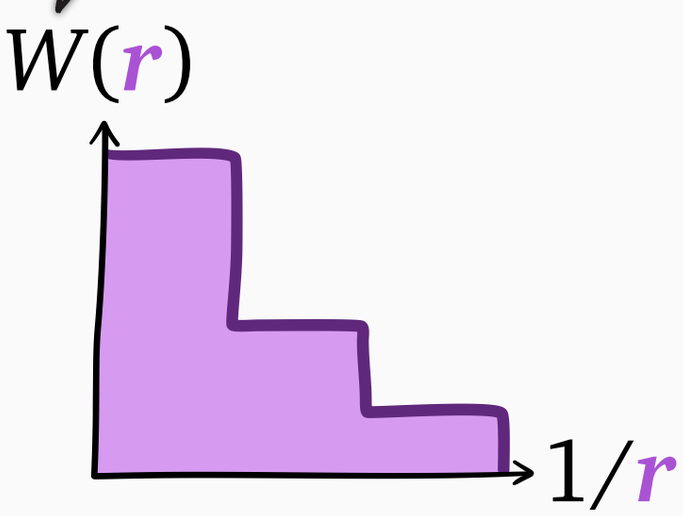

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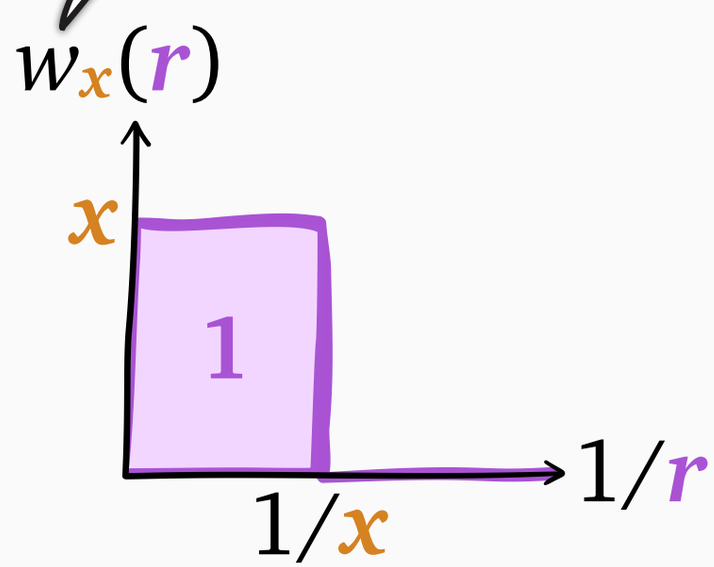

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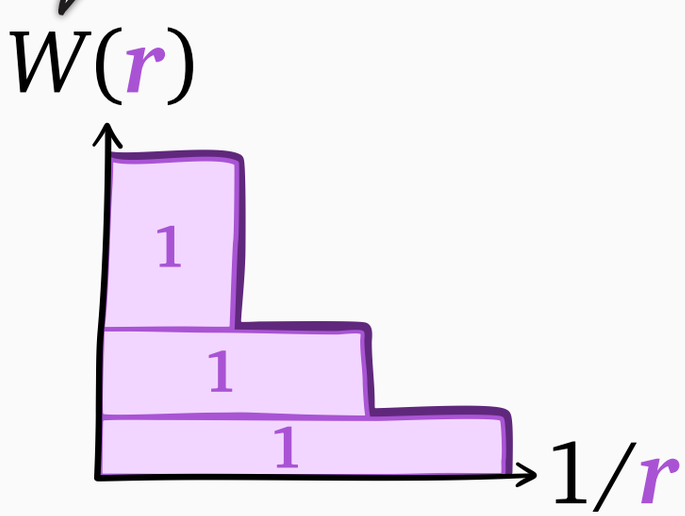

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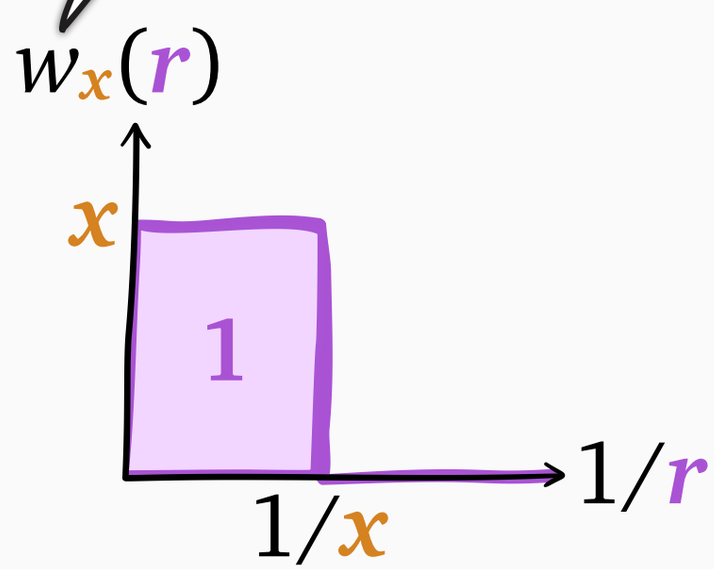

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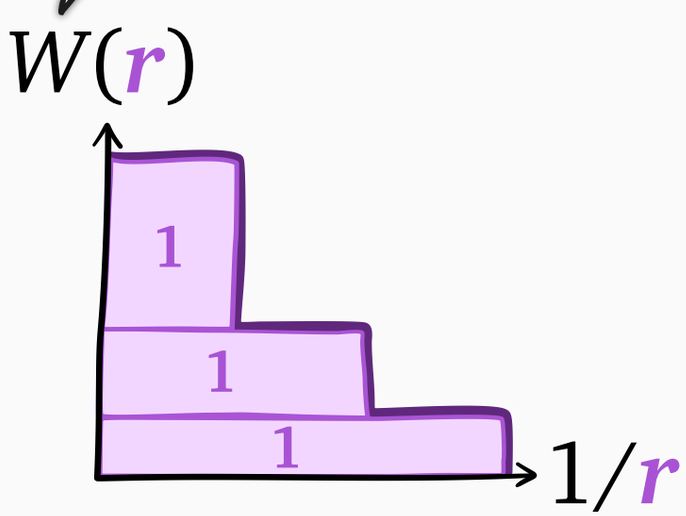
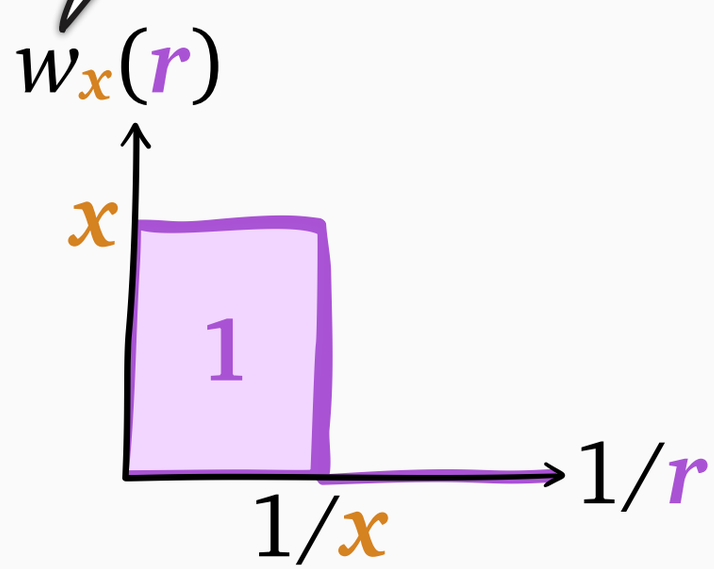
From r-work to number of jobs N

Goal: integral $= N$



Suffices: integral $= 1$



NEW!

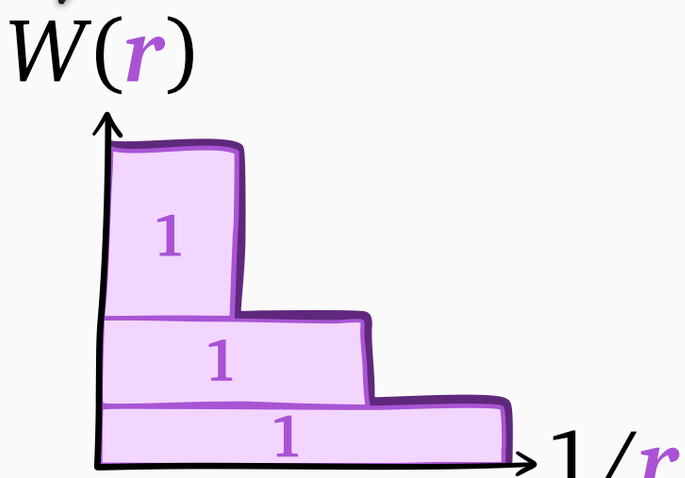
Theorem:

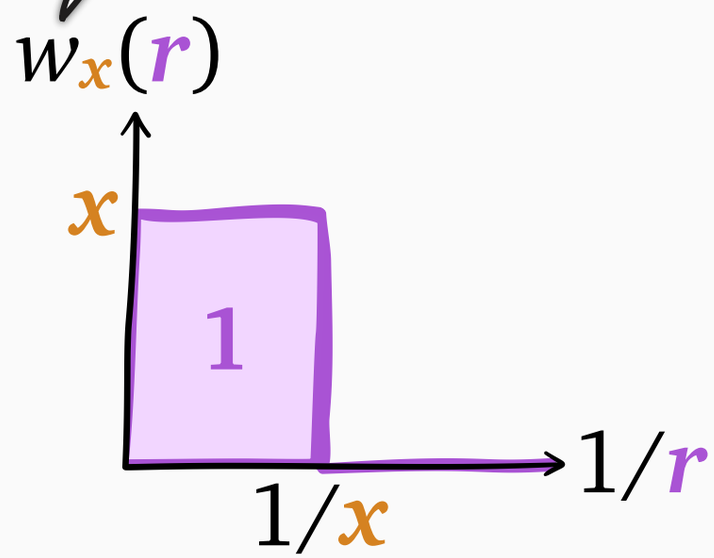
$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

From r-work to number of jobs N

Goal: integral $= N$

Suffices: integral $= 1$





NEW!

WINE

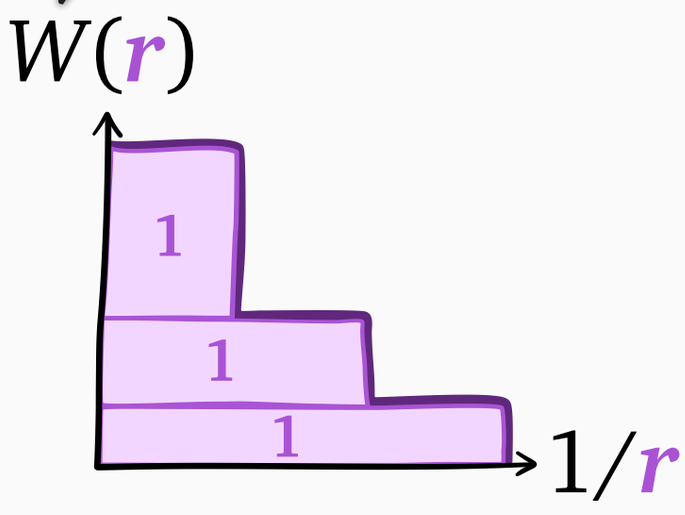
Theorem:

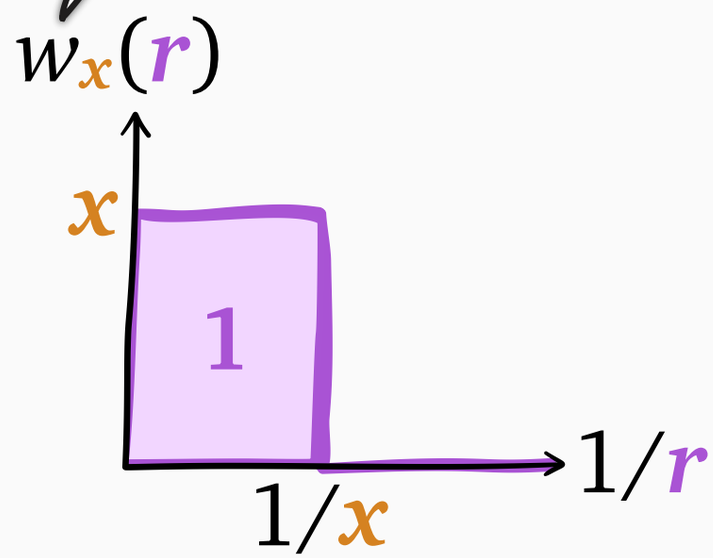
$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

From r-work to number of jobs N

Goal: integral $= N$

Suffices: integral $= 1$





NEW!

Theorem:

$$N = \int_0^\infty \frac{W(r)}{r^2}\,\mathrm{d}r$$

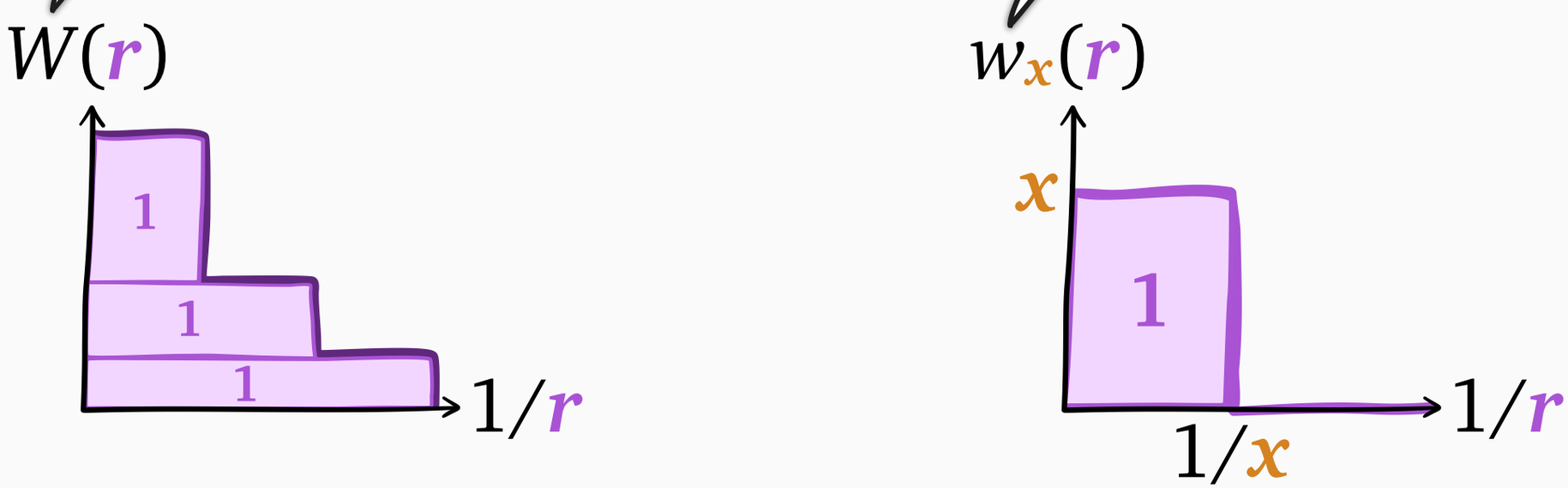
uses **rank** = rem. size

WINE

From r-work to number of jobs N

Goal: integral $= N$

Suffices: integral $= 1$



NEW!

Theorem: under *any* policy,

$$N = \int_0^\infty \frac{W(r)}{r^2} \mathrm{d}r$$

uses **rank** = rem. size

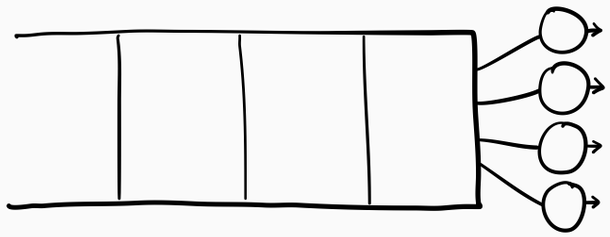
WINE

Impact of WINE

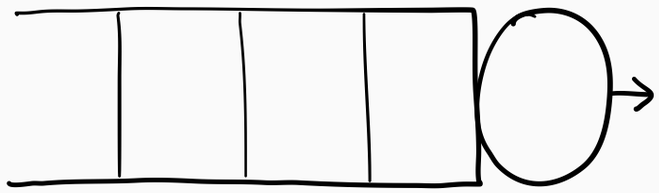
Impact of WINE

multiserver systems

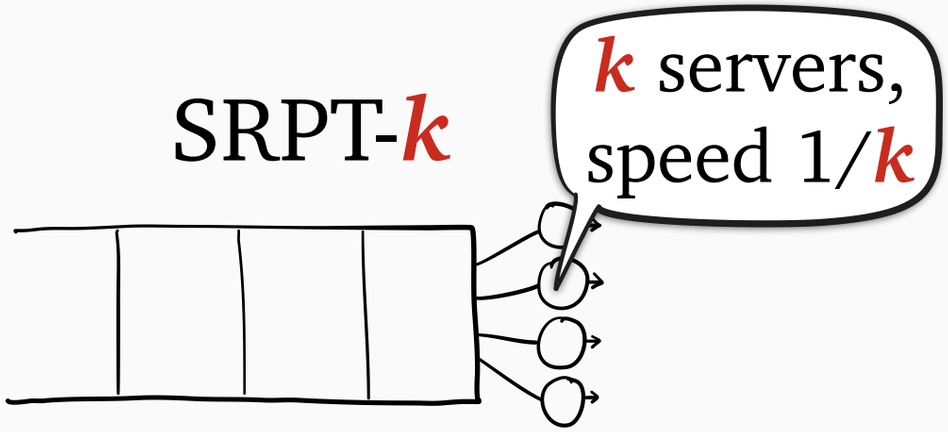
SRPT-k



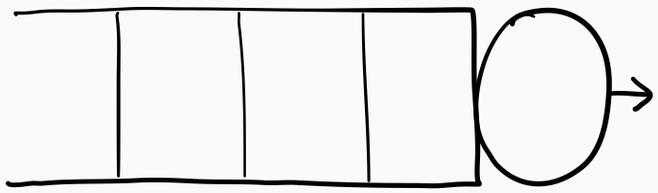
SRPT-1

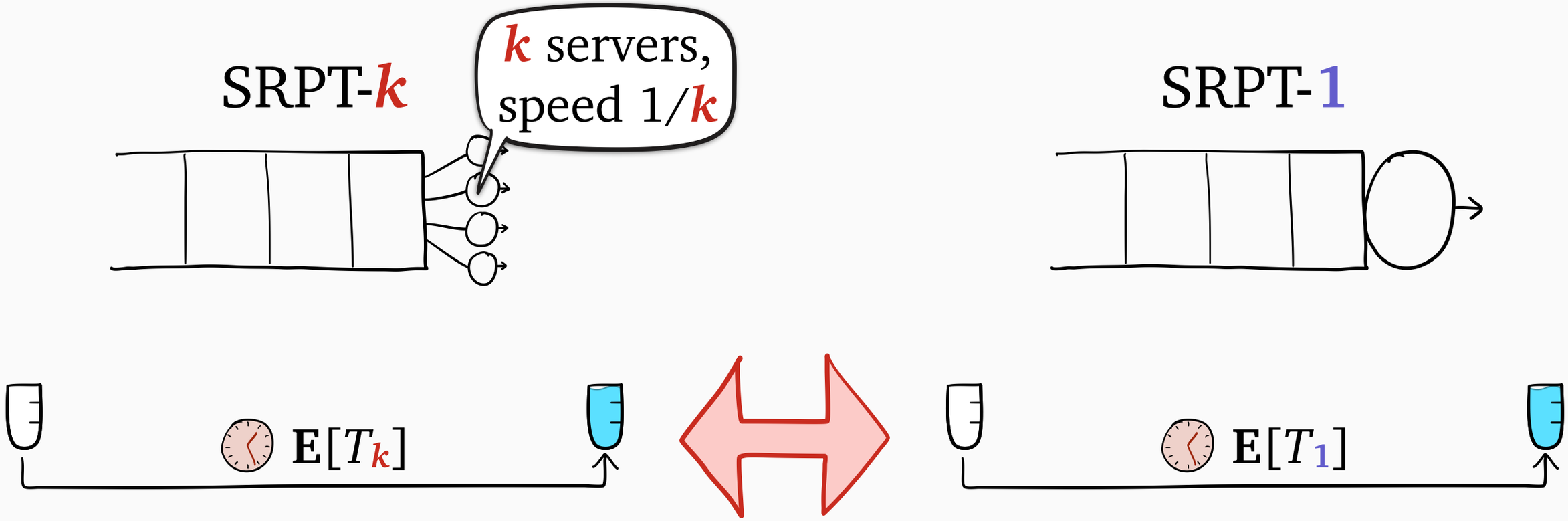


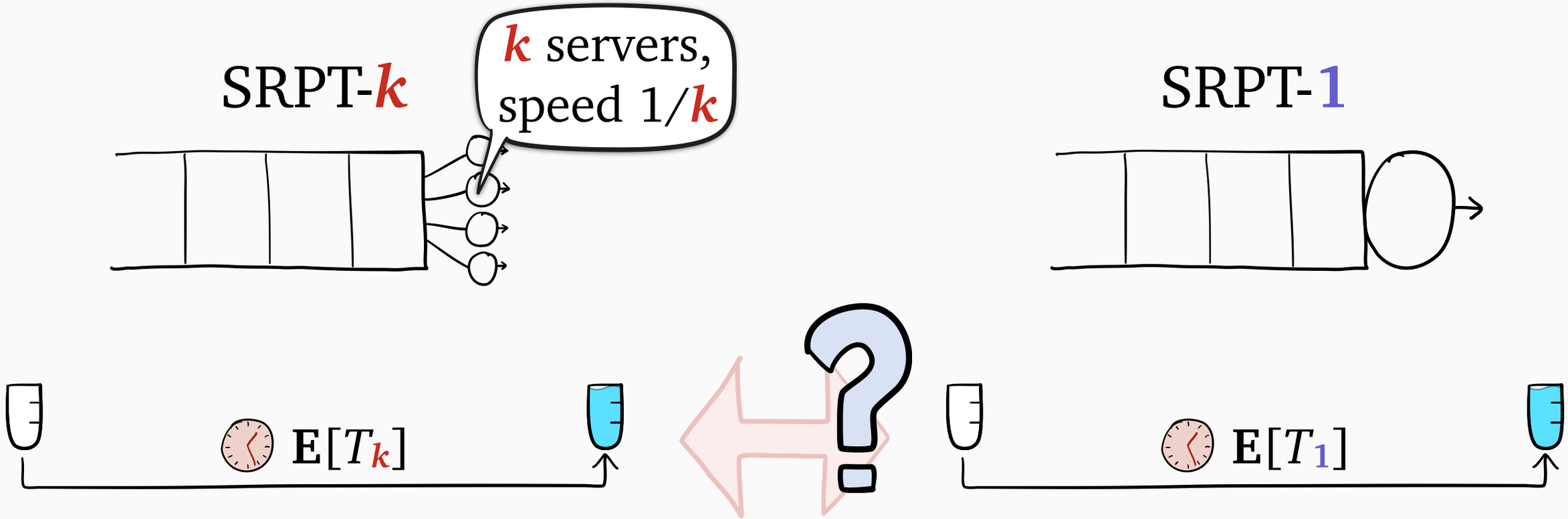
SRPT-k



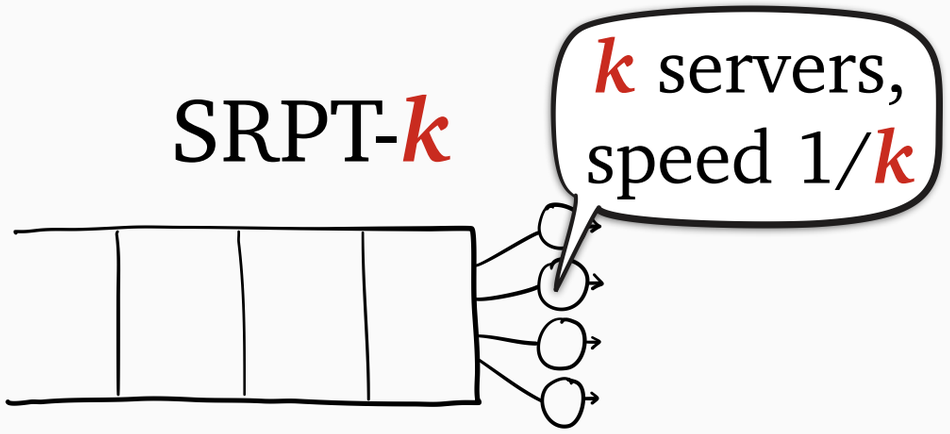
SRPT-1



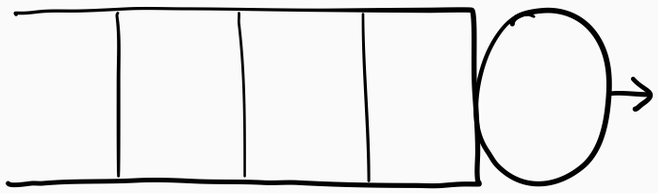

SRPT-k
k servers, speed $1/k$
SRPT-1
$\mathbf{E}[T_k]$
$\mathbf{E}[T_1]$


SRPT-k
k servers, speed $1/k$
SRPT-1
$\mathbf{E}[T_k]$
$\mathbf{E}[T_1]$

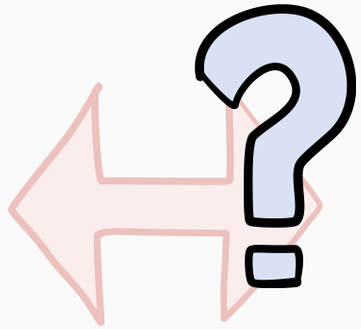
SRPT-k


k servers,
speed $1/k$

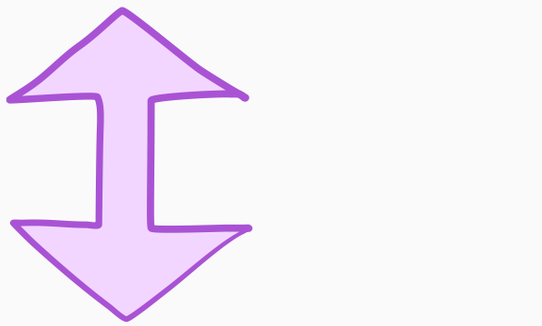
SRPT-1

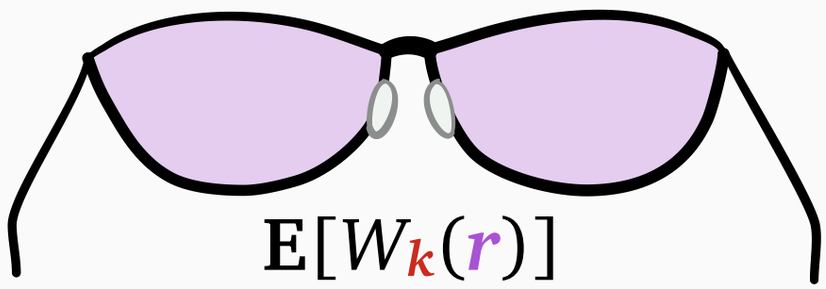


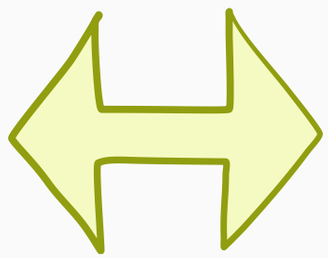

$\mathbf{E}[T_k]$

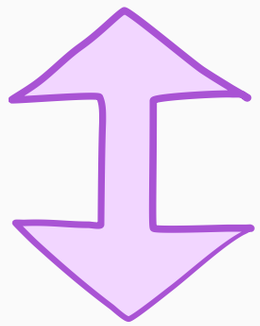


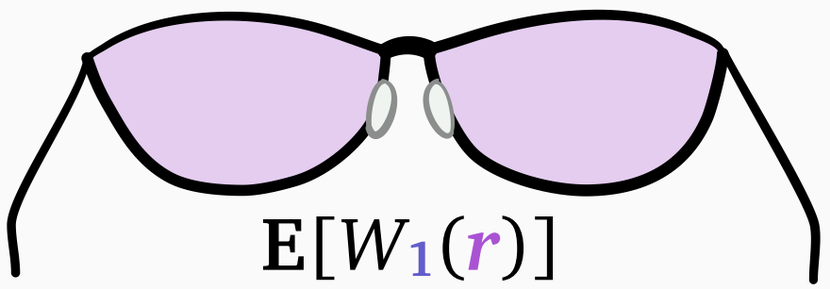

$\mathbf{E}[T_1]$



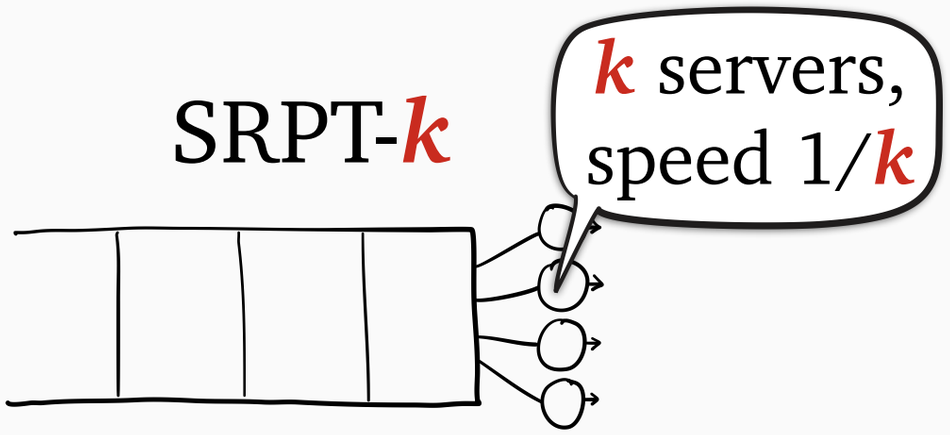

$\mathbf{E}[W_k(r)]$



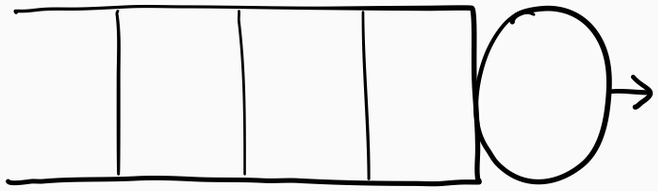



$\mathbf{E}[W_1(r)]$

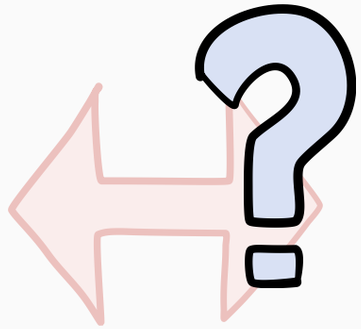
SRPT-k


k servers,
speed $1/k$

SRPT-1

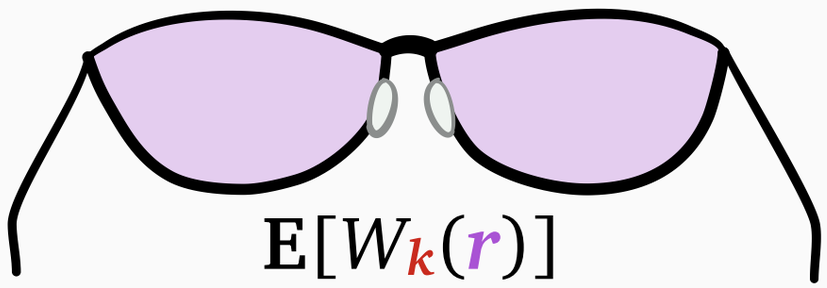


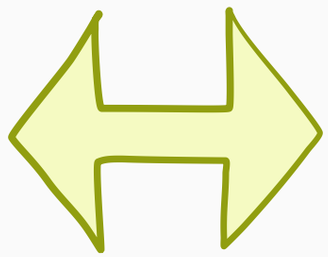

$\mathbf{E}[T_k]$



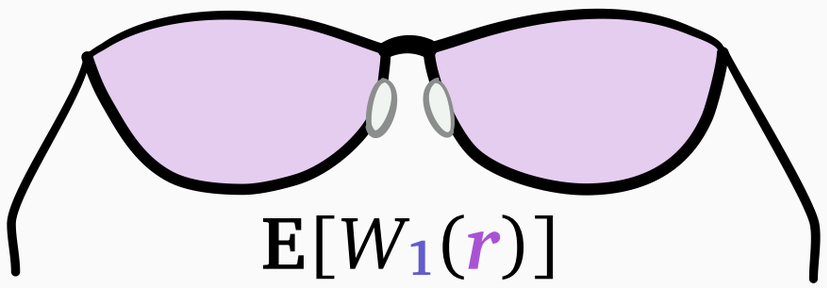

$\mathbf{E}[T_1]$


WINE


$\mathbf{E}[W_k(r)]$




WINE


$\mathbf{E}[W_1(r)]$

SRPT-k

k servers, speed $1/k$

SRPT-1

$\mathbf{E}[T_k]$

$\mathbf{E}[T_1]$

WINE

WINE

$\mathbf{E}[W_k(r)]$

$\mathbf{E}[W_1(r)]$

Lemma: r-work decomposition

SRPT-k

k servers, speed $1/k$

SRPT-1

$\mathbf{E}[T_k]$

$\mathbf{E}[T_1]$

WINE

WINE

$\mathbf{E}[W_k(r)]$

$\mathbf{E}[W_1(r)]$

NEW!

Lemma: r-work decomposition

SRPT-k

k servers, speed $1/k$

SRPT-1

$\mathbf{E}[T_k]$

$\mathbf{E}[T_1]$

WINE

WINE

$\mathbf{E}[W_k(r)]$

$\mathbf{E}[W_1(r)]$

NEW!

Lemma: r-work decomposition

Impact of WINE

multiserver systems

Impact of WINE

multiserver systems

First analysis of

- SRPT-***k***
- **Gittins-*k***

Impact of **WINE**

multiserver systems

noisy size estimates

First analysis of

- SRPT-***k***
- **Gittins-*k***

Impact of **WINE**

multiserver systems

- *First analysis* of
  - SRPT-***k***
  - **Gittins**-***k***

noisy size estimates

compare **noisy-info** ***r***-work
to **perfect-info** ***r***-work

WINE References

[7] Grosof, Scully, and Harchol-Balter (2018). "SRPT for Multiserver Systems." *Perform. Eval.* (PERFORMANCE 2018). **Winner: PERFORMANCE 2018 Best Student Paper Award.**

[8] Grosof, Scully, and Harchol-Balter (2019). "Load Balancing Guardrails: Keeping Your Heavy Traffic on the Road to Low Response Times." *Proc. ACM Meas. Anal. Comput. Syst.* (SIGMETRICS 2019). **Winner: SIGMETRICS 2018 Outstanding Student Paper Award.**

[9] Scully, Grosof, and Harchol-Balter (2021). "Optimal Multiserver Scheduling with Unknown Job Sizes in Heavy Traffic." *Perform. Eval.* (PERFORMANCE 2020 issue).

[10] Scully, Grosof, and Harchol-Balter (2020). "The Gittins Policy is Nearly Optimal in the M/G/k under Extremely General Conditions." *Proc. ACM Meas. Anal. Comput. Syst.* (SIGMETRICS 2021). **Winner: 2022 INFORMS George Nicholson Student Paper Competition.**

[11] Scully (2021). "Bounding Mean Slowdown in Multiserver Systems." *SIGMETRICS Perform. Eval. Rev.* (MAMA 2021).

[12] Scully, Grosof, and Mitzenmacher (2022). "Uniform Bounds for Scheduling with Job Size Estimates." *13th Innovations in Theoretical Computer Science Conference* (ITCS 2022).

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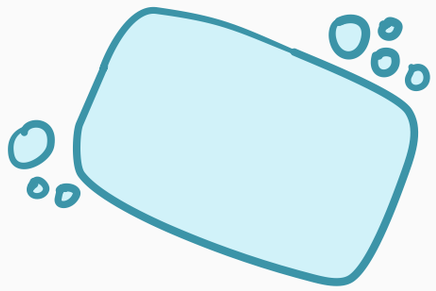
Outline: two new tools



SOAP

new unifying theory of single-server scheduling

→ unknown sizes

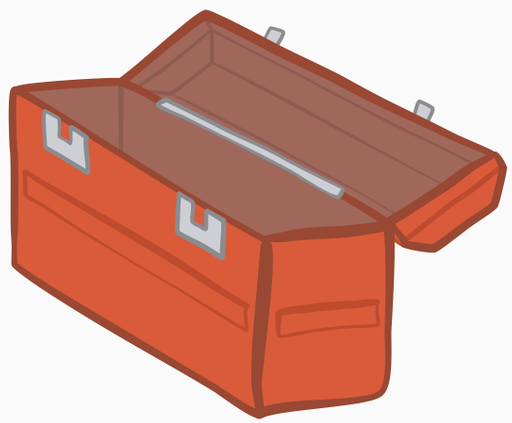
→ preemption limitations

WINE

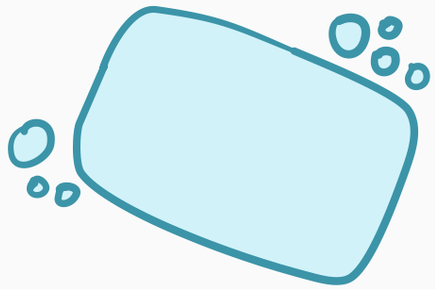
new queueing identity to complement Little's Law

→ multiserver systems

→ noisy size estimates



Outline: two new tools



SOAP

new unifying theory of single-server scheduling

→ unknown sizes

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WINE

new queueing identity to complement Little's Law

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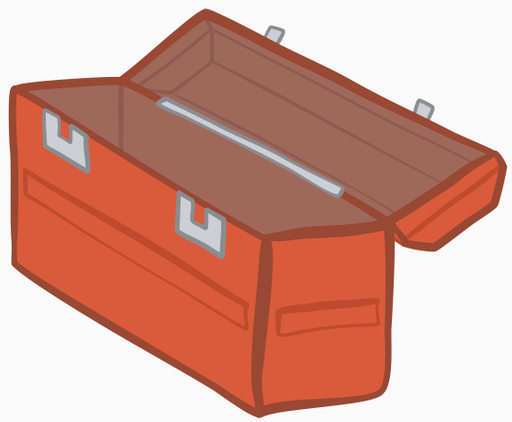


SOAP &WINE

SOAP &WINE

Thesis statement?

Thesis statement?

Yes, queueing theory can!

Past:
Analyze policies one-by-one

Past:

Analyze policies one-by-one

Present:

Analyze all **rank**-function-based policies at once

MLPS, SMART

Past:

Analyze policies one-by-one

Present:

Analyze all **rank**-function-based policies at once

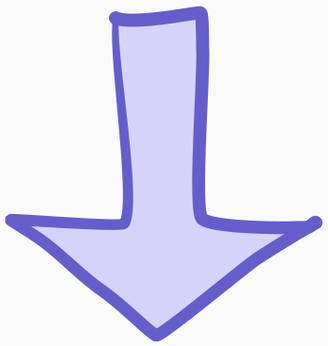
MLPS, SMART

Past:

Analyze policies one-by-one

Present:

Analyze all **rank**-function-based policies at once



Future:

Other policy classes?

Past:

Scheduling in M/G/1, only FCFS in M/G/k

Past:

Scheduling in M/G/1, only FCFS in M/G/k

Present:

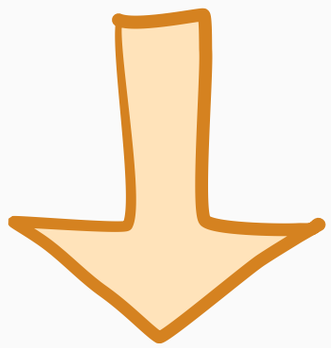
Scheduling in M/G/k

Past:

Scheduling in M/G/1, only FCFS in M/G/k

Present:

Scheduling in M/G/k



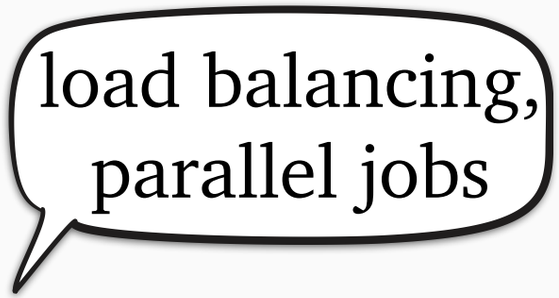
Future:

Scheduling in complex multiserver architectures?

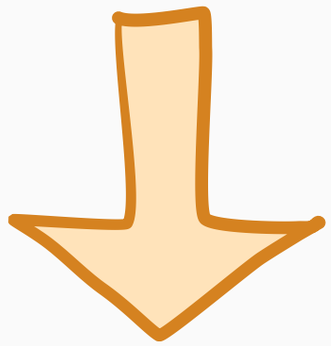
Past:

Scheduling in M/G/1, only FCFS in M/G/k

Present:



Scheduling in M/G/k



Future:

Scheduling in complex multiserver architectures?

Past:

Gittins for specific uncertainty models, one-by-one

Past:

Gittins for specific uncertainty models, one-by-one

Present:

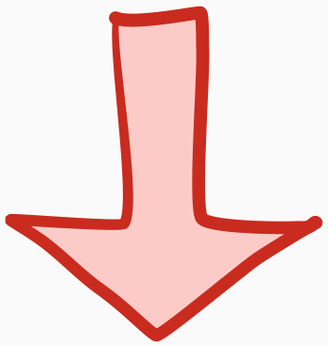
Gittins for any service-based uncertainty model

Past:

Gittins for specific uncertainty models, one-by-one

Present:

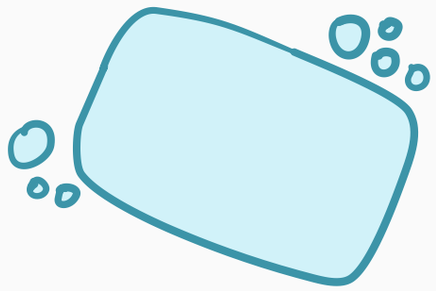
Gittins for any service-based uncertainty model



Future:

Underspecified models? “Restless” models?

Yes, queueing theory can!



SOAP

new unifying theory of single-server scheduling

→ unknown sizes

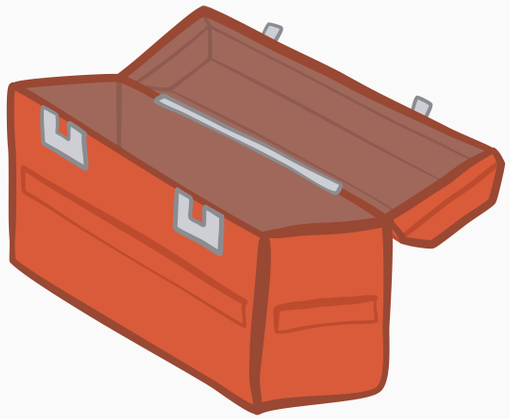
→ preemption limitations

WINE

new queueing identity to complement Little's Law

→ multiserver systems

→ noisy size estimates



SOAP def. and non-examples

Preemption checkpoints

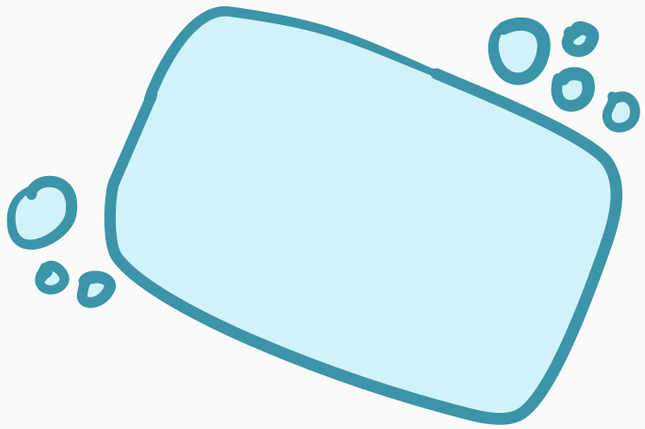
Limited priority levels

Tail decay: heavy vs. light

***r*-work decomposition**

SRPT-***k*** and **Gittins-*k*** $\mathbf{E}[T]$

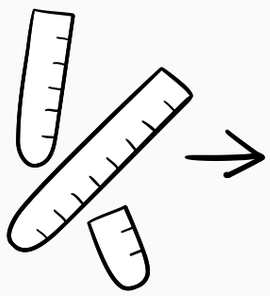
back



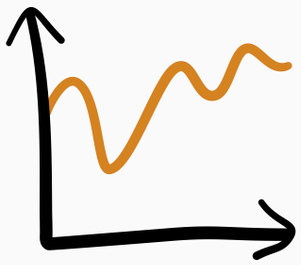
SOAP

Schedule **O**rdered by **A**ge-based **P**riority

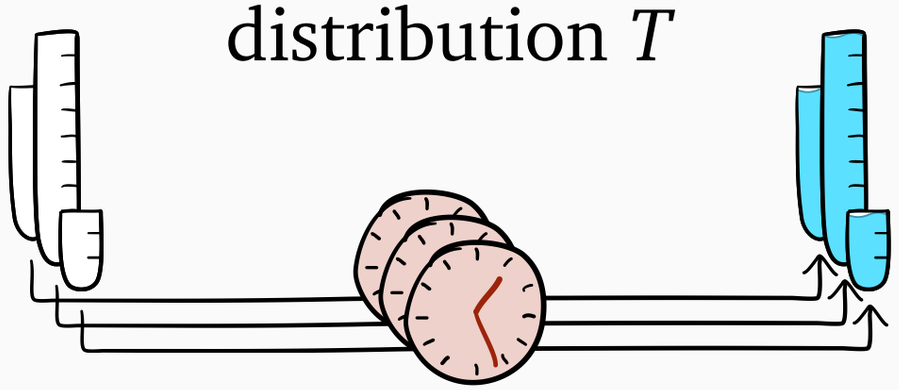
stochastic arrival process λ, S



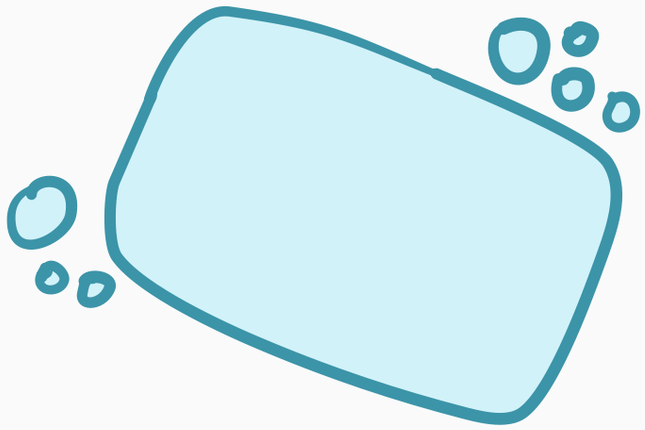
rank function



response time distribution T



back



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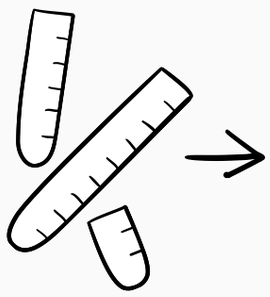

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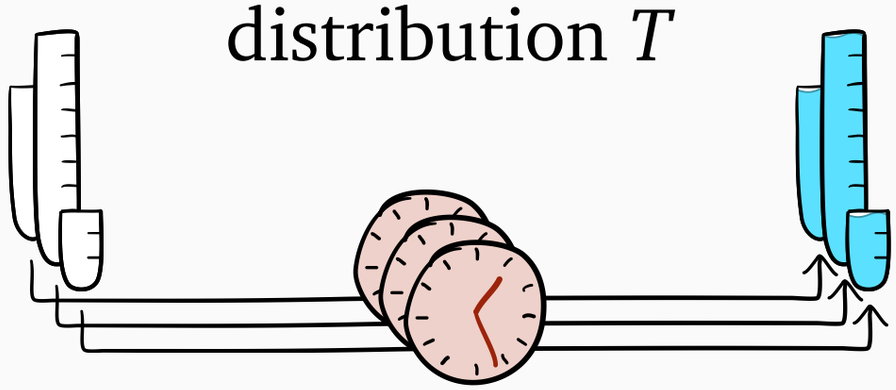

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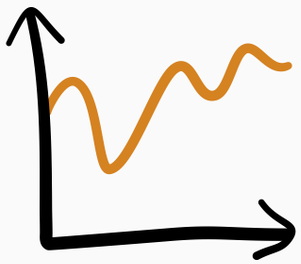
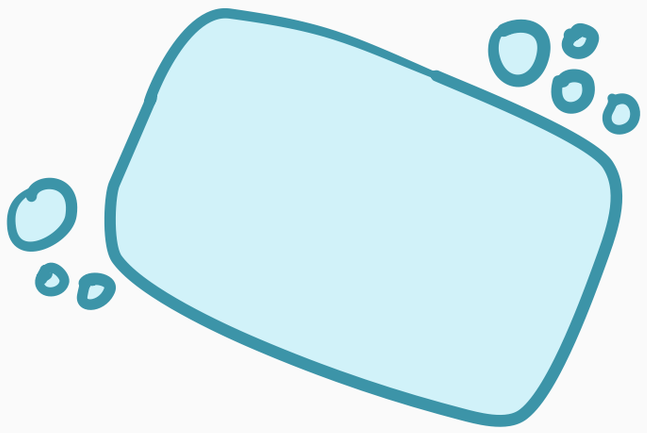


(label, **age**) ↦ **rank**

where labels are static

back

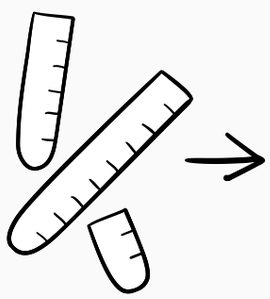


SOAP

Schedule **O**rdered by **A**ge-based **P**riority

label dist. L,
(*L*, *S*) pairs i.i.d.

stochastic arrival process λ, S



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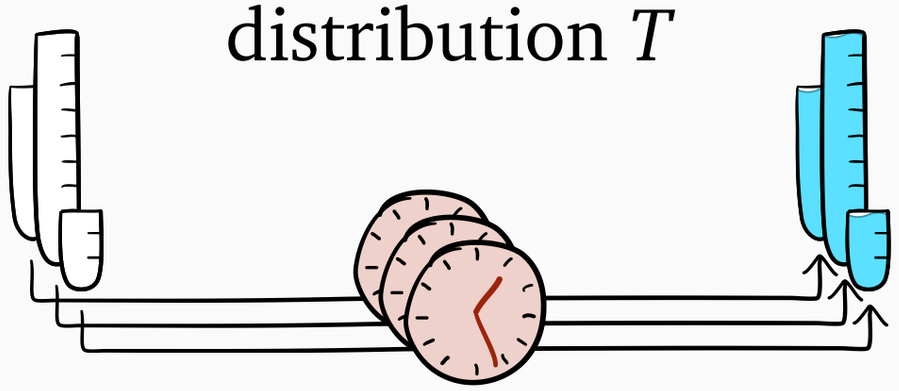

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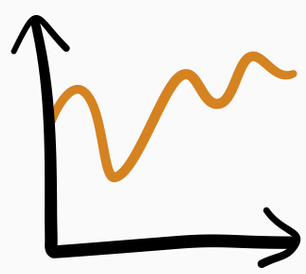

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back

What's *not* a **SOAP** policy?

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Scheduling not based on each job having a **rank**

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Job's **rank** changes while it's not in service

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Tiebreaking not FCFS (or LCFS)

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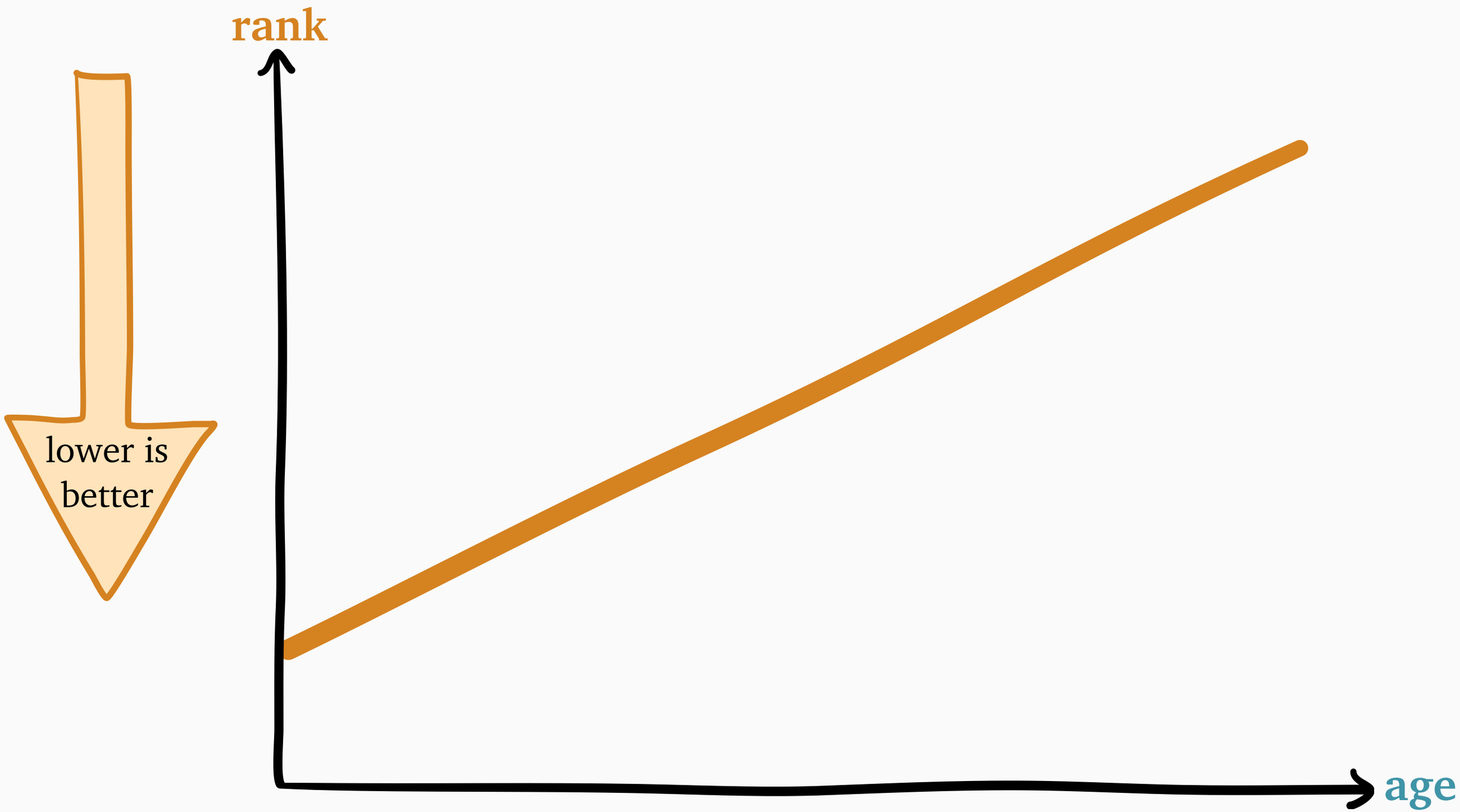
Tiebreaking not FCFS (or LCFS)

PS: can't break ties by sharing (or randomizing)

back

Preemption checkpoints

Can only preempt only at *checkpoint* ***ages***



back

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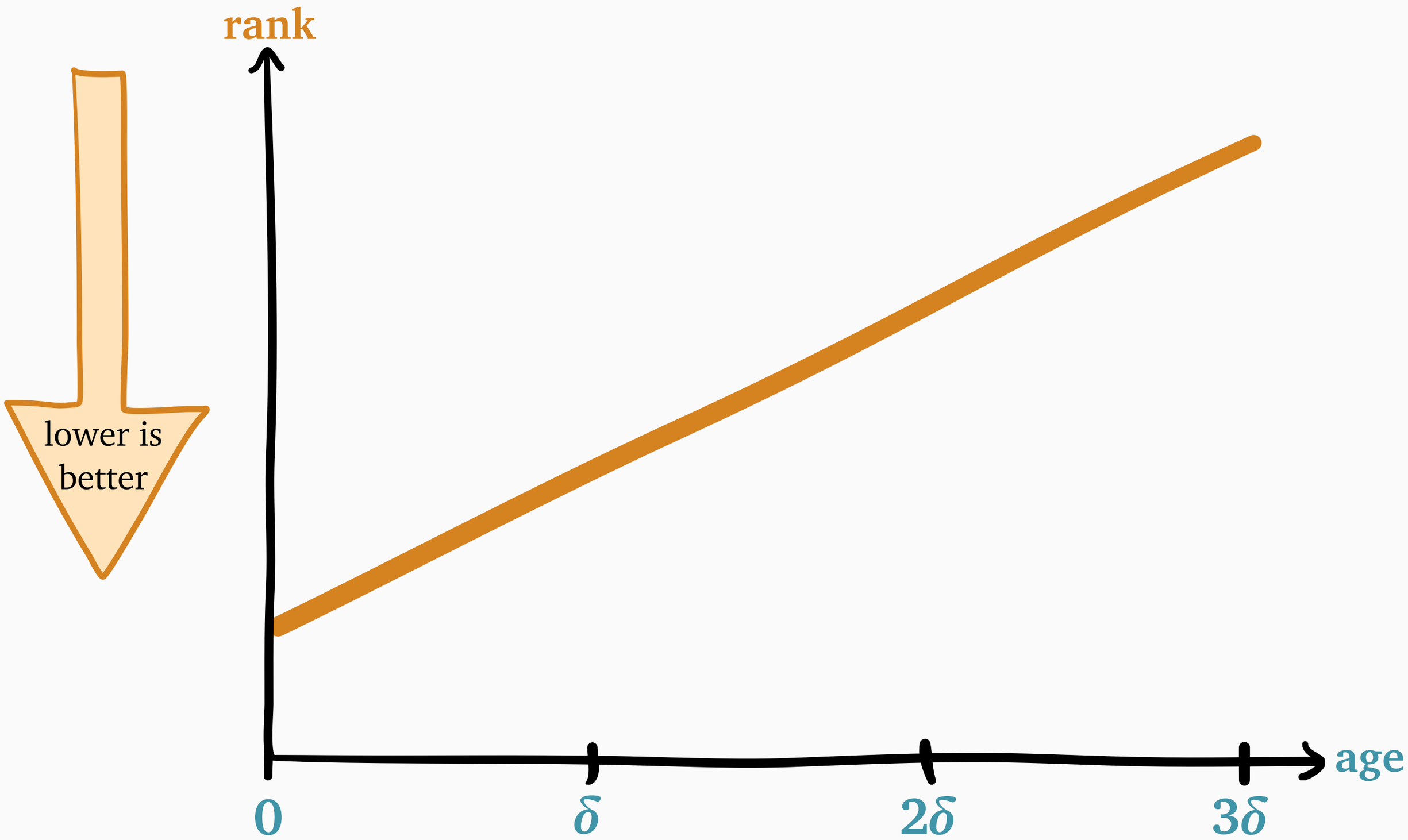

back

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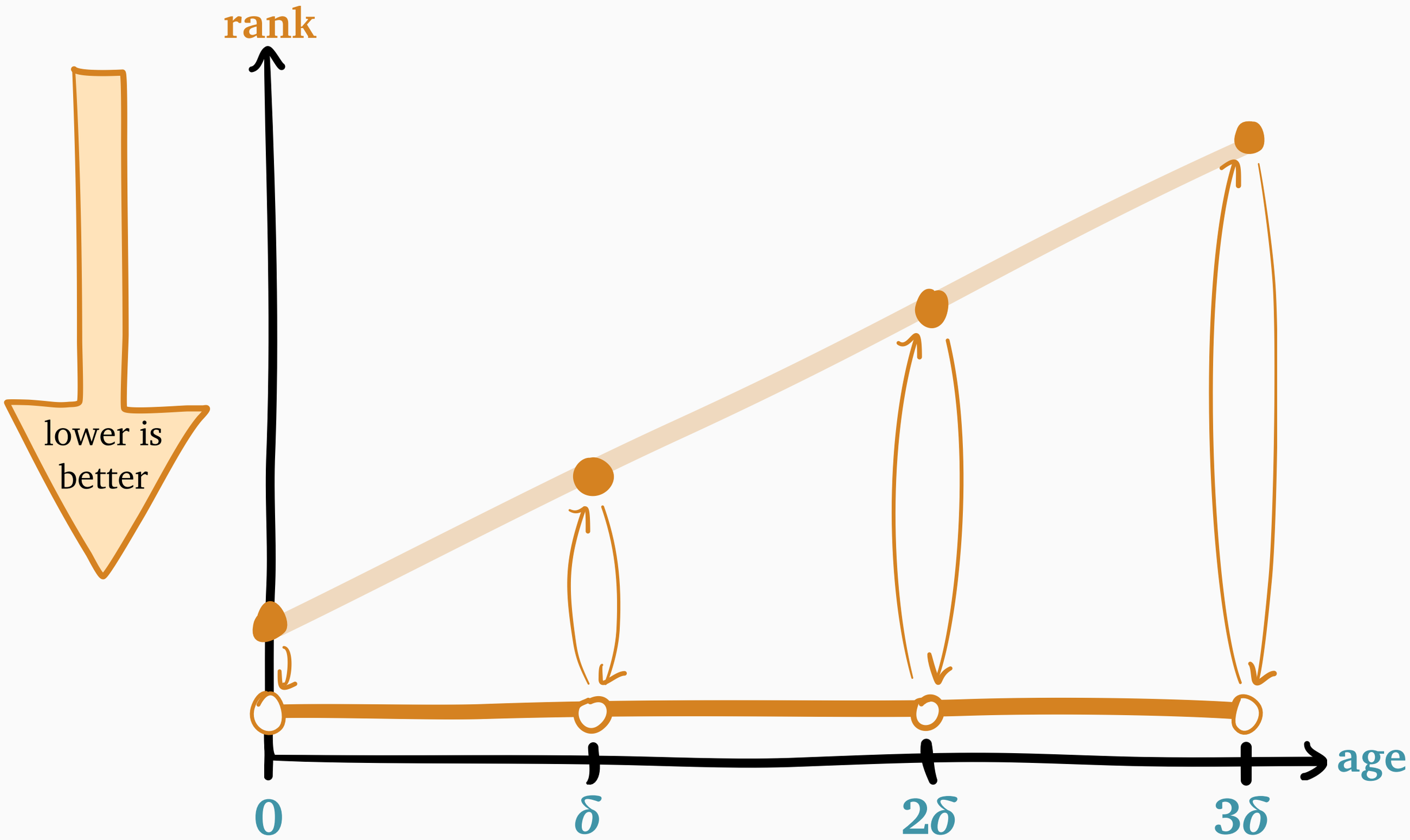

back

Checkpoint frequency?

Suppose each checkpoint incurs an overhead γ

back

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What is the optimal gap δ between checkpoints?

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Checkpoint frequency?

Suppose each checkpoint incurs an overhead γ

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optimal packet size?

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Checkpoint frequency?

Suppose each checkpoint incurs an overhead γ

What is the optimal gap δ between checkpoints?

- large δ: less overhead

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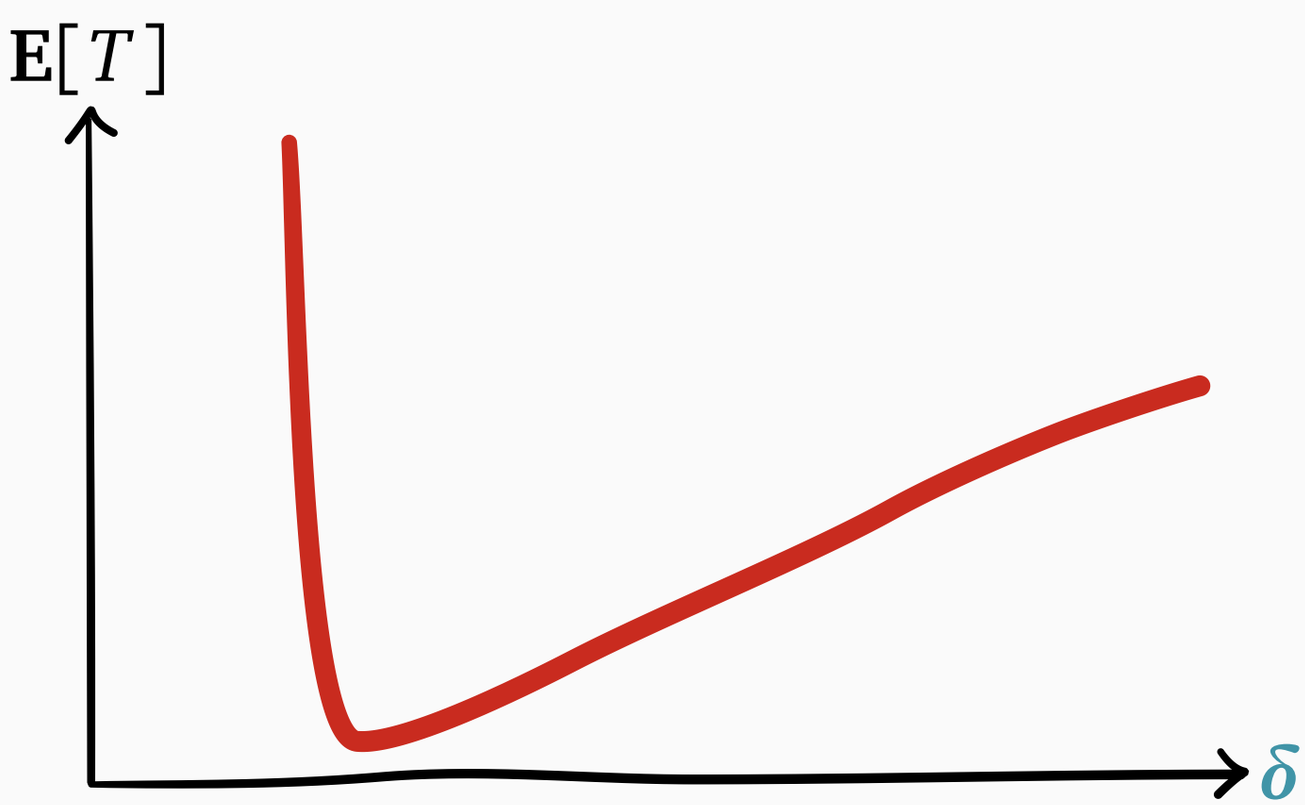

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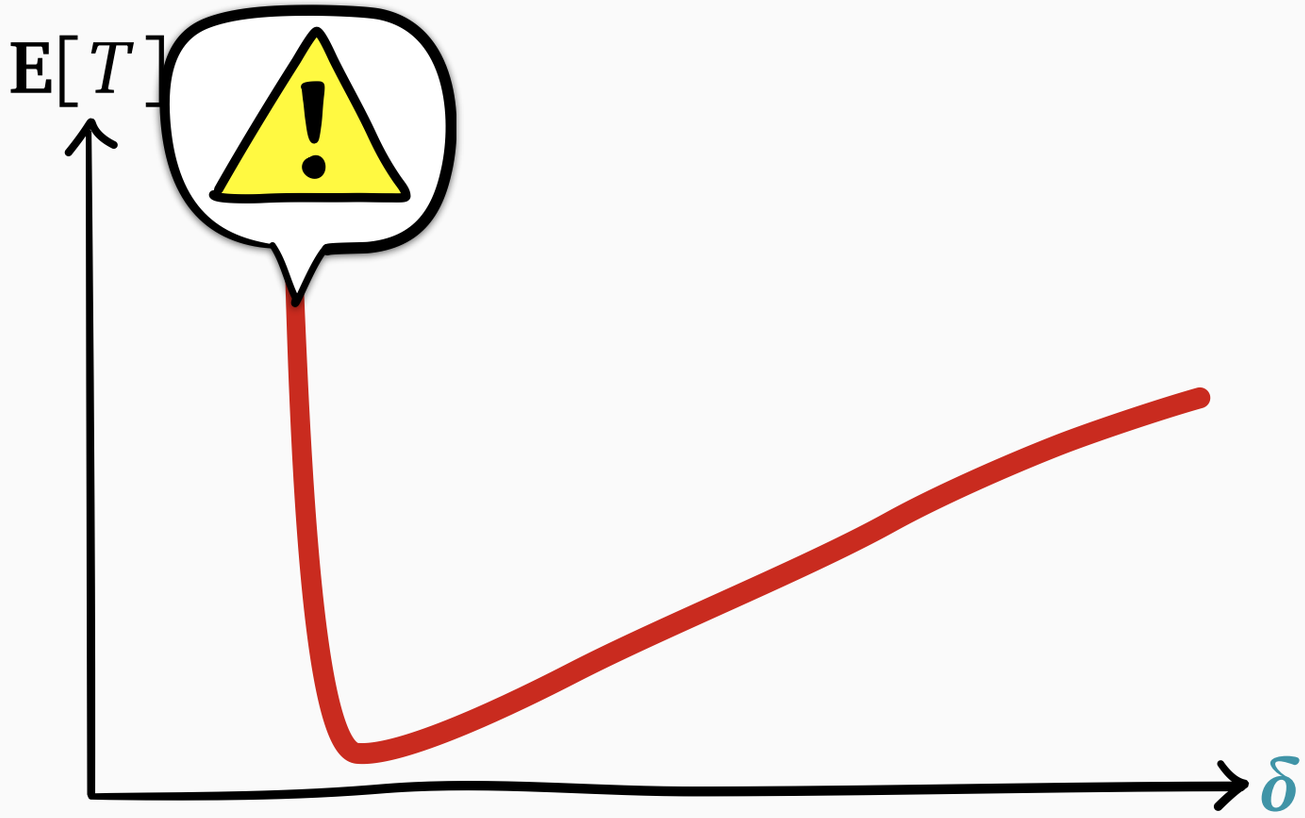

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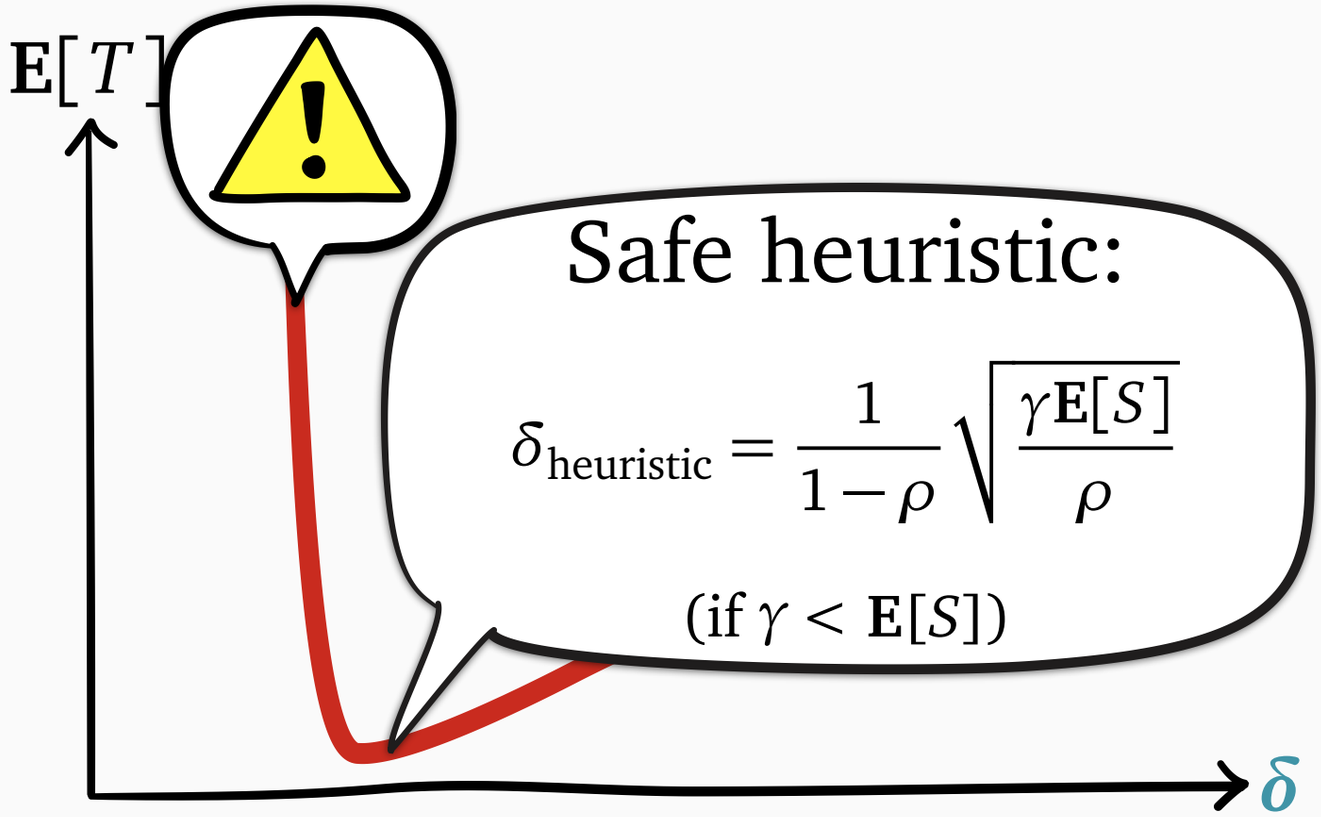
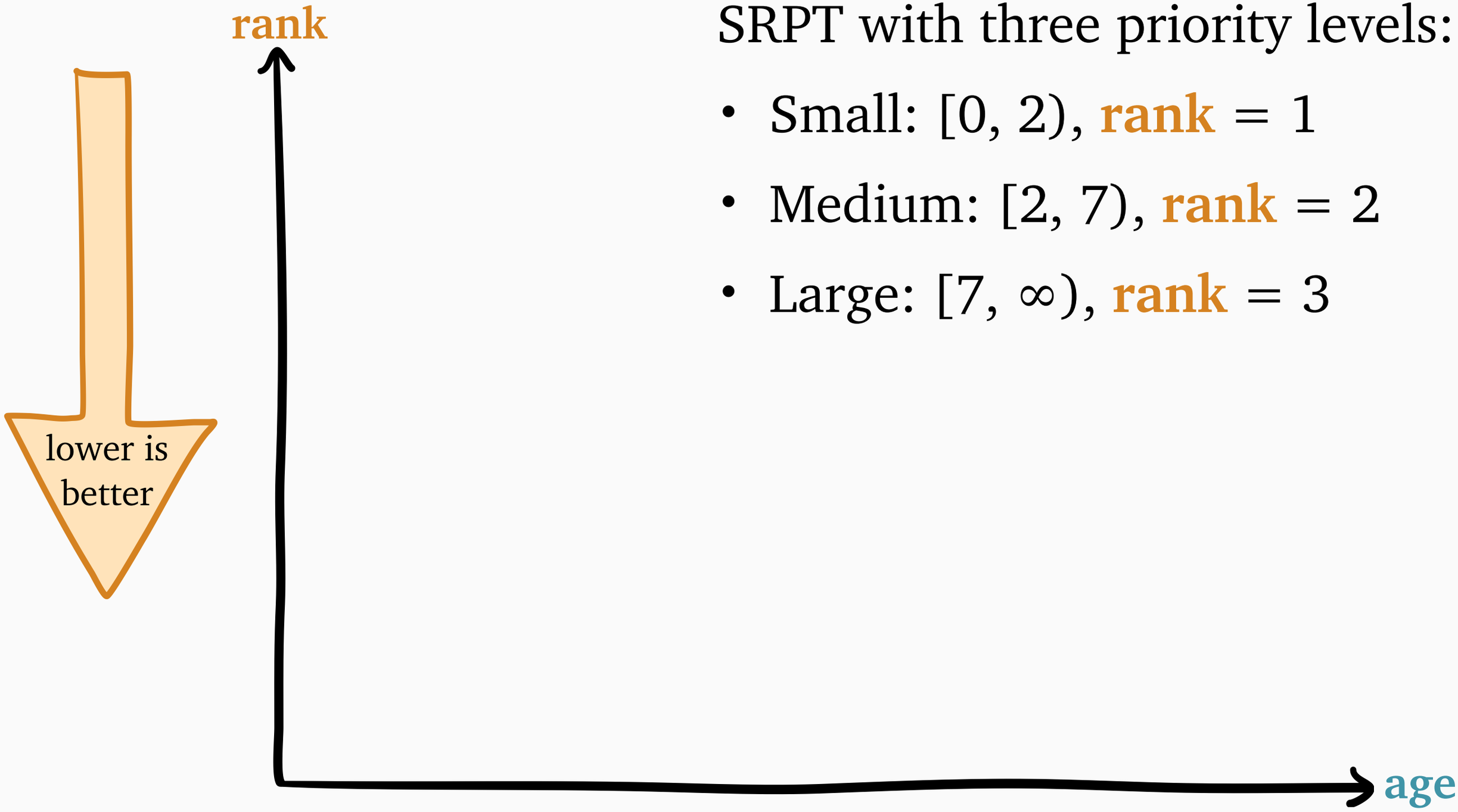


back

Limited priority levels

back

Limited priority levels

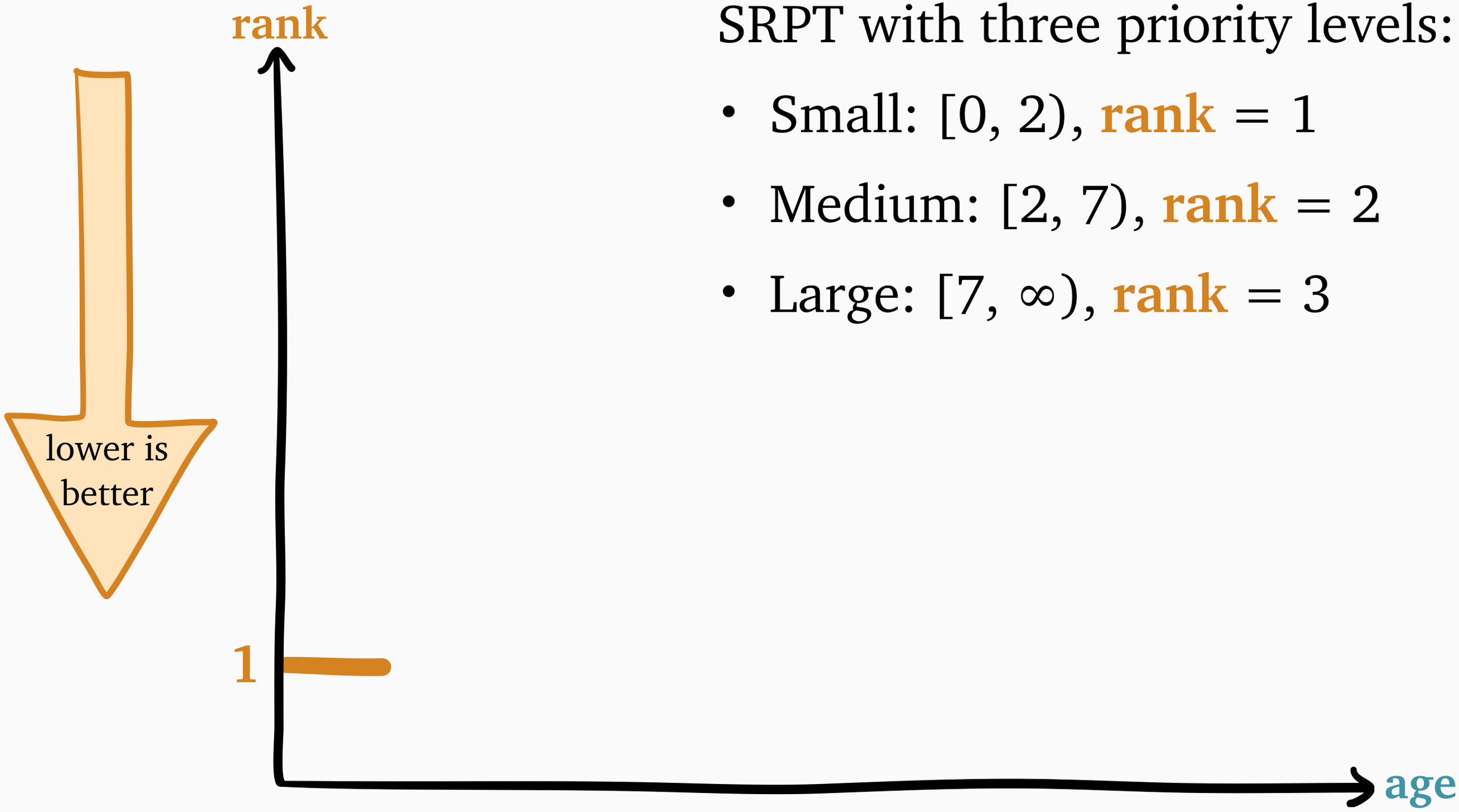


SRPT with three priority levels:

- Small: $[0, 2)$, **rank** $= 1$
- Medium: $[2, 7)$, **rank** $= 2$
- Large: $[7, \infty)$, **rank** $= 3$

back

Limited priority levels



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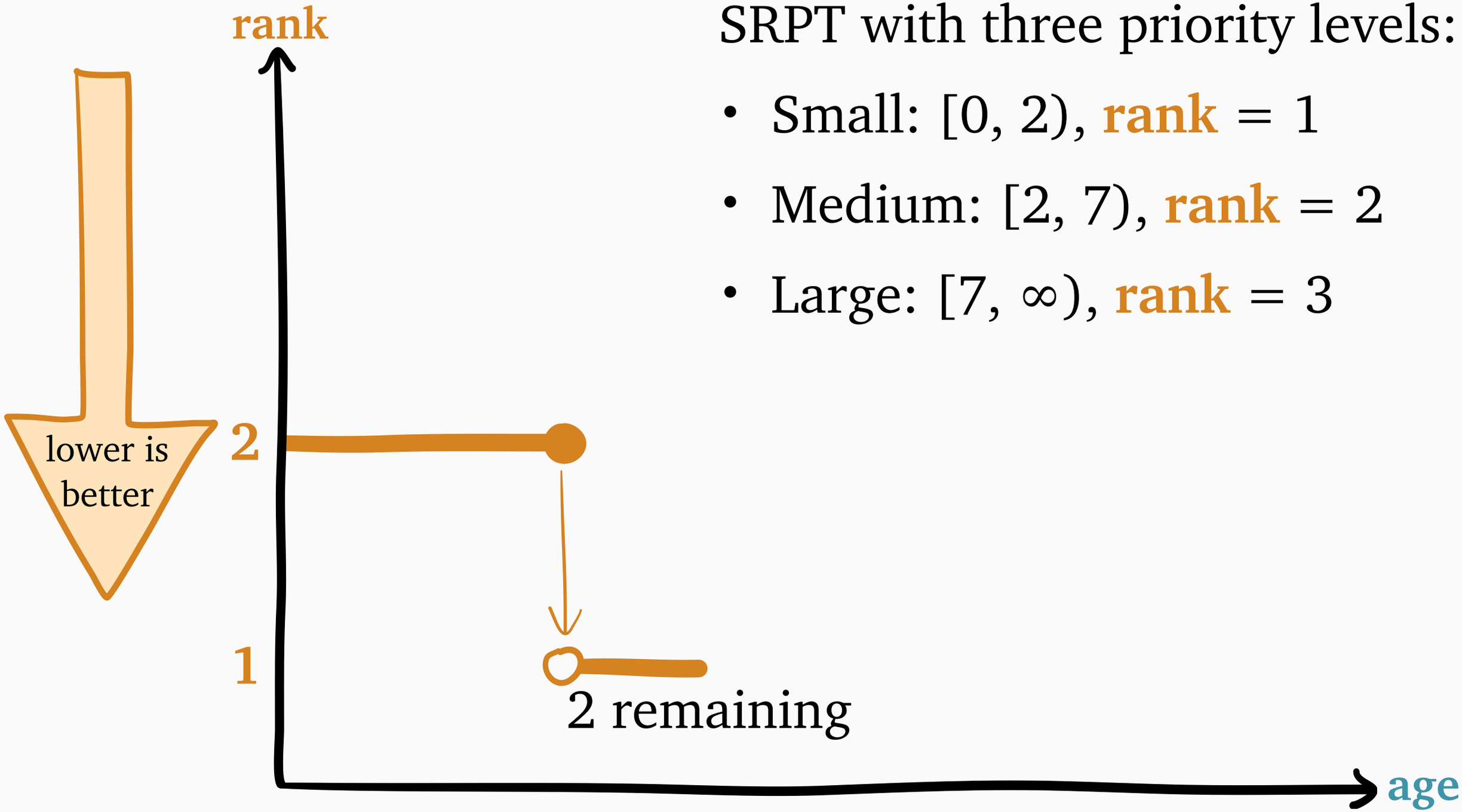

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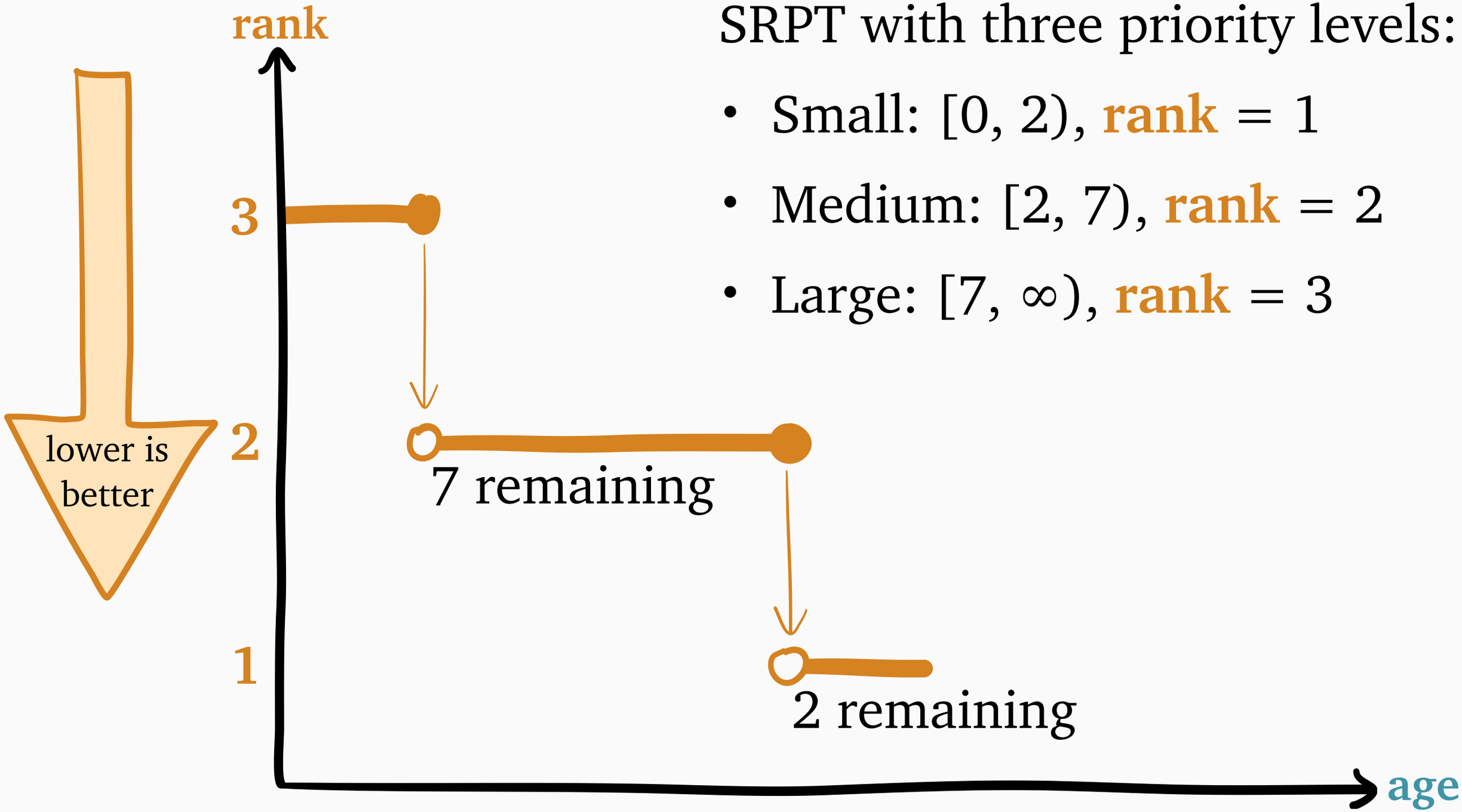

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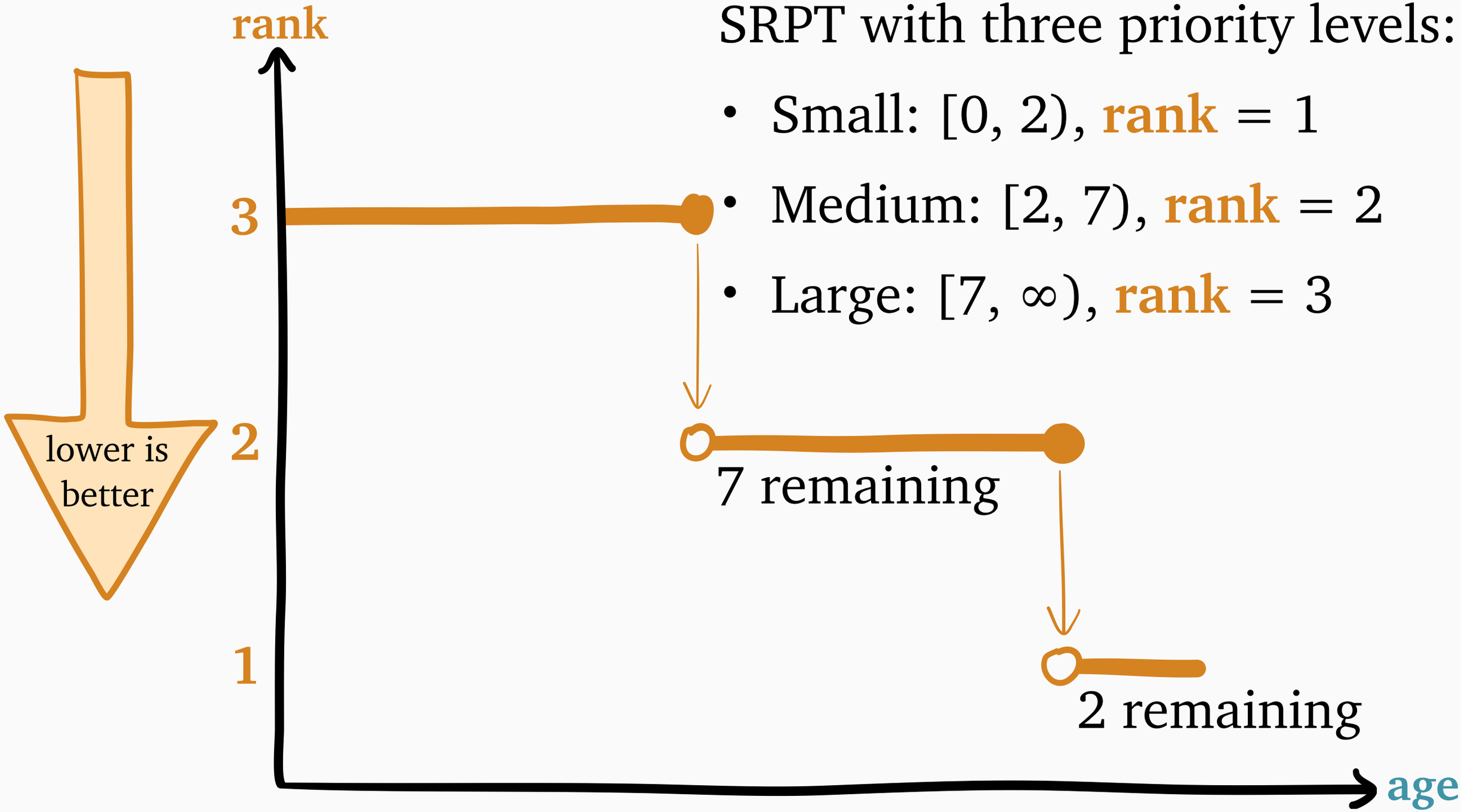

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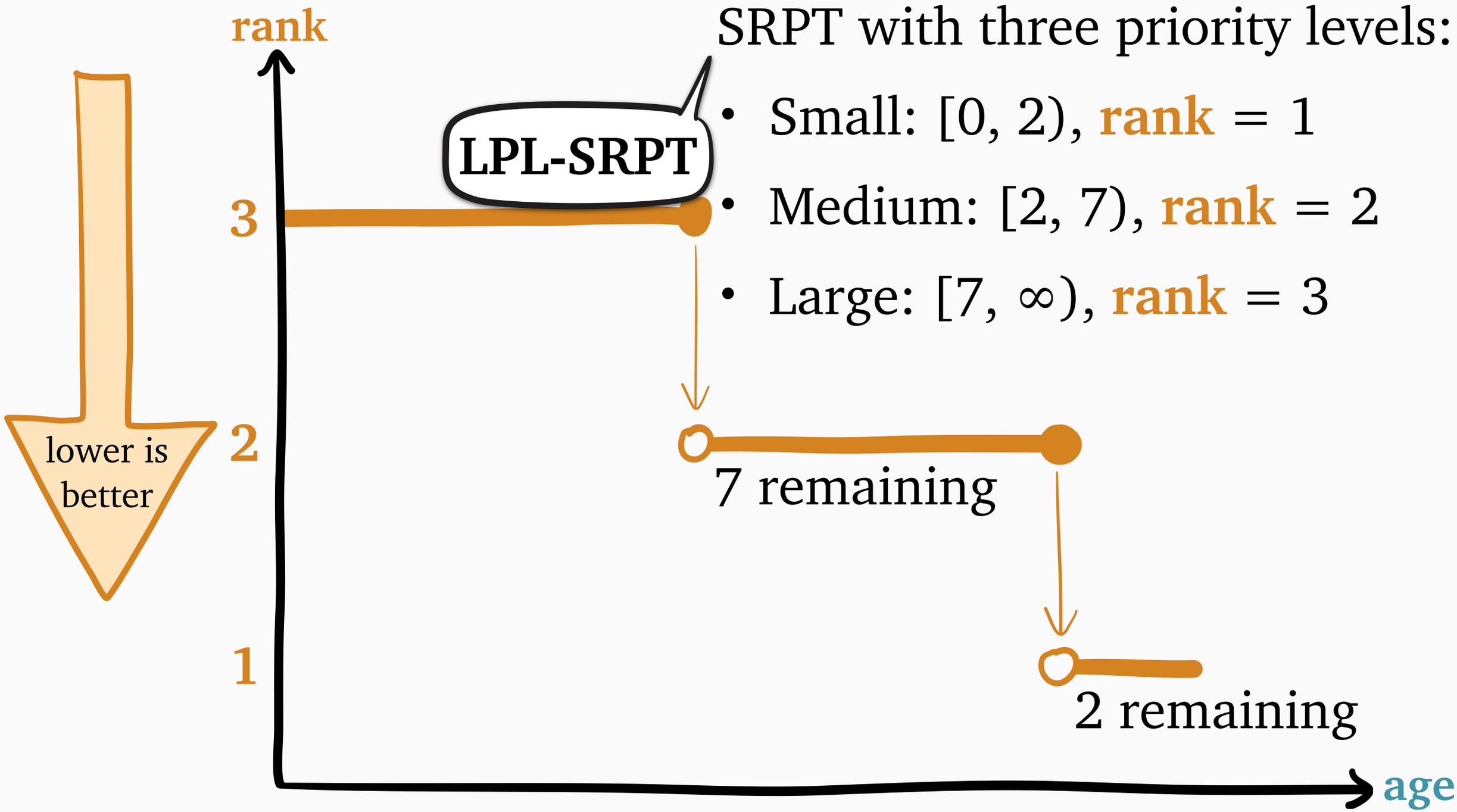

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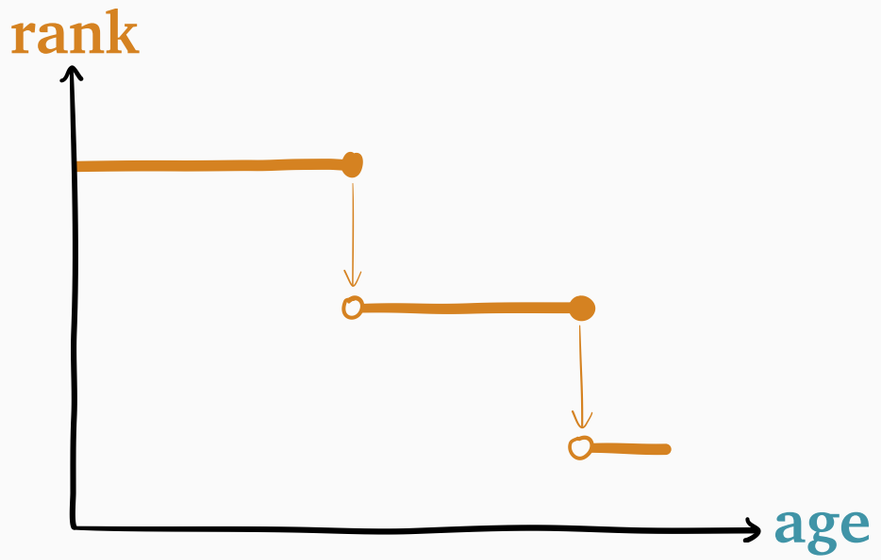
back

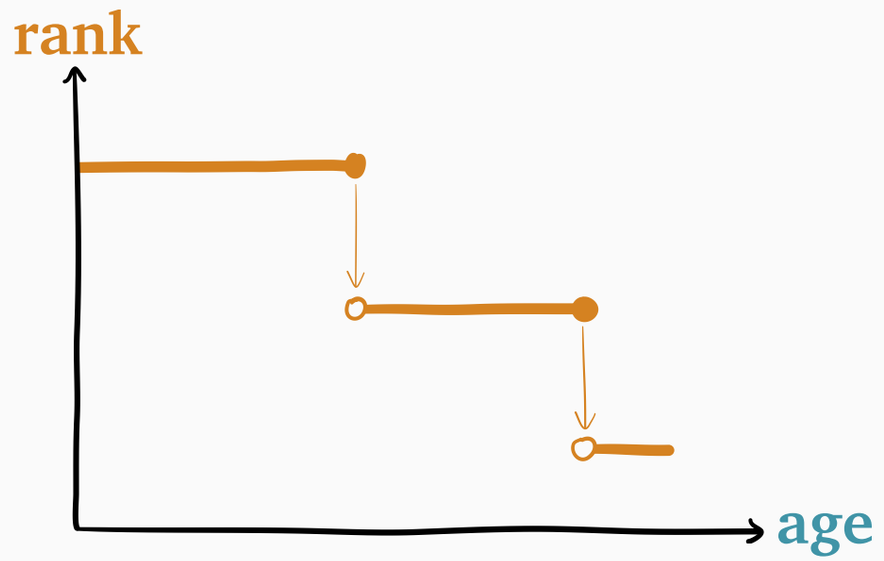
LPL-SRPT questions



- How many levels do we need?
- How do we choose size cutoffs?
- Can we do better than LPL-SRPT?

back

LPL-SRPT questions



High var: 5-ish

Low var: 2-ish

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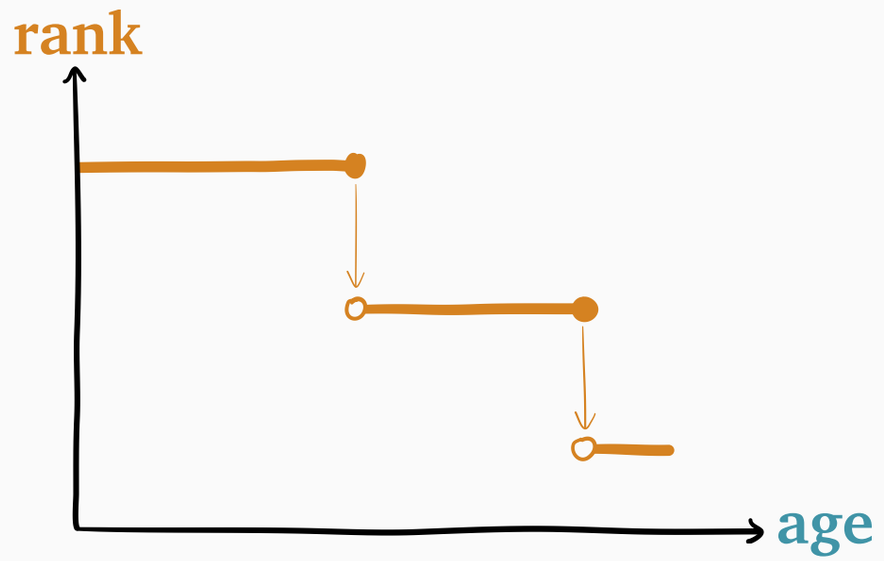
LPL-SRPT questions



High var: 5-ish

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Balancing load is a good heuristic

- How many levels do we need?
- How do we choose size cutoffs?
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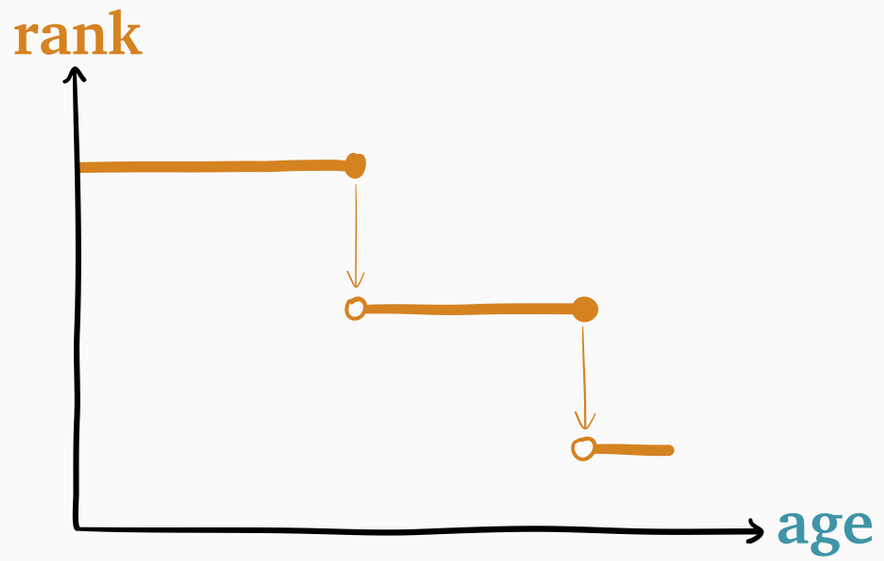
LPL-SRPT questions



High var: 5-ish

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- How many levels do we need?
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Balancing load is a good heuristic

Yes! Constant rank better

back

	Heavy-tailed	*Light-tailed*
Prior work		
New results		

✓ = optimal
? = intermediate
✗ = pessimal

back

	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	
	✓ SRPT	
	✓ LAS	
New results		

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	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
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	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
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New results	✓ LAS with checkpoints, constant gap	

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	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
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not always optimal with growing gap

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Prior work	✗ FCFS	✓ FCFS
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	✓ **Gittins**, (**M-**)**SERPT**	

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Prior work	✗ FCFS	✓ FCFS
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New results	✓ LAS with checkpoints, constant gap	?✗ Non-FCFS **SOAP**
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	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
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New results	✓ LAS with checkpoints, constant gap	?✗ Non-FCFS **SOAP**
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	✓ SRPT	✗ SRPT
	✓ LAS	✗ LAS
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	✓ **Gittins**, (**M-**)**SERPT**	✓?✗ **Gittins**, (**M-**)**SERPT**

for some S, **Gittins** = FCFS

✓ = optimal

? = intermediate

✗ = pessimal

back

	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
	✓ SRPT	✗ SRPT
	✓ LAS	✗ LAS
New results	✓ LAS with checkpoints, constant gap	?✗ Non-FCFS **SOAP**
	✓ **Gittins**, (**M-**)**SERPT**	✓?✗ **Gittins**, (**M-**)**SERPT**

for some S, **Gittins** = FCFS

for some S, **Gittins** = LAS

✓ = optimal

? = intermediate

✗ = pessimal

back

	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
	✓ SRPT	✗ SRPT
	✓ LAS	✗ LAS
New results	✓ LAS with checkpoints, constant gap	?✗ Non-FCFS **SOAP**
	✓ **Gittins**, (**M-**)**SERPT**	✓?✗ **Gittins**, (**M-**)**SERPT**
		✓? Modified **Gittins**

✓ = optimal
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✗ = pessimal

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	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
	✓ SRPT	✗ SRPT
	✓ LAS	✗ LAS
New results	✓ LAS with checkpoints, constant gap	?✗ Non-FCFS **SOAP**
	✓ **Gittins**, (**M-**)**SERPT**	✓?✗ **Gittins**, (**M-**)**SERPT**
		✓? Modified **Gittins**

✓ = optimal
? = intermediate
✗ = pessimal

$\mathbf{E}[T]$ within $1+\varepsilon$ factor of optimal

back

	Heavy-tailed	*Light-tailed*
Prior work	✗ FCFS	✓ FCFS
	✓ SRPT	✗ SRPT
	✓ LAS	✗ LAS
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	✓ **Gittins**, (**M-**)**SERPT**	✓?✗ **Gittins**, (**M-**)**SERPT**
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$\mathbf{E}[T]$ within $1+\varepsilon$ factor of optimal

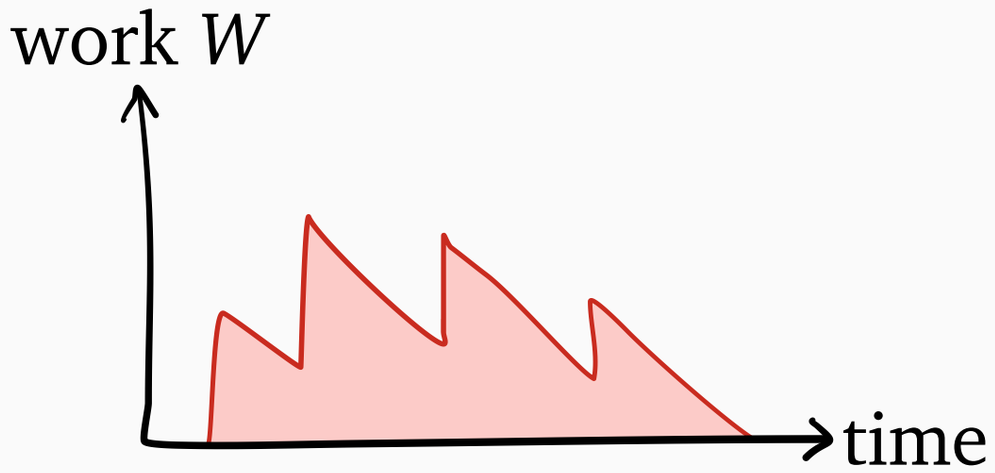
[Scully, Kreveld, Boxma, Dorsman, and Wierman, SIGMETRICS 2020]
[Scully and Kreveld, arXiv 2021]

back

Work decomposition

back

Work decomposition



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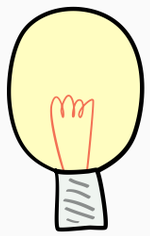
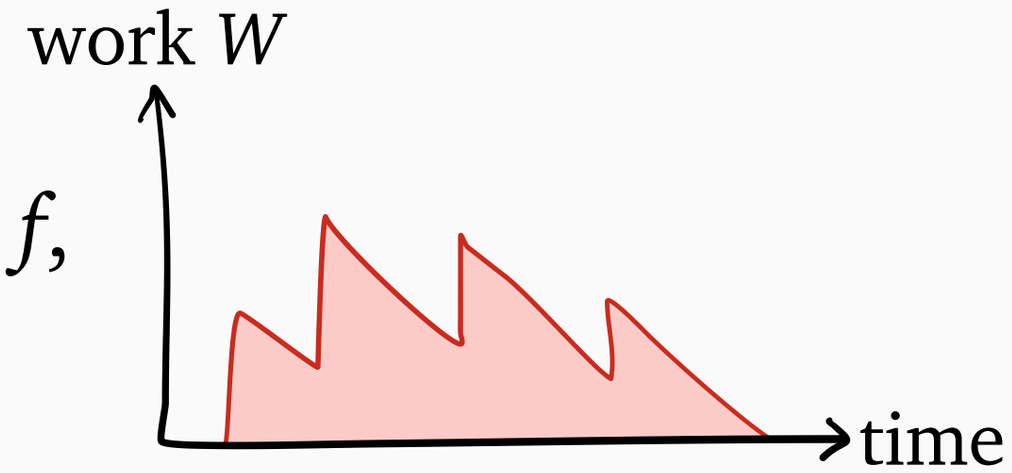
Work decomposition



In steady-state system, for any f,
$\mathbf{E}[\, f(W)]$ constant w.r.t. time



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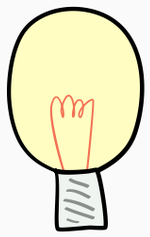

In steady-state system, for any f,
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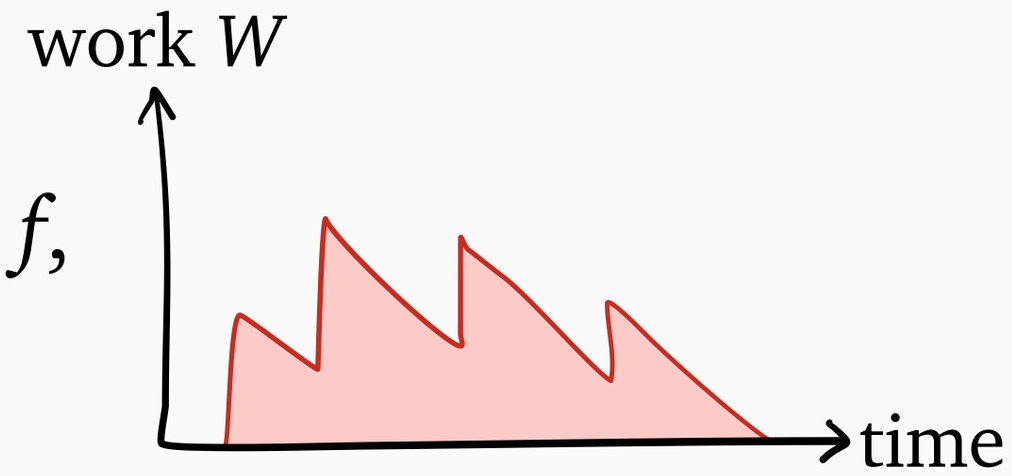
we use
$f(w) = w^2$



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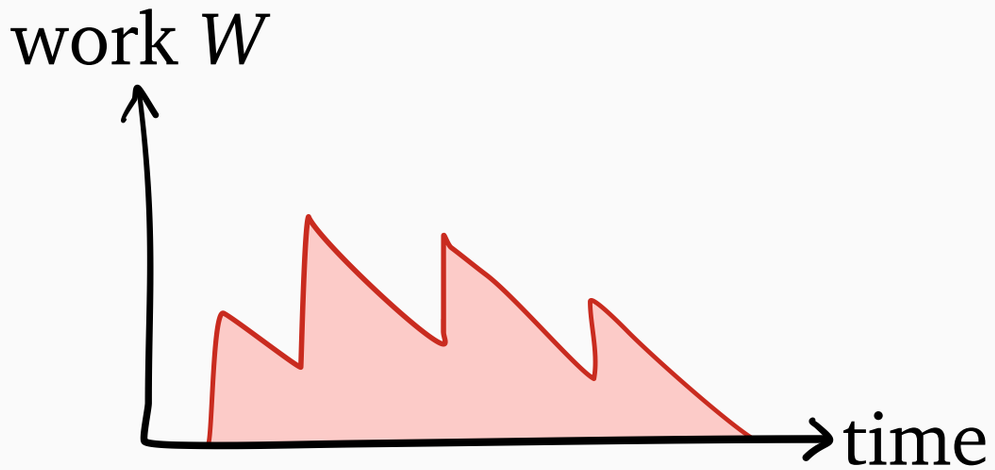

$$\mathbf{E}[W^2 \text{ decrease rate}] = 2\mathbf{E}[BW]$$

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Work decomposition

B = service rate, a.k.a. fraction of servers busy

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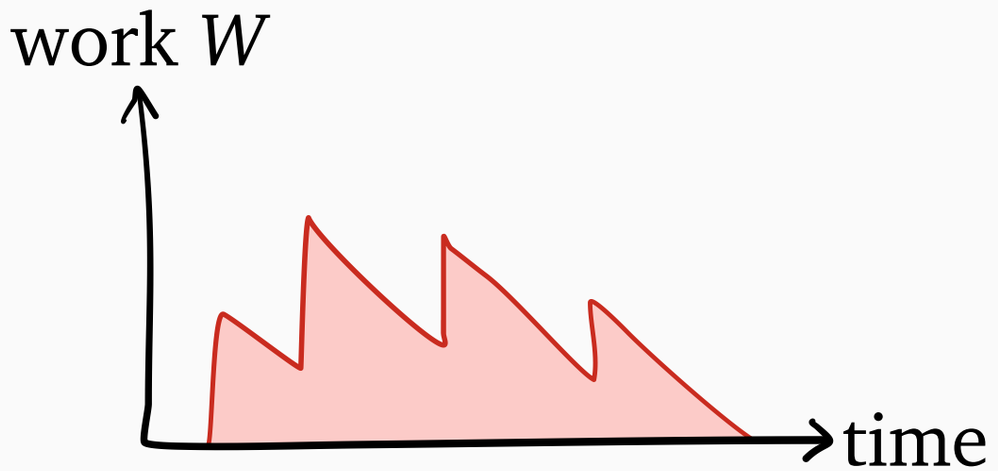

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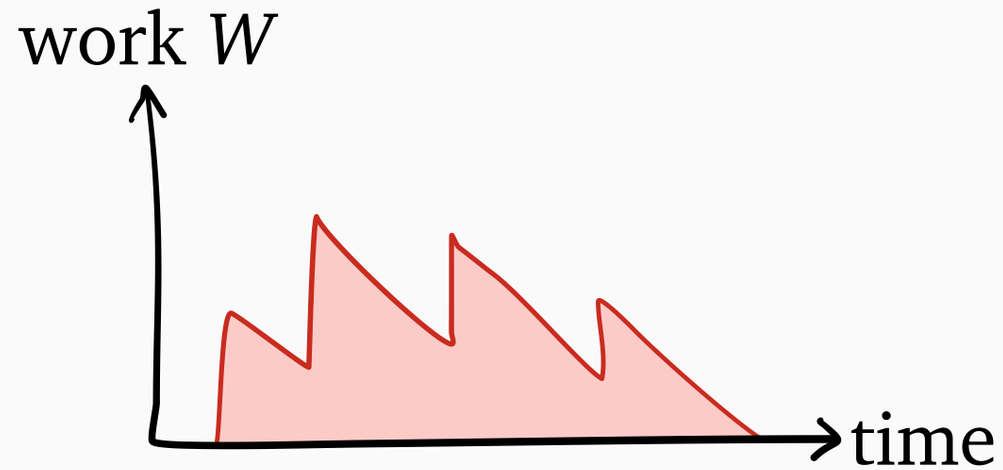

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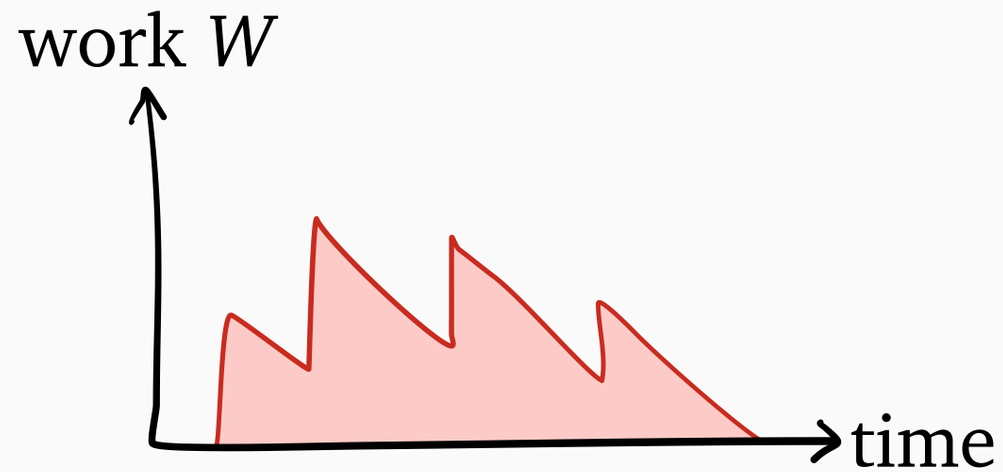

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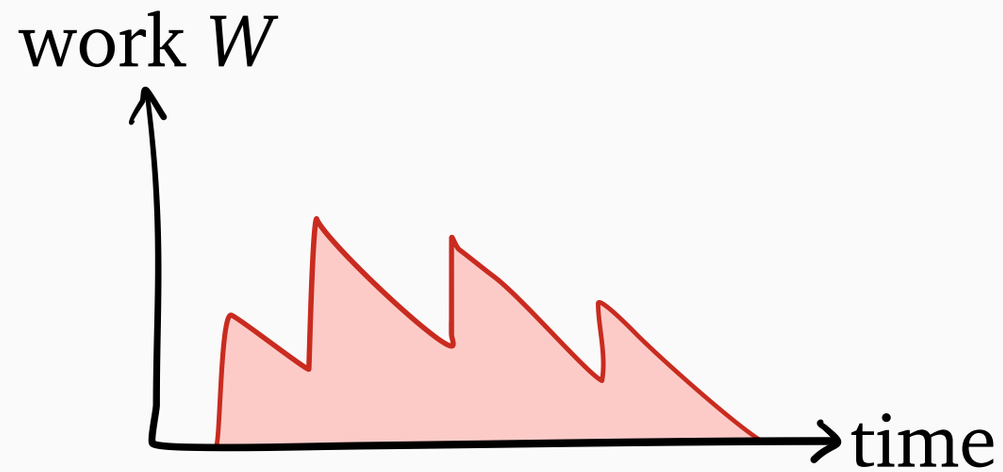

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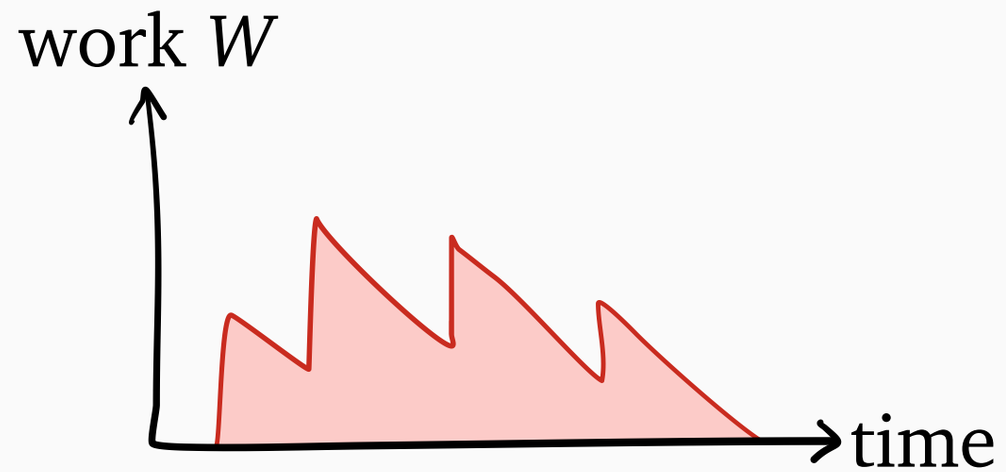

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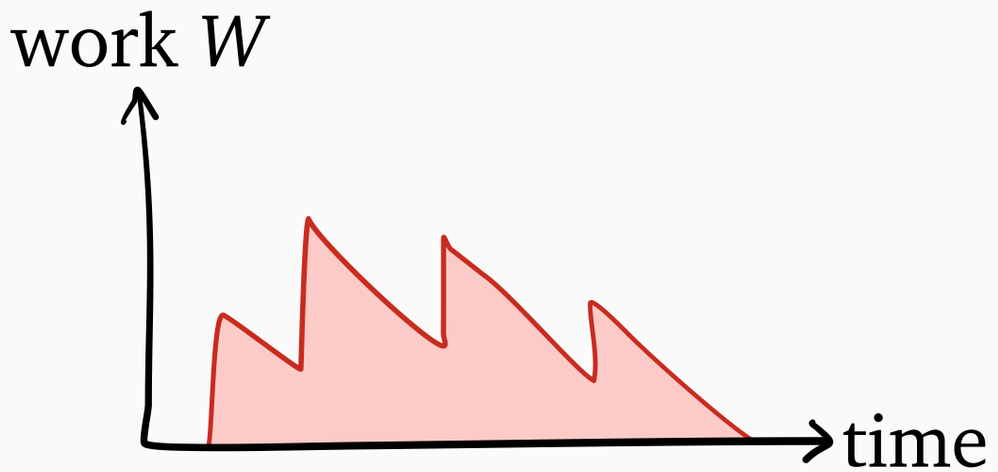

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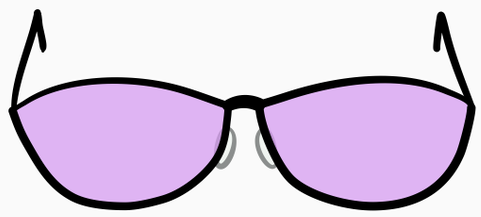

Similar story with r-work

NEW!

back

$$\mathbf{E}[W_k] = \mathbf{E}[W_1] + \frac{\mathbf{E}[(1 - B_k)W_k]}{1 - \rho}$$

back

Suppose $S \leq s_{\max}$ with probability 1

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$\leq (k-1)s_{\max}$

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Suppose $S \leq s_{\max}$ with probability 1

$\mathbf{E}[B] = \rho$ $\leq (k-1)s_{\max}$

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"work of $\leq k-1$ jobs"

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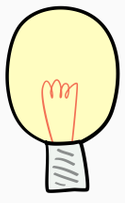

Single job's r-work is at most r

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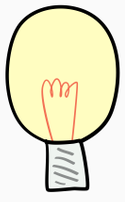

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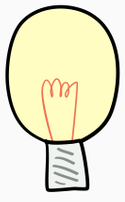

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can improve

back

SRPT-k and Gittins-k $\mathbf{E}[T]$

WINE gives *first analysis* of

- SRPT-k
- Gittins-k

back

SRPT-k and Gittins-k $\mathbf{E}[T]$

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Theorem: for SRPT and Gittins,

$$\mathbf{E}[T_k] \leq \mathbf{E}[T_1] + (k-1) \cdot O\left(\log \frac{1}{1-\rho}\right)$$

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Corollary: SRPT and Gittins minimize $\mathbf{E}[T]$ in heavy traffic (in their respective settings)